Philippe WS CSC 349

Lab 10

Problem 1

Heaviest Item First Counter Example:

Greedy Solution Value: $\mathrm{f}(s_a) = v_1$ | Optimal Solution Value: $\mathrm{f}(s^*) = v_2$

Item	Value	Weight
1	v_1	w_1
2	v_2	w_2

Capacity: *C*

Restrictions:
$[v_2 = v_1 * c + 1] \wedge [w_2 < w_1 \leq C]$

$\frac{\mathrm{f}(s*)}{\mathrm{f}(s_a)}$ cannot be bounded by any arbitrary value of c since $\frac{\mathrm{f}(s*)}{\mathrm{f}(s_a)} > c$. Thus, the *Heaviest Item First* approach is not a c approximation of the 0-1 Knapsack Problem for any arbitrary c.

Most Valuable Item First Counter Example:

Greedy Solution Value: $\mathrm{f}(s_a) = v_1$ | Optimal Solution Value: $\mathrm{f}(s^*) = v_2 + v_3$

Item	Value	Weight
1	v_1	w_1
2	v_2	w_2
3	v_3	w_3

Capacity: *C*

Restrictions:
$[v_2 + v_3 = v_1 * c + 1] \wedge$
$[v_2 < v_1] \wedge [v_3 < v_1] \wedge$
$[w_2 + w_3 \leq C] \wedge [w_1 \leq C] \wedge$
$[v_2 < v_1] \wedge [v_3 < v_1]$

$\frac{\mathrm{f}(s*)}{\mathrm{f}(s_a)}$ cannot be bounded by any arbitrary value of c since $\frac{\mathrm{f}(s*)}{\mathrm{f}(s_a)} > c$. Thus, the *Most Valuable Item First* approach is not a c approximation of the 0-1 Knapsack Problem for any arbitrary c.

Problem 4

Proof: C vertex cover in G => V-C a clique in G'

Proof by contraposition

1. If (v,w) ∉ E' then (v,w) ∈ E. (By definition of the complement graph)
2. However, v, w ∈ V-C which means that v and w are not in C.
3. Thus, the edge v-w is not covered by a vertex contained in C.
4. This is a contradiction of the premise that C is a vertex cover

Proof: C vertex cover in G => V-C a clique in G'

Proof by contraposition

1. Assume C is not considered a vertex cover,
2. Thus, there is an edge v-w in E such that both are not contained in C
3. Thus, verticies v and w are contained in V-C
4. However, the edge v-w is not contained in E', since it is in E
5. This shows V-C is not a clique

Problem 5

–Define T: G→G' by V→V' construct E' by checking all pairs of vertices in V, if the edge is in E discard, if the edge is not in E then add to E', this clearly takes only polynomial time since there are less than $|V|^2$ pairs of vertices

Proof: Min Vertex Cover polynomial reducible to Max Clique

1. To solve the Min Vertex Cover Problem map Construct G' as above and solve the Max clique problem on G', call this C. Then the Min Vertex cover in G is V-C.
2. This is a vertex cover by the previous theorem stated in problem 4.
3. It is the min vertex cover since if any vertex cover was smaller it would correspond to a larger clique than C in G'. Which would contradict that C was a solution to the Max Clique problem on G'

Proof: Max Clique polynomial reducible to Min Vertex

1. Then to solve the Max Clique Problem map Construct G' as above and solve the Min Vertex Cover problem on G', call this C'. Then the Max Clique cover in G is V-C'
2. This is a Max Clique by the previous theorem stated in problem 4.
3. It is the max clique since if any vertex cover was larger it would correspond to a smaller vertex cover than C in G'. Which would contradict that C was a solution to the Min Vertex Cover problem on G'