Embedded Model Predictive Control

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https://youtu.be/PY0-h5LEUiY

Abstract

We study methods for accelerating Model Predictive Control (MPC) solutions. Our project features:

- An efficient implementation of Accelerated Dual Gradient Projected algorithm (GPAD)
- An improvement over an existing GPAD algorithm
- A benchmark with off-the-shell Interior Point solvers on a constrained and ill-conditioned tracking MPC problem.

Introduction

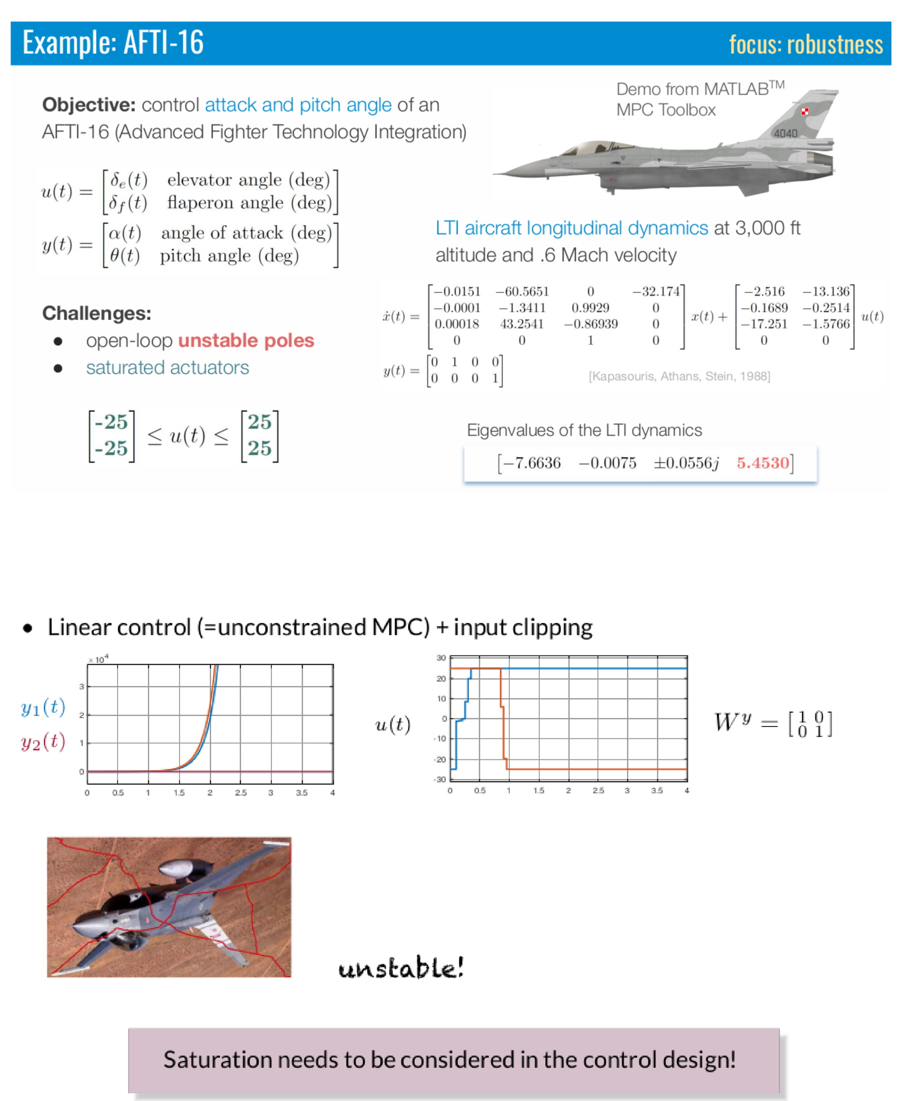
We formulate a convex-embedded MPC problem as follows:

$$\begin{aligned} \min_{x,u} \ & \textstyle\sum_{k=0}^{N-1} \left(x_{k+1}^T Q x_{k+1} + u_k^T R u_k\right) + x_N^T Q_N x_N \\ \text{s.t.} \ & x_{k+1} = A x_k + B u_k + f \\ & F x_k + G u_k \leq d \\ & F_N x_N \leq d_N \\ & x_0 = x_{\text{init}} \end{aligned}$$

We rewrite the problem in the following form:

$$\begin{aligned} \underset{z}{\text{minimize}} \ & \tfrac{1}{2} z^T H z \quad H \text{ is diagonal} \\ s.t. \ & A_e z = b_e \quad A_e \text{ is sparse} \\ & A_i z \leq b_i \quad A_i \text{ is sparse} \end{aligned}$$

AFTI-16 Model with reference tracking



Dual Projected Gradient Approach

We consider the Lagrangian and dual functions:

$$L(z, \lambda) = \frac{1}{2} z^T H z + \lambda^T (A_i z - b_i)$$

$$g(\lambda) = \inf_{z \mid A_e z = b_e} \frac{1}{2} z^T H z + \lambda^T (A_i z - b_i)$$

$$Ł_{inner}(z, \nu) = \frac{1}{2} z^T H z + \lambda^T (A_i z - b_i) + \nu^T (A_e z - b_e)$$

We solve the KKT system by block elimination:

$$\begin{bmatrix} H & A_e^T \\ A_e & 0 \end{bmatrix} \begin{bmatrix} z \\ \nu \end{bmatrix} = \begin{bmatrix} -A_i^T \lambda \\ b_e \end{bmatrix}$$

$$z^*(\lambda) = -H^{-1} A_e^T \left(-H_A^{-1}\left(b_e + A_e H^{-1} A_i^T \lambda\right)\right) - H^{-1} A_i^T \lambda$$

$$\lambda_{k+1} \leftarrow \left(\lambda_k + \alpha \nabla_\lambda g(\lambda_k)\right)_+$$

Dual Projected Gradient Algorithm

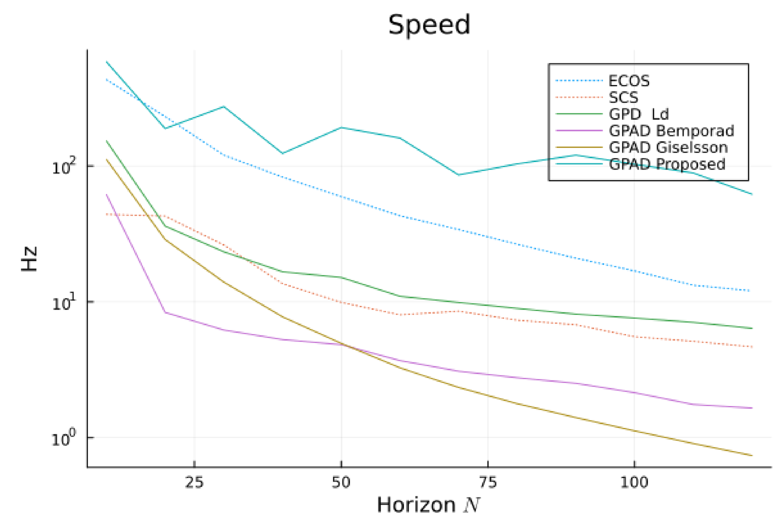
Initialize λ_0
for $k = 0$ to $K_{\max}$ **do**
 $z_k = \underset{z \in \{z \mid A_e z = b_e\}}{\arg\min} L(z, \lambda_k)$
 $\lambda_{k+1} = \left(\lambda_k + \frac{1}{L} \nabla_\lambda L(z_k, \lambda_k)\right)_+$

Accelerated++ Dual Projected Gradient

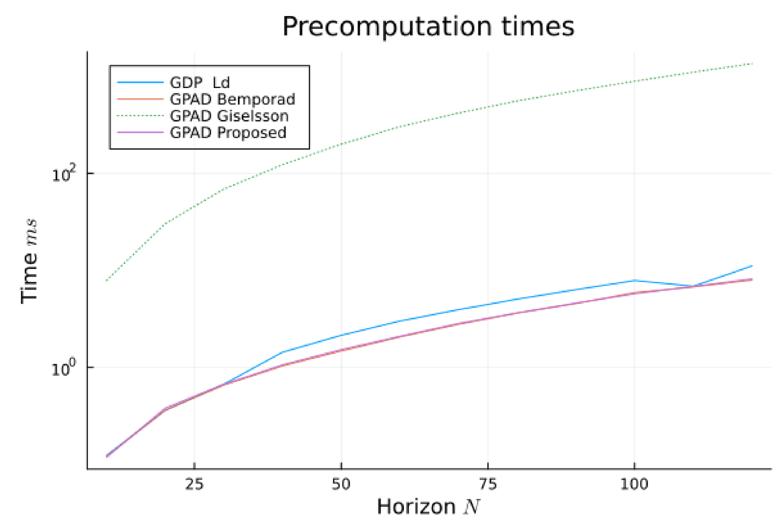
Initialize λ_0
Initialize $\theta_0 = \theta_{-1} = 1$
Set $H_A = A_e H^{-1} A_e^T$
Compute H_A^{-1}
Set $Z_0 = H^{-1} A_e^T H_A^{-1} b_e$
Set $Z_1 = H^{-1} A_e^T H_A^{-1} A_e H^{-1} A_i^T - H^{-1} A_i^T$
Compute $\mathbf{L}^{-1} = \text{diag}\left(A_i H^{-1} A_i^T\right)^{-1}$
for $k = 0$ to $K_{\max}$ **do**
 $\beta_k = \frac{\theta_k (1 - \theta_{k-1})}{\theta_{k-1}}$
 $\omega_k = \lambda_k + \beta_k (\lambda_k - \lambda_{k-1})$
 $z_k = Z_0 + Z_1 \omega_k$
 $\lambda_{k+1} = \left(\omega_k + \mathbf{L}^{-1} (A_i z_k - b_i)\right)_+$
 $\theta_{k+1} = \frac{\sqrt{\theta_k^4 + 4\theta_k^2} - \theta_k^2}{2}$

Experiment Results

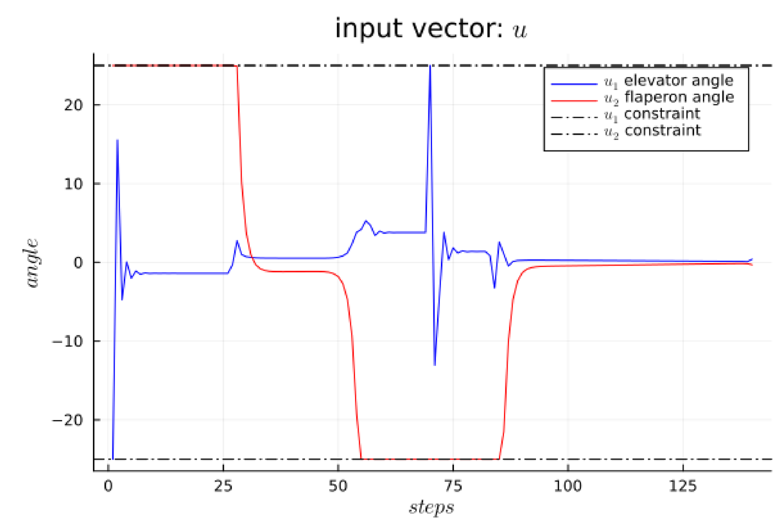
The code is written in Julia. The simulation platform uses an Intel(R) Core(TM) i9-9900K CPU @ 3.60GHz. We benchmark different GPAD algorithms with ECOS, an off-the-shell interior-point solver designed specifically for embedded applications.

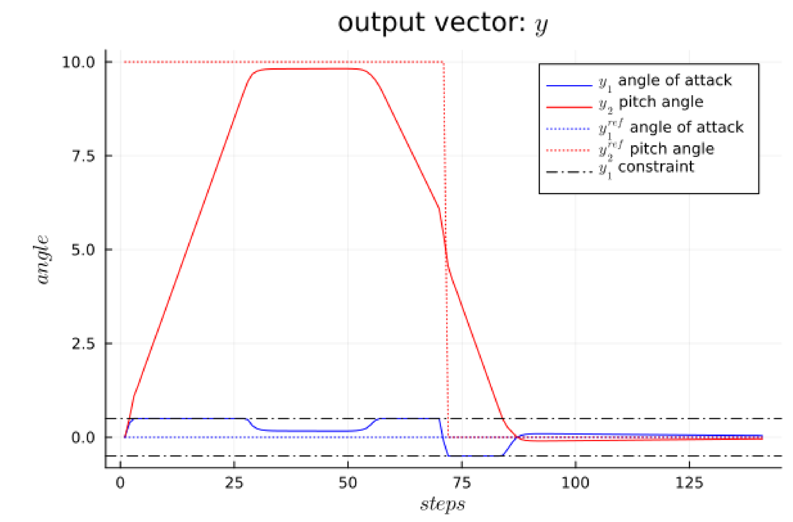


We deal with a tracking problem. Vectors b_i, b_e change with every new tracking trajectory. Moreover, A and B matrices may change between subsequent MPC problem setups. They are typically linearized versions of a nonlinear model. As a consequence, the pre-computations can also be a bottleneck.



The GPAD Giselsson algorithm requires pre-computing a $\mathbf{L}^{-1}$ matrix by solving an SDP problem every time we update A and B matrix. This update is required in tracking problems where we repeatedly linearize nonlinear models. We propose a faster approximation of $\mathbf{L}^{-1}$.





N_{horizon}	10	40	100	120
$(n_{\text{vars}}, m_{\text{constraints}})$	$(64, 128)$	$(244, 488)$	$(604, 1208)$	$(724, 1448)$
ECOS ms	2.3 ms	12 ms	59 ms	82 ms
GPAD* ms	1.7 ms	8 ms	10 ms	17 ms
GPAD* precomp	6%	12%	50%	50%
GPAD* iters	262	441	120	204
$f_{\text{opt}}^{\text{gpad}^*} - f_{\text{opt}}^{\text{ecos}}$	$1e^{-7}$	$4.5e^{-5}$	$8e^{-3}$	$8e^{-3}$

Conclusion

We derived an Accelerated Dual Projected Gradient implementation running at 588 Hz for a limited time horizon $N = 10$ and 100 Hz for $N = 100$. The proposed algorithm is faster than off-the-shell Interior Point solvers like ECOS. We exploited problem structure taking advantage of sparse matrices, solved KKT by block elimination, performed pre-computations, and used acceleration methods. We update gradient with different step sizes in different directions avoiding hyper-parameter tuning thanks to a simple formula when previous publications relied on solving an SDP problem.

Code: https://github.com/PhilippeW83440/empc

Future Work

We plan to work on a custom Interior Point method.

References

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