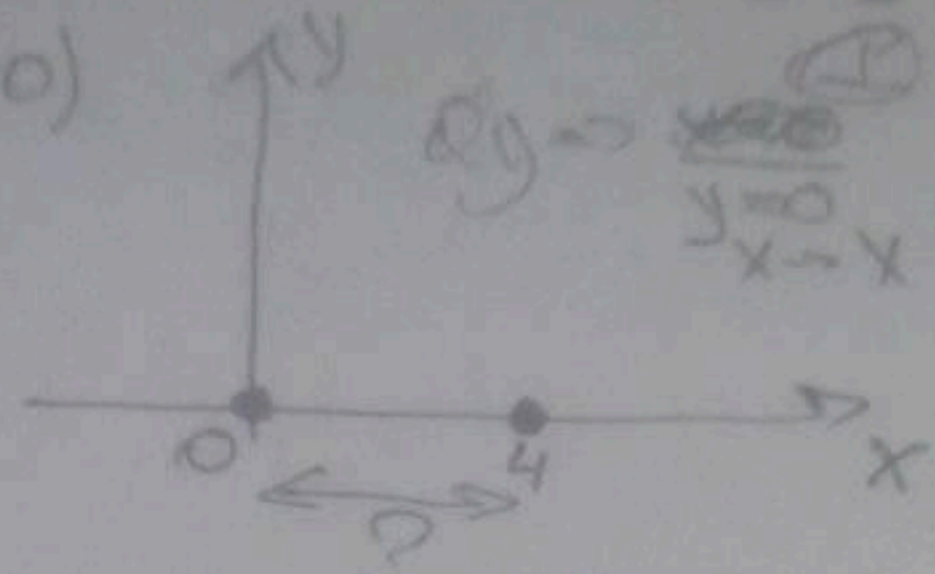
4.14 Oy F(4;0)



$d=\sqrt{(4-0)^2+0} \Rightarrow$

$d \Rightarrow 4$

$M(x;y) \Rightarrow \quad Oy \cdot M = B \cdot M$

$\sqrt{(x-0)^2+(y-0)^2} = \sqrt{(x-4)^2+(y-0)^2}$

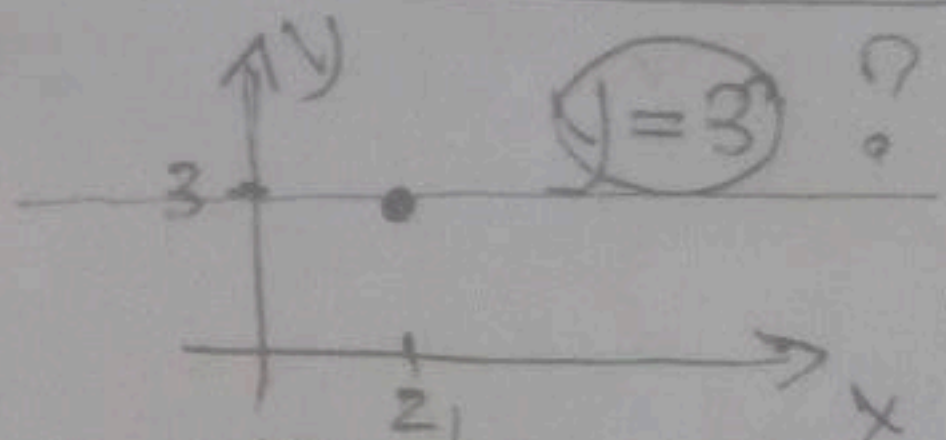
$x^2 = x^2-8x+16+y^2$

$y^2 = 8(x-2)$

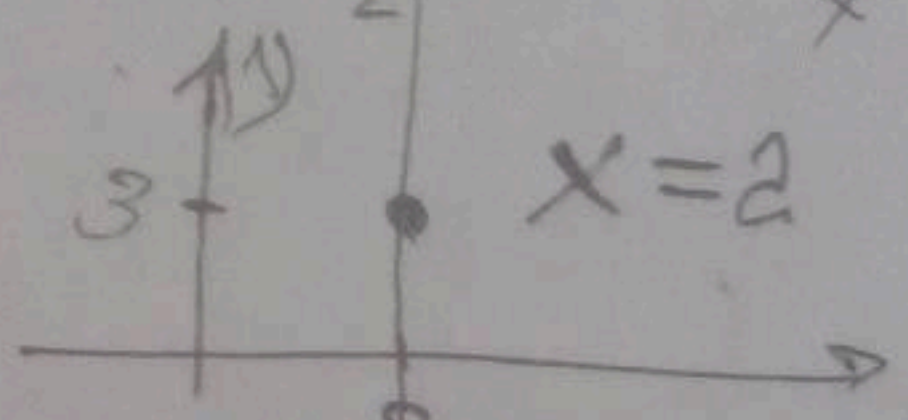
4.15 A(2;3)

а) $\parallel Ox$

$y=kx+b$



б) $\parallel Oy$



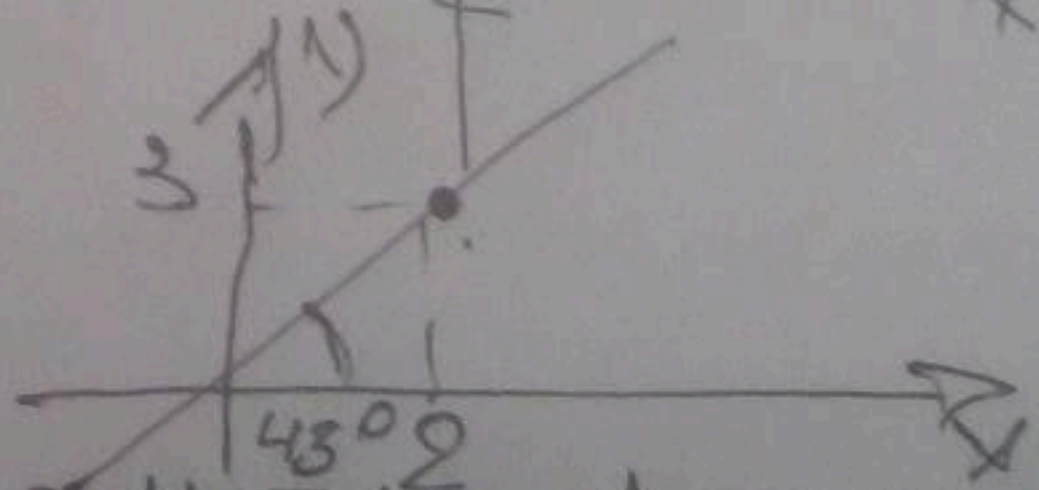
в) $\angle 45^\circ Ox$

$y=kx+b$

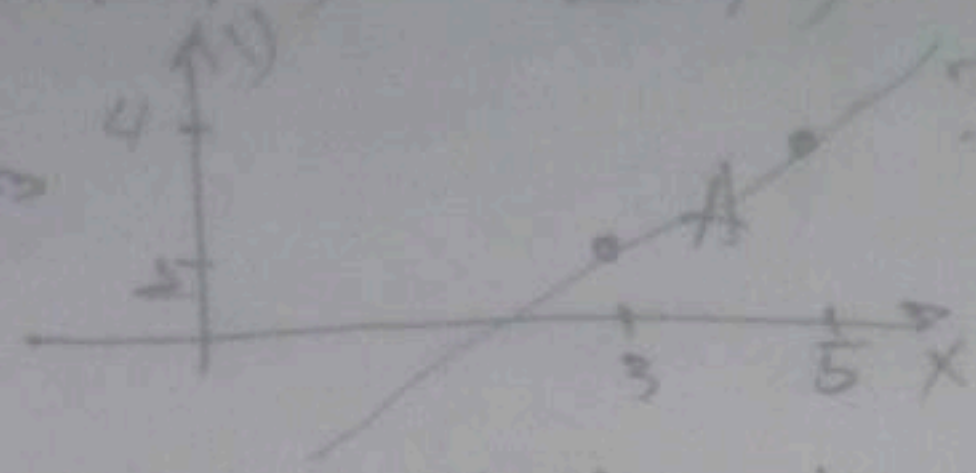
$k=\operatorname{tg} 45^\circ = 1$

$y-y_1=k(x-y_1)$

$y-3=x-2 \Rightarrow y=x-1$



4.16 a) A(3, 1) B(5, 4)



$$k = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{4-1} \Rightarrow$$

$$k \Rightarrow \frac{2}{4} = 0,5$$

$$(y - y_1) = k(x - x_1)$$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$y - 1 = 0,5(x - 3)$$

$$y = \frac{1}{2}x - \frac{3}{2} + 1 \Rightarrow \quad y = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}(x - 1)$$

$$\frac{y-1}{4-1} = \frac{x-3}{5-3} \Rightarrow$$

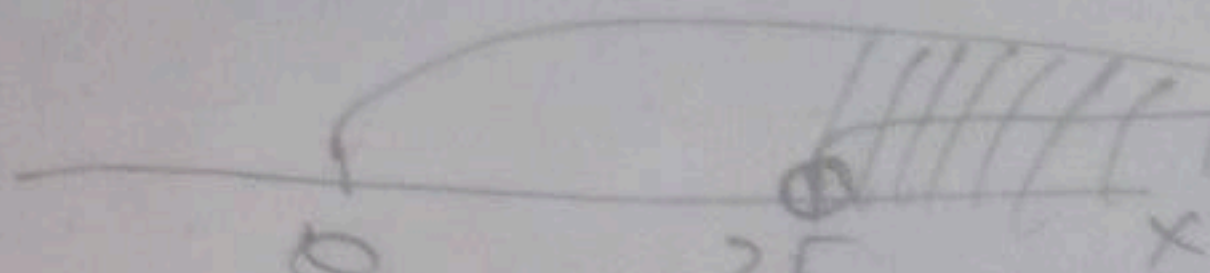
$$3x - 9 = 2y - 2$$

$$3x - 2y - 7 = 0$$

(5.8) (5)

$y = \sqrt{x} + \lg(2x-5)$

$$\begin{cases} x \geq 0 \\ 2x - 5 > 0 \end{cases} \Rightarrow \begin{cases} x \geq 0 \\ x > 2{,}5 \end{cases}$$



$x \in (2{,}5; +\infty)$

(5.11) $y = \sqrt{3}\sin x + \cos x$

$y = 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right)$

$\sin\frac{\pi}{3}\sin x + \cos\frac{\pi}{3}\cos x$

$\cos\left(\frac{\pi}{3} + x\right) \leq 2$

$|y| \leq 2$

$\cos\frac{\pi}{6}\sin x + \sin\frac{\pi}{6}\cos x$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

7.28

$(u \cdot v)' = u'v - v'u$ (1)

$y = x^3 \cdot \ln^2 x$

$y' = 3x^2 \cdot \ln^2 x + x^3 \cdot 2\ln x \cdot \frac{1}{x}$

$y' = 3x^2 \ln^2 x + \frac{x^2}{2} \ln x$

7.29 $y = (xe^{2x} + 3)^5$

$y' = 5(xe^{2x} + 3)^4 \times \left(e^{2x}(1 + 2x)\right)$

$(xe^{2x} + 3)' = e^{2x} + x \cdot e^{2x} \cdot 2 = e^{2x} + 2xe^{2x} = e^{2x}(1 + 2x)$

$(xe^{2x})' = e^{2x} + x \cdot (e^{2x})' = e^{2x} + 2xe^{2x}$

$= e^{2x}(2x + 1)$

7.43 $y = \ln(x + \sqrt{x^2+12})$ $x_0 = 2$

$$y' = \frac{1}{x+\sqrt{x^2+12}} \cdot \left(1 + \frac{1 \cdot 2x}{2\sqrt{x^2+12}}\right) =>$$

$$y' = \frac{1}{x+\sqrt{x^2+12}} \cdot \left(1 + \frac{x}{\sqrt{x^2+12}}\right)$$

$$y'(x_0) = \frac{1}{2+\sqrt{16}} \cdot \left(1 + \frac{2}{\sqrt{16}}\right) = \frac{1}{6} \cdot \left(1 + \frac{1}{2}\right) =$$

$$= \frac{1}{6} \cdot \frac{3}{2} = \frac{1}{4}$$

7.52

$y = x^n$ $\quad y' = n x^{n-1}$

$y'' = n(n-1) x^{n-2}$

$y''' = n(n-1)(n-2) x^{n-3}$

$$y^{(k)} = \prod_{i=0}^{k} (n-i) * x^{n-k}$$

$$n! = n \cdot (n+1)(n+2)(n+3) \cdot \ldots = \prod_{k=1}^{n} k$$

8.26

$y = x^3 - 2x^2 - 7x + 4$

$y' = 3x^2 - 4x - 7$

$y' = 0$

$3x^2 - 4x - 7 = 0$

$D = 16 - 4 \cdot 3 \cdot (-7) =$

$= 100$

$x_{1,2} = \frac{4 \pm \sqrt{10}}{2 \cdot 3} = \frac{[14, -6]}{6}$

$x_1 = \frac{14}{6} = \frac{7}{3}$ $x_2 = -1$

$y_2 = -1 - 2 + 7 + 4 = 8$

$y_1 = \left(\frac{7}{3}\right)^3 - 2\left(\frac{7}{3}\right)^2 - 7 \cdot \frac{7}{3} + 4$

$= 304/27$

$y(\frac{7}{3}) \Rightarrow$ min $\Rightarrow y = -284/27$

$y(-1) \Rightarrow$ max $\quad y = 8$

№32

$y = 3x^2 - 6x$

$y' = 6x - 6$

$y'(\frac{1}{2}) = -3$

$y'(2) = 6$

$y(1) = -3$

$y(0) = 0$

$y(3) = 27 - 27 = 0$

min / max

$[0; 3]$

$y' = 0$

$6x - 6 = 0$

$x = 1$

min $x = 1$

max $x = 3$

№37 точки перегиба и выпуклость

$y = 2x^2 + \ln x$

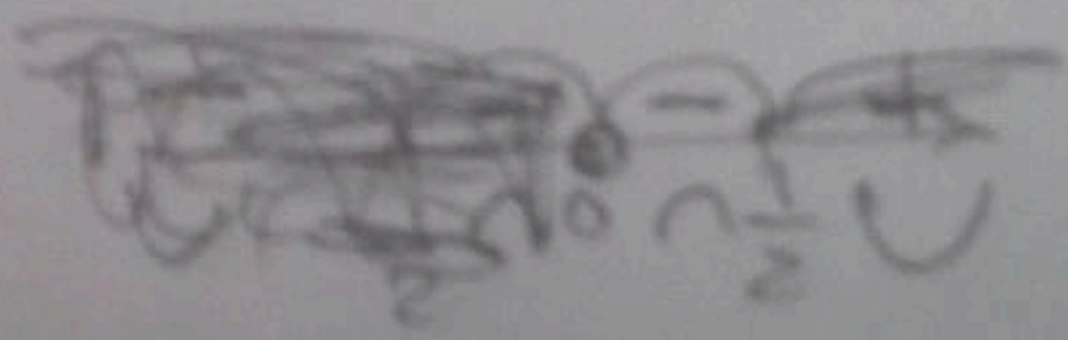
$y' = 4x + \frac{1}{x} \Rightarrow y'' = 4 - \frac{1}{x^2}$

$\begin{cases} 4 - \frac{1}{x^2} > 0 \\ x \neq 0 \\ x > 0 \end{cases} \Rightarrow \begin{cases} \frac{1}{x^2} < 4 \\ x \neq 0 \\ x > 0 \end{cases}$

$x = \pm \frac{1}{2}$

$x > 0$

$x \neq 0$



8.41

$$y = \frac{3-4x}{2-5x} \Rightarrow y = \frac{ax+b}{cx+d}$$

горизонті асимптота:

$$y = \frac{a}{c} \Rightarrow y = -\frac{4}{5}$$

вертикаль асимптота

$$x = -\frac{?}{5}$$