Exercise: Vectors, Lines and Planes

1. Let

$$\vec{A} = \langle 3, 4\rangle \qquad \vec{B} = \langle -3, 0\rangle \qquad \vec{C} = \langle -3, 2, -4\rangle$$
$$\vec{D} = \langle 6, -4, 8\rangle \qquad \vec{E} = \langle 1, 3, 2\rangle \qquad \vec{F} = \langle 2, -3, 1\rangle$$

   1.1 Find $\vec{B} - 3\vec{A}$

   1.2 Find $\| -4\vec{B}\|\vec{C} + 5\vec{F}$

   1.3 Find the vector of length 2 that has the same direction as $\vec{D}$.

   1.4 Show that the vector $\vec{C}$ and $\vec{D}$ are parallel.

   1.5 Find the initial point of $\vec{E}$, if the terminal points is $(4, -3, 1)$.

2. Let $\vec{v} = 10\vec{i} + 11\vec{j} - 2\vec{k}$ and $\vec{u} = 3\vec{j} + 4\vec{k}$. Find

   2.1 $\vec{v} \cdot \vec{u}$, $\|\vec{v}\|$, $\|\vec{u}\|$.

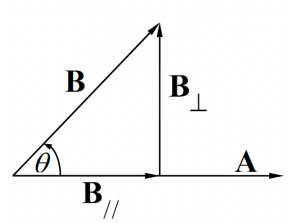
   2.2 Cosine of the angle between $\vec{u}$ and $\vec{v}$.

   2.3 The scalar component of $\vec{u}$ in the direction of $\vec{v}$ ($\text{Comp}_{\vec{v}}\vec{u}$).

   2.4 The projection vector of $\vec{u}$ along $\vec{v}$ ($\text{proj}_{\vec{v}}\vec{u}$).

3. Find the angle between $\vec{u} = \sqrt{3}\vec{i} - 7\vec{j}$, $\vec{v} = \sqrt{3}\vec{i} + \vec{j} - 2\vec{k}$

4. For general vectors $\vec{A}$ and $\vec{B}$, the vectors $\vec{B}_{\perp}$ and $\vec{B}_{/\!/}$ are defined as in the below diagram:



   If $\vec{A} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{B} = 5\vec{j} - 3\vec{k}$

   Find $\vec{B}_{/\!/}$ and $\vec{B}_{\perp}$.

5. Let $\vec{A} = \langle 1, 2, 0 \rangle, \vec{B} = \langle 2, -1, 5 \rangle$ and $\vec{C} = \langle 7, 3, 1 \rangle$. Find

   5.1 $\vec{A} \times (\vec{B} \times \vec{C})$

   5.2 $\vec{A} \times (\vec{C} - 2\vec{B})$

   5.3 $4\vec{C} \times (3\vec{A} \times 2\vec{A})$

   5.4 $\frac{1}{10}\left[\left[\frac{1}{5}(\vec{A} \times \vec{B})\right] \times (\vec{A} \times \vec{C})\right]$

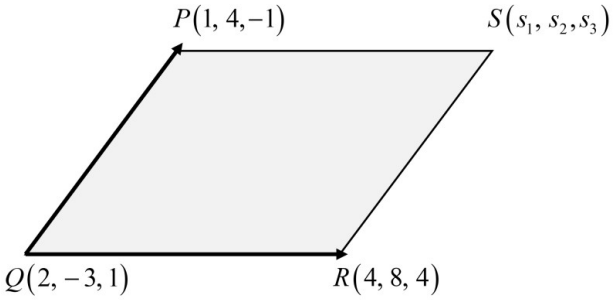
6. Let $\vec{A} = \langle 3, 1, 1 \rangle, \vec{B} = \langle -1, 2, 1 \rangle$ and $\vec{C} = \langle 4, -8, -4 \rangle$. Which vectors are perpendicular to each other? And which vectors are parallel?

7. Determine whether the given vectors $\vec{A}, \vec{B}$ and $\vec{C}$ lie in the same plane or not?

   7.1 $\vec{A} = \langle -2, 5, 2 \rangle, \vec{B} = \langle 3, 0, -2 \rangle, \vec{C} = \langle 1, 5, 0 \rangle$

   7.2 $\vec{A} = \langle -3, 1, -2 \rangle, \vec{B} = \langle 2, 2, 7 \rangle, \vec{C} = \langle 1, -1, -1 \rangle$

8. Let $P(1, 4, -1), Q(2, -3, 1), R(4, 8, 4)$ and S be the vertices of the parallelogram $PQRS$. Find the coordinates of the point S.



(NOT TO SCALE)

9. Let

$$\vec{A} = \vec{i} + 5\vec{j} - 3\vec{k} \qquad \vec{B} = -4\vec{i} + \vec{j} - 2\vec{k}$$
$$\vec{C} = 7\vec{i} - \vec{k} \qquad \vec{D} = 4\vec{j} + 3\vec{k}$$

   Find

   9.1 $\| -2\vec{D} \| (\vec{A} + 3\vec{B}) \cdot \vec{D}$

   9.2 The angle between vector $\vec{B}$ and $\vec{C}$.

   9.3 The volume of the rectangular cuboid with $\vec{A}, \vec{B}$ and $\vec{C}$ are adjacent sides.

   9.4 The vector of length $\sqrt{5}$ and orthogonal to $\vec{C}$ and $\vec{D}$.

10. Let

$$A(0,2,2), \quad B(8,8,-2), \quad C(9,12,6), \quad D(2,0,4), \quad E(5,-1,3)$$

Answer the following questions:

10.1 The vector $\overrightarrow{AB}$ is orthogonal to vector $\overrightarrow{CD}$ or not?

10.2 If $\|k\overrightarrow{CB}\| = 9$. Find the value of k.

10.3 Find the area of the triangle A, B, C.

10.4 Find the equation of the plane passing through points A, D, E.

10.5 Find the symmetric equations of the line passing through the points E and parallel to $\overrightarrow{DC}$.

11. Find symmetric equations of the line passing through the point $(3,0,1)$ and parallel to the line.

$$x = y + 2, \quad z = 4.$$

12. Find an equation of the plane that passes through the point $(1,-3,6)$ and perpendicular to the planes $3x + y - z = 3$ and $x - y + 3z = 6$.

13. Determine whether the line and plane are parallel or perpendicular?

13.1 $x = 4 + 2t, \quad y = -t, \quad z = 3 - 4t, \quad 3x + 2y + z - 5 = 0.$

13.2 $x = -1 + 2t, \quad y = 4 + t, \quad z = 1 - t, \quad -2x - y + z = 12.$

14. Find the acute angle of intersection of the planes $x + y - 2z = 5$ and $3y - 4z = 6$.

15. Find the distance between the point and the plane.

15.1 $(1,-2,3); \quad 2x - 2y + z = 5$

15.2 $(0,1,5); \quad 3x + 7y - 2z - 5 = 0$