Philippe WS CSC 349

Count Analysis

a.) Recurrence Relation
   a. Since the algorithm is essentially Merge Sort with addition, which is not the domineering operation, the recurrence relation is:

$$\mathrm{T}(n) = 2\left(\mathrm{T}\left(\frac{n}{2}\right)\right) + n - 1, \qquad \mathrm{T}(1) = 0$$

b.) Closed Form Solution

$$\mathrm{T}(n) = 2\left(\mathrm{T}\left(\frac{n}{2}\right)\right) + n - 1, \qquad \mathrm{T}(1) = 0$$

$$\mathrm{T}(n) = 2\left(2\left(\mathrm{T}\left(\frac{n}{2^2}\right)\right) + \frac{n}{2} - 1\right) + n - 1$$

$$\mathrm{T}(n) = 2^2\left(\mathrm{T}\left(\frac{n}{2^2}\right)\right) + n - 2 + n - 1$$

$$\mathrm{T}(n) = 2^2\left(\mathrm{T}\left(\frac{n}{2^2}\right)\right) + 2n - 3$$

$$\dots$$

$$\mathrm{T}(n) = 2^k * \mathrm{T}\left(\frac{n}{2^k}\right) + kn - \sum_{i=0}^{k-1} 2^i$$

$$\frac{n}{2^k} = 1 \rightarrow n = 2^k \rightarrow k = \log_2(n)\,, \qquad \sum_{i=0}^{k-1} 2^i = 2^{i+1} - 1 \rightarrow 2^k - 1$$

$$\mathrm{T}(n) = 2^{\log_2(n)}\left(\mathrm{T}\left(\frac{n}{2^{\log_2 n}}\right)\right) + n(\log_2(n)) - (\,2^{\log_2(n)} - 1)$$

$$\mathrm{T}(n) = n\big(\mathrm{T}(1)\big) + n\log_2(n) - n + 1$$

$$\mathrm{T}(n) = n * (0) + n\log_2(n) - n + 1$$

Closed Form Solution

$$\mathrm{T}(n) = n\log_2(n) - n + 1$$