

# PRACTICAL CONSIDERATIONS IN EXPERIMENTAL COMPUTATIONAL SENSING

by

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As members of the Dissertation Committee, we certify that we have read the dissertation prepared by Phillip K. Poon titled Practical Considerations in Experimental Computational Sensing and recommend that it be accepted as fulfilling the dissertation requirement for the degree of Doctor of Philosophy.

Date: 12 December 2016

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Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copies of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

Date: 12 December 2016

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SIGNED: Phillip K. Poon

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## ABSTRACT

Implementing computational optical sensors often comes with various issues that many traditional sensors may not encounter.

## CHAPTER 1

## Introduction

This chapter introduces the reader to the concepts of *isomorphic sensing*, *multiplex sensing*, *indirect-imaging*, *task-specific sensing*, *compressive sensing* and *computational sensing*. It also provides the motivation for the need to address the practical issues in experimental computational sensing.

A *measurement* is a process that converts a physical phenomena to a collection of data. The signal-of-interest is the physical phenomena that we are interested in quantifying. We will call the collection of data the measurement data. Often the measurement is *isomorphic*. Isomorphic sensing is the concept that a sensor's measurement data resembles the signal-of-interest. Isomorphic sensing is called *traditional sensing*. In isomorphic sensing the analog hardware, analog-to-digital converter (ADC), and processing algorithms are all separate components, see Figure 1.1. Computational sensing is the concept that a joint design of the sensor, often though *coding* of the analog signal, with inversion algorithms can exceed the performance of an isomorphic sensor. *Computational sensors* exist at the intersection of these processes, see Figure 1.2 [1]. While isomorphic sensors can provide flexible sensing in multiple applications, a computational sensor's joint design can lead to performance increases. Throughout this chapter and the rest of this dissertation, we will provide many examples that highlight the differences between computational and isomorphic sensing.

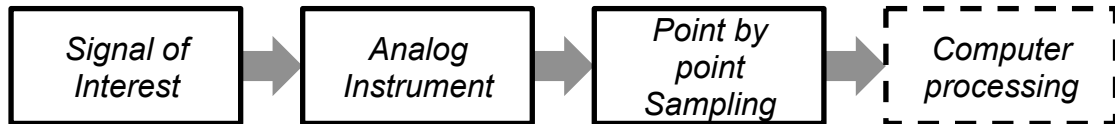


Figure 1.1: A systems view of a traditional sensing scheme. The signal-of-interest is incident upon the analog instrument. The analog instrument forms an isomorphism of the signal which is then periodically sampled point-by-point through an ADC device. Once the signal is in digital form, post-processing algorithms are often used to perform various tasks such as noise reduction, detection, and classification. Notice that the analog instrument, sampling scheme, and processing are all separated.

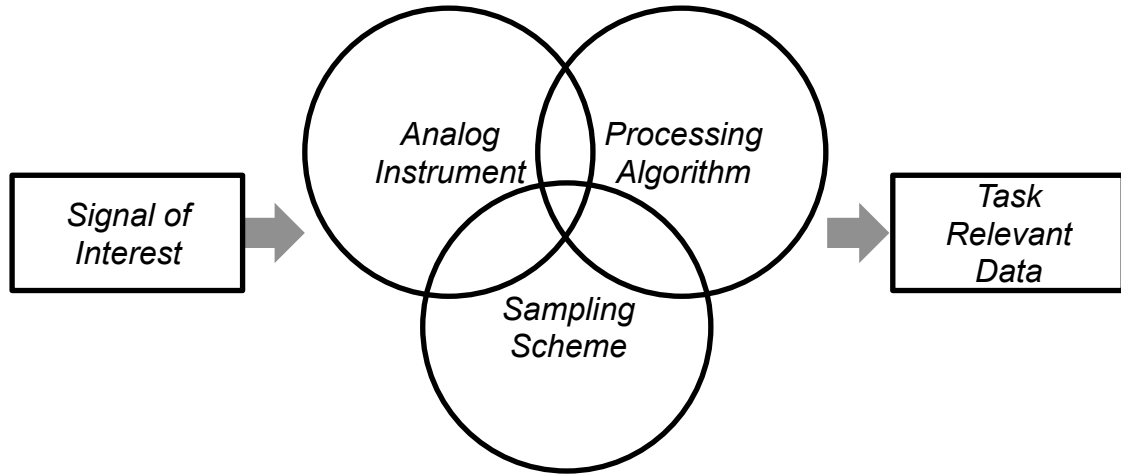


Figure 1.2: A systems view of a computational sensing scheme. The signal-of-interest is incident upon the analog instrument.

Rather than a rigorous discussion, this chapter will discuss some of the major developments and concepts in the field of computational sensing on an intuitive level. This will familiarize the reader with important terminology and techniques common in the field of computational sensing. A rigorous discussion with mathematical formalism of the concepts is presented in chapter 2. This chapter will also discuss some of the challenges I and many other experimentalists and engineers have faced when developing computational sensing prototypes. I will close this chapter with a brief look ahead to the rest of the dissertation.

### 1.1 Isomorphic Sensing

In Greek, the word isomorphic loosely translates to equal in form. Traditional sensors perform isomorphic sensing. In the context of this dissertation, an isomorphic sensor is any sensor which attempts to produce measurement data that resembles the signal-of-interest. In this paradigm, the analog instrument, sampling scheme, and post-processing algorithms are separate components and processes.

We will discuss three important examples of isomorphic sensors: the pinhole camera, the photographic camera and the optical spectrometer (which I will just call a spectrometer from now on, even though there are many instruments called spectrometers that not concerned with optical spectra). These sensors have had major roles throughout the history of optics and in the physical sciences, so it is natural

to use them as examples of computational optical sensing. Therefore it is important we first understand the isomorphic version of these sensors.

In the photographic camera, the signal-of-interest is the object's intensity distribution. This can be the scatter or emitted light from a person, a tree or a distant group of stars. The analog instrument consists of the lenses which are designed and fabricated to produce an intensity distribution (optical image) that looks like the object at the focal-plane array (FPA). The more that the image resembles the object the better the optics. The FPA then samples and quantizes the image and produces a digital representation of the object's intensity distribution, the measurement data. If one is interested in performing a task such as detection or classification, the measurement data is often is post-processed to perform those tasks.

There are two major sub-systems in the photographic camera which determine how well it performs: the optics and the FPA. Ideally, the optics (the analog instrument in this case) will produce a point spread function (PSF) which is infinitely small in diameter. For example, in a task such as the detection of a star from several neighboring stars in the night sky, if the PSF is much larger than the center to center separation of the two stars in the optical image, it will be quite difficult to detect. A careful reader will note that this is the same argument used by Lord Raleigh in proposing his resolution criterion [2]. Even if the PSF is small enough, the FPA must sample at a fine enough pixel-to-pixel spacing, called the *pixel pitch*, to accurately reproduce the intensity variations at the scale which is pertinent to the task. Intuitively, this makes sense because if the image of the stars are imaged onto a single pixel, then one cannot ever hope to be able to accurately detect the star without some other prior or side information.

The pinhole camera consists of a small hole and a box which prevents any light except from the pinhole to enter, see Figure 1.3. The pinhole camera is useful for imaging in parts of the electromagnetic spectrum and particles for which there is no direct analog to refractive lens or reflective mirror. Like the photographic camera, the pinhole camera also attempts to produce a small PSF for better spatial resolution and a human observer, film, or FPA can be used to measure the object scene. As the hole size decreases, the spatial resolution increase at the cost of reducing the signal strength.

In the spectrometer, the signal of interest is the spectrum of the object. The optics are designed to take the incoming light and separate various wavelength components, see Figure 1.4. The part of the spectrometer which is used to physically

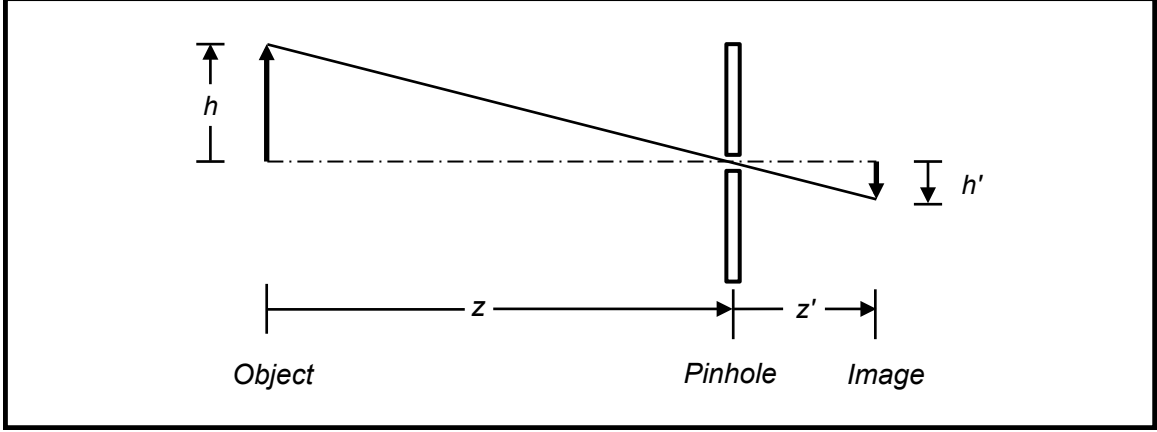


Figure 1.3: A pinhole camera.

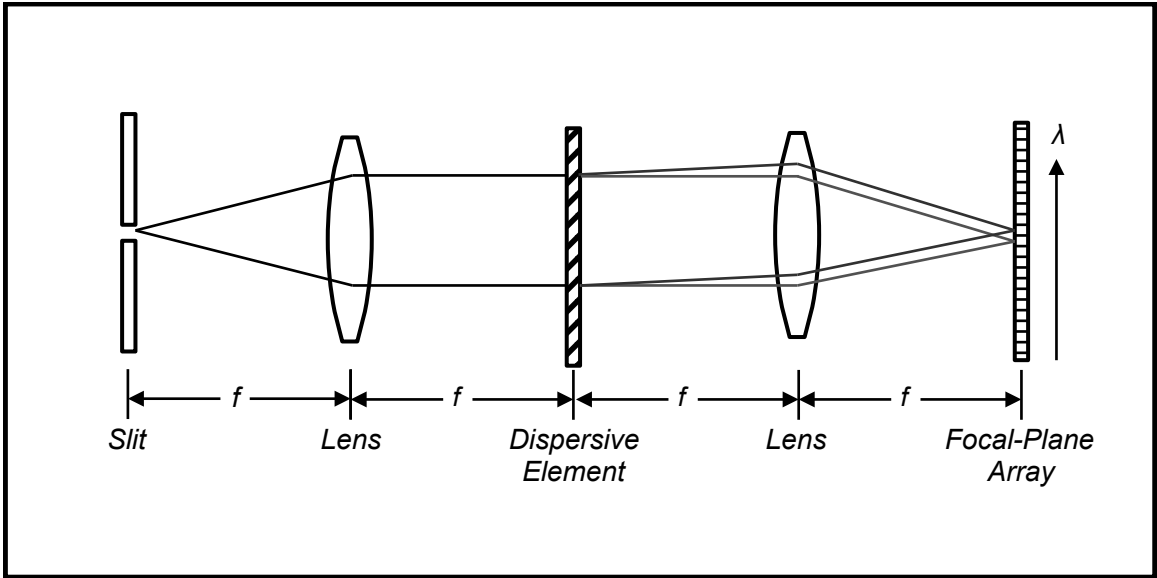


Figure 1.4: An isomorphic slit spectrometer with a 4F configuration.

isolate the wavelengths is called a *monochromator*. The result is a spectral intensity as a function of position at the FPA. The FPA and post-processing algorithms are used in the same manner as the photographic camera, which is to sample the optical spectrum creating a digital version of it and to perform various tasks on the measurement data. For now, we will concentrate on the slit spectrometer, which measures the spectrum at a single point on the object.

In the spectrometer, one of the important performance metrics is *spectral resolution*, which we denote  $\delta\lambda$ . The spectral resolution is the smallest difference in wavelength the instrument can discern. Large spectral resolutions can degrade the spectrometers ability to discern important parts of the spectrum. Similarly with the camera, the FPA must have a pixel pitch which is small enough in order to correctly sample the variations in the spectrum.

The point-by-point nature of isomorphic sensing is both a strength and a source of weakness.

The strength comes from the straightforward and intuitive architecture of the isomorphic sensor. Each subsystem: the optics, the focal-plane array (FPA), and the post-processing can be designed and constructed separately as long as they meet their individual specifications. As long as the signal-to-noise ratio (SNR) is sufficient and the sampling rate is high enough, we are guaranteed to recover the signal.

One of the weaknesses of the isomorphic approach is the the ability to measure low SNR signals. Because the signal-of-interest is sampled in a completely parallel fashion at each exposure, each pixel contributes a certain amount of noise. If the noise dominates, the measurement fidelity decreases often forcing the operator to increase the exposure time. For weak signals, the exposure time can become prohibitive and for temporally dynamic signals this may lead to a loss of resolution. Indeed, one of the major engineering trade-offs faced by traditional spectrometer designers is that when one attempts to increase the light collection (increased slit-width) the spectral resolution  $\delta\lambda$  degrades. Similarly, in the pinhole camera, there is a throughput versus spatial resolution trade-off, increasing the size of the pinhole degrades the PSF.

It would be easy to assume that with the recent revolution in machine learning and statistical signal processing combined with the dramatic increase in computing power that we could simply post-process poor measurements and obtain useful data. However, this isn't possible due to the an important theorem in information theory called the *data processing inequality* [3]. In layman's terms, it means "garbage in, garbage out".

Another weakness of isomorphic sensing is that the separation of the analog instrument, the sampling scheme, and the data processing algorithms lead to increased size, weight and power-cost (SWAP-C). As we mentioned in the photographic camera, the optics must be designed to produce a small PSF. For demanding applications, the optical design and fabrication can be the most expensive component of the sensor. While FPA prices in the visible have fallen, FPAs in certain parts of the electromagnetic spectrum can be quite expensive or non-existent [4, 5].

In many cases, the signal is redundant and high resolution sampling becomes a waste of resources, such as data storage and communications bandwidth. A good example is in photography where often the post-processing takes the digital image and applies a compression algorithm which looks for patterns in the signal and



reduces the file size, discarding much of the sample data [6].

The isomorphic sensor has served humanity well, however with all the weakness that I have discussed, I will now begin to discuss some of major techniques in computational sensing that can be used to address some or all of the issues that I just stated.

## 1.2 Development of Multiplexing in Sensing

*Multiplexing* in sensing allows each measurement sample to be a combination of multiple points of the signal-of-interest. Multiplexing is a powerful tool that can be exploited by the sensor designer to eliminate or relax design trade-offs.

A simple example which illustrates the usefulness of multiplex sensing is weighing objects. In this example, we are given a 100 sheets of paper. Let's assume that the scale's measurement error is insignificant. Isomorphic measurement sensing means one would need to measure each sheet of paper individually. Requiring 100 measurements.

Now, let's say the scale's measurement error is on the order of the weight of the sheet. Measuring each sheet individually produces a large measurement error. In order to reduce the error to an acceptable SNR we need to make several measurements per sheet to reduce the error to an acceptable level.

However, we can be a little smarter. We can measure all 100 sheets at the same time. Since the weight of all 100 sheets is much larger than the measurement error of the scale, we can dramatically increase the precision of the measurement. If we can assume that each sheet is the same weight, then we are done.

The weighing problem is analogous to the spectroscopy example. As discussed earlier in section 1.1, there is trade-off between light collection and spectral resolution. Increasing the slit-width to increase the amount of light has the effect of degrading the spectral resolution  $\delta\lambda$ . Around the late 1940's and early 1950's, several important papers and inventions demonstrated the effectiveness of multiplexing in spectroscopy. At the time the FPA was non-existent, so in the slit spectrometer shown in Figure 1.4, where the FPA is pictured, there was actually another slit. To record the intensity at each spectral channel, either the dispersive element or the exit slit had to be mechanically translated, making the measurements even slower by a factor of  $N_\lambda$ , the number of spectral channels of interest.

Golay was the first to propose multiplexing the slit spectrometer by creating a

pattern of binary (1's and 0's) entrance and exit slits [7]. The idea borrowed heavily from communications theory which is concerned with the reliable transmission of information over a noisy channel. In the Golay multi-slit spectrometer, the patterns of entrance and exit slits are matched based on mathematically useful properties, this is similar to coding and decoding signals in communications. In communications theory the process of structuring the data from the source to the receiver is referred to as *coding*. Similarly, in computational sensing, the transmission of information between an object signal-of-interest and the sensor is considered a coding problem [8]. In the multi-slit spectrometer, the entrance slits act to code the spectrum while the exit slits decoded the coded spectrum. Intuitively, the ability to use multiple entrance and exit slits increases the optical throughput of the spectrometer. Golay's idea dramatically increased the optical throughput without degrading the spectral resolution.

Fellgett devised an alternative approach to multiplex spectroscopy, the Fourier Transform spectrometer [9]. He is generally credited with discovering the multiplex advantage, so it is often called the Fellgett advantage instead.

Another example that is pertinent to this dissertation is coded aperture imaging. Coded aperture imaging can be thought of as the multiplexed version of a pinhole camera. As mentioned earlier in section 1.1, there is a trade-off between the throughput and spatial resolution. However in many fields, such as high-energy particle imaging, refractive lenses and reflective mirrors are non-existent or underdeveloped. By using multiple pinholes the throughput is increased without sacrificing spatial resolution. However, the pattern of the pinholes, which is the code, in this case must be carefully designed in order for the reconstruction to be feasible. Fenimore, Canon, and Gottesman were the first to create an elegant solution to coded aperture design called uniformly redundant arrays [10, 11].

In summary, multiplexing has the ability to eliminate classic trade-offs in isomorphic sensors: signal strength or resolution. Modern researchers are still actively developing novel ways to implement multiplexing to increase resolution and sensitivity in the spatial domain [12, 13], spectral domain [14, 15], and temporal domain [16, 17]. However, multiplexing is not without its own set of challenges. As we mentioned, the coding must often be designed to obtain feasible signal reconstruction. We now discuss inverse problems in computational sensing.

### 1.3 Forward Models and Inverse Problems

In the computational sensing community, a model explains the mapping of the signal-of-interest to the measurement data is called the *forward model*. The problem of taking the observed data and calculating a reconstruction of the signal-of-interest or task-specific parameters is called the *inverse problem*.

As you can imagine, solving inverse problems of isomorphic measurements, when one is concerned with reconstruction of the signal-of-interest, tend to be straight forward. In the weighing problem, the measurement is also the reconstruction. In the slit spectrometer, where the forward model can be simply the continuous to discrete mapping of the spectrum. The spectrum is the interpolated measurement.

Of course, we can begin to add various levels of complexity to the forward model to account for various physical aspects of the sensor, such as the fact the FPA can't measure certain wavelength regions or the noise in our measurements. But again, assuming proper sampling and enough SNR, the reconstruction of the isomorphic signal is the measurement. This simplicity is one reason why isomorphic sensing still dominates at the consumer level despite all of the drawbacks I discussed earlier in section 1.1.

However, the multiplexing of signal information forces us to develop computational steps to solving the inverse problem. In the multiplexed weighing problem, a significant complication occurs when each sheet of paper has a different weight. Now solving the inverse problem is not as straight forward. A single measurement of all 100 sheets in this case is an underdetermined problem, since we have 1 equation and 100 unknowns. What we can do is try measuring different combinations of the 100 sheets, each new combination provides us with a new equation to work with reducing the error. Naively, we might assume that we can randomly choose 100 unique combinations and solve 100 equations using the algebra we were taught in high school. This works fine when there is no measurement error. However, in the presense of noise, in many applications including the weighing problem, random combinations are not the best way to conduct the coding. They are sub-optimal in terms of reconstruction error. This lead many to begin working on optimal coding strategies of signals for sensing and is major topic in this dissertation.

In fact, the idea of multiplexing was not invented by Golay or Fenimore, what they contibuted was the creation of effective ways to code and decode their signal-of-interest, making their sensors more appealing to a broader community of scientists

and engineers.

In summary, the forward model of a sensor is essentially accounting for the physics which govern the measurement. While the solving the inverse problem is a mathematical problem which attempts to either reconstruct the object or to calculate task-specific data from the measurement data. Unfortunately, not all multiplexing forward models codes have mathematically elegant inversion steps. Often the physics of the situation force non-isomorphic measurements which require a computational step to solve the inverse problem.

#### 1.4 Indirect Imaging

While Golay, Fennimore, and others were leveraging multiplexing to eliminate trade-offs in traditional sensors, an entirely disparate group of researchers were working on imaging techniques for which there was no isomorphic analog. In these cases the physics of the sensing modality prevents a point-by-point sampling of the signal-of-interest. Indirect imaging refer to sensing schemes which include X-ray Computed Tomography (CT), Single-Photon Emission Computed Tomography (SPECT), Positron Emission Tomography (PET), Magnetic Resonance Imaging (MRI) and certain forms of sonic and radio wave imaging all require a data-processing or reconstruction step to solve an inverse problem [18].

Perhaps one of the most successful early examples of indirect imaging and the rise of inverse problems in sensing is the development of radar. While early radar was concerned with the detection and distance of an object, development of imaging radar began after World War II. Imaging radar and specifically Synthetic Aperture Radar (SAR) can use time delay information combined with the doppler effects and interference patterns of coherent radio waves to create high resolutions of terrain and buildings.

In medicine, a common imaging modality is X-Ray CT. In X-Ray CT, computational inversion is required to reconstruct a 2 or 3-dimensional function from 1 or 2-dimensional measurement data. The forward model can be simple: In a collimated beam architecture with a 1-dimensional detector array, we say that each sample from each pixel on the array is proportional to the total number of x-ray photons that have not been absorbed by the object [19] plus noise. The inversion of course is not straight forward. The culmination of the work related to the inversion techniques and the actual prototype resulted in the Nobel Prize for Physiology or

Medicine in 1979 [20].

Indirect imaging is a subfield of computational sensing. Due to the medical or military applications of these computational sensors, there has been an intense push to reduce measurement time and improve task-specific and reconstruction results. Many of the techniques from other subfields of computational sensing have been brought to bear for indirect imaging [21, 22].

We have discussed the development of multiplex sensing and indirect imaging and how the ideas from both subfields are analogous in terms of producing a non-isomorphic measurement. However a major step in practical implementation of computational sensing is being able to obtain the measurements in a quick, reliable and efficient manner. Computational sensing as a field would not exist without the most important invention in optics and photonics of the 20th century.

### 1.5 The Digital Imaging Revolution

In 1969 Boyle and Smith invented the Charge-Coupled Device (CCD) [23]. The CCD is the first integrated circuit device which using a 2-dimensional arrangement of pixels which could reliably convert an intensity distribution to a digital signal. The CCD is a type of FPA.

The CCD was a major breakthrough for entire fields and industries who depended on the reliable sampling, storage and transmission of optical signals. Until then one either had to use film or bulky tubes that required an electron beam to be scanned across an image scene, such as the Image orthicon [24]. For their invention, Boyle and Smith both received the Nobel Prize in Physics in 2009 [25]

The invention of the digital camera by Sasson followed shortly after [26]. Several years later the first digital spectrometer was invented. The exit slit was replaced by the CCD, which allowed for instant and simultaneous measurement of the entire spectrum in a compact architecture [27].

The development of the Complementary Metal–Oxide–Semiconductor (CMOS) FPA was also important. While in scientific settings, it could not rival the quality of the CCD, its cheaper cost brought digital imaging to the consumer level. Other technology like the digital computers and computer networking also provided major contributions to the democratization of imaging and optical sensing. While scientific grade optical instrumentation was and is still expensive, the researcher could at least capture, process, and share measurement data with significantly less effort. Without

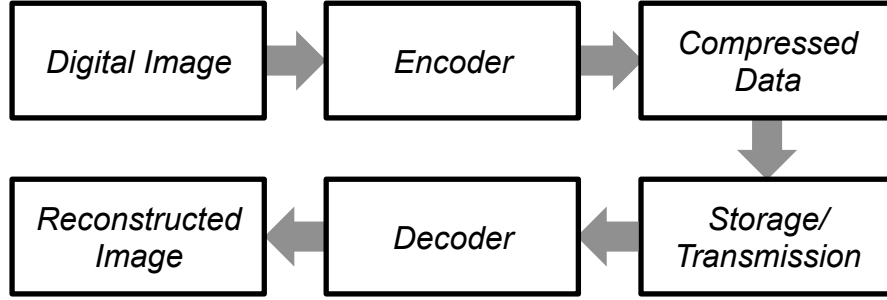


Figure 1.5: An general flowchart for image and data compression techniques.

it, the field of computational sensing would not exist.

Algorithms for efficient and reliable storage and transmission of digital images became more important. Over time the pixel count continued to increase and the sheer volume of digital image and video data being generated and transmitted over networks began to outpace improvements in storage and transmission capacity. While many engineers developed new technology to combat the hardware limitations of storage and transmission. This also led to a renewed effort by researchers to develop more efficient image and video compression algorithms [28, 29].

While a multitude of compression techniques exist, they all follow the same basic process, see Figure 1.5. Once the CCD samples the optical signal and produces the measurement data, the encoder uses the compression algorithm to look for redundancies in the data and produce a lower dimensional representation of the image. The compressed data can then either be stored or transmitted or both. The decoder solves the inverse problem of reconstructing the image.

As mentioned in at the beginning of this chapter, computational sensing lies at the intersection of the design of the analog hardware, sampling schemes, and processing algorithms. Many of the algorithms used in computational sensing are the same as or inspired by the techniques used by the image processing community. This is because a major effort of computational sensing is the desire to make more resource efficient measurements.

## 1.6 Compressive Sensing

Traditionally, in order to increase the resolution of a sensor, one had to increase the number of measurements. This means that the SWAP-C must also increase. A camera with just a few megapixels FPA costs less than one with hundreds of

megapixels. The cost of designing the optics will also need to scale to provide enough optical resolution. In a perfect world, we could capture all the information we need from just a few measurements.

Conventional signal processing dictates that accurate reconstruction of the signal-of-interest is highly improbable. If we have a discrete signal we need at least as many measurements as there are signal elements to solve the inverse problem. If the number of measurements is less, then the inverse problem is underdetermined. Fortunately, a signal acquisition technique called compressive sensing allows us to design sensors that solve these types of highly underdetermined inverse problems.

As discussed earlier, much of the data being generated by sensors are redundant. Images, spectra, video, and audio data of real-world signals tend to exhibit patterns or redundancies that can be exploited. This allows a compression algorithm to significantly reduce the amount of data needed to represent the signal.

There is a class of compression algorithms called lossy [30]. In lossy compression, not only are redundancies exploited but data that is deemed insignificant to the signal quality is discarded. Only the most important part of the signal is kept as part of the compressed representation of the original signal. When the signal is uncompressed, the amount of data is less than the original measured data. The difference in quality is often unnoticeable to a human observer. In both lossless and lossy compression, the goal is to obtain a *sparse* representation of the signal. A sparse representation means that the signal can be well approximated with only a few non-zero elements in a representation basis. A representation basis is a basis in which the signal-of-interest is sparse. For example, most natural images are sparse in the Fourier basis. The representation basis is typically not the native basis of the signal-of-interest, i.e. pixel number or spectral channel.

In 2006, David Donoho argued that traditional sensors tend to produce vast amounts of measurement data, but often the majority of data is redundant and discarded in the compression step. He proposed that sensors can be designed to directly measure the most relevant data in a signal, suggesting a measurement scheme that can measure a compressed form of the signal [31]. This is the idea behind compressive sensing sometimes known as *compressive sampling*. If the measurements are compressive then it should be possible to significantly reduce the number of measurements to accurately reconstruct the signal.

Note that there is a subtle but powerful distinction between compressive sensing and the traditional approach of sensing and then compressing. In the traditional

approach, compression algorithms operate as a post-processing step. Therefore, a traditional compression algorithm will have access to the entire signal to look for redundancies and convert it into a sparse representation. In compressive sensing, we attempt the compression directly and therefore do not have access to the entire uncompressed signal. The algorithms must assume that the signal has a sparse representation.

The question of how to actually measure or code the analog signal to directly obtain compressed data is also important. Fortunately, random coding tends to work well in many instances when the signal has a sparse representation. Much of the work in this dissertation will discuss other types of coding schemes that can be used to outperform random codes.

The idea of compressive sensing seems to be similar to the concept of multiplex sensing. However, there is an important distinction to be made. In compressive sensing, the aim is to obtain the relevant information in as few measurements as possible. In multiplexing, the goal is to overcome limitations mainly due to lack of SNR. Many compressive sensing schemes also employ multiplexing.

One useful example of compressive sensing versus traditional sensing is the single pixel camera [12]. The single pixel camera is a multiplexing camera architecture that uses time sequential random measurements and recovers the image in significantly less measurements ( $=\text{number of exposures} \times \text{pixels}$ ) than the conventional camera, see Figure 1.6. Another example is the Coded Aperture Snapshot Spectral Imaging (CASSI) architecture [32], which can reconstruct a spectral data cube in significantly less FPA exposures than a traditional spectral imaging architecture.

Another important distinction is between reconstruction and task-specific sensing. Task-specific sensing tends to refer to measurement techniques that attempt to directly perform tasks such as detection, classification, and estimation without the intermediate step of reconstructing the high-dimensional signal. Compressive sensing is useful not just of overcoming resolution limitations in reconstruction but for reducing measurement resources for task-specific sensing. For example, in facial recognition the goal is detection of an individual person. Reconstruction of the face image is simply an intermediate step, therefore, one can develop a compressive sensing scheme that is optimal for direct facial detection, skipping the step of image reconstruction [33].

Multiplex sensing, compressive sensing, and task-specific sensing are considered subfields of computational sensing. Computational sensors offer significant advan-



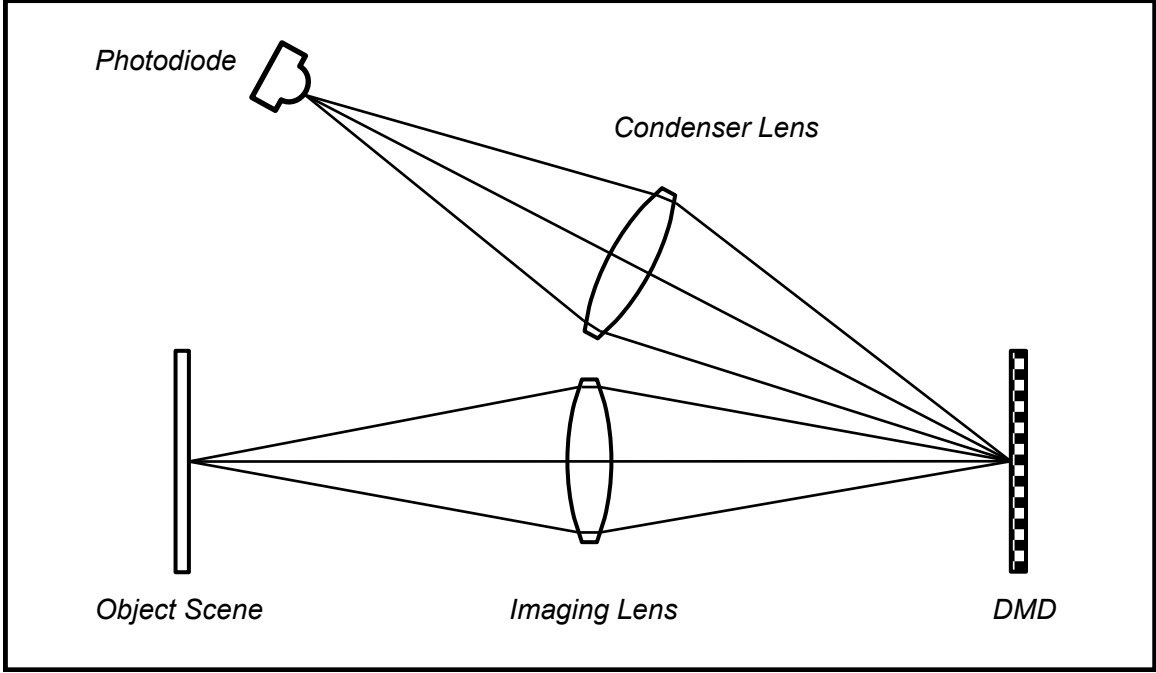


Figure 1.6: A single pixel camera architecture.

tages that allow us to overcome classic engineering trade-offs in sensor design. However, computational sensing has its own unique set of engineering problems that we will now discuss.

### 1.7 Practical Considerations in Computational Sensing

So far we have discussed the development of computational sensing techniques and how they are used to ameliorate trade-offs in traditional sensor design. Computational sensing as a field is continuing to grow at a rapid pace. The number of journal publications related to computational sensing has steadily increased every year since 2008 [34]. There is now a major OSA meeting dedicated to computational sensing called Computational Optical Sensing and Imaging (COSI) [35] and textbooks dedicated to its study and development [8, 36]. It is important to recognize that while it is a powerful approach to radically new sensor architectures, challenges remain in our ability to design and demonstrate experimental practical computational sensors.

One of the major obstacles in computational sensing is calibration. Calibration is the act of quantifying the sensor's measurements in order to produce an accurate forward model. While many traditional sensors also require calibration, computational sensors tend to be more sensitive to calibration error.

There are two main reasons for this. The first reason is simply due to the fact that non-isomorphic measurements require a computational step to solve an inverse problem. The algorithms rely on accurate knowledge of the forward model to separate measurement data due to the instrument and data due to the signal-of-interest.

The second reason is due to the lack of redundancy in a compressive sensor's measurement. The redundancies that are typically deemed wasteful in traditional sensing, can also be used by post-processing algorithms to solve the inverse problem in a robust fashion. If the calibration is wrong for a few measurement samples, one can still use error-correcting algorithms to recover the compromised data. In a compressive measurement, these redundancies are exploited in the physical measurement process [37]. Only a few numbers are used to represent many. If the few numbers are misinterpreted due to poor calibration, the inversion algorithm's performance will suffer. We will illustrate in this dissertation the calibration challenges in several computational sensors.

Since, calibration has become a major drawback in *Compressive sensors*. A consumer cannot be expected to spend time calibrating everytime the instrument is physically bumped or the air pressure changes. In high dimensional compressive sensors like hyperspectral imagers, the calibration time can last hours.

Another hurdle in the implementation of practical computational sensing is the need for prior knowledge. For example, in compressive imaging we have to assume the signal is sparse in some basis. Fortunately most realistic objects are sparse or compressible with modern representation bases, such as the wavelet basis. However, sometimes one needs to image something that is difficult to represent with any current basis, such as the intensity pattern due to laser speckle. One needs an alternative representation basis, so training sets are often used to generate a new basis in which the signal can be represented as sparse. One must generate many different example training data so a particular signal-of-interest can have a sparse representation in that basis. This becomes time and computationally expensive. Many algorithms require some prior knowledge of the statistics of the signal and noise ahead of time for optimal performance. A good example of this is the AFSSIC, a computational spectral classifier which requires knowledge of standard deviation of the noise's probability density function to perform spectral classification in the least number of measurements as possible.

Reducing the number of detector elements is often the goal in computational

sensing. A notable example is the single pixel camera is an architecture. However, the single pixel camera requires several time sequential measurements. Each measurement displays a different Digital Micro-Mirror Display (DMD) pattern to create a randomly encoded measurements [12]. The drawback to this architecture is that one must point the camera at the object scene until enough measurements have been collected for proper reconstruction. A complication arises when this architecture is used to image temporally varying object scenes. One must display the DMD patterns even faster and reduce the exposure time to keep up. As a result the SNR may begin to degrade.

A possible way to mitigate this issue is to do all the encoding in parallel. However, a completely parallel approach would require a lens, a DMD (or coded aperture), and a detector pixel for each measurement. Since each lens uses a different entrance pupil, this will mean that each will have a slightly different view of the object scene. This drastically scales the complexity of the architecture and algorithms. In this dissertation, I will discuss a compromise to parallel coding, in two different computational sensors, by using a common entrance pupil and a CCD.

Much of the Often, theoretically optimal measurements take advantage of codes which are simultaneously positive and negative. In reality, with incoherent light we are unable to make negative measurements. One is often forced into situations where one must record two sets of measurements and subtract one set of measurements from the positive set of measurements. This means an additional noise term is added to each effective measurement.

Algorithms engineered to solve the inverse problems often do not account for the non-negativity of many physical situations. For example, in spectral unmixing where the problem is to solve for concentration of each material's spectrum. A non-negative fractional abundance that sums to one is a physical requirement. However, there is a serious lack of sparsity promoting algorithms that are able to enforce both the non-negativity and the sum to one constraint.

A major issue in both the theoretical and experimental compressive sensing community is a lack of code design schemes. For the most part random measurements techniques are dominant because they obey a key theoretical result which guarantees reconstruction with high probability. Intuitively, designed, meaning non-random, measurement codes which can take into account prior knowledge of the physical limitations of the sensing task and statistical assumptions of the signal-of-interest should be able to outperform random measurements. For example, we

will show that in certain applications we can use Principal Component Analysis (PCA) to outperform random measurements. In one application, we will demonstrate that simply designed codes outperform random codes in low SNR environments. However, as of yet there is no adaptive measurement package one can simply take “out-of-the-box” and begin applying them to computational sensing problems and outperform random measurements on a consistent basis.

These issues and others will be discussed in a case study manner throughout this dissertation. I will now discuss the organization of this dissertation and the three separate computational sensors I have built and how I have tried to mitigate and resolve some of the practical issues associated with computational sensing.

## 1.8 Dissertation Overview

I used this chapter to cover the major concepts of computational sensing. More importantly, I provided the major topic of discussion for this dissertation, the challenges that remain towards developing practical computational sensors. I will use the next chapter as an opportunity to provide a more detailed and mathematically rigorous look at the various coding schemes that are popular in computational sensing, as well as the ones I have used. This includes a discussion of the mathematical methods that I and my collaborators developed and deployed in the AFSSI-C: Bayes rule, Log-Likelihood Ratios, and the Maximum a Posteriori decision technique. A more rigorous treatment of compressive sensing will also be provided which includes several important results from the compressive sensing community. I will also talk about several optimization algorithms and estimation techniques that have been used during the development of the three computational sensors in my work.

Chapter 3 will introduce a new system architecture for compressive target tracking—the Static Computational Optical Undersampled Tracker (SCOUT). This system is designed to overcome a variety of challenges that typically arise in traditional optical sensing approaches. It provides several advantages over the single-pixel camera architecture, notably it can capture many simultaneous encodings of the field-of-view with a single camera exposure. I will present experimental results that validate the performance of the proposed architecture.

Chapter 4 will discuss an important experimental prototype which demonstrates adaptive spectral image classification. The system is a major experimental advance-

ment compared to current computational spectral imaging architectures which focus on reconstruction. Furthermore, the ability to perform adaptive measurements, where each code is designed based on the history of prior measurement data allows this instrument to outperform traditional spectral imaging architectures by a factor of 100 in low SNR scenarios.

Chapter 5 will discuss current efforts to experimentally demonstrate compressive spectral unmixing. Often in spectral imaging, the spectra present in the field-of-view of each pixel is actual a mixture of several spectra, a mixed spectrum. Spectral unmixing is the task of inferring the fraction of each constituent spectrum in the mixed spectrum. I employ a unique architecture for computational spectral sensing which uses no diffraction grating or refracting prism but relies on the wavelength dependent nature of the birefringence in an LCOS. We will demonstrate the effectiveness of various compressive sensing measurement schemes to outperform a traditional sensor in significantly less measurements.

Chapter 6 will summarize the dissertation and provide my perspective on how the field should approach the issues in practical computational optical sensing.

## CHAPTER 2

### Formalism

This chapter introduces the reader to the more rigorous concepts and mathematical background that will be required to fully understand the material presented in the later chapters of this dissertation.

A rigorous discussion of multiplexing and signal-to-noise ratio will be discussed, as well as various coding schemes used in various notable computational sensors as well as the ones in this dissertation.

Since the AFSSI-C relies on a variation of Principal Component Analysis (PCA) and a Bayesian algorithm for coding design we will discuss some of the fundamentals of Bayesian probability and the Log-Likelihood Ratios.

The measurement process implements a mapping from the

#### 2.1 Isomorphic Sensing

$$\mathbf{g} = \mathbf{H}\mathbf{r} \tag{2.1}$$

where  $\mathbf{g}$  is the measurement vector.

#### 2.2 Multiplexing

##### 2.2.1 The Multiplex Advantage

Fellgett was the first to note that multiplex measurements compared to isomorphic measurements improve the signal-to-noise ratio on the order of  $\sqrt{N_\lambda}$

##### 2.2.2 Coding Schemes

A good example is the Hadamard matrix based code [38]. The Hadamard code is appealing because reconstruction is simply the transpose of the original matrix.

## 2.3 Principal Component Analysis

## 2.4 Bayesian Rules and Log-Likelihood Ratios

## 2.5 Compressive Sensing

### 2.5.1 The Nyquist-Shannon Sampling Theorem

The Nyquist-Shannon Sampling Theorem states one must sample a signal with a sampling rate that is at least twice the maximum frequency of the signal to prevent aliasing [39].

### 2.5.2 Sparsity, Incoherence, and the Restricted Isometry Property

At first glance, compressive sensing is unintuitive. Going back to the weighing example, if we have hundreds of items we can group the items together to increase the precision of our measurements. However, we still have to take as many measurements as there are items, to solve for their weight. If the number of measurements is less than the number of objects, then a unique solution is impossible. If we have some sort of helpful information, we can significantly reduce the number of solutions, discarding away solutions that are inconsistent with this prior information.

In compressive sensing, this helpful knowledge is called *sparsity*. A signal is sparse if only a few elements of the signal are non-zero. For compressive sensing to work, the signal-of-interest itself doesn't need to be sparse. If one only needs to describe it with a few representation vectors relative to the native dimensionality of the signal. Note that the sign Sparsity is quite similar to how compressible a signal is. Most real world signals aren't sparse, they are compressible. However, we know from lossy compression techniques that we can throw away most of the data, that much of the information is not sparse. The reconstruction algorithms rely on the assumption of sparsity to correctly estimate the original signal from the small number of measurements [1].

The actual question of how to code the analog signal-of-interest to produce the most compressive measurements as possible is still an active area of research [1]. Many in the compressive sensing community refer to the codes as the measurement basis. This interpretation of the coding scheme allows us to think of the measurement data as projections of the signal-of-interest onto a set of vectors. Incoherence is the concept that the measurement basis and the representation basis should have

low correlation with each other. The more incoherent—lower correlation—the two bases are, the higher the probability of successful reconstruction of compressive measurements. Random matrices have a high probability of being incoherent with any basis [40], which is why they are so popular in experimental prototypes [] since it simplified the task of having to design the measurement basis. However, as we will see random measurements do not always lead to the best reconstruction results since they are agnostic to the signal-of-interest. The choice of the measurement basis can be a difficult task depending on the application, however it has

coherence, However, mathematicians have shown that the measurement basis should have a low maximum correlation with the representation basis, the basis in which the object signal has a sparse representation. This allows the signal to get scheme relies of the coherence of the representation basis and the measurement basis. Up till now the representation basis has been the canonical Dirac basis.

Quite surprising, Other than sparsity, when other knowledge of the signal can be made often random measurements are the optimal measurements.

### 2.5.3 Inversion

#### 2.5.3.1 Least Squares

Suppose

$$\mathbf{g} = \mathbf{H}\mathbf{r} \quad (2.2)$$

Given  $\mathbf{g}$  and  $\mathbf{H}$  we want to solve for  $\mathbf{r}$ . If the matrix is full rank then we can simply multiply both sides of equation 2.2 by  $\mathbf{H}^{-1}$

$$\mathbf{H}^{-1}\mathbf{g} = \mathbf{H}^{-1}\mathbf{H}\mathbf{r} = \mathbf{I}\mathbf{r} = \mathbf{r} \quad (2.3)$$

If  $\mathbf{H}$  is not full rank then its inverse does not exist. However we can try to find a solution  $\hat{\mathbf{r}}$  that minimizes the least squared error. This is called the *Least Squares Solution* also known as the *Least Squares Estimator*, *Ordinary Least Squares* and by many other names. We define the squared error as

$$\|\mathbf{e}\|^2 = \|\mathbf{H}\mathbf{r} - \mathbf{g}\|^2 \quad (2.4)$$

To minimize the error, we take the derivative of equation 2.4 with respect to  $\mathbf{r}$  and set it equal to zero and solve for  $\mathbf{r}$ . The full derivation which shows each step is given in Appendix A. The least squares estimate:



$$\hat{\mathbf{r}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{g} \quad (2.5)$$

2.5.3.2 L0 and L1 Norm Minimization

2.5.3.3 LASSO and sparsity regularization

$$\mathfrak{F}\left\{\int_a^b g(\xi, \eta) h(x - \xi, y - \eta) d\xi d\eta\right\}$$

## CHAPTER 3

### Static Computational Optical Undersampled Tracker

This chapter introduces the reader to the Static Computational Optical Undersampled Tracker.

## CHAPTER 4

### Adaptive Feature Specific Spectral Imaging-Classifier

This chapter introduces the reader the AFSSI-C.

## CHAPTER 5

### Computational Spectral Unmixing

This chapter introduces the reader the computational spectral unmixer.

## CHAPTER 6

### Conclusion

This chapter concludes the dissertation.

## APPENDIX A

## Derivation of the Least Squares Estimator

Suppose

$$\mathbf{g} = \mathbf{H}\mathbf{r} \quad (\text{A.1})$$

Given  $\mathbf{g}$  and  $\mathbf{H}$  we want to solve for  $\mathbf{r}$ . If the matrix is full rank then we can simply multiply both sides of equation A.1 by  $\mathbf{H}^{-1}$

$$\mathbf{H}^{-1}\mathbf{g} = \mathbf{H}^{-1}\mathbf{H}\mathbf{r} = \mathbf{I}\mathbf{r} = \mathbf{r} \quad (\text{A.2})$$

If  $\mathbf{H}$  is not full rank then its inverse does not exist. However we can try to find a solution  $\hat{\mathbf{r}}$  that minimizes the squared error. This is called the *Least Squares Solution* also known as the *Least Squares Estimator*, *Ordinary Least Squares* and by many other names. We define the squared error as

$$\|\mathbf{e}\|^2 = \|\mathbf{H}\mathbf{r} - \mathbf{g}\|^2 \quad (\text{A.3})$$

To minimize the error, we take the derivative of equation A.3 with respect to  $\mathbf{r}$  and set it equal to zero and solve for  $\mathbf{r}$ . Note that the equation A.3 can be expanded in terms of an inner product

$$\|\mathbf{e}\|^2 = \|\mathbf{H}\mathbf{r} - \mathbf{g}\|^2 = \sum_{i=1}^N e_i^2 = \mathbf{e}^T \mathbf{e} = (\mathbf{H}\mathbf{r} - \mathbf{g})^T (\mathbf{H}\mathbf{r} - \mathbf{g}) \quad (\text{A.4})$$

The transpose is distributive

$$(\mathbf{H}\mathbf{r} - \mathbf{g})^T = (\mathbf{H}\mathbf{r})^T - \mathbf{g}^T \quad (\text{A.5})$$

The transpose of a product of matrices equals the product of their transposes in reverse order

$$(\mathbf{H}\mathbf{r})^T = \mathbf{r}^T \mathbf{H}^T \quad (\text{A.6})$$

So equation A.4 becomes

$$\begin{aligned} \|\mathbf{e}\|^2 &= (\mathbf{r}^T \mathbf{H}^T - \mathbf{g}^T)(\mathbf{H}\mathbf{r} - \mathbf{g}) \\ &= \mathbf{r}^T \mathbf{H}^T \mathbf{H} \mathbf{r} - \mathbf{r}^T \mathbf{H}^T \mathbf{g} - \mathbf{g}^T \mathbf{H} \mathbf{r} + \mathbf{g}^T \mathbf{g} \end{aligned} \quad (\text{A.7})$$

We can see that the two middle terms  $\mathbf{r}^T \mathbf{H}^T \mathbf{r} = \mathbf{g}^T \mathbf{H} \mathbf{r}$  because they are just scalars.

$$\|\mathbf{e}\|^2 = \mathbf{r}^T \mathbf{H}^T \mathbf{H} \mathbf{r} - 2\mathbf{g}^T \mathbf{H} \mathbf{r} + \mathbf{g}^T \mathbf{g} \quad (\text{A.8})$$

To find the least squares solution, take the gradient with respect to  $\mathbf{r}$  and set it equal to zero.

It should be noted that there are two different notations for writing the derivative of a vector with respect to a vector  $\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$ . If the numerator  $\mathbf{y}$  is of size  $m$  and the denominator  $\mathbf{x}$  of size  $n$ , then the result can be laid out as either an  $m \times n$  matrix or  $n \times m$  matrix, i.e. the elements of  $\mathbf{y}$  laid out in columns and the elements of  $\mathbf{x}$  laid out in rows, or vice versa. They are both correct and equal, which leads to confusion when switching back in forth. I will write both to reduce confusion.

Clearly the gradient of the third term in equation A.8 w.r.t  $\mathbf{r}$  is 0, so it goes away. We first tackle the first term on the right hand side in equation A.8

$$\frac{\partial}{\partial \mathbf{r}} \mathbf{r}^T \mathbf{H}^T \mathbf{H} \mathbf{r} \quad (\text{A.9})$$

Let  $\mathbf{K} = \mathbf{H}^T \mathbf{H}$ . Since  $\mathbf{K}$  is symmetric, we can use the identity

$$\frac{\partial}{\partial \mathbf{r}} \mathbf{r}^T \mathbf{K} \mathbf{r} = 2\mathbf{r}^T \mathbf{K} = 2\mathbf{K}^T \mathbf{r} \quad (\text{A.10})$$

since  $\mathbf{K} = \mathbf{K}^T$  then

$$\frac{\partial}{\partial \mathbf{r}} \mathbf{r}^T \mathbf{H}^T \mathbf{H} \mathbf{r} = 2\mathbf{r}^T \mathbf{H}^T \mathbf{H} = 2\mathbf{H}^T \mathbf{H} \mathbf{r} \quad (\text{A.11})$$

and the gradient of the middle term in equation A.8 is simply  $-2\mathbf{H}^T \mathbf{g}$  so

$$\frac{\partial}{\partial \mathbf{r}} \|\mathbf{e}\|^2 = 2\mathbf{H}^T \mathbf{H} \mathbf{r} - 2\mathbf{g}^T \mathbf{H} \quad (\text{A.12})$$

setting it equal to zero and solving for  $\mathbf{r}$  gives the least squares estimate

$$\hat{\mathbf{r}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{g} \quad (\text{A.13})$$

## Glossary

**Coding** In the context of computational sensing, coding is the process of modifying or modulating an analog signal during measurement. Coding is often used to reduce degeneracy in the measurement data. In the context of spectroscopy, the spectral filters act to code the spectrum. . 18

**Compressive sampling** *See* compressive sensing. 23

**Compressive sensing** A sensing technique that attempts to directly a compressive or sparse representation of the signal-of-interest during the measurement. Compressive sensors attempt to uses significantly less measurements than the dimensionality of the signal-of-interest. Compressive sensing relies on non-linear optimization algorithms to perform reconstruction or task-specific sensing from highly underdetermined inverse problems. These algorithms often rely on sparsity to avoid overfitting.. 12, 23, 40

**Compressive sensors** *See* compressive sensing. 26

**Computational sensing** Any sensing technique in which the sensor uses indirect-imaging, multiplex sensing, compressive sensing, or task-specific sensing.. 12, 40

**Computational sensor** *See* computational sensing. 12

**Data processing inequality** An theorem from information theory that proves the information of a signal cannot be increased via a local physical operation.. 16

**Forward model** A numerical model, typically an equation, that explains the mapping of the analog signal-of-interest to the measurement data.. 19

**Indirect-imaging** An imaging sensor that attempts to reconstruct an image of an object using non-isomorphic measurements. A computational step is required to reconstruct the object signal-of-interest. X-Ray CT and SAR are examples types of indirect-imaging. . 12, 40

**Inverse problem** The problem of taking the measurement data and calculating a reconstruction of the signal-of-interst or task-specific parameters. In computational sensing, computer algorithms are used to solve inverse problems.. 19

**Isomorphic** *See* isomorphic sensing. 12

**Isomorphic sensing** An isomorphic sensor is any sensor that attempts to produce measurement data that resembles the signal-of-interest. An isomorphic measurement is a measurement that resembles the signal-of-interest. An isomorphic measurement can be described as a one-to-one mapping from the signal-of-interest to the measurement, and is represented in matrix notation with an identity matrix. Isomorphic sensing is synonomous with traditional sensing.. 12, 40, 41



**Measurement** A process that converts a physical phenomena to a collection of data. 12

**monochromator** An optical instrument that transmits a selectable narrow wavelength band of light chosen from a wider range of wavelengths available at the input.. 15

**multiplex sensing** A multiplexing sensor is any sensor that attempts to combine the physical phenomena of the analog signal-of-interest in to a few or one analog-to-digital sample to overcome limits due to signal-to-noise ratio. .The measurement data that does resemble the signal-of-interest. A matrix representation of a multiplex measurement will not look like an identity matrix. 12, 40, 41

**Multiplexing** *See* multiplex sensing. 17

**Pixel pitch** The center to center distance between pixels on a focal-plane array such as a CCD or CMOS image sensor.. 14, 15

**Sparse** A vector or matrix in which most of the elements are zero.. 23

**Spectral resolution** The smallest the smallest difference in wavelength the instrument or sensor can discern.. 15

**Task-specific sensing** A sensor that does not attempt to reconstruct the signal-of-interest to perform a signal processing task such as detection, estimation, and classification. The AFSSI-C is an example of a task-specific sensor. . 12, 40

**Traditional sensing** *See* isomorphic sensing. 12

## Acronyms

- ADC** analog-to-digital converter. 9, 12
- AFSSI-C** Adaptive Feature Specific Spectral Imaging-Classfier. 4, 5, 26, 28, 30, 35, 41
- CASSI** Coded Aperture Snapshot Spectral Imaging. 24
- CCD** Charge-Coupled Device. 21, 22, 27
- CMOS** Complementary Metal–Oxide–Semiconductor. 21
- COSI** Computational Optical Sensing and Imaging. 25
- CT** Computed Tomography. 20, 40
- DISP** Duke Imaging and Spectroscopy Program. 5
- DMD** Digital Micro-Mirror Display. 27
- FPA** focal-plane array. 14–17, 19, 21, 22, 24
- LCOS** Liquid Crystal on Silicon. 5, 29
- LENS** Laboratory for Engineering Non-Traditional Sensors. 4, 5
- MRI** Magnetic Resonance Imaging. 20
- PCA** Principal Component Analysis. 28
- PET** Positron Emission Tomography. 20
- PSF** point spread function. 14, 16
- SAR** Synthetic Aperture Radar. 20, 40
- SCOUT** Static Computational Optical Undersampled Tracker. 5
- SLM** Spatial Light Modulator. 5
- SNR** signal-to-noise ratio. 16, 17, 19, 24, 27–29
- SPECT** Single-Photon Emission Computed Tomography. 20
- SWAP-C** size, weight and power-cost. 16, 22

## Symbols

$\delta\lambda$  Spectral Resolution. 15–17

$\mathbf{g}$  The Measurement Vector. 30

$N_\lambda$  The Number of Spectral Channels. 17

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