15.24

$z = \ln(x^2+2y^3)$

$z'_x = \frac{2x}{x^2+2y^3}$ $\quad z'_y = \frac{6y^2}{x^2+2y^3}$

15.25

$z = (1+x^2)^y$

$z'_x = y(1+x^2)^{y-1} \times 2x$

$z'_y = (1+x^2)^y \cdot \ln(1+x^2)$

15.31

$z = 2xy - 4x - 2y$

$z'_x = 2y - 4$ $\quad z'_y = 2x - 2$

$\begin{cases} 2y - 4 = 0 \\ 2x - 2 = 0 \end{cases} \Rightarrow \begin{cases} y = 2 \\ x = 1 \end{cases}$

$z''_{xx} = 0$ (A) $\quad z''_{yy} = 0$ (B) $\quad z''_{xy} = z''_{yx} = 2$ (C)

$AC - B^2$ — экст. не выч.

15.37

$z = x^2 - y^2$

$\begin{cases} z'_x = 2x \\ z'_y = -2y \end{cases}$ кр: $x = 0$, $y = 0$

$f\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \ldots \frac{f(x_1, y_1) + f(x_2, y_2)}{2}$

$x_1, y_1 \to (0; 0)$

$x_2, y_2 \to (1; 1)$ $\quad z\left(\frac{1}{2}; \frac{1}{2}\right) = \frac{1}{4} - \frac{1}{4} = 0$

$\frac{z(0;0) + z(1;1)}{2} = \frac{0}{2} = 0$

$(x_1, y_1) \Rightarrow (2; 3)$ $\quad z(2;3) = 9 - 16 = -7$

$(x_2, y_2) \Rightarrow (4; 5)$ $\quad \frac{z(2;3) + z(4;5)}{2} =$

$\Rightarrow \frac{(4-9) + (16-25)}{2} =$

$= -7$

не выпукла

15.39 $y = ax + b$

x	0,5	1,0	1,5	2,0	2,5	3,0
y	0,7	1,7	1,6	3,1	3,6	4,6

$$S = \sum_{i}^{n} (y(x) - y)^2 = \sum_{i}^{6} (a x_i + b - y_i)^2$$

$$D = n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 = 136,\overline{3} - 110,25 = \underline{26,25}$$

$$\begin{cases} \left(\sum x_i^2\right) a + \left(\sum x_i\right) b = \sum x_i y_i \\ \left(\sum x_i\right) \cdot a + n \cdot b = \sum y_i \end{cases}$$

$$\begin{cases} 136,\overline{3}\, a + 10,\overline{5}\, b = 33,\overline{4}\overline{3} \\ 10,\overline{5}\, a + 6\, b = 15,3 \end{cases}$$

$$b = \frac{15,3 - 10,\overline{5}\, a}{6} \Rightarrow a = -0,226667$$

$$b = 2,15333$$

$$y = -0,226667x + 2,15333$$

упр. 5
стр 288

$$A = \frac{1}{D}\left(N\sum xy - \sum x \sum y\right)$$

$$B = \frac{1}{D}\left(\sum x^2 \sum y - \sum x \sum xy\right)$$

$$D = N\sum x^2 - \left(\sum x\right)^2$$

$$\begin{cases} A\sum x^2 + B\sum x = \sum xy & \times -\frac{1}{\sum x^2} \\ A\sum x + NB = \sum y & \times \frac{1}{\sum x} \end{cases}$$

$$\begin{cases} -A - B\frac{\sum x}{\sum x^2} = -\frac{\sum xy}{\sum x^2} \\ A + NB\frac{\sum x}{\sum x} = \frac{\sum y}{\sum x} \end{cases} +$$

$$\begin{cases} -A - B\frac{\sum x}{\sum x^2} = -\frac{\sum xy}{\sum x^2} \\ 0 + B\left(\frac{N}{\sum x} - \frac{\sum x}{\sum x^2}\right) = \frac{\sum y}{\sum x} - \frac{\sum xy}{\sum x^2} \end{cases}$$

$$\begin{cases} A = \frac{1}{\sum x^2}\left(\sum xy - B\sum x\right) \\ B = \frac{\sum y \sum x^2 - \sum x \sum xy}{\sum x \sum x^2} \times \frac{\sum x \sum x^2}{N\sum x^2 - \left(\sum x\right)^2} \end{cases}$$

$$B = \frac{\sum y \sum x^2 - \sum x \sum xy}{N\sum x^2 - \left(\sum x\right)^2}$$

5

$$A = \frac{\sum xy}{\sum x^2} - \frac{\sum x}{\sum x^2} \times \frac{\sum y \sum x^2 - \sum x \sum xy}{N \sum x^2 - (\sum x)^2} =$$

$$= \frac{N \sum x^2 \sum xy - \cancel{\sum xy (\sum x)^2} - \sum x \sum y \sum x^2 + \cancel{\sum x \sum x \sum xy}}{\sum x^2 (N \sum x^2 - (\sum x)^2)} =$$

$$= \frac{N \cancel{\sum x^2} \sum xy - \sum x \sum y \cancel{\sum x^2}}{\cancel{\sum x^2} (N \sum x^2 - (\sum x)^2)} =$$

$$A = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - (\sum x)^2}$$

$$B = \frac{\sum y \sum x^2 - \sum x \sum xy}{N \sum x^2 - (\sum x)^2}$$

ответ получен

5.2 a)
стр. 303

x	-3	-1	1	3
y	15	5	1	5

6

$$f(x) = Ax^2 + Bx + C$$

$$A\sum x_i^4 + B\sum x_i^3 + C\sum x_i^2 = \sum y_i x_i^2$$

$$A\sum x^3 + B\sum x^2 + C\sum x_i = \sum yx$$

$$A\sum x^2 + B\sum x + C\cdot n = \sum y$$

$$\sum_i x_i = 0 \quad \sum_i x_i^2 = 20 \quad \sum_i x_i^3 = 0 \quad \sum_i x_i^4 = 164$$

$$\sum_i x_i^2 y = 186 \quad \sum_i x_i y = -34 \quad \sum_i y = 26$$

$$\begin{cases} 164A + 20C = 186 \\ 20B = -34 \\ 20A + 4C = 26 \end{cases}$$

$$A = \frac{7}{8}$$

$$B = -\frac{17}{10}$$

$$C = \frac{17}{8}$$

$$f(x) = \frac{7}{8}x^2 - \frac{17}{10}x + \frac{17}{8}$$

Упражн. 3 стр. 504

а) $f(x) = Ce^{Ax}$

$X = x$

$Y = \ln(y)$

$C = e^B$

$X = x$: $y = Ce^{AX}$

$\ln(y) = Ax + \ln C$

$Y = \ln(y)$: $\ln(y) = Ce^{Ax}$

$\ln(y) = \ln e^{Ce^{Ax}}$

$y = e^{Ce^{Ax}} \Rightarrow \ln y = Ce^{Ax}$

$C = e^B$: $y = e^B e^{Ax} \Rightarrow e^{Ax+B}$

$\ln y = Ax + B$

б) $f(x) = Cx^A$

$X = \ln(x)$: $\Rightarrow y = C \cdot \ln(x)^A$

$\ln y = \ln C + A\ln(\ln x)$

$C = e^B$: $y = e^B x^A$

$\ln y = B + A\ln(x)$

$Y = \ln(y)$: $\ln(y) = \ln e^{Cx^A}$

$y = e^{Cx^A} \Rightarrow \ln y = C \cdot x^A$

в)

x	1	2	3	4	5
y	0,6	1,9	4,3	7,6	12,6

$y = Ax + B$

$\sum x^2 = 55$ $\quad \sum xy = 477$

$\sum x = 15$ $\quad \sum y = 14,4$

$$\begin{cases} 55A + 15B = 47{,}7 \\ 25A + 5B = 14{,}4 \end{cases}$$

$A \Rightarrow 0{,}45$

$B \Rightarrow 1{,}53$

$$y = 0{,}45x + 1{,}53$$

Упражн. 6
стр. 305

$P(t) < L$

$$P(t) = \frac{L}{1+Ce^{At}}$$

a) (0; 200) (1; 400) (2; 650) (3; 850) (4; 950) $\Rightarrow L = 1000$

A - ? C - ?

x	0	1	2	3	4
y	200	400	650	850	950

$$y \cdot (1 + Ce^{Ax}) = 1000$$

$$1 + Ce^{Ax} = 1000 \frac{1}{y} \Rightarrow C \to e^{B}$$

$$e^{B+Ax} = 1000 \frac{1}{y} - 1 \qquad Y = \ln\left(\frac{1000-y}{y}\right)$$

$$Y = Ax + B = \ln(1000-y) - \ln y$$

$\sum x^2 = 30$ $\sum x = 10$ $\sum xy = 3050$ $\sum y = 3050$

$$\begin{cases} 30A + 10B = 3050 \\ 10A + 5B = 3050 \end{cases} \Rightarrow \begin{matrix} A \to 195 \\ B \to 220 \end{matrix}$$

$$Y = 195x + 220 = \ln \frac{1000-y}{y}$$

$$P = e^{195x+220} = \frac{1000-y}{y}$$

$$y \cdot P + y = 1000$$

$$y(P+1) = 1000$$

$$y = \frac{1000}{e^{195x+220}+1}$$

Упр. 6

б) $y = \frac{5000}{1 - Ce^{Ax}}$

$\Leftarrow$

$Y = \ln\left(\frac{5000}{y} - 1\right)$

$\ln\left(\frac{5000}{y} - 1\right) = Ax + \ln(C)$

$Y = Ax + \ln C$

$\sum x^2 = 30 \quad \sum x = 10$

$\sum xy = 27500 \quad \sum y = 9500$

$\begin{cases} 30A + 10\ln C = 27500 \\ 10A + 5\ln C = 9500 \end{cases}$

$A \to 820 \qquad \ln C \to 320$

$C = e^{320}$

$\ln\left(\frac{5000}{y} - 1\right) = 820x + e^{320}$

$\cancel{e^{820x + 320} + e}$

$y = \frac{e^{820x + e^{320}} + 1}{5000}$

упр. 18 а)

(1,1,7) (1,2,9) (2,1,10) (2,2,11) (2,3,12)

x	1	1	2	2	2
y	1	2	1	2	3
z	7	9	10	11	12

$$z = f(x, y)$$

$$S = \sum_{i}^{5} \left(f(x, y) - z\right)^2 = \sum_{0}^{5} \left(Ax + By + C - z\right)^2$$

$$S'_A = \sum 2(Ax + By + C - z)x = 0$$

$$S'_B = \sum 2(Ax + By + C - z)y = 0$$

$$S'_C = \sum 2(Ax + By + C - z) = 0$$

$$A = \frac{12}{5} \quad B = \frac{6}{5} \quad C = \frac{19}{5}$$

$$z = \frac{12}{5}x + \frac{6}{5}y + \frac{19}{5}$$

$$\begin{cases} A\sum x^2 + B\sum xy + C\sum x - \sum zx = 0 \\ A\sum xy + B\sum y^2 + C\sum y - \sum zy = 0 \\ A\sum x + B\sum y + nC - \sum z \end{cases}$$

$$\sum x^2 = 14 \quad \sum x = 8 \quad \sum xy = 15$$

$$\sum y^2 = 19 \quad \sum y = 8 \quad \sum zx = 82$$

$$\sum zy = 93$$

$$\sum z = 49$$

$$\begin{cases} 14A + 15B + 8C - 82 = 0 \\ 15A + 19B + 8C - 93 = 0 \\ 8A + 8B + 5C - 49 = 0 \end{cases}$$