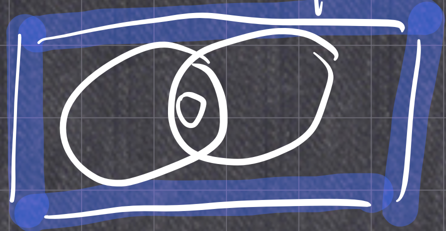
Orthogonal Projektion ✓

$$V = U + U^{\perp}$$

$$\dim(n) \quad \dim(r) \quad \dim(n-r)$$

$v \to \lambda u_1 + \mu u_2$ (Zerlegung)



Damit: Erhalte die Lösung x von

$\|b - Ax\| = \min.$ als Lösung von

$$\boxed{A^T A x = A^T b}$$

记42

$$Ax = b$$

A^T

Anwendung von 空间映射

[考法]

① Bestimme die orthogonale Projektion $u = P_U(v)$

U: 空间

v: 分解对象 [向量]

wobei $v = \begin{pmatrix}1\\2\\3\end{pmatrix}$

$U = \left\langle \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\2\\2\end{pmatrix} \right\rangle$

$\|v - u\|$ min.

$\Rightarrow u_0 = A x_0$. (拆)

$$\underset{2\times3}{A^T}\, v = \underset{2\times3}{A^T}\,\underset{3\times2}{A}\,\underset{2\times1}{x_0}$$

$$A = \begin{pmatrix}1 & 2\\0 & 2\\1 & 2\end{pmatrix}$$

$A^T A = (2\ 3)$

$$A^T v = \underbrace{\begin{pmatrix}1&0&1\\1&1&1\end{pmatrix}}_{\cancel{3\times 1}\ 2\times 3}\underbrace{\begin{pmatrix}1\\2\\3\end{pmatrix}}_{3\times 1} = \begin{pmatrix}4\\6\end{pmatrix}$$

2×2

$$\Rightarrow \overset{2\times 2}{\begin{pmatrix}2&2\\2&3\end{pmatrix}} \underset{2\times 1}{x_0} = \overset{2\times 1}{\begin{pmatrix}4\\6\end{pmatrix}}$$

用EKM

写作：$\left(\begin{array}{cc|c}2&2&4\\2&3&6\end{array}\right)$

$$\Rightarrow \left(\begin{array}{cc|c}1&1&2\\2&3&6\end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c}1&1&2\\0&1&2\end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|c}1&0&0\\0&1&2\end{array}\right)$$

$$x_0 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$A x_0 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

3×2 $\quad 2 \times 1$ $\quad 3 \times 1$.

$$= \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$d = \left\| \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\| = \left\| \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\| = \sqrt{2}$$

② Lösung überbestimmte LGS

满足a.b

Das Problem: $Ax = b$

≠ 或 b 且

eigentlich nicht lösbar

$$\begin{cases} x + y = 2 \\ x + y = 1.2 \\ 2x - y = 4. \end{cases}$$

$\Rightarrow$ #Gleichungen > #Vari

a

ABER: Ersatzlösung

从 $Ax=b$

变为 $\|b-Ax\| = \min.$

$\Downarrow$ Das Überrest (Residuum)

$$A^Tb = A^TAx$$

Bsp:

$$x = 1$$
$$x + y = 2$$
$$y = 3$$

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

$$x = ?$$

$$b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\|b - Ax\| = \min.$$

$$A^TAx = A^Tb$$

计算：

$$A^T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\underset{2\times 3}{A^T}\ \underset{3\times 2}{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\underset{2\times 3}{A^T}\ \underset{3\times 1}{b} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\underset{2\times 1}{} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 2 & 1 & 3 \\ 1 & 2 & 5 \end{array}\right)$$

$$\begin{pmatrix} 1 & 2 & 5 \\ 2 & 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 5 \\ 0 & -3 & -7 \end{pmatrix}$$

$$y = \frac{7}{3}$$

$$x = 5 - 2 \times \frac{7}{3} = 5 - \frac{14}{3} = \frac{1}{3}$$

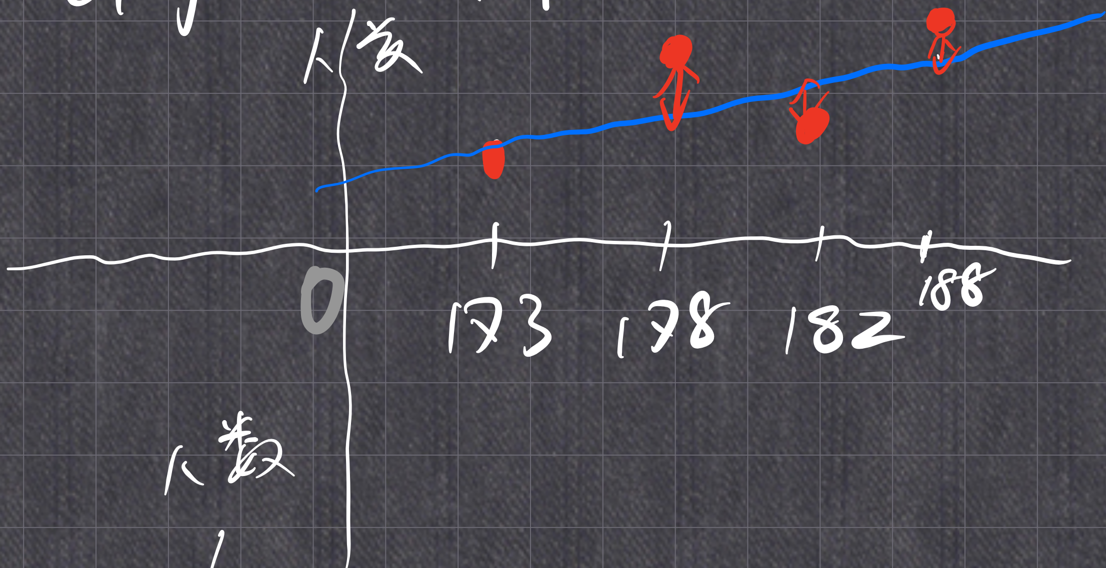
$$\begin{pmatrix} x = \frac{1}{3} \\ y = \frac{7}{3} \end{pmatrix}$$

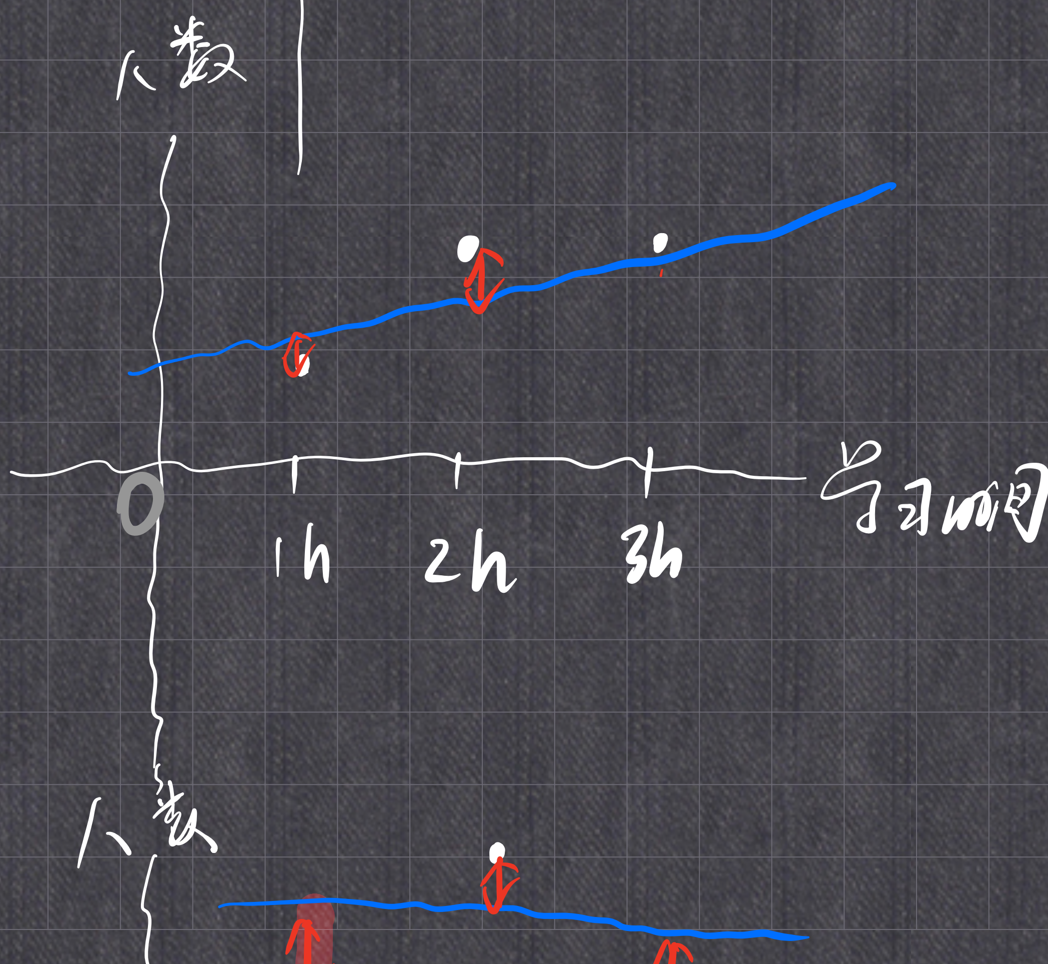
③ 川西想子 MATLAB

Methode der Quadrat

und zwar

Gegeben: „Punktwert"

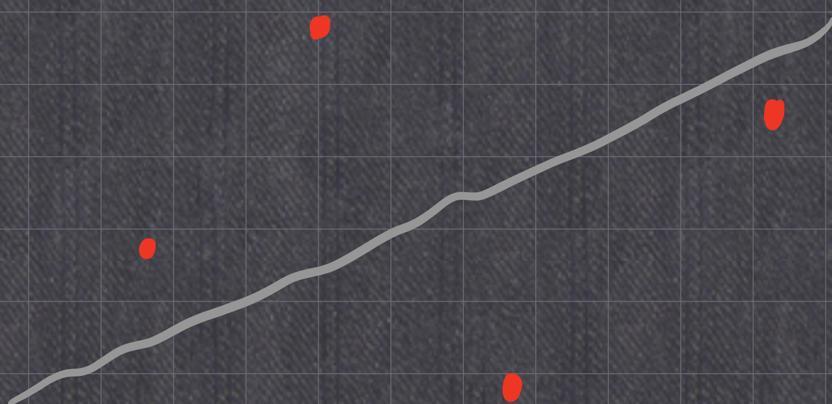




统计方式

为了规避符号，采用平方d.

方差



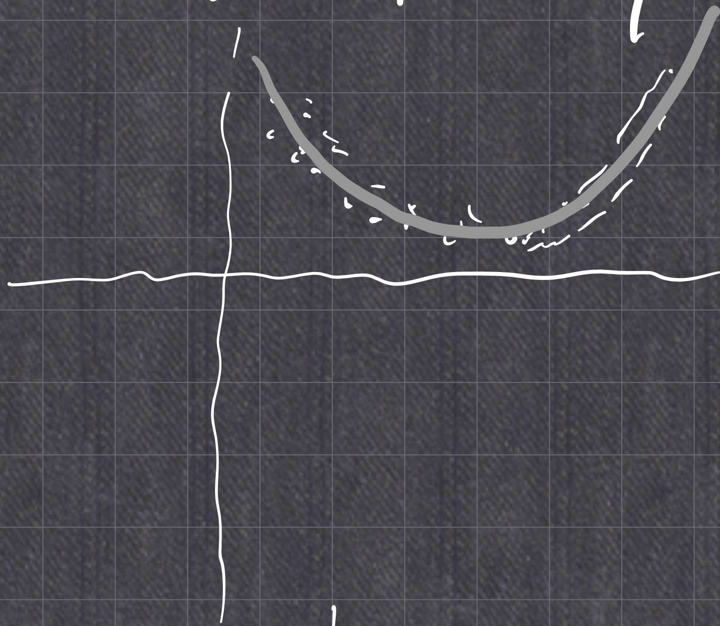
l:

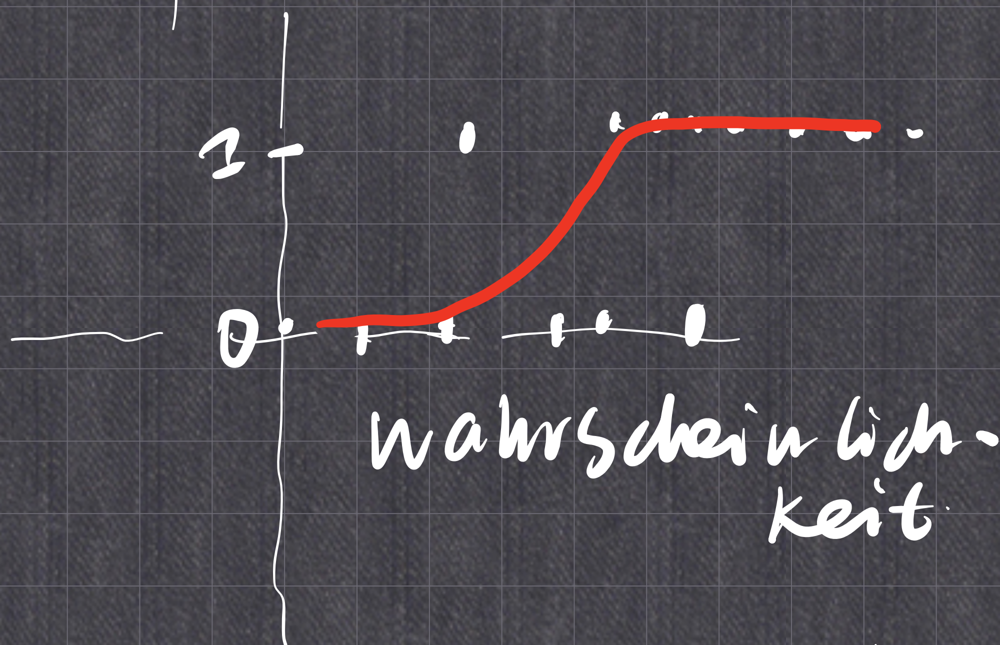
linear Ausgleichgerade

(Regressionsgerade)

$l: y = \beta_0 + \beta_1 x$

当然: $y = \beta_0 + \beta_1 x + \beta_2 x^2$.





$\Rightarrow$ beste Approximation

durch Logistische Funktion

Basisfunktionen:

$f_1 \quad f_2 \cdots f_r$

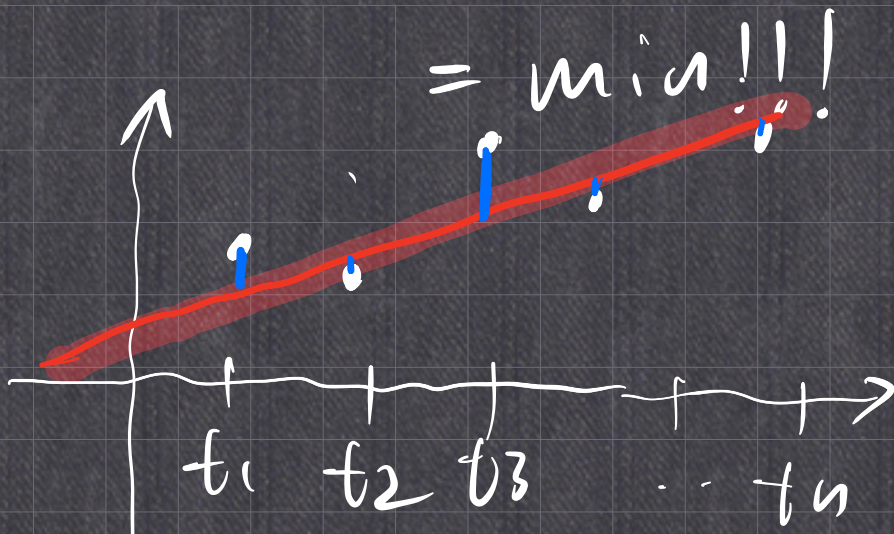
如: $g = p_0 x^0 + p_1 x^1$

$$\begin{cases} f_1 = x^0 \\ f_2 = x^1 \end{cases}$$

从更高维度,不止局限于 $x^0, x^1, x^2 \ldots$

Dann: Minimiere

$$(g(t_1) - f(t_1))^2 + \cdots + (g(t_n) - f(t_n))^2$$



一般X轴是时间

$$A = \begin{pmatrix} f_1(t_1) & \cdots & f_n(t_1) \\ & \vdots & \\ f_1(t_n) & \cdots & f_n(t_n) \end{pmatrix}$$

$$\begin{pmatrix} 1 & 174 \\ 1 & 175 \\ 1 & 180 \\ 1 & 200 \end{pmatrix}$$

$$\boxed{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

(147.77) (181.80) (170.80)

$$A = \begin{pmatrix} 1 & 173 & 173^2 \\ 1 & 182 & 182^2 \\ 1 & 170 & 170^2 \end{pmatrix}$$

$f_1 = 1 \quad f_2 = (x) \quad f_3 = x^2.$

$$B = \begin{pmatrix} 70 \\ 90 \\ 80 \end{pmatrix}$$

$$Ax = B$$

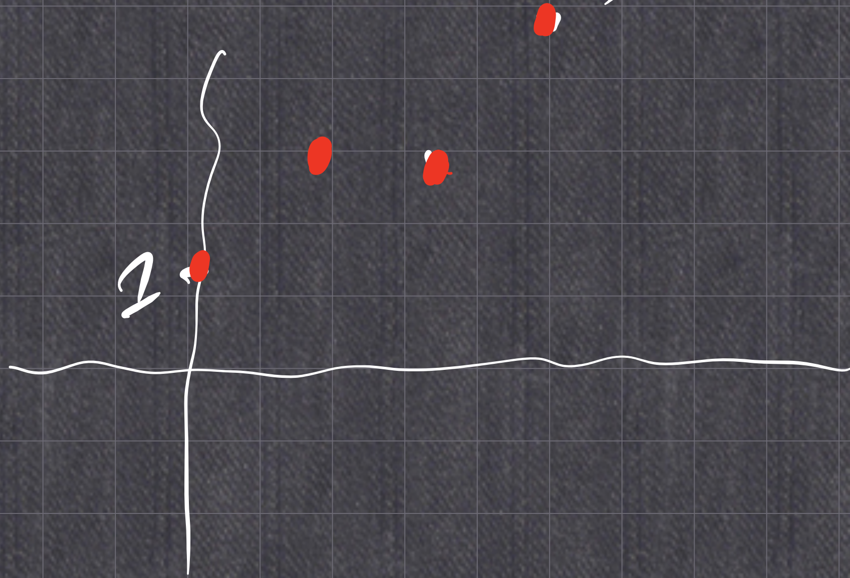
即解所解方程的系数

$A\underset{\text{系数}}{\hat{x}} = b$ 是最好的

$\Rightarrow \|b - Ax\| =$ 方差和

≈7 min.

Bsp: (0,1) (1,2) (2,2)
(3,4) (4,6)



f Basis: $f_1 = 1$

$f_2 = x$

过程: $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ 2 \\ 2 \\ 4 \\ 6 \end{pmatrix}$

$$A x = b$$

$5\times 2 \quad 2\times 1 \quad 5\times 1$

$$\| b - Ax \| = \min.$$

$$A^T b = A^T A x$$

$$A^T = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix}$$

$2\times 5 \qquad 5\times 2.$

$$= \begin{pmatrix} 5 & 10 \\ 10 & 30 \end{pmatrix}$$

$$A^T b = \begin{pmatrix} 15 \\ 32 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 5 & 10 & 15 \\ 10 & 30 & 42 \end{array}\right)$$

解得：$\beta_0 = 0.6$

$\beta_1 = 1.2$

$$y = 0.6 + 1.2x$$