

1 Introduction

Many machine learning tasks involve high dimensional data in order to accurately represent their complex nature. The problem is that reinforcement learning is not scaling up well to accommodate the high dimensional data. For example, an usual approach for exploring a space with help of RL would be to count the exact states that have been visited already and through that calculate their individual relative probability. Now an even exploration can be achieved, when at each state the action is chosen that yields the lowest probability. The predicament that arises in high dimensional states is that they cannot be counted. In a real space the probability for each state would be zero and that can severely limit the scope of the exploration. So the dimensionality of the state itself has to be reduced to be able to work with the data

Two common methods to reduce the dimensionality of said data are the PCA and Deep Learning, i.e. autoencoders. In this thesis we want to look at both option and see how efficiently and evenly either approach explores the working space of a robot's endeffector.

2 Preliminaries

2.1 Markov Decision Process

The Markov Decision Process (MDP) is specified as a sequence of states and actions which adhere to the Markov property. In other words, which action a is chosen at state s is only depending on the state s and not its history. It is mathematically defined as a 5-tuple $M = (S, A, T, R, p_\theta)$, where

- S is the set of states
- A is the set of actions
- T is the transition probability function, such that $T(s, s', a) = p(s'|s, a)$, where $s, s' \in S$ and $a \in A$
- R is the reward function, which assigns a reward r to every transition caused by action a
- p_θ is the initial distribution, which tells how likely it is to start in a certain state

The goal now is to find a policy π , which tells the probability $\pi(s, a)$ that action a is chosen at state s . This is called an optimization problem and can for example be solved with reinforcement learning.

2.2 Reinforcement Learning

Reinforcement learning is a field of machine learning, in which an agent shall learn a policy to maximize some reward. It is supposed to do that on its own,

meaning the entire process is unsupervised, The environment in which the agent acts is usually formulated as a Markov Decision Process. The environment has a set of states S , the agent chooses from a set of actions A in each state and hence gets a reward r . The selection which action is chosen at which state is called policy and can be modeled as:

- $\pi : A \times S \mapsto [0, 1]$
- $\pi(s, a) = p(a|s)$

The aim now is to find a policy π , which maximizes the sum of all rewards. Because of this it is necessary to have knowledge of the accumulative reward starting at any state s . The value function V_π determines exactly that. It returns the expected accumulative reward when policy π was followed. It is defined as follows:

$$V^\pi(s) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s], \quad (1)$$

where $\gamma \in [0, 1]$, so that the effect of earlier states is counted less and less. It is called the discount. The maximum possible value of V^π is defined as:

$$V^*(s) = \underset{\pi}{argmax} V^\pi(s) \quad (2)$$

When V^* has a solution, it is the best solution for the problem. Thus it is apparently sufficient to merely examine the state values to reach optimality. However, it can be useful to also consider the action values. The function which does this, is defined as:

$$Q^\pi(s) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r_t | s_0 = s, a_0 = a] \quad (3)$$

It returns the accumulative reward, starting at state s and choosing action a .

Now, under the condition that π^* is the optimal policy, optimality can be achieved, when at each state s the next action is chosen from $Q^{\pi^*}(s, \cdot)$.

2.3 Problems in exploration

The aim of this thesis is, as mentioned previously, to find an optimal exploration strategy in continuous state-action space. Which means that preferably no single state occurs more than once. In the discrete case, We can measure that by counting how often a single state has been occupied and from that derive the state's probability of occurrence. Now whenever the exploration strategy needs to decide which action to do next, the available action, which reaches the state with the lowest probability, is chosen. But that is only practicable when the state is discrete. However, since the state of the robot is not in fact in discrete space – but in continuous space – we run into a problem. The probability

that exactly this single state exists is infinitesimal. The result can be that the exploration will concentrate on a very tiny area within the search space, because even the smallest change of one of the state variables yields a probability of zero, for the state was not yet occupied. This is aggravated immensely by the increase of dimensions. The solutions to these kinds of problems are twofold. Firstly, not the probability of each single state is calculated but their combined density and furthermore, the dimensions of the state are being reduced to an amount that is more manageable to work with.

2.3.1 Dimensionality Reduction

Dimensionality reduction is the process of transforming a given data set from a high-dimensional space into a low-dimensional space. This transformation has to be executed in such a way that the low dimensional representation of the data still retains the key properties of the original high-dimensional data in order to be able to properly work with it. There exist several ways of how to reduce the dimensions of a given data set. Two of them are discussed in detail in this thesis: Principal Component Analysis and dimensionality reduction through a neural network called autoencoder. Both these approaches are explained more thoroughly in the following chapters.

2.3.2 PCA

The Principal Component Analysis (PCA) is a method to reduce the dimensionality of a dataset by decorrelating its data and increase its variance. It does this by transforming the data to so called principal components. The principal components are then ordered in a way, in which the first few of them hold the basic information value of the original data set and account for the most variance.

The first step is to compute the covariance matrix of the entire dataset. The reason for this is to see, whether there are any relations between different variables. The formula to calculate the covariance between two random variables X and Y is:

$$Cov(X, Y) = \frac{1}{n} \sum_{i=1}^n (x_i - \mathbb{E}(X))(y_i - \mathbb{E}(Y)), \quad (4)$$

where n denotes the size of each random variable and $x_i \in X$ and $y_i \in Y$. $\mathbb{E}(X)$ and $\mathbb{E}(Y)$ are referring to the means of X and Y , respectively. The covariance matrix for a 3-dimensional dataset would look like this:

$$\begin{pmatrix} Cov(X, X) & Cov(X, Y) & Cov(X, Z) \\ Cov(Y, X) & Cov(Y, Y) & Cov(Y, Z) \\ Cov(Z, X) & Cov(Z, Y) & Cov(Z, Z) \end{pmatrix}$$

Since the covariance of a variable with itself is its variance, the diagonal of the matrix consists of the variance of each random variable. And since furthermore, the covariance is commutative, the matrix is mirrored at the diagonal.

The second step in the PCA is to calculate the eigenvectors and eigenvalues of the covariance matrix. This is done to ascertain the principal components. The formula for calculating the eigenvalues of a matrix is the following:

$$\det(\mathbf{A} - \lambda \mathbf{I}) = 0, \quad (5)$$

where \mathbf{A} denotes the matrix, \mathbf{I} the identity matrix and λ the eigenvalues to be calculated.

So in the case of the 3-dimensional covariance matrix, the calculation would look like this:

$$\text{Cov} \left(\begin{pmatrix} \text{Cov}(X, X) - \lambda & \text{Cov}(X, Y) & \text{Cov}(X, Z) \\ \text{Cov}(Y, X) & \text{Cov}(Y, Y) - \lambda & \text{Cov}(Y, Z) \\ \text{Cov}(Z, X) & \text{Cov}(Z, Y) & \text{Cov}(Z, Z) - \lambda \end{pmatrix} \right) = 0$$

For each eigenvalue λ there is a corresponding eigenvector v . They can be calculated with the following formula:

$$(\mathbf{A} - \lambda_i \mathbf{I})v_i = 0 \quad (6)$$

where $i = 1, \dots, m$ and m denotes the number of eigenvalues.

Since the goal is to reduce the dimensionality of the dataset, the next step is to sort the eigenvectors (principal components) and only keep the k of them, which account for the most variance. But because the eigenvectors simply indicate the direction of the new axes, the sorting is done based on the corresponding eigenvalues. So the k eigenvectors are chosen that possess the highest eigenvalues. These are then used to form a new $d \times k$ dimensional matrix \mathbf{W} .

Now the last step is to multiply this matrix \mathbf{W} with the original dataset to project it into the lower dimensional subspace. The formula for this is:

$$\mathbf{S} = \mathbf{O} \times \mathbf{W}, \quad (7)$$

where \mathbf{S} denotes the projection in the subspace and \mathbf{O} the original dataset.

Sometimes the last step is to also whiten the data to make it even less correlated and to give all data points the same variance. This is done by dividing every dimension (column) of the matrix \mathbf{W} by the square root of its corresponding eigenvalue.

2.3.3 Autoencoder

The special kind of neural networks used in this thesis is called autoencoder. Its purpose is to reduce the dimension of a given input, then increase it again and reconstruct the original information.

An autoencoder is usually comprised of two main parts: The encoding part and the decoding part. Since autoencoders are usually symmetric in nature, the decoder has the same amount of layers as the encoder. It incorporates the hidden layer(s) and the output layer (see figure ?).

The idea is that the encoder subsequently reduces the dimensionality of the input through several different layers to a compressed representation of the input. Following that the decoder increases the dimensionality of the now compressed input until the input and the output have the same dimension. Given that procedure, an autoencoder is classified as an unsupervised learning model.

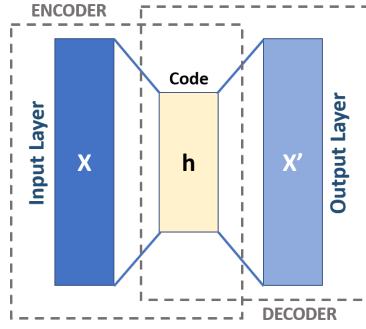


Figure 1: Simple Autoencoder: Placeholder

The two parts of an autoencoder can be mathematically represented as follows:

- The encoder is a function $f : X \mapsto H$ that maps the n -dimensional input array $X \in \mathbb{R}^n$ to the compressed representation $H \in \mathbb{R}^m$ with m dimensions, such that $m < n$.
- The decoder is a function $g : H \mapsto Y$ that maps the m -dimensional compressed representation $H \in \mathbb{R}^m$ to $Y \in \mathbb{R}^n$

The autoencoder itself can be represented as the concatenation of these two functions:

- Choose f and g , such that $f, g = \underset{f, g}{\operatorname{argmin}} |X - g(f(X))|$.

An autoencoder often only has just one hidden layer, but it does not have to be limited to it. More layers can bring certain advantages in some situations like reducing the computational costs¹ or decrease the amount of training data². In this thesis an autoencoder with three hidden layers is used (see figure ?).

¹Wikipedia:Platzhalter

²Wikipedia:Platzhalter

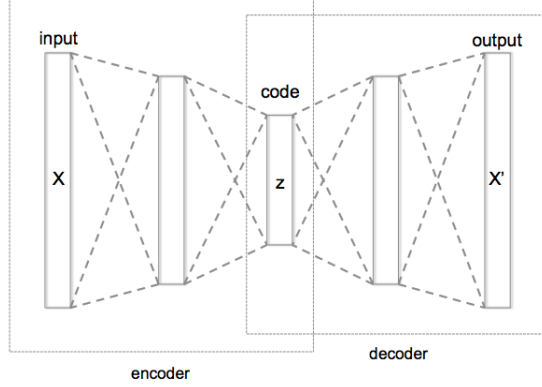


Figure 2: Simple Autoencoder: Platzhalter

2.4 Density Estimation

Density estimation is a way to estimate the probability density function of given data. The probability density function describes the relative likelihood that a specific sample (here one dimension of the reduced state-action-pair) occurs anywhere in the search radius. An example of that is the normal distribution $X \sim \mathcal{N}(\mu, \sigma^2)$, where μ is the mean and σ^2 the variance. The probability that a state occurs that is less than one σ away from μ is 68.26 percent. Now the exploration strategy does not have to choose the next state with the lowest probability, but with the lowest probability density. However, this estimation has to be computed for every dimension of the state.

2.5 Optimization Problems

2.5.1 What is an Optimization Problem

An optimization problem is the problem of finding the best solution from all available solutions. It can be mathematically represented as:

$$f^*(x) = \underset{x}{\operatorname{argmin}} f(x) \quad (8)$$

2.5.2 Evolution Strategy

An evolution strategy is a technique that is primarily used for optimization problems. As the name suggests, this technique tries to find the optimal solution for a problem through the methods of evolution: Mutation and selection. Several implementations exist that differ in what they mutate and what they select. But the general process is, it generates a set of candidate solution which it analyzes

on the basis of a fitness or an objective function. The proposed solutions that yield the best fitness values are then used to generate the next generation of candidate solutions. This process only ceases until a predefined criteria has been met.

2.5.3 The CMA-ES

One kind of evolution strategies proposes new candidate solutions by randomly sampling from a multivariate normal distributions with μ and a fixed Σ . Each generation the current mean is updated based on the best candidates from the previous generation. But because the Σ is fixed and with that the search radius, one shortcoming is that when Σ is inadequately chosen, the search can be rather slow for Σ is too small or even worse, the search gets stuck in a local optimum.

The **Covariance Matrix Adaption Evolution Strategy** (CMA-ES) is a special kind of an evolution strategy that overcomes this issue, since it not only updates the mean μ every generation but also the covariance matrix Σ . As is illustrated in figure ?, this modification allows for a large search radius in the beginning and thus a fast convergence to the optimum and a smaller search radius towards the end for finetuning the found optimum.

The general procedure of the CMA-ES can be presented as follows:

- Create multivariate normal distribution $X \sim \mathcal{N}(\mu, \Sigma)$ (The initial values are usually $\mu_0 = 0$ and $\Sigma_0 = I$)
- Sample N points from X , such that $Y = (y_1, \dots, y_N)$ with $y_i \in X \forall i = 1 \dots N$
- Evaluate all samples from Y with a previously defined fitness function f , such that $F = (f(y_1), \dots, f(y_N)) \forall y_i \in Y$
- From F choose the M samples with the best fitness value (i.e. the highest or the lowest) and calculate the new mean μ and the new covariance matrix Σ

This procedure is repeated until a termination criteria has been met, for example a certain amount of generations have passed or a certain threshold was surpassed.

3 Related Works

4 Proposed Methods

4.1 Interpolating trajectory

When a new action is chosen, it is necessary to enable a smooth transition between the current action and the new action that is supposed to be taken.

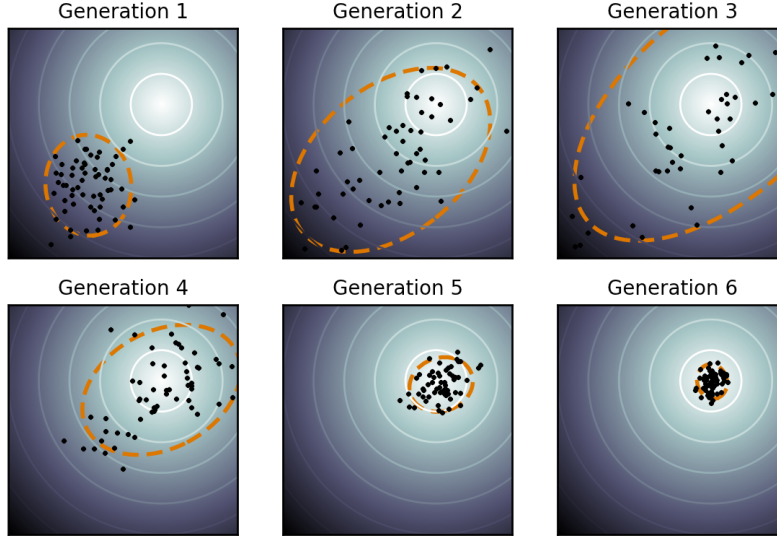


Figure 3: CMA-ES: Placeholder

In this example it means that a smooth transition between the current and the new velocity has to be found, because otherwise an abrupt change could harm the robot’s mechanical parts. Since a new action is chosen every second and the simulation works in 50 ms steps, about 20 points between the current and the new velocity have to be interpolated. The formula that was used to achieve that is the following:

$$1 - \sin(t * \frac{\pi}{2})^3 * V_c + \sin(t * \frac{\pi}{2})^3 * V_n, \quad (9)$$

where V denotes the velocity and t the time that has passed within the one second interval in 50 millisecond steps, i.e. 50ms, 400ms or 850ms and

4.2 Selection of new Actions

Three different ways of generating a new set of actions are presented here. The first method is to just simply sample a new action from a normal distribution, the second option is to calculate a new set of actions with the help of an autoencoder and the CMA-ES and the last option involves sampling through PCA.

4.2.1 Sampling from normal distribution

Sampling a new action from a normal distribution $X \sim \mathcal{N}(\mu, \sigma)$ is the simplest method computation wise and also the quickest. A random value is drawn from

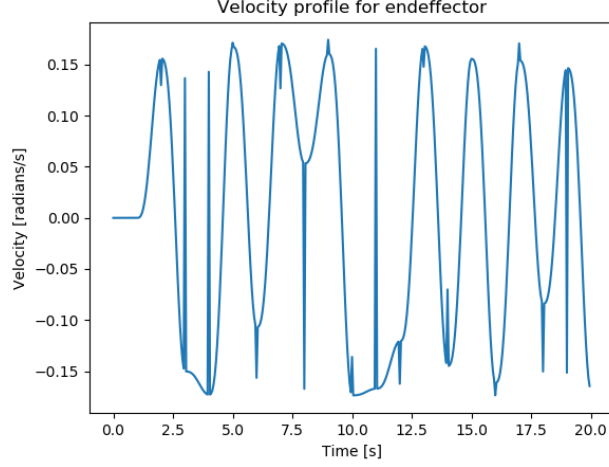


Figure 4: Velocity profile for endeffector for one episode

the normal distribution, where μ is equal to zero and σ is equal to $\frac{\text{constraint}}{\sqrt{2}}$ (see figure ?, line 11). In this thesis the constraint is referring to the angular speed of the joint and is set to ten degrees. This process is repeated for every joint, seven in this thesis (see figure ?, line 4).

After an action is sampled for each joint, it is checked whether the actions satisfy all the constraints. If even one of them is violated, the entire process is started anew (see figure ?, line 10).

Algorithm 1 Base Line

```

1: procedure MAIN
2:    $EPISODES \leftarrow N$ 
3:    $ITERATIONS \leftarrow M$ 
4:    $K \leftarrow \text{Number of joints}$ 
5:    $constraint \leftarrow \text{Max joint velocity}$ 
6:   Start environment
7:
8:   for  $i \leftarrow 1$ ,  $EPISODES$  do
9:     for  $j \leftarrow 1$ ,  $ITERATIONS$  do
10:      while  $constraint$  is not satisfied do
11:        Sample new set of actions  $\{a_{k,j}\}_{k=1}^K$  with  $\mathcal{N}(0, \frac{constraint}{\sqrt{2}})$ 
12:        Interpolate trajectory between  $\{a_{k,j-1}\}_{k=1}^K$  and  $\{a_{k,j}\}_{k=1}^K$ 
13:        Reset environment
14:
```

Figure 5: Pseudocode Baseline

4.2.2 Sampling through an Autoencoder and the CMA-ES

Another way of sampling a new set of actions is through the CMA-ES and an autoencoder. It has already been explained how they work and what they are used for individually, but now we use them in conjunction. The process itself also uses density estimation as an evaluation method for RL.

The first part is to set up the autoencoder itself. In this thesis a two layer approach has been chosen. The first layer reduces the 21 dimensional input vector to a 16 dimensional vector. This vector is reduced further to five dimensions in the second layer (see figure ?).

For the activation function PReLU was used. After the autoencoder has been setup, it is trained after each episode with all the to this point acquired data for twenty epochs (see figure ?,line 17).

The second step is the actual sampling of the actions with the help of the CMA-ES. In each iteration of the program the CMA-ES is initialized with the last action (the current velocity), a standard deviation of 0.25, a population size of 16 and a maximum amount of iterations of 50 (see figure ?, line 10). That means that the CMA-ES runs at max 50 iterations and that each iteration 16 sets of candidate solutions are being sampled by the algorithm. Each of those sets gets concatenated with the current state and propagated through the autoencoder to obtain the compressed representation. The compressed representation of the current state-actions-pairs are then evaluated on the compressed representation of the state-actions-pairs of the last N episodes. The set of actions that yields the best result (lowest density) is then chosen as the base for the next iteration of the CMA-ES. This is repeated until no better result can be obtained or 50 iterations are reached, The best overall result is the chosen as the new action (see figure ?, line 11).

4.2.3 Sampling from PCA and CMA-ES

The second option in which density estimation is used as a evaluation method for RL and new actions are sampled with the CMA-ES, is the approach in which the input vector is compressed in its dimensionality with the help of PCA.

In each iteration the data of the last N episodes is compressed in its dimensionality using the PCA (see figure ?, line 14). The sets of actions that are proposed by the CMA-ES are also compressed using the same method. The density estimation of this compressed representation of the sets of actions is then evaluated on the density estimation of the compressed representation of the data of the last N episodes. The set of actions that yields the best result (lowest density) is then chosen as the base for the next iteration of the CMA-ES. This is then repeated, as it was in the autoencoder approach as well, until the best result has been found or 50 iterations have been surpassed (see figure ?,

Algorithm 2 Autoencoder

```

1: procedure MAIN
2:    $EPISODES \leftarrow N$ 
3:    $ITERATIONS \leftarrow M$ 
4:    $K \leftarrow \text{Number of joints}$ 
5:    $constraint \leftarrow \text{Max joint velocity}$ 
6:   Initialize Autoencoder
7:
8:   for  $i \leftarrow 1, EPISODES$  do
9:     for  $j \leftarrow 1, ITERATIONS$  do
10:      while constraint not satisfied  $\&$  max iteration not reached do
11:         $\triangleright X$  refers to set of candidate solution proposed by CMA-ES
12:         $\{a_{k,j}\}_{k=1}^K \leftarrow \underset{x \in X}{\text{argmin density}}(AE(\{s_{k,j}, a_{k,x}\}_{k=1}^K))$ 
13:        Interpolate trajectory between  $\{a_{k,j-1}\}_{k=1}^K$  and  $\{a_{k,j}\}_{k=1}^K$ 
14:        Add current state and action  $\{s_{k,j}, a_{k,j}\}_{k=1}^K$  to replay buffer  $R$ 
15:        Reset environment
16:
17:      Train Autoencoder
18:      Save hidden layer data

```

Figure 6: Pseudocode Autoencoder

line 9f).

Algorithm 3 PCA

```

1: procedure MAIN
2:    $EPISODES \leftarrow N$ 
3:    $ITERATIONS \leftarrow M$ 
4:    $K \leftarrow \text{Number of joints}$ 
5:    $constraint \leftarrow \text{Max joint velocity}$ 
6:
7:   for  $i \leftarrow 1, EPISODES$  do
8:     for  $j \leftarrow 1, ITERATIONS$  do
9:       while constraint not satisfied  $\&$  max iteration not reached do
10:         $\triangleright X$  refers to set of candidate solution proposed by CMA-ES
11:         $\{a_{k,j}\}_{k=1}^K \leftarrow \underset{x \in X}{\text{argmin density}}(AE(\{s_{k,j}, a_{k,x}\}_{k=1}^K))$ 
12:        Interpolate trajectory between  $\{a_{k,j-1}\}_{k=1}^K$  and  $\{a_{k,j}\}_{k=1}^K$ 
13:        Add current state and action  $\{s_{k,j}, a_{k,j}\}_{k=1}^K$  to replay buffer  $R$ 
14:        Fit PCA with state-action-pairs from replay buffer  $R$ 
15:        Reset environment
16:

```

Figure 7: Pseudocode PCA

5 Experimental Setup

5.1 Environment Setup

The robot that was used throughout this thesis is the kuka lbr iiwa 7 r800 fabricated by the company called KUKA AG. To simulate the behavior of said robot, the software CoppeliaSim by Coppelia Robotics was used. The code itself is written in Python.

5.2 Task formulation as an RL problem

A state is defined as a 14-dimensional array. The first seven entries are the joint positions of the robot measured in Radian. The last seven values are the current velocity of each joint of the robot measured in Radian per second. The action is a seven dimensional array, which holds the new joint velocities that the robot should reach. Together both state and action form the state-action-pair, which is mentioned throughout the thesis.

One second in the simulation consists of 20 50 millisecond steps. In this thesis this is referred to as one iteration. At the start of the iteration a new action is chosen and at the end of the iteration the state velocity should be equal to the action.

100 iteration make up one episode. The different approaches were tested with a different number of episodes. The baseline and the PCA approach both ran for 200 episodes and the autoencoder approach ran for 500 episodes.

6 Results

For the presentation two kinds of plots are available. Firstly the plot that shows all the positions where a specific joint has been during all iterations and all episodes in Cartesian (x,y,z) coordinates. The second shows a density estimation of the position of a joint in radian. It illustrates how often a specific angle has been occupied.

The autoencoder has an additional kind of plots. It shows the density estimation of the compressed representation of the input layer.

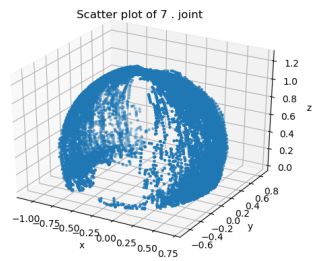
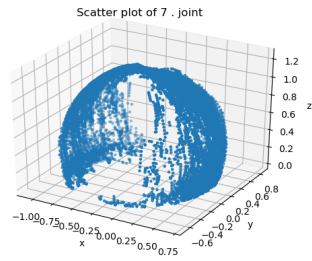
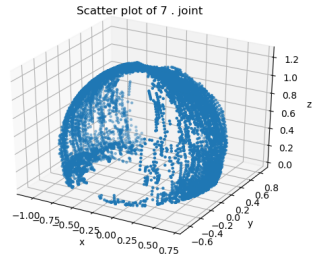
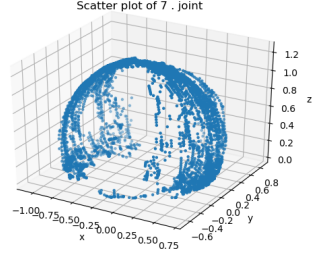
Only a part of the the research is shown here directly. Other plots can be found in the appendix.

6.1 Baseline Exploration

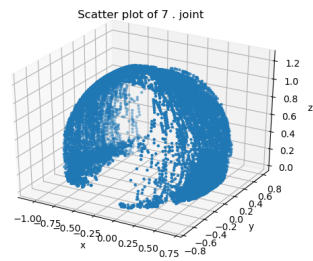
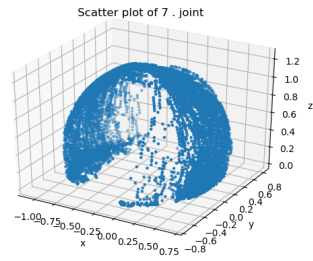
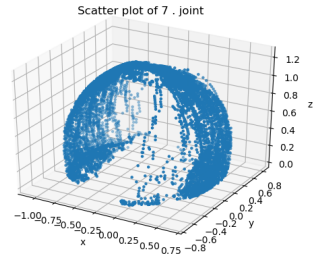
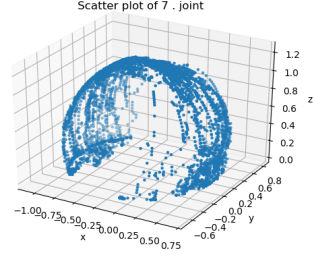
First we want to take a look on how the baseline exploration fared in exploring the working space of the robot. As a quick reminder, the next action in this approach was randomly sampled from a normal distribution. In figure ? we can see the result of that sampling method. The plots show all the positions the endeffector has occupied after 60, 120, 160 and 200 episodes for two examples in Cartesian Coordinates. We can see that only a small portion of the entire working space has been explored by the robot's endeffector. If we also compare the progression of the exploration after certain episodes, we can also observe, than the explored room itself is only shrinking slowly, since the endeffector keeps visiting the same coordinates.

The effect of the sampling through the normal distribution can be seen in figure ? as well. It shows plots of the density of the robot's inherent endeffector

position in Radians for the same two examples as in figure ?. As we can see, the density looks similar to a normal distribution. Most of the density is centered around 0 degree (the mean of the normal distribution). Additionally we can see that the density quickly fades to zero and never reaches the constraint that the joint has (3.054326 Radian in this case). The same can be observed for all other joints as well (see Appendix).

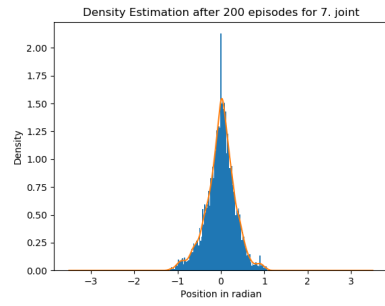
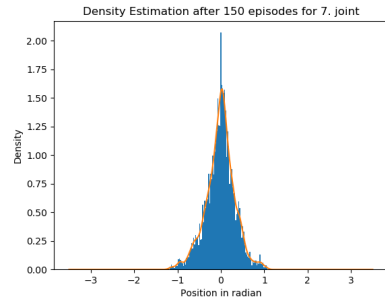
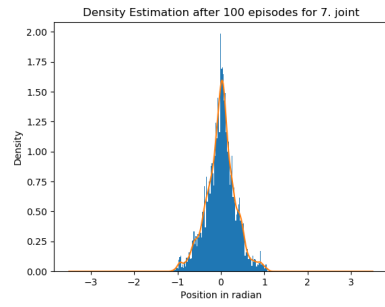
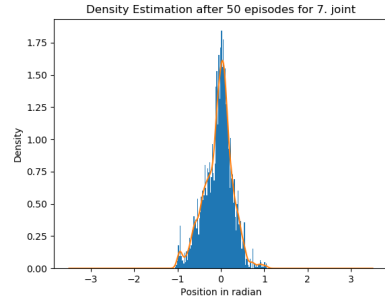


Plot of endeffector of one simulation
after 60, 120, 160 and 200 episodes

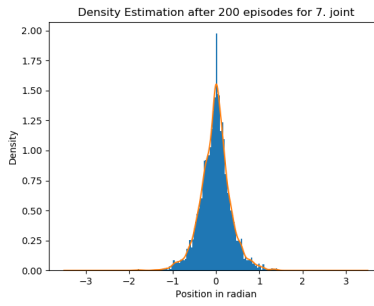
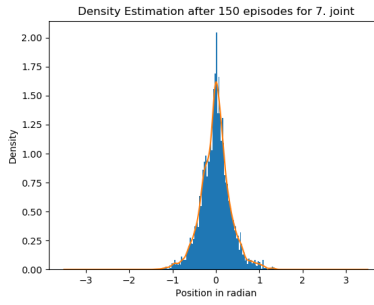
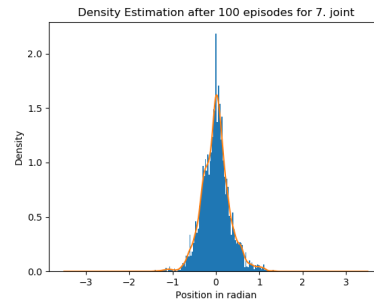
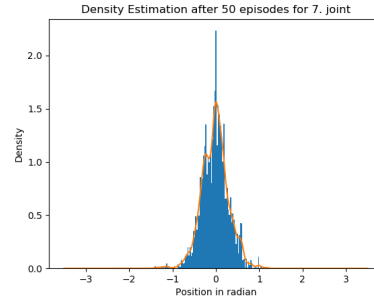


Plot of endeffector of another simulation
after 60, 120, 160 and 200 episodes

Figure 8: Comparison of the endeffector positions after different episodes



Plot of density of endeffector position of one simulation after 60, 120, 160 and 200 episodes



Plot of density of endeffector position of another simulation after 60, 120, 160 and 200 episodes

Figure 9: Comparison of the endeffector position density after different episodes

6.2 Autoencoder exploration

6.2.1 Training of the autoencoder

At the end of each episode the autoencoder is trained with all the to this point acquired state-action-pairs. What setting were used, you can see in the ??? chapter. From figure ? we can see that the loss is roughly between 0.05 and 0.35.

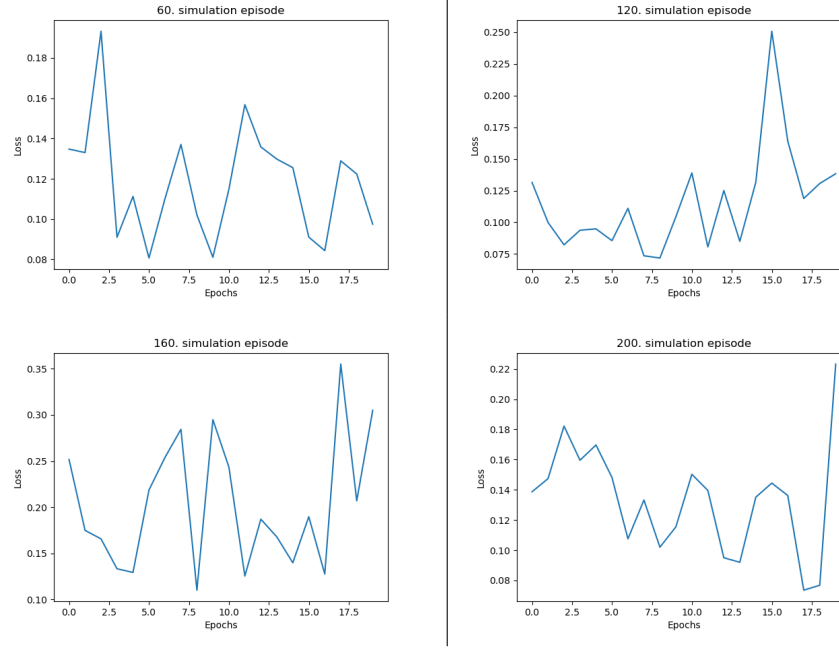


Figure 10: Comparison of the loss of the autoencoder after 60, 120, 160 and 200 episodes

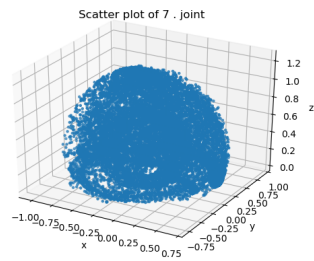
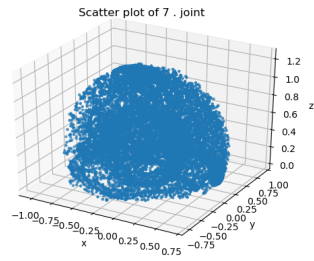
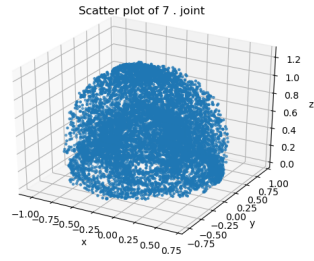
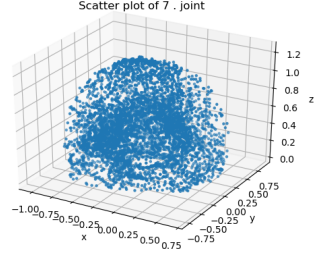
6.2.2 Exploration of the working space

Now we take a look how the autoencoder approach fared in exploring the working space of the robot. Again as a reminder, the autoencoder approach used said autoencoder to reduce the state-action-pair in its dimensionality and thus, with the help of the CMA-ES, allowed to choose the next action, based on the lowest density. Figure ? shows the robot's endeffector position after 60, 120, 160 and 200 episodes for two examples in Cartesian coordinates. In contrast to the baseline approach, we can easily see that robot explored far more area of the working space. Even after 120 episodes, we can already observe that the autoencoder approach has explored the area quite evenly. Furthermore it seems that this method does not keep visiting the same coordinates again and again,

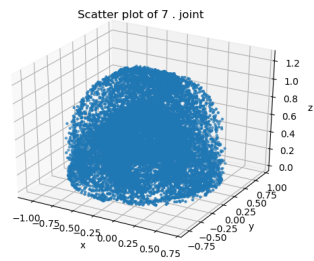
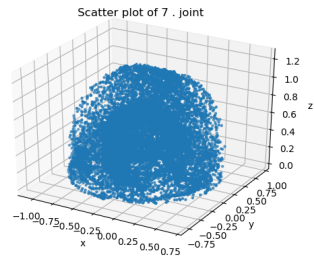
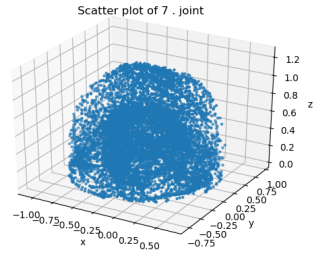
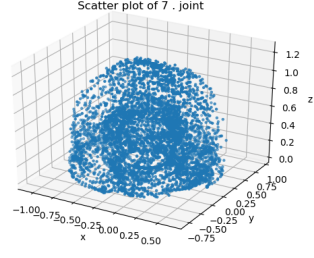
since with increasing episodes the plots become more densely packed (compare the plots after 120 and 200 episodes).

The same can be observed in figure ? as well. It shows the plots of the density of the endeffector's position after 60, 120, 160 and 200 episodes. We can see that in contrast to the baseline approach, the density is not as centered around the mean. It is more evenly distributed between the positive and negative constraint of the joint. Nevertheless, we can also observe that the endeffector (as well as the other joints, see Appendix) seem to often occupy the fringe position, that is the position that is equal to the constraint. It seems that in the sampling process it often is the case that the lowest density can be reached, when one or more joints are locked in place and only the other joints are moving then.

The evolution of the density distribution can be seen in figure ?. It shows the density of the entire data (state action pairs) after 1, 50, 100 and 150 episodes. Since the goal of this approach is the minimization of exactly this density, the first the plots (after 1 and after 50 episodes) shows what is expected. The density decreased more or less significantly. But when we look at the density after 100 episodes, something unanticipated happened. The density increased. But if we now compare that to the figure ? and figure ?, we can see that already after 100 episodes, the approach has explored the working space quite evenly, which makes it harder for the process to find an action that leads to a position that has not yet been occupied and therefore has low density. Consequently the algorithm will oftentimes choose an action that will lead to an already visited position, what increases the total density.

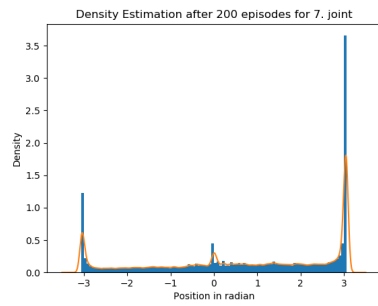
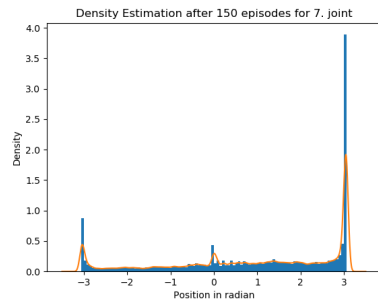
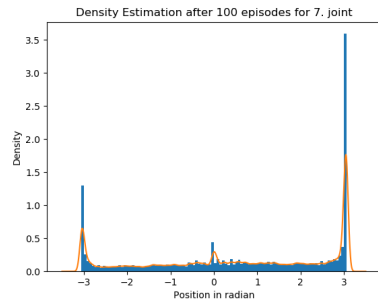
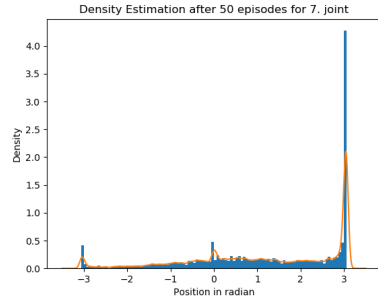


Plot of endeffector of one simulation
after 60, 120, 160 and 200 episodes

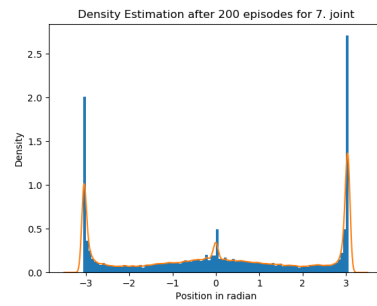
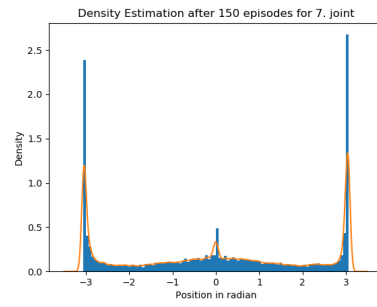
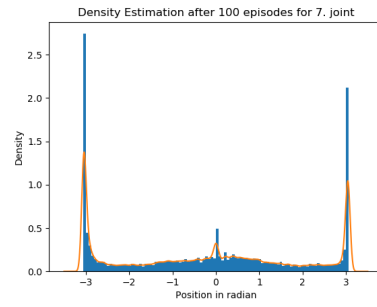
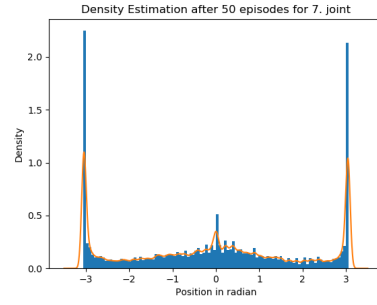


Plot of endeffector of another simulation
after 60, 120, 160 and 200 episodes

Figure 11: Comparison of the endeffector position after a different episodes

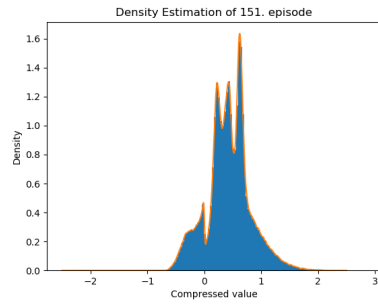
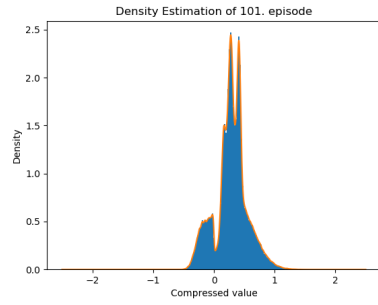
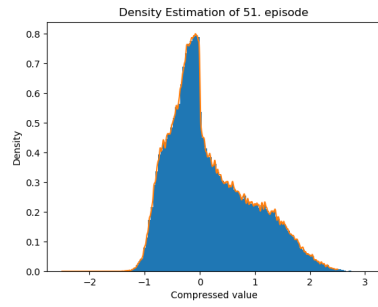
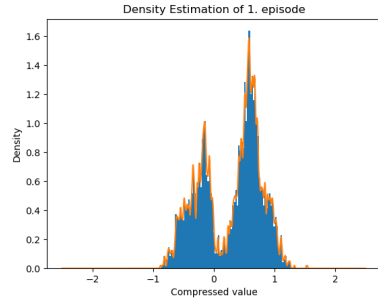


Plot of density of endeffector position of one simulation after 60, 120, 160 and 200 episodes

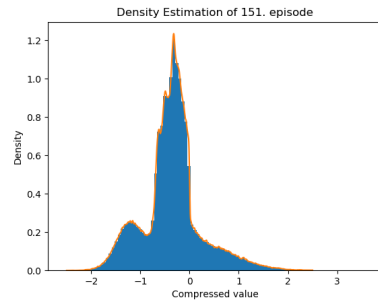
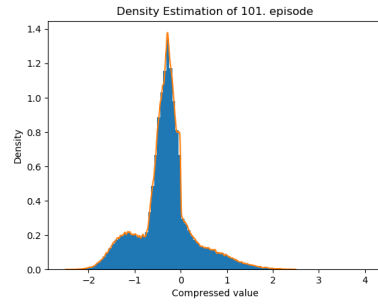
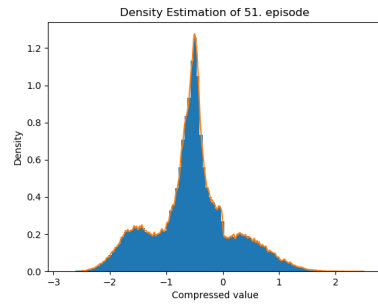
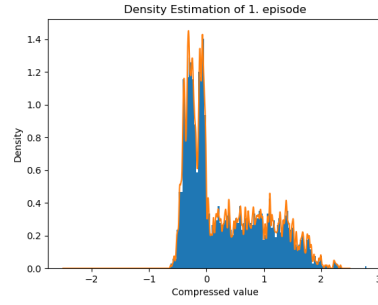


Plot of density of endeffector position of one simulation after 60, 120, 160 and 200 episodes

Figure 12: Comparison of the density of the position of the endeffector



Plot of density of endeffector position of one simulation after 60, 120, 160 and 200 episodes



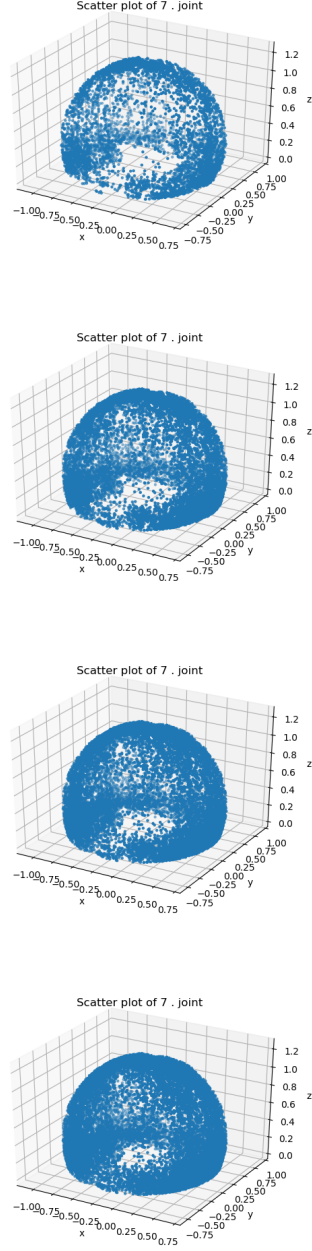
Plot of density of endeffector position of one simulation after 60, 120, 160 and 200 episodes

Figure 13: Comparison of the density of the position of the endeffector

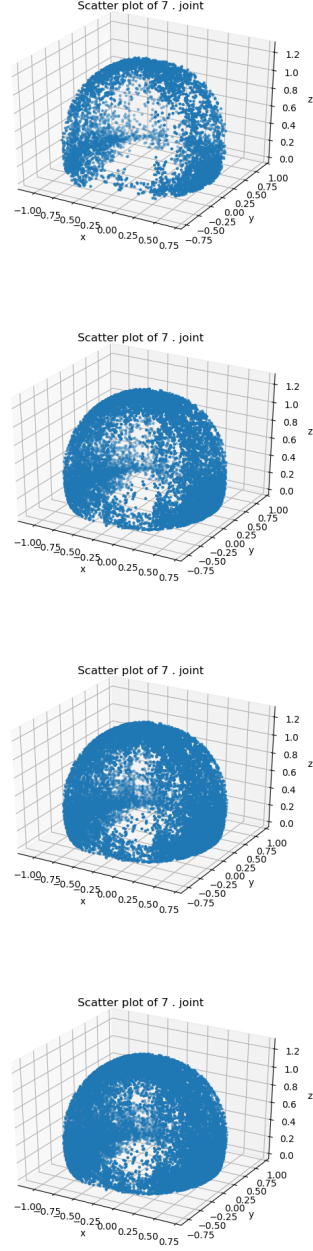
6.3 PCA exploration

Let us now look at the last approach of this thesis. Again, as a reminder, the PCA-approach reduced the dimensionality of the state-action-pair and thus, with the help of the CMA-ES, allowed to choose the next action based on the lowest density. In figure ? we can see the result of that approach. It shows the plots of the position of the robot's endeffector after 60, 120, 160 and 200 episodes for two examples in Cartesian coordinates. In contrast to the baseline approach we can once more see that the PCA approach seems to explore the space much more quickly and evenly. Nevertheless, when we take a closer look and compare it to the results of the autoencoder approach, we can observe that the PCA approach is nowhere close to explore the same area. The entire interior of the half sphere was apparently not visited by the endeffector.

The same conclusion can be drawn from figure ?. It shows the density of the endeffector's inherent position in Radian. Since the density does not spread out evenly between the positive and negative joint constraint and since the same can be observed for all other joints (see Appendix), we can deduce that not the entire working space of the robot has been visited, given that not the entire range of the joints has been used.

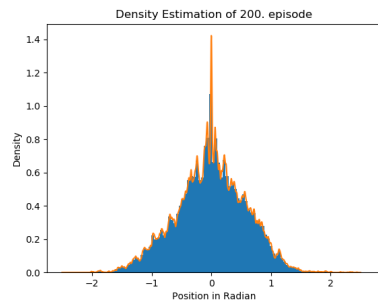
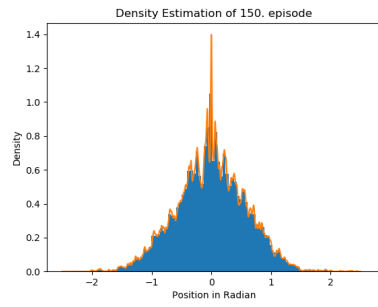
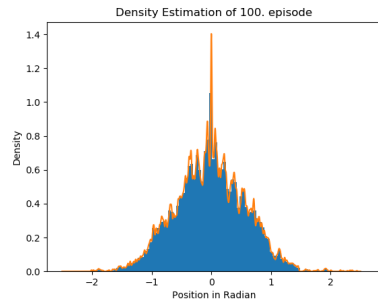
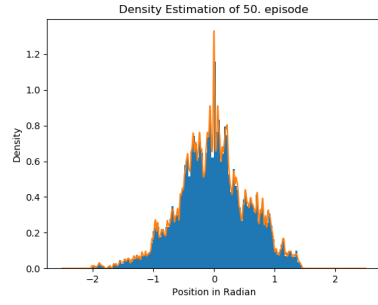


Plot of endeffector of one simulation
after 60, 120, 160 and 200 episodes

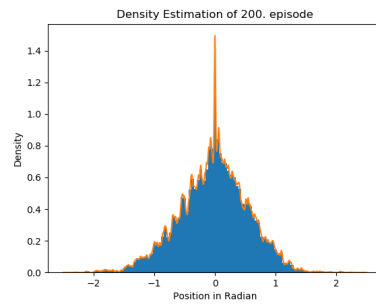
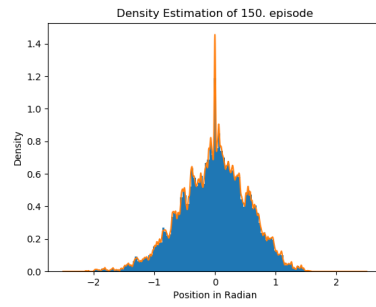
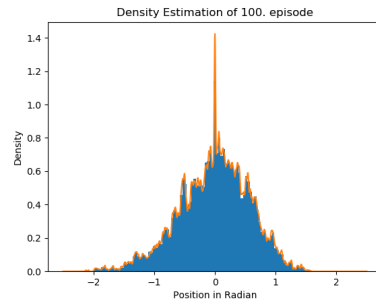
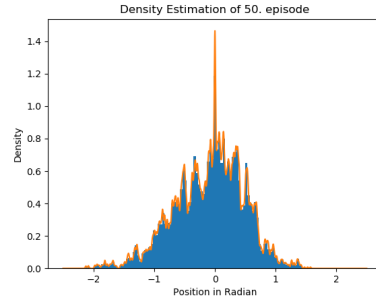


Plot of endeffector of another simulation
after 60, 120, 160 and 200 episodes

Figure 14: Comparison of the endeffector position after a different episodes



Plot of density of endeffector position of one simulation after 60, 120, 160 and 200 episodes



Plot of density of endeffector position of one simulation after 60, 120, 160 and 200 episodes

Figure 15: Comparison of the density of the position of the endeffector

6.3.1 Additonal

Each approach has been tested for at least two different simulations with the same settings for 200 episodes. The autoencoder approach however has been conducted for 500 episodes. You can see the additional data in the figure ? below.

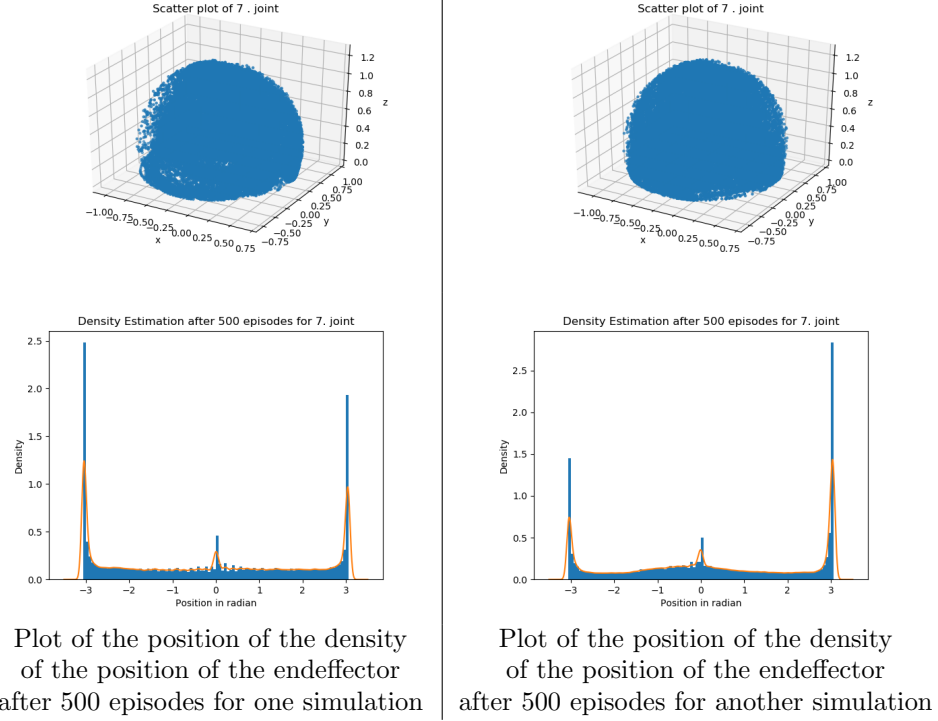


Figure 16: Comparison of position and the density of the position of the endeffector

7 Conclusion

This thesis presented three approaches for exploring the working space of a robot: Sampling from a normal distribution, sampling with the CMA-ES with density reduction through an autoencoder and sampling with CMA-ES with density reduction through PCA. We can evaluate these approaches through two different methods.

The first would be the time required to complete all 200 episodes. Here the random sampling from a normal distribution took the least and a constant amount of time, because the calculation was the same in each iteration and

the computational cost did not rise with an increase in episodes. So it took roughly $200 \times 100 \times 1$ seconds for the program to finish. The PCA approach was the second fastest. The most time intensive computation here was the fitting of the PCA with the data. To limit the computational cost, this data was restricted to the last 20 episodes. So, after these 20 episodes, each iteration took about 1.5 seconds. Hence the entire time is roughly $200 \times 100 \times 1.5$ seconds. The autoencoder approach the most amount of time. In each iteration the existing autoencoder had to be evaluated on the collected data. Like with the previous approach, to limit computational time, this data has been restricted to the last 20 episodes. After these an iteration took about 1.5 seconds. Thus, the computational time during the episode is similar to PCA approach. But after each episode it was also necessary to train the autoencoder. That was done with all the to this point acquired data. Hence the computational time increased with the increase of the episodes. While in the first few episodes the training did not take more than a few seconds, in the later episodes the same training took almost an hour.

The first method for evaluation is how well the space has been explored. Each approach ran for 200 episodes, which means that the amount of occupied points is the same. Now we can look how evenly they are distributed though out the space. For that we can look onto the figures from the previous chapter, namely figure 1, 2 and 3. Here we can see that the random sampling from a normal distribution fared the worst. Only a chunk of the entire space has been explored and it seems that the robot's endeffector keeps visiting the same spots over and over. The other two approaches seem to explore the space much more evenly. But we can clearly see, when we compare figure 2 and figure 3 that the autoencoder approach was exploring the working space of the robot more extensively than the PCA approach, for the interior of the sphere has not been well explored in the latter.