Philippe WS CSC 349

Lab 7

<u>Document 1, Problem 3</u>

Counter Examples

a.) Capacity: W = 4

Id	Weight	Value
1	4	6
2	2	4

b.) Capacity: W = 4

Id	Weight	Value
1	4	4
2	3	3

c.) Capacity: W = 4

Id	Weight	Value	Ratio
1	3	12	4
2	2	7	3.5

Problem

A.) V(W): A function representing the maximum value of a knapsack of capacity W obtained from a set of items n, with each item i having weight w_i, value v_i, and infinite occurrences,

B.) $V(W) = \max_{1 \leq i \leq n \;\&\&\; w(i) < W} \{ V(W-w_i) + v_i \}$, $V(0) = 0$.

C.) The table is a 1D array containing the maximum values of knapsacks of capacity 1 to n;

D.) *Pseudo Code Filling Table*

```
//items[n], the list of n items, where each item object contains its id, weight, and value.
//W, the maximum capacity of the knapsack we are trying to fill.

fillKnapsackTable( items[n], W ){

        resultTable[W];
        for( j = 1 to W ){

                sackCurrentMax = resultTable[j-1];
                possibleMax = max_{1 ≤ i ≤ n && w(i) < W}{ items[i].value +

                                                resultTable[j-items[i].weight] };

                if(possibleMax > sackCurrentMax)
                        sackCurrentMax = possibleMax;

                resultTable[j] = sackCurrentMax;

        }

}
```

E.) Pseudo Code Traceback

```
traceBack(resultTable, items[n]){

        tracebackArray = []

        i = resultTable[resultTable.length-1];

        while( i > 0){

                while j = 0 to N

                        if( resultTable[i] == resultTable[i - items[j].weight]
                                                                + items[j].value)

                                tracebackArray.add( items[j].item )

                                i = resultTable[i – items[j].weight]

                                break;

    }

    return tracebackArray;

}
```

F.) Since iterating through the list of items is constant time, then there are a constant C number of operations for any knapsack of capacity W. Thus, the time complexity is O(C*W) or O(n).

Document 1, Problem 4

A.) C(i) gives the optimal cost of consulting business operations for a period of i months.

B.) $C(i) = \min\{ C(i-1) + M + SF_i, C(i-1) + NY_i \}$ if (location == NY) ||

$\min\{ C(i-1) + M + NY_i, C(i-1) + SF_i \}$ if (location == SF)

C.) A 1D array containing the optimal consulting business operations costs up to index/month i.

D.) Pseudo Code Iterative Table Fill

```
//Inputs: SF[i] the set of costs each month for SF. NY[i] the set of costs each month for
//NY[i]. M, the cost to move between the two cities.
minBusinessCost( SF[i], NY[i], M){

        result[][] = [2][i];

        result[0] = 0; result[1] = min{ SF[1], NY[1] };

        bool inSF = SF[1] <= NY[1] ? inSF = true : inSF = false;
```

```
        cities[0] = null;

        for (n = 1 to i){

                if( inSF ){
                        if( result[n-1] + M + NY[n] < result[n-1] + SF[n]  ){
                                result[0][n] = result[0][i-1] + M + NY[n];
                        }else{
                                result[0][n] = result[0][n-1] + SF[n];

                                inSF = false;
                        }

                }else{
                        if( result[n-1] + M + SF[n] < result[n-1] + NY[n] ){
                                result[0][n] = result[0][n-1] + M + SF[n];
                                inSF = true;
                        }else{
                                result[0][n] = result[0][n-1] + NY[n];
                        }

                }

                result[1][n] = inSF;

        }

        return result;
}
```

E.) Pseudo Traceback

```
traceback( result[2][i] ) {

        //Since in the second dimension we stored the true or false value of whether or not
        //we were in city 1, simply follow that and prescribe the values as necessary to
        //some sort of string array.

        string[] cities = [i];
        for( n = 1 to i ){

                cities[n-1] = result[1][i] ? “SF” : “NY”;
        }

        return cites;

}
```

F.) There are a constant number of comparisons per iteration of filling out the table, thus, for any number of i months, there are C comparisons. Thus C*i, and complexity of $\theta(n)$

Document 2, Problem 6

A.) Opt(m_i) is a function representing the maximum profit that can be obtained from the set of valid restaurants placed at further m_i.

B.) Opt(m_i) = max{ Opt(Cl(m_i-k)) + p_i, Opt(m_{i-1}) }, Opt(0) = 0; Cl(x) is the closest valid restaurant location contained in the set m_n to the value distance x.

C.) The table is a one-dimensional array contain the maximum value possible for i restaurants, where i is the count of restaurant locations and index in the array.

D.) Pseudo code Table Filling

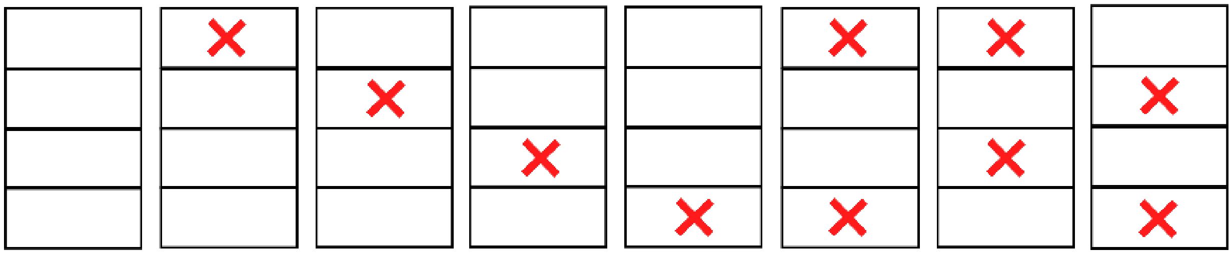
```
optimalFireStonePlacement( m[n], p[n], int k ){

        Cl[n]; //An array containing the closest compatible previous restaurant of any
        //placement i ≤ n.

        resultTable[n];

        resultTable[0] = 0;

        for(i = 1 to n){

                resultTable[i] = max{ resultTable[ Cl[i] ] + p[i], resultTable[i-1];

        }

}
```

E.) Pseudo Code Traceback

```
traceback(Opt[n], p[j]){

        locations = [];

        largestRevenue = Opt[n];

        for( j = n to 1){

                if( Opt[j] > Opt[j-1] && Opt[j] == largestRevenue){

                        locations.add( j );

                        largestRevenue -= p[j];

                }

        }

        return locations;

}
```

Document 2, Problem 7

A.)



B.) $Opt(k, i) = Opt(k-1, i) + \max_{x \leq |Cmp(i)|}\{ S(Cmp(i)) \}$,

Where S(x) is the is set of scores derived from the set of all column patterns. Cmp(x) is the list of all column patterns compatible with pattern i.

$Opt(0, x) = -\infty$, for $1 \leq x \leq 8$.

Pseudo Code Table Fill

```
optimalPebblePlacement( Board[4][n] ){

        //An array with 8 rows 2 columns, containing all possible configurations of
        //columns. Either the row number of the pebble placement or -1 if not placed.

        Config[8][2];


        //An array of lists containing all compatible patterns for any pattern i. Could also
        //be represented as an 8x8 compatibility matrix.

        Cmp[8][~];

        //Initialize the result table.

        resultTable[7][n];

        for( i = 0 to 7 )

                resultTable[i][0] = 0;

        //Create table

        for ( col = 1 to n ){

                for( p = 0 to 7 ){

                        max = 0;

                        val1 =  Board( Config[p][0], Config[p][1] );
```

```
                    for( i  = 0 to Cmp[p].length ){
                         val2 = resultTable[i][col-1];
                         if(val1 + val2 > max) max = val1 + val2;
                    }
                    resultTable[p][col] = max;
               }
          }
          return resultTable;
     }
```

Pseudo Code Traceback

```
traceback( resultTable[p][n], Cmp[p] ){
     patternChoices[n];
     maxVal = max_{i=0-7} { resultTable[i][n-1] };
     maxIndex = resultTable.getIndex(Val);
     patternChoice[n-1] = maxIndex;
     for( i = n-1 to 1 ){
          for( p in Cmp(maxIndex) ){
               if(maxValue < resultTable[p][i] ) {
                    maxIndex = p;
                    maxValue = resultTable[p][i];
               }
          }
          patternChoices[i-1] = p;
     }
}
```