14. 11. 2023

Practice Exercises (in class)

(1) $\int \frac{5x-4}{(x+1)(2x-1)}\,dx$ $= \frac{A}{x+1} + \frac{B}{2x-1}$

$$\frac{5x-4}{(x+1)(2x-1)} = \frac{A(2x-1) + B(x+1)}{(x+1)(2x-1)}$$

$$5x-4 = A(2x-1) + B(x+1) \quad *$$

Since this undefines at $x = -1, \frac{1}{2}$

plug $x = -1$ into $*$:

$$-9 = -3A$$

$$A = 3$$

plug $x = \frac{1}{2}$ into $*$:

$$-\frac{3}{2} = \frac{3}{2}B$$

$$B = -1$$

$$\therefore \frac{5x-4}{(x+1)(2x-1)} = \frac{3}{x+1} - \frac{1}{2x-1}$$

$$\int \frac{5x-4}{(x+1)(2x-1)} = \int \frac{3}{x+1} - \frac{1}{2x-1}\,dx$$

$$= 3\ln|x+1| - \frac{1}{2}\ln|2x-1| + C$$

$$\boxed{\int \frac{1}{Cx \pm C}\,dx = \frac{1}{C}\ln|Cx+C| + C}$$

(2) $\int \frac{x^2+18x+5}{(x+1)(x-2)(x+3)}\,dx$

$$\frac{x^2+18x+5}{(x+1)(x-2)(x+3)} = \frac{A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

$$\therefore x^2+18x+5 = A(x-2)(x+3) + B(x+1)(x+3) + C(x+1)(x-2)$$

undefines at $x = -1, 2, -3$

$x = -1$:

$$1 - 18 + 5 = A(-3)(+2)$$

$$-6A = -12$$

$$A = 2$$

$x = 2$:

$$45 = 15B$$

$$B = 3$$

$x = -3$:

$$-40 = 10C$$

$$C = -4$$

$$\frac{x^2+18x+5}{(x+1)(x-2)(x+3)} = \frac{2}{x+1} + \frac{3}{x-2} - \frac{4}{x+3}$$

$$\int \frac{x^2+18x+5}{(x+1)(x-2)(x+3)}\,dx = 2\ln|x+1| + 3\ln|x-2| - 4\ln|x+3| + C$$

Try these for the practice before exam

(3) $\int 2x\sqrt{1+x^2}\,dx$

(4) $\int \frac{(\ln x)^2}{x}\,dx$

(5) $\int \frac{\tan^{-1}x}{1+x^2}\,dx$

(6) $\int x^4 \ln x\,dx$

(7) $\int \ln\sqrt{x}\,dx$

} Tabular 3rd step

Homework (14. 11. 2023)

(1) $I \;\; \frac{11-15x}{(3x-1)(3x+2)}\,dx$

$$\frac{11-15x}{(3x-1)(3x+2)} = \frac{A}{3x-1} + \frac{B}{3x+2}$$

$$\frac{11-15x}{(3x-1)(3x+2)} = \frac{A(3x+2) + B(3x-1)}{(3x-1)(3x+2)}$$

$$11 - 15x = A(3x+2) + B(3x-1)$$

undefines at $x = \frac{1}{3}, \frac{-2}{3}$

$x = \frac{1}{3}$:

$$6 = 3A + 0$$

$$A = 2$$

$x = -\frac{2}{3}$:

$$21 = -3B$$

$$B = -7$$

$$\int \frac{11-15x}{(3x-1)(3x+2)}\,dx = \int \frac{2}{3x-1} - \frac{7}{3x+2}\,dx$$

$$= \frac{2}{3}\ln|3x-1| - \frac{7}{3}\ln|3x+2| + C$$

(2) $\int \frac{4x^2-23x+19}{(x-6)(x+1)(x+3)}\,dx$

$$\frac{4x^2-23x+19}{(x-6)(x+1)(x+3)} = \frac{A}{x-6} + \frac{B}{x+1} + \frac{C}{x+3}$$

(x+1)

$$= \frac{A(x+1)(x+3) + B(x-6)(x+3) + C(x-6)}{(x-6)(x+1)(x+3)}$$

$$4x^2-23x+19 = A(x+1)(x+3) + B(x-6)(x+3) + C(x-6)(x+1)$$

undefines at $x = 6, -1, -3$

$144 - 138 + 19 = 63A$ $\quad x = 6:$

$A = \frac{25}{63}$

$x = -1:$ $\quad 4 + 23 + 19 = -14B$

$46 = -14B$

$B = \frac{-23}{7}$

$x = -3:$ $\quad 124 = 18C$

$C = \frac{62}{9}$

$$\int \frac{4x^2-23x+19}{(x-6)(x+1)(x+3)}\,dx = \int \frac{25/63}{x-6} - \frac{23/7}{x+1} + \frac{62/9}{x+3}\,dx$$

$$= \frac{25}{63}\ln|x-6| - \frac{23}{7}\ln|x+1| + \frac{62}{9}\ln|x+3| + C$$

(3) $\int \cos(3-7x)\,dx$

Let $u = 3-7x$

$du = -7\,dx$

$-\frac{du}{7} = dx$

$$\int \cos(3-7x)\,dx = \frac{-1}{7}\int \cos u\,du$$

$$= -\frac{1}{7}\sin u + C = \frac{-\sin(3-7x)}{7} + C$$

(4) $\int e^{3x-2}\,dx$

Let $u = 3x-2$

$du = 3\,dx$

$dx = \frac{du}{3}$

$$\int e^{3x-2}\,dx = \frac{1}{3}\int e^u\,du$$

$$= \frac{1}{3}e^u + C$$

$$= \frac{1}{3}e^{3x-2} + C$$

(5) $\int -2x\sin(1-x^2)\,dx$

Let $u = 1-x^2$

$du = -2x\,dx$

$$\int \sin(1-x^2)\,(-2x)\,dx = \int \sin u\,du$$

$$= -\cos u + C$$

$$= -\cos(1-x^2) + C$$

(6) $\int \frac{\sin(3x)}{9-\cos(3x)}\,dx$

let $u = 9-\cos(3x)$

$\frac{du}{3} = \sin(3x)\,dx$

$$\int \frac{\sin(3x)}{9-\cos(3x)}\,dx = \frac{1}{3}\int \frac{1}{u}\,du$$

$$= \frac{1}{3}\ln|u| + C$$

$$= \frac{1}{3}\ln(9-\cos 3x) + C$$

(7) $\int 2x^2\sqrt{x^3+1}\,dx$

Let $u = x^3+1$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$u^{\frac{1}{2}+1}$

$u^{\frac{3}{2}}$

$$\int 2x^2\sqrt{x^3+1}\,dx = \frac{2}{3}\int \sqrt{u}\,du$$

$$= \frac{2}{3}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{4}{9}(x^3+1)^{3/2} + C$$

(8) $\int \sin^4 x \cos x \, dx$

Let $u = \sin x$

$du = \cos x \, dx$

$$\int \sin^4 x \cos x \, dx = \int u^4 \, du = \frac{u^5}{5} + C = \frac{\sin^5 x}{5} + C$$

(9) $\int \frac{x-4}{x^2-8x+3} \, dx$

Let $u = x^2 - 8x + 3$

$du = 2x - 8 \, dx$

$\frac{du}{2} = (x-4) \, dx$

$$\int \frac{x-4}{x^2-8x+3} \, dx = \frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|x^2-8x+3| + C$$

(10) $\int (e^{3x}+4)^5 e^{3x} \, dx$

Let $u = e^{3x} + 4$

$\frac{du}{3} = e^{3x} \, dx$

$$\int (e^{3x}+4)^5 e^{3x} \, dx = \frac{1}{3} \int u^5 \, du = \frac{1}{18} u^6 + C = \frac{(e^{3x}+4)^6}{18} + C$$

(11) $\int \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx$

Let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} \, dx \Rightarrow 2\,du = \frac{1}{\sqrt{x}} \, dx$

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}} \, dx = 2 \int \cos u \, du = 2 \sin u + C = 2 \sin\sqrt{x} + C$$

(12) $\int \frac{x}{e^{2x}} \, dx \to x e^{-2x}$

Choose: $u = x$, $dv = e^{-2x} \, dx$

	u	dv
$+$	x	e^{-2x}
$-$	1	$-\frac{e^{-2x}}{2}$
$+$	0	$\frac{e^{-2x}}{4}$

$$\int x e^{-2x} \, dx = \frac{-x e^{-2x}}{2} - \frac{e^{-2x}}{4} + C$$

(13) $\int x^3 \ln(x^2) \, dx$

Choose: $u = \ln(x^2)$

$dv = x^3 \, dx$

	u	dv
$+$	$\ln x^2$	x^3
$-\int$	$\frac{2}{x}$	$\frac{x^4}{4}$

$$\int x^3 \ln x^2 \, dx = \frac{x^4 \ln x^2}{4} - \int \frac{x^3}{2} \, dx = \frac{x^4 \ln x^2}{4} - \frac{x^4}{8} + C$$

(14) $\int \frac{\ln x}{\sqrt{x}} \, dx$

Choose: $u = \ln x$

$dv = \frac{1}{\sqrt{x}} \, dx$

	u	dv
$+$	$\ln x$	$x^{-\frac{1}{2}}$
$-\int$	$\frac{1}{x}$	$2x^{\frac{1}{2}}$

$$\int \frac{\ln x}{\sqrt{x}} \, dx = 2\sqrt{x} \ln x - \int 2x^{-\frac{1}{2}} \, dx = 2\sqrt{x} \ln x - 4\sqrt{x} + C$$

(15) $\int \frac{\ln(x+1)}{x^2}\,dx$

Choose : $u = \ln(x+1)$

$dv = x^{-2}\,dx$

	u	dv
+	$\ln(x+1)$	x^{-2}
−	$\int \frac{1}{x+1}$	$-x^{-1}$

$$\int \frac{\ln(x+1)}{x^2}\,dx = \frac{-\ln(x+1)}{x} + \int \frac{1}{x(x+1)}\,dx$$

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$

$$\int \frac{1}{x(x-1)} = \int \frac{1}{x} - \frac{1}{x+1}\,dx$$

$$= \ln|x| - \ln|x+1| + C$$

$$\int \frac{\ln(x+1)}{x^2}\,dx = \frac{-\ln(x+1)}{x} + \ln|x| - \ln|x+1| + C$$

(16) $\int e^{-2x}\sin 3x\,dx$

Choose : $u = \sin 3x$

$dv = e^{-2x}\,dx$

	u	dv
+	$\sin 3x$	e^{-2x}
−	$3\cos 3x$	$\frac{-e^{-2x}}{2}$
+	$\int -9\sin 3x$	$\frac{e^{-2x}}{4}$

$$\int e^{-2x}\sin 3x\,dx = \frac{-e^{-2x}\sin 3x}{2} - \frac{3e^{-2x}\cos 3x}{4} - \int \frac{9e^{-2x}\sin 3x}{4}\,dx$$

$$\int e^{-2x}\sin 3x\,dx = \frac{-2e^{-2x}\sin 3x}{13} - \frac{3e^{-2x}\cos 3x}{13} + C$$

Homework (15.11.2023)

(1) $\int \frac{3x^2+1}{x(2x^2+1)}\,dx$

$$\frac{3x^2+1}{x(2x^2+1)} = \frac{A}{x} + \frac{Bx+C}{2x^2+1}$$

$$= \frac{A(2x^2+1) + Bx^2 + Cx}{x(2x^2+1)}$$

$$3x^2+1 = A(2x^2+1) + Bx^2 + Cx$$

Let $x = 0$, then;

$1 = A$

$x = 1$, then;

$4 = 3 + B + C$

$B + C = 1$ — (1)

$x = -1$, then;

$4 = 3 + B - C$

$B - C = 1$ — (2)

$B + \cancel{C} = 1$

$B - \cancel{C} = 1$

$2B = 2$

$B = 1 \Rightarrow C = 0$

$$\int \frac{3x^2+1}{x(2x^2+1)}\,dx = \int \frac{1}{x} + \frac{x}{2x^2+1}\,dx$$

$$\int \frac{x}{2x^2+1}\,dx$$

$u = 2x^2+1$

$du = 4x\,dx$

$\frac{du}{4} = x\,dx$

$$= \ln|x| + \frac{1}{4}\ln|2x^2+1| + C$$

(2) $\int \frac{2x^3+4x^2-2x+4}{(x^2-1)^2}\,dx$

$\rightarrow ((x+1)(x-1))^2 = (x+1)^2(x-1)^2$

$$= \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$= \frac{A(x+1)(x-1)^2 + B(x-1)^2 + C(x-1)(x+1)^2 + D(x+1)^2}{(x+1)^2(x-1)^2}$$

$$2x^3+4x^2-2x+4 = \longrightarrow$$

$$\longrightarrow A(x+1)(x-1)^2 + B(x-1)^2 + C(x-1)(x+1)^2 + D(x+1)^2$$

Let $x = -1$, then;

$-2+4+2+4 = 4B$

$B = 2$

$x = 1$, then;

$2+4-2+4 = 4D$

$D = 2$

$x = 0$, then;

$4 = A + 2 - C + 2$

$A = C$

$x = 2$, then;

$16+16-4+4 = 3A + 2 + 9C + 18$ $\hookrightarrow C = A$

$12 = 12A$

$A = 1 = C$

$$\int \frac{2x^3+4x^2-2x+4}{(x^2-1)^2}\,dx = \int \frac{1}{x+1} + \frac{2}{(x+1)^2} + \frac{1}{x-1} + \frac{2}{(x-1)^2}\,dx$$

$$= \ln|x+1| - \frac{2}{x+1} + \ln|x-1| - \frac{2}{x-1} + C$$

25. 11. 2023

Find the area enclosed by the following equations by using the integral.

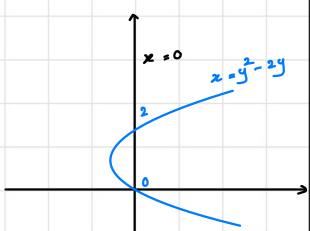
(1) Y-axis, $x = y^2 - 2y$ $\Rightarrow dy$

$x = 0$

$0 = x = y^2 - 2y$

$y^2 - 2y = 0$

$y = 0$ (or) 2

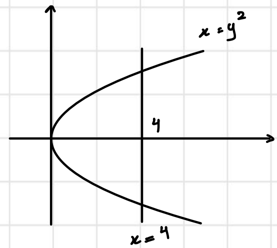


$$\text{Area} = -\int_0^2 y^2 - 2y\,dy = -\left(\frac{y^3}{3} - y^2\right)\Big|_0^2$$

$$= -\left(\frac{8}{3} - 4\right)$$

$$= +\frac{4}{3}$$

(2) $y^2 = x$, $x = 4$



$4 = x = y^2$

$y^2 = 4$

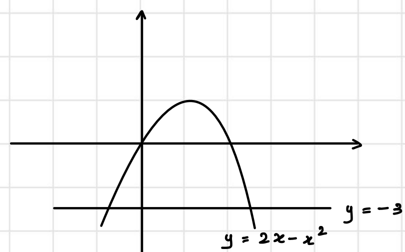
$y = \pm 2$

$$\int_{-2}^{2} 4 - y^2\,dy = \left(4y - \frac{y^3}{3}\right)\Big|_{-2}^{2}$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = \frac{32}{3}$$

(3) $y = 2x - x^2$, $y = -3$



$-3 = y = 2x - x^2$

$-3 = 2x - x^2$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1$ or $x = 3$

$$\int_{-1}^{3} 2x - x^2 + 3\,dx$$

$$= \left(x^2 - \frac{x^3}{3} + 3x\right)\Big|_{-1}^{3}$$

$$= \left(9 - \frac{27}{3} + 9\right) - \left(1 + \frac{1}{3} - 3\right)$$

$$= \frac{32}{3}$$

(4) $x = 3y - y^2$, $x = 3 - y$

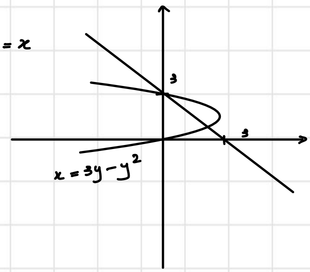
$3y - y^2 = 3 - y = x$

$3 - y = 3y - y^2$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 1$ (or) $y = 3$



$$\int_1^3 3y - y^2 - 3 + y \; dy$$

$$= \left(2y^2 - \frac{y^3}{3} - 3y\right)\Big|_1^3$$

$$= (18 - 9 - 9) - \left(2 - \frac{1}{3} - 3\right)$$

$$= 0 - \left(-\frac{1}{3} - 1\right) = \frac{4}{3}$$

(5) Y-axis, $y^2 - 4x - 4 = 0$

$\downarrow$ $x = 0$; $\hookrightarrow \dfrac{y^2 - 4}{4} = x$

$0 = \dfrac{y^2 - 4}{4}$

$y^2 - 4 = 0$

$y = \pm 2$

$$-\int_{-2}^{2} \frac{y^2}{4} - 1 \; dy = -\left(\frac{y^3}{12} - y\right)\Big|_{-2}^{2}$$

$$= -\left[\left(\frac{2}{3} - 2\right) - \left(-\frac{2}{3} + 2\right)\right]$$

$$= -\left(-\frac{4}{3} - \frac{4}{3}\right)$$

$$= \frac{8}{3}$$

(6) $x = y^2$, $x = y$

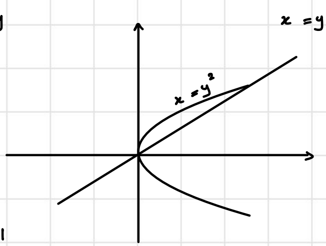
$y^2 = x = y$

$y^2 = y$

$y^2 - y = 0$

$y(y-1) = 0$

$y = 0$ (or) $y = 1$



$$\text{Area} = \int_0^1 y - y^2 \; dy$$

$$= \left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right) - 0$$

$$= \frac{1}{6}$$

(7) $y^2 = 8x$, $x^2 = 4y$ $\longrightarrow$ $x = \pm\sqrt{4y}$

only $+\sqrt{4y}$ is needed

$\hookrightarrow \dfrac{y^4}{64} = x^2$

$\dfrac{y^4}{64} = x^2 = 4y$

$y^4 = 256y$

$y^4 - 256y = 0$

$y(y^3 - 256) = 0$

$y = 0$ (or) $y^3 = 256$

$y = 6.35$

$$\text{Area} = \int_0^{6.35} 2\sqrt{y} - \frac{y^2}{8} \; dy$$

$$= \left(\frac{4y^{\frac{3}{2}}}{3} - \frac{y^3}{24}\right)\Big|_0^{6.35}$$

$$= \left(\frac{4}{3}(6.35)^{\frac{3}{2}} - \frac{(6.35)^3}{24}\right) - 0$$

$$= 10.667 \approx \frac{32}{3}$$

(8) $x^2 - 5x + y = 0$, $y = x$

$\hookrightarrow y = 5x - x^2$

$x = y = 5x - x^2$

$x = 5x - x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ (or) $x = 4$

$$\text{Area} = \int_0^4 5x - x^2 - x \, dx$$

$$= \left(2x^2 - \frac{x^3}{3}\right)\Big|_0^4$$

$$= \left(32 - \frac{64}{3}\right) - 0$$

$$= \frac{32}{3}$$

3. 12. 2023

Improper Integral

(1) $\int_3^\infty \frac{1}{x^4} \, dx$

$$= \lim_{t\to\infty} \int_3^t \frac{1}{x^4} \, dx$$

$$= \lim_{t\to\infty} \frac{-1}{3x^3}\Big|_3^t$$

$$= \lim_{t\to\infty} \cancel{\frac{-1}{3t^3}}^{\,0} + \frac{1}{3 \cdot 3^3}$$

$$= 0 + \frac{1}{3^4} = \left(\frac{1}{3}\right)^4$$

(2) $\int_1^\infty \frac{\ln x}{x} \, dx$

$$\int \frac{\ln x}{x} \, dx = \int \ln x \cdot \frac{1}{x} \, dx$$

Let $u = \ln x$

$du = \frac{1}{x} dx$

$$= \int u \, du$$

$$= \frac{u^2}{2} + C$$

$$= \frac{(\ln x)^2}{2} + C$$

$$\int_1^\infty \frac{\ln x}{x} \, dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x} \, dx$$

$$= \lim_{t\to\infty} \frac{(\ln x)^2}{2}\Big|_1^t$$

$$= \lim_{t\to\infty} \cancel{\frac{(\ln t)^2}{2}}^{\,\infty} - \cancel{\frac{(\ln 1)^2}{2}}^{\,0}$$

$$= \infty \quad (\text{diverges})$$

(3) $\int_{-\infty}^{-1} \frac{1}{\sqrt[3]{x}} \, dx$

$$= \lim_{t\to-\infty} \int_t^{-1} x^{-\frac{1}{3}} \, dx$$

$$= \lim_{t\to-\infty} \frac{3x^{2/3}}{2}\Big|_t^{-1}$$

$$= \lim_{t\to-\infty} \left[\frac{3(-1)^{2/3}}{2} - \frac{3(t)^{2/3}}{2}\right]$$

$$= \frac{3}{2} - \infty = \infty \quad (\text{diverges})$$

(4) $\int_0^\infty \frac{1}{e^x + e^{-x}} \, dx$

$$\int \frac{1}{e^x + e^{-x}} \, dx = \int \frac{1}{e^x + \frac{1}{e^x}} \, dx$$

$$= I \frac{1}{\frac{e^{2x}+1}{e^x}} \, dx$$

$$= I \frac{e^x}{(e^x)^2 + 1} \, dx$$

Let $u = e^x$

$du = e^x dx$

$$= I \frac{1}{u^2+1} \, du$$

$$= \tan^{-1} u + C$$

$$= \tan^{-1}(e^x) + C$$

$$\int_0^\infty \frac{1}{e^x + e^{-x}} \, dx = \lim_{t\to\infty} \int_0^t \frac{1}{e^x + e^{-x}} \, dx$$

$$= \lim_{t\to\infty} \tan^{-1} e^x \Big|_0^t$$

$$= \lim_{t\to\infty} \left(\tan^{-1} e^t - \tan^{-1} e^0\right)$$

$1 = \tan(?)$

$\frac{\sin ?}{\cos ?}$

$$= \lim_{t\to\infty} \tan^{-1} e^t - \frac{\pi}{4} \Rightarrow \boxed{\begin{array}{l} 1 = \tan\left(\frac{\pi}{4}\right) \\ \tan^{-1}(1) = \frac{\pi}{4} \end{array}}$$

$\tan^{-1}(\infty) = ?$

$\infty = \tan(?)$

$\frac{\sin}{\cos} \Rightarrow \frac{\pi}{2}$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{2\pi - \pi}{4} = \frac{\pi}{4}$$

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(5) $\displaystyle\int_0^1 x \ln x \, dx$

$\displaystyle\int x \ln x \, dx = ?$

By Tabular Method:

choose $u = \ln x$, $dv = x\,dx$

	u	dv
$+$	$\ln x$	x
$\int -$	$\frac{1}{x}$	$\frac{x^2}{2}$

$$\int x \ln x \, dx = \frac{x^2 \ln x}{2} - \int \frac{x^{\cancel{2}}}{2\cancel{x}}\, dx$$

$$= \frac{x^2 \ln x}{2} - \int \frac{x}{2}\, dx$$

$$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$$

$$\int_0^1 x \ln x \, dx = -\int_0^1 x \ln x \, dx$$

$$= -\left(\frac{x^2 \ln x}{2} - \frac{x^2}{4}\right)\Bigg|_0^1$$

$$= \frac{x^2}{4} - \frac{x^2 \ln x}{2}\Bigg|_0^1$$

$$= \left(\frac{1}{4} - \frac{0}{2}\right) - 0$$

$$= \frac{1}{4}$$

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(6) $\displaystyle\int_1^e \frac{1}{x \ln x}\, dx$

$\displaystyle\int \frac{1}{x \ln x}\, dx = ?$ Let $u = \ln x$

$du = \frac{1}{x}\, dx$

$$= \int \frac{1}{\ln x} \cdot \frac{1}{x}\, dx = \int \frac{1}{u}\, du = \ln|u| + C$$

$$\int \frac{1}{x \ln x}\, dx = \ln(\ln x) + C$$

$$\int_1^e \frac{1}{x \ln x}\, dx = \lim_{t \to 1^+} \int_t^e \frac{1}{x \ln x}\, dx$$

$$= \lim_{t \to 1^+} \ln(\ln x)\Big|_t^e$$

$$= \lim_{t \to 1^+} \ln(\ln e) - \overset{-\infty}{\ln(\ln t)}$$

$$= \infty \text{ (diverges)}$$

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(7) $\displaystyle\int_0^{\pi} \frac{\sin x}{1 + \cos x}\, dx$

$\displaystyle\int \frac{\sin x}{1 + \cos x}\, dx = ?$

Let $u = 1 + \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$$\int \frac{1}{1 + \cos u} \cdot \sin x\, du = \int \frac{1}{u} - du$$

$$= -\ln|u| + C$$

$$= -\ln|1 + \cos x| + C$$

$$\int_0^{\pi} \frac{\sin x}{1 + \cos x}\, dx = \lim_{t \to \pi^-} \int_0^t \frac{\sin x}{1 + \cos x}\, dx$$

$$= \lim_{t \to \pi^-} -\ln|1 + \cos x|\Big|_0^t$$

$$= \lim_{t \to \pi^-} -\ln|\cos x + 1|\Big|_0^t$$

$$= \lim_{t \to \pi^-} -\underset{-\infty}{\ln|\cos(t) + 1|} + \ln 2$$

$$= \infty \text{ (diverges)}$$

---

(8) $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x^2 - 1}}\, dx$

$$= \lim_{t \to \infty} \int_1^t \frac{1}{x\sqrt{x^2 - 1}}\, dx$$

$$= \lim_{t \to \infty} \sec^{-1}(x)\Big|_1^t$$

$$= \lim_{t\to\infty} \sec^{-1}(t) - \cancel{\sec^{-1}(1)} \to 0 \qquad a = ?$$

$\sec^{-1}(\infty) = a$ $\qquad 1 = \sec(a)$

$\infty = \sec(a)$ $\qquad = \frac{1}{\cos(a)} \to 0$

$= \frac{1}{\cos(a)}$

$= \frac{1}{\cos(\frac{\pi}{2})}$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Trapezoidal Rule

$$\Delta x = \frac{b-a}{n}, \quad x_k = k\Delta x$$

$$\int_a^b f(x_n)\,dx = \frac{b-a}{2n}\left(f(x_0) + 2\left(f(x_1) + f(x_2) + f(x_3) + f(x_4)\right) + f(x_5)\right)$$

(1) $\int_0^1 \cos\sqrt{x}\,dx$

$f(x_0) = f(0) = \cos\sqrt{0}$

$f(x_1) = f(0.2) = \cos\sqrt{0.2}$

$f(x_2) = f(0.4) = \cos\sqrt{0.4}$

$f(x_3) = f(0.6) = \cos\sqrt{0.6}$

$f(x_4) = f(0.8) = \cos\sqrt{0.8}$

$f(x_5) = f(1) = \cos\sqrt{1}$

$$\int_0^1 \cos\sqrt{x}\,dx \approx \frac{0.2}{2}\left(1 + 2(0.9 + 0.81 + 0.71 + 0.63) + 0.54\right)$$

$= 0.746$

(2) $\int_0^1 \frac{1}{x^3+1}\,dx$

$$\int_0^1 \frac{1}{x^3+1}\,dx \approx \frac{1-0}{2.5}\left(f(0) + 2\left(f(0.2) + f(0.4) + f(0.6) + f(0.8)\right) + f(1)\right)$$

= do it yourself

(3) $\int_0^1 e^{x^2}\,dx$

$f(x_0) = f(0) = 1$

$f(x_1) = f(0.2) = 1.04$

$f(x_2) = f(0.4) = 1.17$

$f(x_3) = f(0.6) = 1.43$

$f(x_4) = f(0.8) = 1.9$

$f(x_5) = f(1) = 2.718$

$$\int_0^1 e^{x^2}\,dx \approx \frac{1-0}{2.5}\left(1 + 2(1.04 + 1.17 + 1.43 + 1.9) + 2.718\right)$$

$= 1.48$

(4) $\int_0^1 \sqrt{\cos x}\,dx$

$f(x_0) = f(0) = 1$

$f(x_1) = f(0.2) = 0.99$

$f(x_2) = f(0.4) = 0.96$

$f(x_3) = f(0.6) = 0.91$

$f(x_4) = f(0.8) = 0.83$

$f(x_5) = f(1) = 0.74$

$$\int_0^1 \sqrt{\cos x}\,dx \approx 0.912$$

(5) $\int_0^1 \frac{1}{\ln(x+1)+1}\,dx$

$f(x_0) = f(0) = 1$

$f(x_1) = f(0.2) = 0.85$

$f(x_2) = f(0.4) = 0.75$

$f(x_3) = f(0.6) = 0.68$

$f(x_4) = f(0.8) = 0.63$

$f(x_5) = f(1) = 0.59$

$$\int_0^1 \frac{1}{\ln(x+1)+1}\,dx \approx 0.741$$

(6) $\int_0^1 e^{\sqrt{x}}\, dx$

$f(x_0) = f(0) = 1$

$f(x_1) = f(0.2) = 1.56$

$f(x_2) = f(0.4) = 1.88$

$f(x_3) = f(0.6) = 2.17$

$f(x_4) = f(0.8) = 2.45$

$f(x_5) = f(1) = 2.718$

$\int_0^1 e^{\sqrt{x}}\, dx \approx 1.98$

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(7) $\int_0^1 2^{x^2}\, dx$

$f(x_0) = f(0) = 1$

$f(x_1) = f(0.2) = 1.03$

$f(x_2) = f(0.4) = 1.12$

$f(x_3) = f(0.6) = 1.28$

$f(x_4) = f(0.8) = 1.56$

$f(x_5) = f(1) = 2$

$\int_0^1 2^{x^2}\, dx \approx 1.298$

---

(8) $\int_0^1 2^{\cos x}\, dx$

$f(x_0) = f(0) = 2$

$f(x_1) = f(0.2) = 1.97$

$f(x_2) = f(0.4) = 1.89$

$f(x_3) = f(0.6) = 1.77$

$f(x_4) = f(0.8) = 1.62$

$f(x_5) = f(1) = 1.45$

$\int_0^1 2^{\cos x}\, dx \approx 1.795$