

# PRACTICAL CONSIDERATIONS IN EXPERIMENTAL COMPUTATIONAL SENSING

by

Phillip K. Poon



A Dissertation Submitted to the Faculty of the

COLLEGE OF OPTICAL SCIENCES

In Partial Fulfillment of the Requirements  
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

2016

THE UNIVERSITY OF ARIZONA  
GRADUATE COLLEGE

As members of the Dissertation Committee, we certify that we have read the dissertation prepared by Phillip K. Poon titled Practical Considerations in Experimental Computational Sensing and recommend that it be accepted as fulfilling the dissertation requirement for the degree of Doctor of Philosophy.

Date: 12 December 2016

---

Amit Ashok

Date: 12 December 2016

---

Rongguang Liang

Date: 12 December 2016

---

Michael E. Gehm

Final approval and acceptance of this dissertation is contingent upon the candidate's submission of the final copies of the dissertation to the Graduate College.

I hereby certify that I have read this dissertation prepared under my direction and recommend that it be accepted as fulfilling the dissertation requirement.

Date: 12 December 2016

---

Dissertation Director: Amit Ashok

## STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at the University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the copyright holder.

SIGNED: Phillip K. Poon

## ACKNOWLEDGEMENTS

Graduate school is an arduous and enlightening experience. It is not difficult by design but by nature it forces one into a state of mind which embraces the edge of knowledge and trek into the unknown. I was fortunate to have many guides who showed me the path, even when there were times when I wandered off to get my bearings. Along the way I encountered many people who not only helped me with the journey but bestowed kindness and friendship, asking for nothing in return.

My main guide along the journey was Professor Michael Gehm. I first met him when I took a graduate level Linear Algebra course which I found particularly challenging. I often went to his office hours asking for help and his ability to be patient and explain concepts from different perspectives is a gift few teachers have. As an advisor, I would like to thank him for all of the help and guidance he has given me over the years. His generosity for funding my graduate studies as well trips to conferences is appreciated. He believed in me more than I believed in myself. I consider him not only as a mentor but as a father figure. If I can become half the scientist that he is, I would consider that a successful career.

I especially want to thank Professor Esteban Vera, who I first met as a postdoctoral researcher in the Laboratory for Engineering Non-Traditional Sensors (LENS) and supervised me for the majority of my graduate studies. Much of the work and results in this dissertation is due to his guidance. Even after he started his professorship in Chile, he was willing to review my data and suggest different methods of analysis. Professor Vera is directly responsible for much of my training as an experimentalist. I consider Professor Vera as an older brother who was always there to protect me from the pitfalls of the graduate school journey.

I also thank Doctor Dathon Golish. His approach to work and life was a calming effect in often stressful times. He made major contributions to the Adaptive Feature Specific Spectral Imaging-Classifer (AFSSI-C) and provided valuable feedback on various research projects and conference presentations.

Thank you Professor Mark Neifeld and Professor Amit Ashok for being my advisor and supervisor during my first year as a PhD student. They were the first to introduce me to many of the techniques and subjects related to computational sensing. They taught me fundamental concepts in optics, statistical signal processing and programming. Many of the results in this dissertation would not have been possible without their teachings.

I've also had many other supervisors along the way whose effort must also be acknowledged: My undergraduate advisor at San Diego State University, Professor Matthew Anderson. Doctor John Crane, who was my supervisor during my internship at the Lawrence Livermore National Laboratory. Professor Joseph Eberly and Professor Gary Wicks who were my advisors at the Institute of Optics.

I would like to formally express gratitude to a number of exceptional teachers throughout my life. Professor Tom Milster who taught Diffraction and Interference and allowed me to be a teaching assistant for that course. Professor Masud Mansuripur, whose course in Electromagnetic Waves was the most elegant and well taught version of the classical nature of light that I have ever had the pleasure to

experience. Professor Jeff Davis, who first ignited my passion for optics while I was an undergraduate physics student at San Diego State University.

I also want to thank several faculty members who committed time from their busy schedules to help with several milestones of my graduate school experience. Special thanks to Professor Julie Bentley, Doctor James Oliver, and Professor Richard Morris who wrote letters of recommendation for me. Appreciation goes to Professor Tom Milster, Professor Harrison Barrett, Professor Russell Chipman, and Professor John Greivenkamp who formed my oral comprehensive exam committee. Thank you to Professor Rongguang Liang who served on my doctoral dissertation committee.

I've had the good fortune to be exposed to amazing groupmates as part of the LENS. David Coccarelli invited our family to spend our first Thanksgiving in North Carolina with him and we had many discussions about college basketball and life. Matthew Dunlop-Gray designed and constructed the AFSSI-C which is the foundation for much the work in this dissertation. Tariq Osman constructed the Static Computational Optical Undersampled Tracker (SCOUT) which is also a major part of this dissertation. Alyssa Jenkins whose combination of sense of humor and intelligence is unmatched. Thank you to Qian Gong, Kevin Kelly, Adriana DeRoos, David Landry, Xiaohan Li, and Dineshbabu Dinakarababu for your friendship. Finally, I consider Wei-Ren Ng as one of my best friends and as a brother. Our time in the LENS group was marked by many late nights spent working in the lab and office. He was generous in sharing his knowledge and gave me the advice that I often did not want to hear but was true.

I would like to thank several members of the Duke Imaging and Spectroscopy Program (DISP) laboratory for their friendship: Patrick Llull, Mehadi Hassan, and Evan Chen. Tsung Han Tsai was not only a colleague but his work on computational polarimetry and spectroscopy using an Liquid Crystal on Silicon (LCOS) Spatial Light Modulator (SLM) was the inspiration which directly lead my idea of using the same architecture for computational spectral unmixing.

Other graduate students, colleagues, and faculty must also be thanked, for at one time or another they all helped me: Basel Salahieh, Vicha Treeaporn, John Hughes, Steve Feller, Myungjun Lee, Sarmad H. Albanna, Professor Lars Furenlid, Doctor Joseph Dagher, Professor Daniel Marks, Professor Janick Roland-Thompson, Mary Pope, Mark Rodriguez, and Amanda Ferris.

Appreciation goes to the all the staff and faculty at the College of Optical Sciences at the University of Arizona. It is one of the most friendly and well run academic departments I have ever had the fortune to be a part of. I hope my career will reflect well upon the college.

Finally, I would like to thank my closest friends that I've met throughout the years. They often provided much needed respites during my journey—Christopher MacGahan, Ricky Gibson, Krista MacGahan, Kristi Behnke, Michael Gehl, Carlos Montances, Matthew Reaves, Vijay Parachuru, Eric Vasquez. Thank you for letting me into your lives and being part of mine.

Last but not least, to my family.

## DEDICATION

For my wife. We moved from city to city. You stuck with me through the highs and lows. You cooked dinner for me when I came home from a long day. You did the chores so I could concentrate on research. You acted as both mother and father to our son while I wrote. You believed in me even when I did not. You sacrificed your dreams and goals so I could accomplish mine.

You're the real Ph.D.

## TABLE OF CONTENTS

LIST OF FIGURES . . . . .	8
LIST OF TABLES . . . . .	9
ABSTRACT . . . . .	10
CHAPTER 1 Introduction . . . . .	11
1.1 Isomorphic Sensing . . . . .	12
1.2 Development of Multiplexing in Sensing . . . . .	16
1.3 Forward Models and Inverse Problems . . . . .	17
1.4 Indirect Imaging . . . . .	19
1.5 The Digital Imaging Revolution . . . . .	20
1.6 From Multiplexing to Compressive Sensing . . . . .	20
1.7 Practical Considerations in Computational Sensing . . . . .	20
1.8 Dissertation Overview . . . . .	20
CHAPTER 2 Formalism . . . . .	21
2.1 Isomorphic Sensing . . . . .	21
2.2 Multiplexing . . . . .	21
2.3 Principal Component Analysis . . . . .	21
2.4 Bayesian Rules and Log-Likelihood Ratios . . . . .	21
2.5 Compressive Sensing . . . . .	21
2.5.1 The Nyquist-Shannon Sampling Theorem . . . . .	21
2.5.2 Sparsity, Incoherence, and the Restricted Isometry Property . . . . .	22
2.5.3 Inversion . . . . .	22
2.5.3.1 L0 and L1 Norm Minimization . . . . .	22
2.5.3.2 LASSO and sparsity regularization . . . . .	22
Acronyms . . . . .	23
Symbols . . . . .	24

## LIST OF FIGURES

1.1	A systems view of a traditional sensing scheme. The signal-of-interest is incident upon the analog instrument. The analog instrument forms an isomorphism of the signal which is then periodically sampled point-by-point through an analog-to-digital converter (ADC) device. Once the signal is in digital form, post-processing algorithms are often used to perform various tasks such as noise reduction, detection, and classification. Notice that the analog instrument, sampling scheme, and processing are all separated. . . . .	11
1.2	A systems view of a computational sensing scheme. The signal-of-interest is incident upon the analog instrument. . . . .	12
1.3	A pinhole camera. . . . .	13
1.4	An isomorphic slit spectrometer with a 4F configuration. . . . .	14



## LIST OF TABLES

## ABSTRACT

Implementing computational optical sensors often comes with various issues that many traditional sensors may not encounter.

## CHAPTER 1

## Introduction

This chapter introduces the reader to the concept of isomorphic sensing and computational sensing and provides the motivation for the need to address the practical issues in experimental computational sensing.

*Isomorphic sensing* is the concept that a sensor's measurements resemble the signal of interest. Isomorphic sensing is often called traditional sensing. In isomorphic sensing the analog hardware, analog-to-digital converter (ADC), and processing algorithms are all separate components, see Figure 1.1. Computational sensing is the concept that a joint design of the sensor, often through coding of the analog signal, with task-specific algorithms can exceed the performance of an isomorphic sensor, see Figure 1.2 [1]. While isomorphic sensors can provide flexible sensing in multiple applications, a computational sensor's joint design naturally lends to performance increases. Throughout this chapter and the rest of this dissertation we will provide many examples that highlight the differences between computational and isomorphic sensing.

Rather than a rigorous discussion, this chapter will discuss some of the major developments and concepts in the field of computational sensing on an intuitive level. This will familiarize the reader with important terminology and techniques common in the field of computational sensing. A rigorous discussion with mathematical formalism of the concepts is given in chapter 2. This chapter will also discuss some

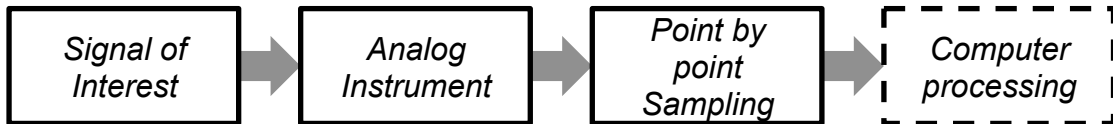


Figure 1.1: A systems view of a traditional sensing scheme. The signal-of-interest is incident upon the analog instrument. The analog instrument forms an isomorphism of the signal which is then periodically sampled point-by-point through an ADC device. Once the signal is in digital form, post-processing algorithms are often used to perform various tasks such as noise reduction, detection, and classification. Notice that the analog instrument, sampling scheme, and processing are all separated.

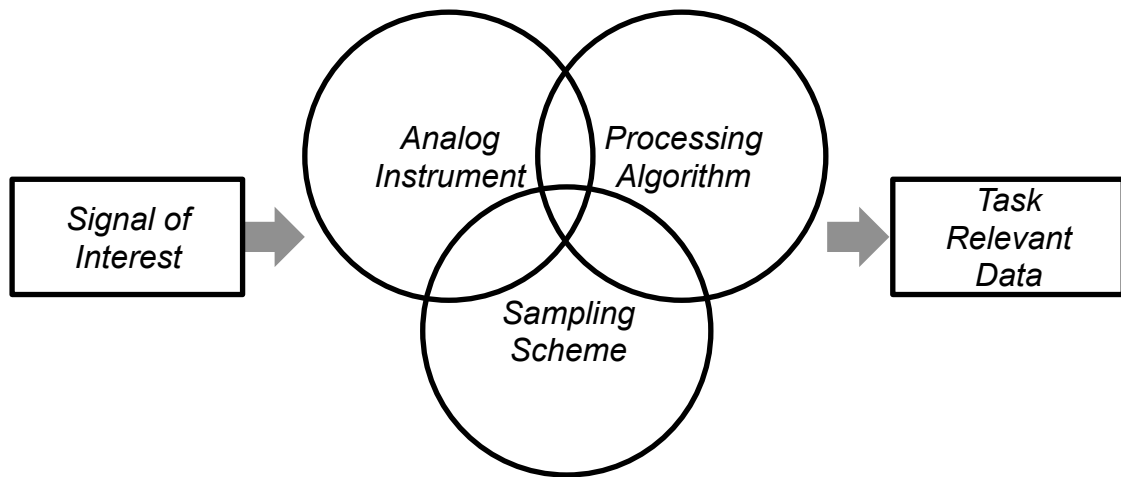


Figure 1.2: A systems view of a computational sensing scheme. The signal-of-interest is incident upon the analog instrument.

of the challenges I and many other experimentalists and engineers have faced when developing computational sensing prototypes. I will close this chapter with a brief look ahead to the rest of the dissertation.

### 1.1 Isomorphic Sensing

In Greek, the word isomorphic loosely translates to equal in form. Traditional sensors perform isomorphic sensing. In the context of this dissertation, an isomorphic sensor is any sensor which attempts to produce an output signal that resembles the signal-of-interest. In this paradigm, the analog instrument, sampling scheme, and post-processing algorithms are separated.

We will discuss three important examples of isomorphic sensors: the pinhole camera, the photographic camera and the optical spectrometer (which I will just call a spectrometer from now on, even though there are many instruments called spectrometers that not concerned with optical spectra). These sensors have had major roles throughout the history of optics and in the physical sciences so it is natural that they have also been the main focus of computational sensing. Therefore it is important we first understand the isomorphic version of these sensors.

In the photographic camera, the signal-of-interest is the object that is being photographed. This can be anything that is scattering or emitting light, a person, a tree or a distant group of stars. The analog instrument consists of the lens which

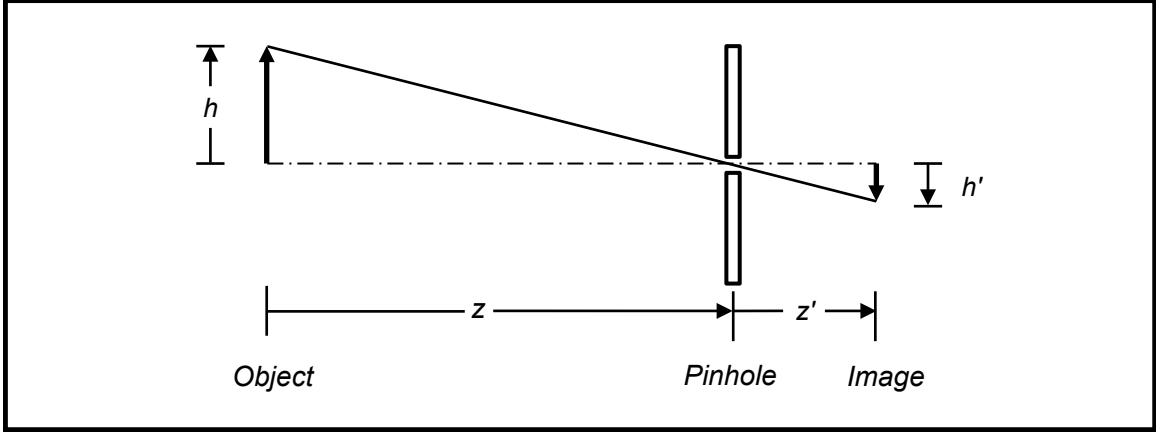


Figure 1.3: A pinhole camera.

is designed and fabricated to produce an image that looks like the object at the focal-plane array (FPA). The more that the image resembles the object the better the optics. The FPA then samples and quantizes the image and produces a digital representation of the object, measurement data. The measurement data is often post-processed to perform such tasks as noise removal or to locate the object.

There are two major sub-systems in the photographic camera which determine how well it performs: the optics and the FPA. Ideally, the optics (the analog instrument in this case) will produce a point spread function (PSF) which is infinitely small in diameter. For example, in a task such as the detection of a star from several neighboring stars in the night sky, if the PSF is much larger than the center to center separation of the two stars in the optical image, it will be quite difficult to detect. A careful reader will note that this is the same argument used by Lord Raleigh in proposing his resolution criterion [2]. Even if the PSF is small enough, the FPA must sample at a fine enough pixel-to-pixel spacing, called the *pixel pitch*, to accurately reproduce the intensity variations at the scale which is pertinent to the task. Intuitively, this makes sense because if the image of both stars and the decreased intensity which signifies a certain amount of separation between the two stars is imaged onto a single pixel, the one cannot ever hope to be able to accurately detect the star without some other prior or side information.

The pinhole camera predates the photographic camera over a thousand years, mostly due to its simplicity. It consists of a small hole and a box which prevents any light except from the pinhole to enter, see Figure 1.3. The pinhole camera is useful for imaging in parts of the electromagnetic spectrum and particles for which there is no direct analog to refractive lens or reflective mirror.

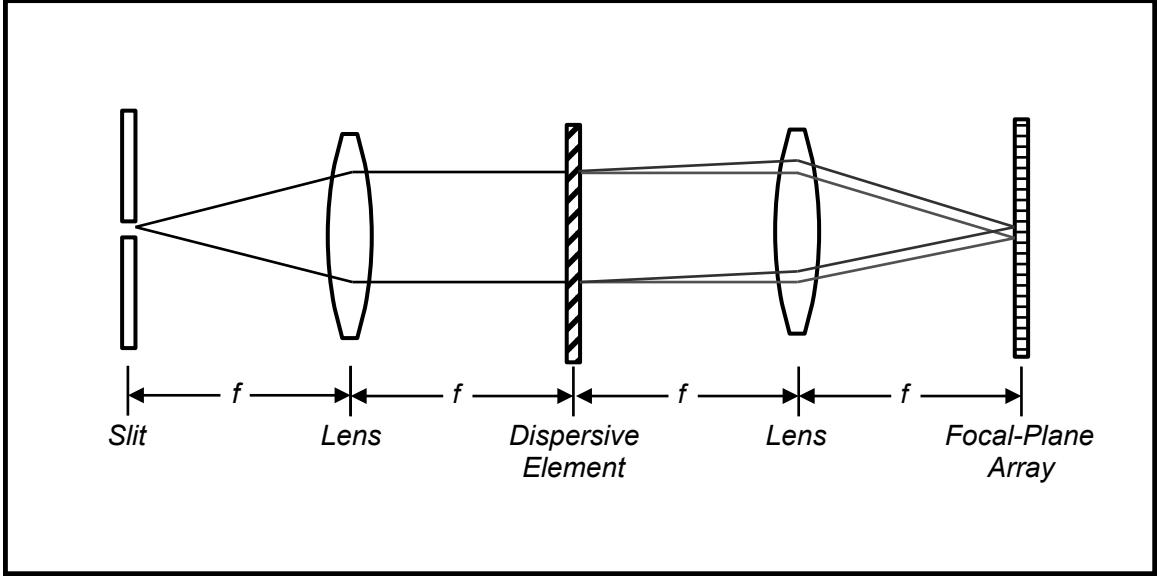


Figure 1.4: An isomorphic slit spectrometer with a 4F configuration.

In the spectrometer, the signal of interest is the spectrum of the object. The optics are designed to take the incoming light and separate various wavelength components, see Figure 1.4. The part of the spectrometer which is used to physically isolate the wavelengths is called a *spectrograph*. The result is a spectral intensity as a function of position at the FPA. The FPA and post-processing algorithms are used in the same manner as the photographic camera, which is to sample the optical spectrum creating digital version of it and to perform various tasks on the measurement data. For now, we will concentrate on the slit spectrometer, which measurements spectrum at a single point on the object.

In the spectrometer, one of the important performance metrics is *spectral resolution*, which we denote  $\delta\lambda$ . The spectral resolution is the smallest difference in wavelength the instrument can discern. Large spectral resolutions can degrade the spectrometers ability to discern important parts of the spectrum. Similarly with the camera, the FPA must have a pixel pitch which is small enough in order to correctly sample the variations in the spectrum.

The point-by-point nature of isomorphic sensing is both a strength and a source of weakness.

The strength comes from the straightforward and intuitive architecture of the isomorphic sensor. Each subsystem: the optics, the focal-plane array (FPA), and the post-processing can be designed and constructed separately as long as they meet their individual specifications. As long as the Signal-To-Noise Ratio (SNR) is sufficient and the sampling rate is high enough we are guaranteed to recover the

signal.

One of the weaknesses of the isomorphic approach however is the the ability to measure low SNR signals. Because the image is sampled in a completely parallel fashion at each exposure, each pixel contributes a certain amount of noise (which is independent of the signal strength). If the noise dominates, the measurement fidelity decreases often forcing the operator to increase the exposure time. For weak signals the exposure time can become prohibitive and for temporally dynamic signals this may lead to a loss of resolution. Indeed, one of the major engineering trade-offs faced by traditional spectrometer designers and users is that when one attempts to increase the light collection (increased slit-width) the spectral resolution  $\delta\lambda$  degrades (increases). Similarly, in the pinhole camera, there is a throughput versus spatial resolution trade-off, increasing the size of the pinhole degrades the PSF.

It would be easy to assume that with the recent revolution in machine learning and statistical signal processing combined with the dramatic increase in computing power that we could simply post-process poor measurements and obtain useful data. However, this isn't possible due to the an important theorem in information theory called the data processing inequality [3]. In layman's term it means "garbage in, garbage out".

Another weakness of isomorphic sensing is that the separation of the analog instrument, the sampling scheme, and the data processing algorithms lead to increased Size, Weight and Power-Cost (SWAP-C). As we mentioned in the photographic camera, the optics must be designed to produce a small PSF. For demanding applications, the optical design and fabrication can be the most expensive component of the sensor. While FPA prices in the visible have fallen recently, FPAs in certain parts of the electromagnetic spectrum can be quite expensive or non-existent [4].

In many cases, the signal is redundant and high resolution sampling becomes a waste of resources such as data storage and communications bandwidth. A good example is in photography where often the post-processing takes the digital image and applies a compression algorithm which looks for patterns in the signal and reduces the file size, discarding much of the sample data [5].

The isomorphic sensor has served humanity well, however with all the weakness that have been discussed I will now begin to discuss some of major techniques in computational sensing that can be used to address some or all of the issues that I just stated.

## 1.2 Development of Multiplexing in Sensing

*Multiplexing* in sensing allows each measurement sample to be a combination of multiple points of the signal-of-interest. Multiplexing is a powerful tool that can be exploited by the sensor designer to eliminate or relax design trade-offs.

A simple example which illustrates the usefulness of multiplex sensing is weighing objects. In this example, we are given a 100 sheets of paper. Let's assume that the scale's measurement error is insignificant. Isomorphic measurement sensing means one would need to measure each sheet of paper individually. Requiring 100 measurements.

Now, let's say the scale's measurement error is on the order of the weight of the sheet. Measuring each sheet individually produces a large measurement error. In order to reduce the error to an acceptable SNR we need to make several measurements per sheet to reduce the error to an acceptable level.

However, we can be a little smarter. We can measure all 100 sheets at the same time. Since the weight of all 100 sheets is much larger than the measurement error of the scale, we can dramatically increase the precision of the measurement. If we can assume that each sheet is the same weight, then we are done.

The weighing problem is analogous to the spectroscopy example. As discussed earlier in section 1.1, there is trade-off between light collection and spectral resolution. Increasing the slit-width to increase the amount of light has the effect of degrading the spectral resolution  $\delta\lambda$ . Around the late 1940's and early 1950's, several important papers and inventions demonstrated the effectiveness of multiplexing in spectroscopy. At the time the FPA was non-existent, so in the slit spectrometer shown in 1.4, where the FPA is pictured, there was actually another slit. To record the intensity at each spectral channel, either the dispersive element or the exit slit had to be mechanically translated, making the measurements even slower by a factor of  $N_\lambda$ , the number of spectral channels of interest.

Golay was the first to propose multiplexing the slit spectrometer by creating a pattern of binary (1's and 0's) entrance and exit slits [6]. The idea borrowed heavily from communications theory which is concerned with the reliable transmission of information over a noisy channel. In the Golay multi-slit spectrometer, the patterns of entrance and exit slits are matched based on mathematically useful properties, this is similar to coding and decoding signals in communications. The entrance slits act to code the spectrum while the exit slits decoded the coded spectrum. Intuitively,



the ability to use multiple entrance and exist slits increases the optical throughput of the spectrometer. Golay’s idea dramatically increased the optical throughput without degrading the spectral resolution.

Fellgett devised an alternative approach to multiplex spectroscopy, the Fourier Transform spectrometer, and was the first to note that multiplex measurements compared to isomorphic measurements improve the signal-to-noise ratio on the order of  $\sqrt{N_\lambda}$  [7]. He is generally credited with discovering the multiplex advantage, so it is often called the Fellgett advantage instead.

Another example that is pertinent to this dissertation is coded aperture imaging. Coded aperture imaging can be thought of as the multiplexed version of a pinhole camera. As mentioned earlier in section 1.1, there is a trade-off between the throughput and spatial resolution. However in many fields, such as high-energy particle imaging, refractive lenses and reflective mirrors are non-existent or underdeveloped. By using multiple pinholes the throughput is increased without sacrificing spatial resolution. However, the pattern of the pinholes, which is the code, in this case must be carefully designed to achieve optimal object reconstruction. Fenimore, Canon, and Gottesman were the first to create an elegant solution to coded aperture design called uniformly redundant arrays [8, 9].

In summary, multiplexing has the ability to eliminate classic trade-offs in isomorphic sensors: signal strength or resolution. Modern researchers are still actively developing novel ways to implement multiplexing to increase resolution and sensitivity in the spatial domain [10, 11], spectral domain [12, 13], and temporal domain [14, 15]. However, multiplexing is not without its own set of problems. We now discuss inverse problems in computational sensing.

### 1.3 Forward Models and Inverse Problems

In the computational sensing community, a model which maps the signal-of-interest to the measurement data is called the *forward model*. The problem of taking the observed data and calculating a reconstruction of the signal-of-interest or task-specific parameters is called the *inverse problem*.

As you can imagine, solving inverse problems of isomorphic measurements, when one is concerned with reconstruction of the signal-of-interest, tend to be straight forward. In the weighing problem, the measurement is also the reconstruction. In the slit spectrometer, where the forward model can be simply the continuous to

discrete mapping of the spectrum. The spectrum is the interpolated measurement.

Of course, we can begin to add various levels of complexity to the forward model to account for various physical aspects of the sensor, such as the fact the FPA can't measure certain wavelength regions or the noise in our measurements. But again, assuming proper sampling and enough SNR, the reconstruction of the isomorphic signal is the measurement. This simplicity is one reason why isomorphic sensing still dominates at the consumer level despite all of the drawbacks I discussed earlier in section 1.1.

However, the multiplexing of signal information forces us to develop computational steps to reconstruct or infer the parameters of the signal that caused the measurements. In the multiplexed weighing problem, a significant complication occurs when each sheet has a different weight. Now solving the inverse problem is not as straight forward. A single measurement of all 100 sheets in this case is an *underdetermined problem* since we have 1 equation and 100 unknowns. What we can do is try measuring different combinations of the 100 sheets, each new combination provides us with a new equation to work with reducing the error. Naively, we might assume that we can randomly choose 100 unique combinations and solve 100 equations using the algebra we were taught in high school. However when there is measurement error, in many applications including the weighing problem, random combinations are not the best way to conduct the coding, they are sub-optimal in terms of reconstruction error. This lead many to begin working on optimal coding strategies of signals for sensing and is major topic in this disseration.

In fact, the idea of multiplexing was not invented by Golay or Fenimore, what they contibuted was the creation of computationally simple ways to code and decode their signal-of-interest, making their sensors more appealing to a broader commu-nicaty of scientists and engineers. A good example is the Hadamard matrix based code [16]. The Hadamard code is appealing because reconstruction is simply the transpose of the original matrix. Unfortunately, not all multiplexing forward models codes have mathematically elegent inversion steps. Often the physics of the situation force non-isomorphic measurements which require a computational step to solve the inverse problem.

## 1.4 Indirect Imaging

While Golay, Fennimore, and others were leveraging multiplexing to eliminate trade-offs in traditional sensors, an entirely disparate group of researchers were working on imaging techniques for which there was no isomorphic analog. In these cases the physics of the sensing modality prevents a point-by-point sampling of the signal-of-interest. These techniques are called indirect imaging.

Perhaps one of the most successful early examples of indirect imaging and the rise of inverse problems in sensing is the development of radar. While early radar was concerned with the detection and distance of an object, development of imaging radar began after World War II. Imaging radar and specifically Synthetic Aperture Radar (SAR) can use time delay information combined with the doppler effects and interference patterns of coherent radio waves to create high resolutions of terrain and buildings.

In medicine, an imaging modality that now common is X-Ray Computed Tomography (CT), one is forced into this situation since the task is to reconstruct a 2 or 3-dimensional function from 1 or 2 dimensional measurement data. The forward model can be simple: In a collimated beam architecture with a 1-dimensional detector array, we say that each sample from each pixel on the array is proportional to the total number of x-ray photons that have not been absorbed by the object [17] plus noise. The inversion of course is not straight forward. The culmination of the work related to the inversion techniques and the actual prototype resulted in the Nobel Prize for Physiology or Medicine in 1979 [18].

In general, non-isomorphic sensing such as indirect imaging schemes which include X-ray CT, Single-Photon Emission Computed Tomography (SPECT), Positron Emission Tomography (PET), Magnetic Resonance Imaging (MRI) and certain forms of sonic and radio wave imaging all require a data-processing or reconstruction step to solve an inverse problem [19]. In summary, the forward model of a sensor is essentially accounting for the physics which govern the measurement while the solving the inverse problem is a mathematical problem which attempts to either reconstruct the object or to calculate task-specific data of the object.

We now turn to what are arguably the two most important inventions of optical sensing in the last several centuries, the digital image sensor and the digital computer.

## 1.5 The Digital Imaging Revolution

Shannon, Nyquist, Witteraker and others established the theory for determining what the pixel spacing must be in order to properly sample the analog signal without losing any information [20, 21, 22].

## 1.6 From Multiplexing to Compressive Sensing

## 1.7 Practical Considerations in Computational Sensing

Decoding the signal, which we can also think of as solving the inverse problem is not necessarily straight forward. Unfortunately the coded of the analog signal and inversion steps are often seperately designed in computational sensing. This becomes especially frequency because the practicing sensor engineer must piece together various techniques from theorestist. This

## 1.8 Dissertation Overview

## CHAPTER 2

### Formalism

This chapter introduces the reader to the more rigorous concepts and mathematical background that will be required to fully understand the material presented in the later chapters of this dissertation.

A rigorous discussion of multiplexing and signal-to-noise ratio will be discussed, as well as various coding schemes used in various notable computational sensors as well as the ones in this dissertation.

Since the AFSSI-C relies on a variation of Principal Component Analysis (PCA) and a Bayesian algorithm for coding design we will discuss some of the fundamentals of Bayesian probability and the Log-Likelihood Ratios.

#### 2.1 Isomorphic Sensing

$$\mathbf{g} = \mathbf{H}\mathbf{r} \tag{2.1}$$

where  $\mathbf{g}$  is the measurement vector.

#### 2.2 Multiplexing

#### 2.3 Principal Component Analysis

#### 2.4 Bayesian Rules and Log-Likelihood Ratios

#### 2.5 Compressive Sensing

##### 2.5.1 The Nyquist-Shannon Sampling Theorem

The Nyquist-Shannon Sampling Theorem states one must sample a signal with a sampling rate that is at least twice the maximum frequency of the signal to prevent aliasing [20].

## 2.5.2 Sparsity, Incoherence, and the Restricted Isometry Property

## 2.5.3 Inversion

### 2.5.3.1 L0 and L1 Norm Minimization

### 2.5.3.2 LASSO and sparsity regularization

## Acronyms

- ADC** analog-to-digital converter. 8, 11
- AFSSI-C** Adaptive Feature Specific Spectral Imaging-Classifer. 4, 5, 21
- CT** Computed Tomography. 19
- DISP** Duke Imaging and Spectroscopy Program. 5
- FPA** focal-plane array. 13–16, 18
- LCOS** Liquid Crystal on Silicon. 5
- LENS** Laboratory for Engineering Non-Traditional Sensors. 4, 5
- MRI** Magnetic Resonance Imaging. 19
- PET** Positron Emission Tomography. 19
- PSF** point spread function. 13, 15
- SAR** Synthetic Aperture Radar. 19
- SCOUT** Static Computational Optical Undersampled Tracker. 5
- SLM** Spatial Light Modulator. 5
- SNR** Signal-To-Noise Ratio. 14–16, 18
- SPECT** Single-Photon Emission Computed Tomography. 19
- SWAP-C** Size, Weight and Power-Cost. 15

## Symbols

$\delta\lambda$  Spectral Resolution. 14–16

$\mathbf{g}$  The Measurement Vector. 21

$N_\lambda$  The Number of Spectral Channels. 16



## REFERENCES

- [1] M. A. Neifeld, A. Mahalanobis, and D. J. Brady, "Task-specific sensing-introduction," *Appl. Opt.*, vol. 45, no. 13, pp. 2857–2858, May 2006. [Online]. Available: <http://ao.osa.org/abstract.cfm?URI=ao-45-13-2857>
- [2] L. Rayleigh, "Investigations in optics, with special reference to the spectro-scope," *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, 1879.
- [3] T. M. Cover and J. A. Thomas, *Elements of information theory*. John Wiley & Sons, 2012.
- [4] I. Noor, O. Furxhi, and E. L. Jacobs, "Compressive sensing for a sub-millimeter-wave single pixel imager," in *SPIE Defense, Security, and Sensing*. International Society for Optics and Photonics, 2011, pp. 80 220K–80 220K.
- [5] D. Taubman and M. Marcellin, *JPEG2000 Image Compression Fundamentals, Standards and Practice: Image Compression Fundamentals, Standards and Practice*. Springer Science & Business Media, 2012, vol. 642.
- [6] M. J. Golay, "Multi-slit spectrometry," *JOSA*, vol. 39, no. 6, pp. 437–444, 1949.
- [7] P. Fellgett, "I.—les principes généraux des méthodes nouvelles en spectroscopie interférentielle-a propos de la théorie du spectromètre interférentiel multiplex," *J. phys. radium*, vol. 19, no. 3, pp. 187–191, 1958.
- [8] E. E. Fenimore and T. Cannon, "Coded aperture imaging with uniformly redundant arrays," *Applied optics*, vol. 17, no. 3, pp. 337–347, 1978.
- [9] S. R. Gottesman and E. Fenimore, "New family of binary arrays for coded aperture imaging," *Applied optics*, vol. 28, no. 20, pp. 4344–4352, 1989.
- [10] M. F. Duarte, M. A. Davenport, D. Takhar, J. N. Laska, T. Sun, K. E. Kelly, R. G. Baraniuk *et al.*, "Single-pixel imaging via compressive sampling," *IEEE Signal Processing Magazine*, vol. 25, no. 2, p. 83, 2008.
- [11] D. Townsend, P. Poon, S. Wehrwein, T. Osman, A. Mariano, E. Vera, M. Stenner, and M. Gehm, "Static compressive tracking," *Optics express*, vol. 20, no. 19, pp. 21 160–21 172, 2012.
- [12] M. E. Gehm, S. T. McCain, N. P. Pitsianis, D. J. Brady, P. Potuluri, and M. E. Sullivan, "Static two-dimensional aperture coding for multimodal, multiplex spectroscopy," *Applied optics*, vol. 45, no. 13, pp. 2965–2974, 2006.
- [13] T.-H. Tsai and D. J. Brady, "Coded aperture snapshot spectral polarization imaging," *Applied optics*, vol. 52, no. 10, pp. 2153–2161, 2013.

- [14] J. Holloway, A. C. Sankaranarayanan, A. Veeraraghavan, and S. Tambe, “Flutter shutter video camera for compressive sensing of videos,” in *Computational Photography (ICCP), 2012 IEEE International Conference on*. IEEE, 2012, pp. 1–9.
- [15] P. Llull, X. Liao, X. Yuan, J. Yang, D. Kittle, L. Carin, G. Sapiro, and D. J. Brady, “Coded aperture compressive temporal imaging,” *Optics express*, vol. 21, no. 9, pp. 10 526–10 545, 2013.
- [16] N. Sloane, T. Fine, P. Phillips, and M. Harwit, “Codes for multiplex spectrometry,” *Applied optics*, vol. 8, no. 10, pp. 2103–2106, 1969.
- [17] J. Radon, “1.1 über die bestimmung von funktionen durch ihre integralwerte längs gewisser mannigfaltigkeiten,” *Classic papers in modern diagnostic radiology*, vol. 5, 2005.
- [18] “The Nobel Prize in Physiology or Medicine, 1979,” [https://www.nobelprize.org/nobel\\_prizes/medicine/laureates/1979/perspectives.html](https://www.nobelprize.org/nobel_prizes/medicine/laureates/1979/perspectives.html), accessed: 2016-08-22.
- [19] H. H. Barrett and K. J. Myers, *Foundations of Image Science*. John Wiley & Sons, 2013.
- [20] C. E. Shannon, “Communication in the presence of noise,” *Proceedings of the IRE*, vol. 37, no. 1, pp. 10–21, 1949.
- [21] H. Nyquist, “Certain Factors Affecting Telegraph Speed,” *Bell System technical journal*, vol. 3, no. 2, pp. 324–346, 1924.
- [22] —, “Certain topics in telegraph transmission theory,” *Unknown Journal*, 1928.