eigenvector

$$A x = \lambda x$$

eigenvalue

$$A = \begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}$$

$$\begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix} \begin{bmatrix} .6 \\ .4 \end{bmatrix} = \begin{bmatrix} .6 \\ .4 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}}_{A} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} .5 \\ -.5 \end{bmatrix}$$

$$A^n \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \left(\frac{1}{2}\right)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A \begin{bmatrix} .8 \\ .2 \end{bmatrix}$$

变成的朋友 ⇒ $\begin{bmatrix} .6 \\ .4 \end{bmatrix}$ (stedy)

$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (steady)

$\boxed{\text{已知一个}A}$

$Ax = \lambda x$ 移项 $(A-\lambda E)x=0$

$(A-\lambda E_n)X=0$

$X \neq \begin{pmatrix}0\\ \vdots \\ 0\end{pmatrix}$ 时 $|A-\lambda E_n|=0$

然后找到 $\lambda_1, \lambda_2 \cdots$

对每个 λ 找到 eigenvektor X.

$$\mathrm{Diag}\underbrace{(A)}_{\text{matrix}} \rightarrow \Lambda = \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \ \lambda_n \end{pmatrix}$$

"Eigenvalue matrix"

$$S = (X_1\ X_2 \cdots X_n)$$

$$AS = S\Lambda$$

证明: $AS = \boxed{A}(X_1\ X_2 \cdots X_n)$

$= (AX_1\ AX_2 \cdots AX_n)$

$= (\lambda_1 x_1 \ \lambda_2 x_2 \cdots \lambda_n x_n)$

$= [x_1 \cdots x_n] \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$

$= S\Lambda$

① $S^{-1}AS = \Lambda$

② $A = S\Lambda S^{-1}$

$A = S \Lambda S^{-1}$

matrix

Eigenvektor

Eigenvalue.

$A^2 = S\Lambda S^{-1} S \Lambda S^{-1}$

$= S(\Lambda^2)S^{-1}$

对 $A=\begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}$ 进行

$A = U\Sigma V^T$ 的确定.

$$AA^T = U\Sigma V^T V \Sigma^T U^T$$
$$= U\,\Sigma\Sigma^T\,U^T$$

$\Downarrow$

$$A^T = \begin{pmatrix} -1 & -1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}$$

$$A\cdot A^T = \begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}$$

2×3 $\quad$ 3×2

2x2的矩阵 $\quad = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

$\Rightarrow U\ \Sigma\Sigma^T\ U^T$

$$\chi_A = \begin{pmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{pmatrix}$$

$$= (2-\lambda)(3-\lambda) = 0$$

$$\lambda_1 = 2 \quad \lambda_2 = 3$$

$$AV = \lambda V$$

$$(A - \lambda E_n)V = 0$$

当 $\lambda_1 = 2$ 时 $A = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}$

$$\begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$2\times2 \quad 2\times1 \quad 2\times1$

当 $\lambda_1 = 3$ 时 $A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

那么. U 作为 $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

U^T 作为 $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Σ其实是 $\sqrt{2}$ $\sqrt{3}$

$$\underset{2\times3}{A} = \underset{2\times2}{U}\ \underset{2\times3}{\Sigma}\ \underset{3\times3}{V^T}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{3} & 0 \end{pmatrix} \begin{pmatrix} ? \end{pmatrix}$$

换个角度出