Exp. Distribution & The Central Limit Theorem

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Synopsis

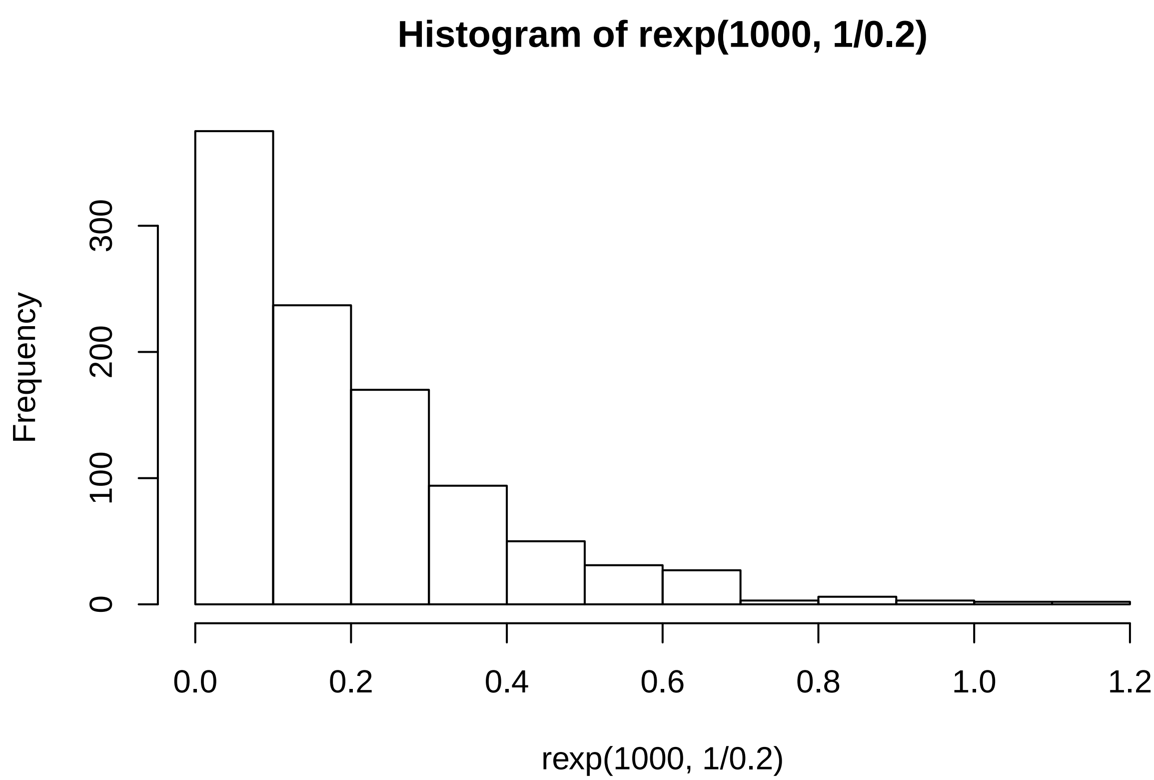
This report attempts to relate the exponential distribution and the Central Limit Theorem. How are they alike, and how do they differ. Can something as non-normal seeming as an exponential curve still follow the Central Limit Theorem.

Analysis

First, lets define the exponential distribution. It is the probability distribution that defines the time between the separate events of a Poisson process. Events occur continuously, separately, and at a constant rate.

Here is the sample of the exponential distribution that we will be working with.

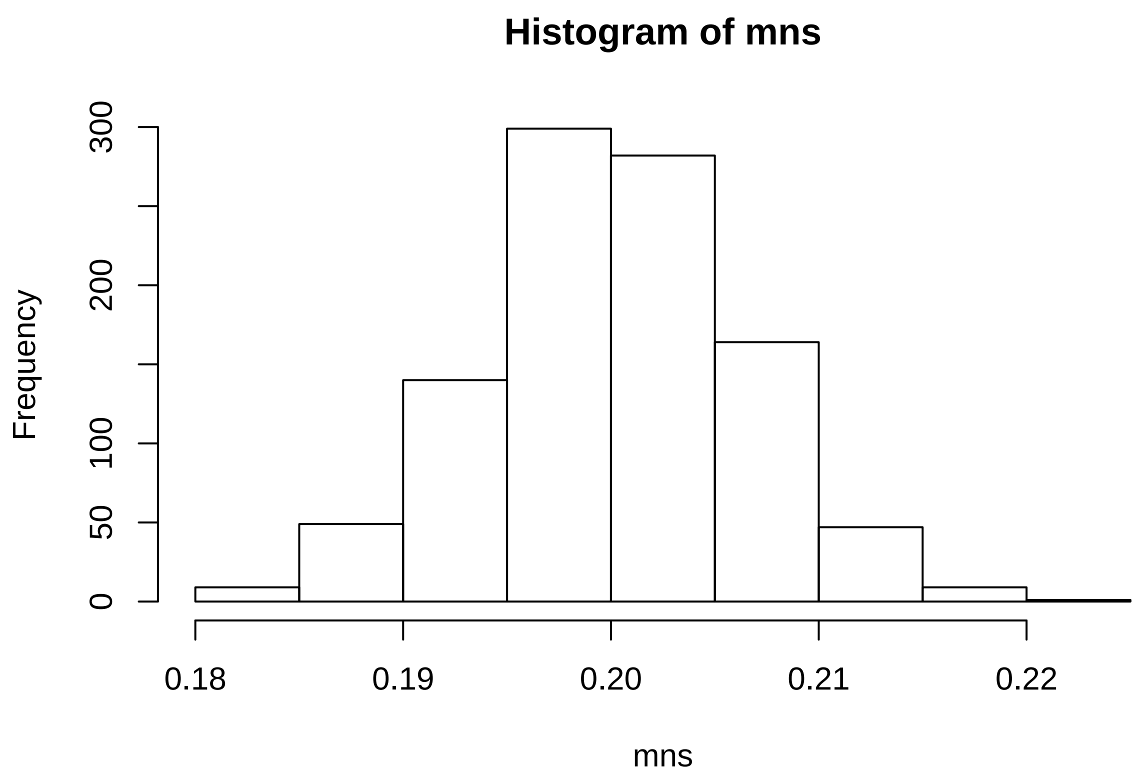
```
hist(rexp(1000, 1/0.2))
```



This report will attempt to show that the distribution of averages of 40 exponentials follows the Central Limit Theorem. The population mean and standard deviation of the exponential distribution are 1/lambda. For this exercise will be using 0.2 for lambda, so the sample mean and sample std should approximate 1/0.2.

Here is a graph of 1000 simulations of 40 exponentials with lambda 0.2.

```r
mns = NULL
for (i in 1 : 1000) mns = c(mns, mean(rexp(1000, 1/0.2)))
hist(mns)
```



It looks as if it is still forming into the bell curve of a normal distribution, contrary to it's seemingly completely non normal disposition.

To support this, let's compare the sample mean and variance, to the theoretical.

They mean should approximately equal

```r
1/0.2
```

```
## [1] 5
```

The variance should approximately equal the standard distribution squared

```r
(1/0.2)^2
```

```
## [1] 25
```

The sample mean is calculated thus.

```r
1/(sum(mns)/length(mns))
```

```
## [1] 4.995179
```

The sample variance is simply the sample mean squared, because the standard distribution is equal to the mean, and the variance is the standard distribution squared.

```r
(1/(sum(mns)/length(mns)))^2
```

```
## [1] 24.95181
```

Result

It seems that the exponential distribution follows the Central Limit Theorem based on these three observations.

1. The distribution of averages of 40 exponentials sample mean approximates the theoretical mean of an exponential distribution.
2. The distribution of averages of 40 exponentials sample variance approximates the theoretical mean of and exponential distribution.
3. The distribution of averages of 40 exponentials, when graphed, seems to follow the bell shaped curve of a normal distribution.

This report therefore concludes that an exponential distribution follows the Central Limit Theorem.