

PRACTICAL CONSIDERATIONS IN EXPERIMENTAL COMPUTATIONAL SENSING

by

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As members of the Dissertation Committee, we certify that we have read the dissertation prepared by Phillip K. Poon titled Practical Considerations in Experimental Computational Sensing and recommend that it be accepted as fulfilling the dissertation requirement for the degree of Doctor of Philosophy.

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SIGNED: Phillip K. Poon

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DEDICATION

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ABSTRACT

Implementing computational optical sensors often comes with various issues that many traditional sensors may not encounter.

CHAPTER 1

Introduction

This chapter introduces the reader to the concept of isomorphic sensing and computational sensing and provides the motivation for the need to address the practical issues in experimental computational sensing.

Isomorphic sensing is the concept that a sensor's measurements resemble the signal of interest. Isomorphic sensing is often called traditional sensing. In isomorphic sensing the analog hardware, analog-to-digital converter (ADC), and processing algorithms are all separate components, see Figure 1.1. Computational sensing is the concept that a joint design of the sensor, often through coding of the analog signal, with task-specific algorithms can exceed the performance of an isomorphic sensor, see Figure 1.2 [1]. While isomorphic sensors can provide flexible sensing in multiple applications, a computational sensor's joint design naturally lends to performance increases. Throughout this chapter and the rest of this dissertation we will provide many examples that highlight the differences between computational and isomorphic sensing.

Rather than a rigorous discussion, this chapter will discuss some of the major developments and concepts in the field of computational sensing on an intuitive level. This will familiarize the reader with important terminology and techniques common in the field of computational sensing. A rigorous discussion with mathematical formalism of the concepts is given in chapter 2. This chapter will also discuss some

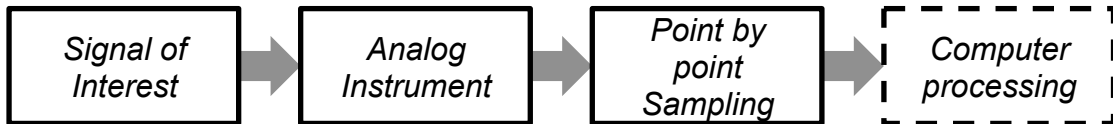


Figure 1.1: A systems view of a traditional sensing scheme. The signal-of-interest is incident upon the analog instrument. The analog instrument forms an isomorphism of the signal which is then periodically sampled point-by-point through an ADC device. Once the signal is in digital form, post-processing algorithms are often used to perform various tasks such as noise reduction, detection, and classification. Notice that the analog instrument, sampling scheme, and processing are all separated.

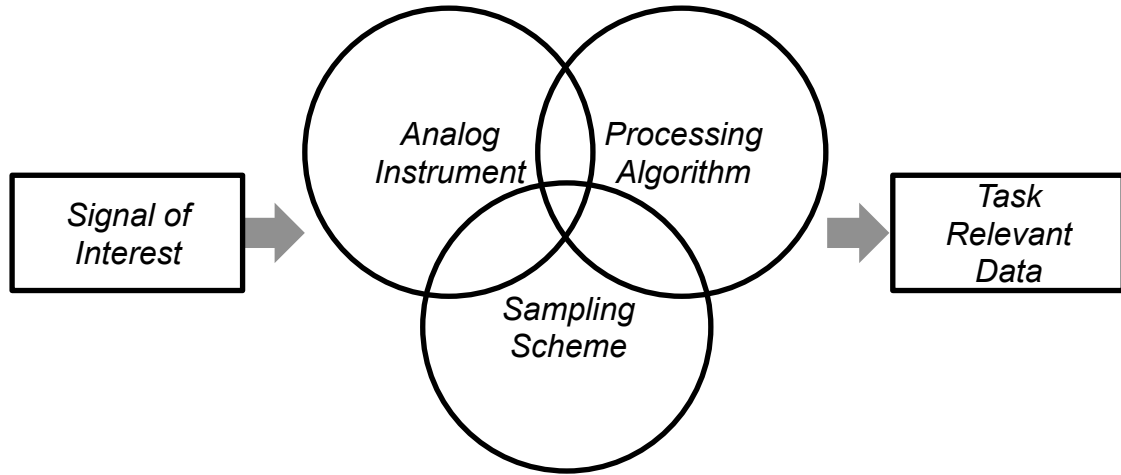


Figure 1.2: A systems view of a computational sensing scheme. The signal-of-interest is incident upon the analog instrument.

of the challenges I and many other experimentalists and engineers have faced when developing computational sensing prototypes. I will close this chapter with a brief look ahead to the rest of the dissertation.

1.1 Isomorphic Sensing

In Greek, the word isomorphic loosely translates to equal in form. Traditional sensors perform isomorphic sensing. In the context of this dissertation, an isomorphic sensor is any sensor which attempts to produce an output signal that resembles the signal-of-interest. In this paradigm, the analog instrument, sampling scheme, and post-processing algorithms are separated.

We will discuss three important examples of isomorphic sensors: the pinhole camera, the photographic camera and the optical spectrometer (which I will just call a spectrometer from now on, even though there are many instruments called spectrometers that not concerned with optical spectra). These sensors have had major roles throughout the history of optics and in the physical sciences so it is natural that they have also been the main focus of computational sensing. Therefore it is important we first understand the isomorphic version of these sensors.

In the photographic camera, the signal-of-interest is the object that is being photographed. This can be anything that is scattering or emitting light, a person, a tree or a distant group of stars. The analog instrument consists of the lens which

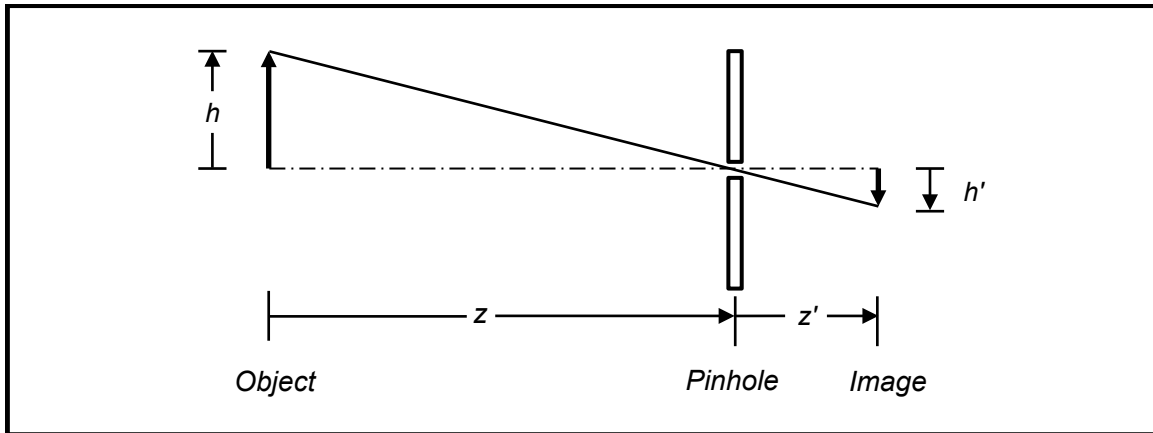


Figure 1.3: A pinhole camera.

is designed and fabricated to produce an image that looks like the object at the focal-plane array (FPA). The more that the image resembles the object the better the optics. The FPA then samples and quantizes the image and produces a digital representation of the object, measurement data. The measurement data is often post-processed to perform such tasks as noise removal or to locate the object.

There are two major sub-systems in the photographic camera which determine how well it performs: the optics and the FPA. Ideally, the optics (the analog instrument in this case) will produce a point spread function (PSF) which is infinitely small in diameter. For example, in a task such as the detection of a star from several neighboring stars in the night sky, if the PSF is much larger than the center to center separation of the two stars in the optical image, it will be quite difficult to detect. A careful reader will note that this is the same argument used by Lord Raleigh in proposing his resolution criterion [2]. Even if the PSF is small enough, the FPA must sample at a fine enough pixel-to-pixel spacing, called the *pixel pitch*, to accurately reproduce the intensity variations at the scale which is pertinent to the task. Intuitively, this makes sense because if the image of both stars and the decreased intensity which signifies a certain amount of separation between the two stars is imaged onto a single pixel, the one cannot ever hope to be able to accurately detect the star without some other prior or side information.

The pinhole camera predates the photographic camera over a thousand years, mostly due to its simplicity. It consists of a small hole and a box which prevents any light except from the pinhole to enter, see Figure 1.3. The pinhole camera is useful for imaging in parts of the electromagnetic spectrum and particles for which there is no direct analog to refractive lens or reflective mirror.

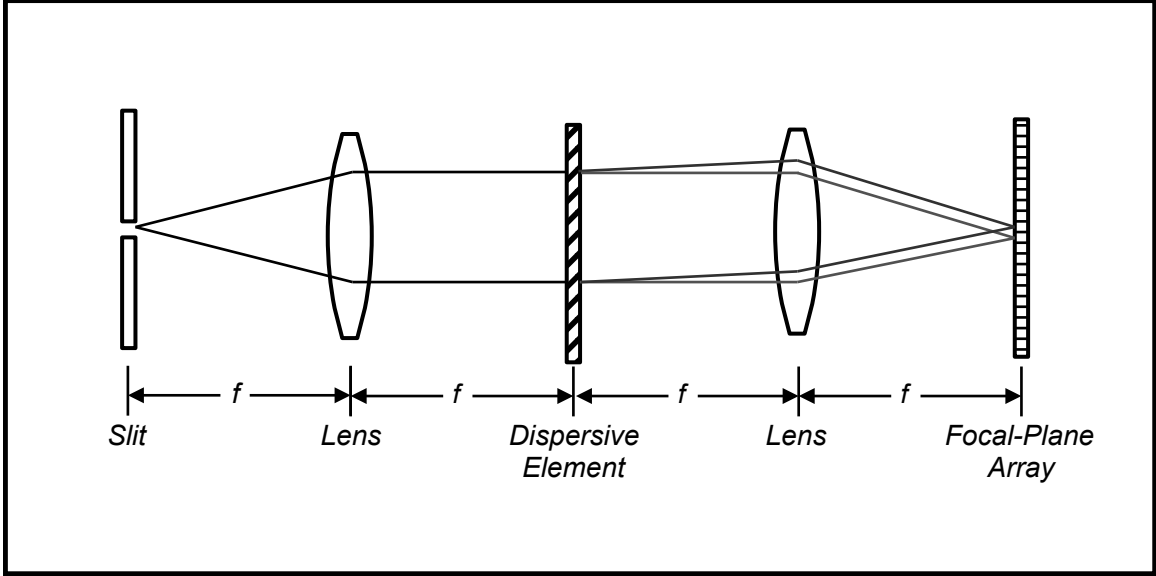


Figure 1.4: An isomorphic slit spectrometer with a 4F configuration.

In the spectrometer, the signal of interest is the spectrum of the object. The optics are designed to take the incoming light and separate various wavelength components, see Figure 1.4. The part of the spectrometer which is used to physically isolate the wavelengths is called a *spectrograph*. The result is a spectral intensity as a function of position at the FPA. The FPA and post-processing algorithms are used in the same manner as the photographic camera, which is to sample the optical spectrum creating digital version of it and to perform various tasks on the measurement data. For now, we will concentrate on the slit spectrometer, which measurements spectrum at a single point on the object.

In the spectrometer, one of the important performance metrics is *spectral resolution*, which we denote $\delta\lambda$. The spectral resolution is the smallest difference in wavelength the instrument can discern. Large spectral resolutions can degrade the spectrometers ability to discern important parts of the spectrum. Similarly with the camera, the FPA must have a pixel pitch which is small enough in order to correctly sample the variations in the spectrum.

The point-by-point nature of isomorphic sensing is both a strength and a source of weakness.

The strength comes from the straightforward and intuitive architecture of the isomorphic sensor. Each subsystem: the optics, the focal-plane array (FPA), and the post-processing can be designed and constructed separately as long as they meet their individual specifications. As long as the Signal-To-Noise Ratio (SNR) is sufficient and the sampling rate is high enough we are guaranteed to recover the

signal.

One of the weaknesses of the isomorphic approach however is the the ability to measure low SNR signals. Because the image is sampled in a completely parallel fashion at each exposure, each pixel contributes a certain amount of noise (which is independent of the signal strength). If the noise dominates, the measurement fidelity decreases often forcing the operator to increase the exposure time. For weak signals the exposure time can become prohibitive and for temporally dynamic signals this may lead to a loss of resolution. Indeed, one of the major engineering trade-offs faced by traditional spectrometer designers and users is that when one attempts to increase the light collection (increased slit-width) the spectral resolution $\delta\lambda$ degrades (increases). Similarly, in the pinhole camera, there is a throughput versus spatial resolution trade-off, increasing the size of the pinhole degrades the PSF.

It would be easy to assume that with the recent revolution in machine learning and statistical signal processing combined with the dramatic increase in computing power that we could simply post-process poor measurements and obtain useful data. However, this isn't possible due to the an important theorem in information theory called the data processing inequality [6]. In layman's term it means "garbage in, garbage out".

Another weakness of isomorphic sensing is that the separation of the analog instrument, the sampling scheme, and the data processing algorithms lead to increased Size, Weight and Power-Cost (SWAP-C). As we mentioned in the photographic camera, the optics must be designed to produce a small PSF. For demanding applications, the optical design and fabrication can be the most expensive component of the sensor. While FPA prices in the visible have fallen recently, FPAs in certain parts of the electromagnetic spectrum can be quite expensive or non-existent [7].

In many cases, the signal is redundant and high resolution sampling becomes a waste of resources such as data storage and communications bandwidth. A good example is in photography where often the post-processing takes the digital image and applies a compression algorithm which looks for patterns in the signal and reduces the file size, discarding much of the sample data [8].

The isomorphic sensor has served humanity well, however with all the weakness that have been discuss I will now begin to discuss some of major techniques in computational sensing that can be used to address some or all of the issues that I just stated.

1.2 Development of Multiplexing in Sensing

Multiplex sensing allows each measurement sample to be a combination of multiple points of the signal-of-interest. Multiplexing is a powerful tool that can be exploited by the sensor designer to eliminate or relax design trade-offs.

A simple example which illustrates the usefulness of multiplex sensing is weighing objects. In this example, we are given a 100 sheets of paper. Let's assume that the scale's measurement error is insignificant. Isomorphic measurement sensing means one would need to measure each sheet of paper individually. Requiring 100 measurements.

Now, let's say the scale's measurement error is on the order of the weight of the sheet. Measuring each sheet individually produces a large measurement error. In order to reduce the error to an acceptable SNR we need to make several measurements per sheet to reduce the error to an acceptable level.

However, we can be a little smarter. We can measure all 100 sheets at the same time. Since the weight of all 100 sheets is much larger than the measurement error of the scale, we can dramatically increase the precision of the measurement. If we can assume that each sheet is the same weight, then we are done. The significant complication occurs when each sheet has a different weight. A single measurement of all 100 sheets is hopeless in this case. We have 1 equation and 100 unknowns.

What we can do is try measuring different combinations of the 100 sheets, each new combination provides us with a new equation to work with reducing the error. Naively we might assume that we can randomly choose 100 unique combinations and solve 100 equations from using the algebra we were taught in high school. However, on average random combinations only work well in high SNR situations. This lead many to begin working on optimal ways to measuring combinations of signals for sensing.

The weighing problem is directly applicable to the spectroscopy example. As discussed earlier in section 1.1, there is trade-off between light collection and spectral resolution. Increasing the slit-width to increase the amount of light has the effect of increasing the spectral resolution $\delta\lambda$.

Around the late 1940's and early 1950's, several important inventions demonstrated the effectiveness of multiplexing in spectroscopy. At the time the electronic FPA were non-existent. In the slit spectrometer shown in 1.4, the FPA was actually another slit. To record the intensity at each spectral channel, either the dispersive

element or the exit slit had to be mechanically translated, making the measurements even slower by a factor of N_λ , the number of spectral channels of interest.

Golay was the first to propose multiplexing the slit spectrometer by creating a pattern of binary (1's and 0's) entrance and exit slits [9]. The idea borrowed heavily from communications theory which is concerned with the reliable transmission of information over a noisy channel. In the Golay multi-slit spectrometer, the patterns of entrance and exit slits are matched based on mathematically useful properties, this is similar to coding and decoding signals in communications. The entrance slits act to code the spectrum while the exit slits decode the coded spectrum.

Intuitively, the ability to use multiple entrance and exit slits increases the optical throughput of the spectrometer. Fellgett devised an alternative approach to multiplex spectroscopy, the Fourier Transform spectrometer, and was the first to note that multiplex measurements compared to isomorphic measurements improve the signal-to-noise ratio on the order of $\sqrt{N_\lambda}$ [10].

Another example that is pertinent to this dissertation is coded aperture imaging. Coded aperture imaging can be thought of as the multiplexed version of a pinhole camera. As mentioned earlier in section 1.1, there is a trade-off between the throughput and spatial resolution. However in many fields, such as high-energy particle imaging, refractive lens and mirrors are non-existent or underdeveloped. By using multiple pinholes the throughput is increased. However, the pattern of the pinholes, which is the code, in this case must be carefully designed to achieve optimal object reconstruction. Fenimore, Canon, and Gottesman were the first to create an elegant solution to coded aperture design called uniformly redundant arrays [11, 12].

As you can see, multiplexing is a central part of computational sensing. As we will see in section 1.5 multiplexing is required to solve highly underdetermined sensing problems. We will now discuss the rise of inverse problems in imaging, and how their parallel development to multiplexed optical sensing contributed to the field of computational sensing. While many in the spectroscopy and high-energy particle imaging community were turning to non-isomorphic techniques to eliminate classical design trade-offs, for other fields, isomorphic sensing is non-existent.

1.3 Inverse Problems in Imaging

What is an inverse problem? And why do they arise in sensing?

It should be noted that often decoding, the task of going from measurement to

signal-of-interest, is not necessarily straight forward. And often the coding of the analog signal and decoding are separately designed in computational sensing. This has become readily apparent because the available components at the experimentalist disposal limit their ability to implement mathematically optimal codes.

We now turn to what is arguably the most important invention of optical sensing in the last century, the electronic image sensor and how it spur the massive growth in computational sensing.

1.4 The Digital Imaging Revolution

Shannon, Nyquist, Wittaker and others established the theory for determining what the pixel spacing must be in order to properly sample the analog signal without losing any information [3, 4, 5].

1.5 From Multiplexing to Compressive Sensing

1.6 Dissertation Overview

CHAPTER 2

Formalism

This chapter introduces the reader to the more rigorous concepts and mathematical background that will be required to fully understand the material presented in the later chapters of this dissertation.

A rigorous discussion of multiplexing and signal-to-noise ratio will be discussed, as well as various coding schemes used in various notable computational sensors as well as the ones in this dissertation.

Since the AFSSI-C relies on a variation of Principal Component Analysis (PCA) and a Bayesian algorithm for coding design we will discuss some of the fundamentals of Bayesian probability and the Log-Likelihood Ratios.

2.1 Isomorphic Sensing

$$\mathbf{g} = \mathbf{H}\mathbf{r} \tag{2.1}$$

where \mathbf{g} is the measurement vector.

2.2 Multiplexing

2.3 Principal Component Analysis

2.4 Bayesian Rules and Log-Likelihood Ratios

2.5 Compressive Sensing

2.5.1 The Nyquist-Shannon Sampling Theorem

The Nyquist-Shannon Sampling Theorem states one must sample a signal with a sampling rate that is at least twice the maximum frequency of the signal to prevent aliasing [3].

2.5.2 Sparsity, Incoherence, and the Restricted Isometry Property

2.5.3 Inversion

2.5.3.1 L0 and L1 Norm Minimization

2.5.3.2 LASSO and sparsity regularization

Acronyms

ADC analog-to-digital converter. 8, 11

AFSSI-C Adaptive Feature Specific Spectral Imaging-Classifer. 4, 5, 18

DISP Duke Imaging and Spectroscopy Program. 5

FPA focal-plane array. 13–16

LCOS Liquid Crystal on Silicon. 5

LENS Laboratory for Engineering Non-Traditional Sensors. 4, 5

PSF point spread function. 13, 15

SCOUT Static Computational Optical Undersampled Tracker. 5

SLM Spatial Light Modulator. 5

SNR Signal-To-Noise Ratio. 14–16

SWAP-C Size, Weight and Power-Cost. 15

Symbols

$\delta\lambda$ Spectral Resolution. 14–16

\mathbf{g} The Measurement Vector. 18

N_λ The Number of Spectral Channels. 17

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