7. Technological progress, unemployment and living standards in the long run

I. Exercise Questions

Readings
Lecture slide set: #9
Macroeconomics ed.9 (Mankiw): *Steady state (8.1), technological progress (9.1), golden rule (8.2)*

Problem 1 (*Solow model*)

Consider an economy with the following Cobb-Douglas-production function:

$$Y = K^{0,5}(LE)^{0,5}$$

where K is the capital stock, L is the employed labour force and E is the efficiency of the economy.

Assume a savings rate of $s = 0,25$, a population growth rate of $n = 0,1$, a depreciation rate of $\delta = 0,3$ and a rate of $g = 0,1$ for the technological progress.

(a) What is the production function per unit of effective labour of the economy?

$$\begin{aligned} F(K, LE) &= Y = K^{0,5}(LE)^{0,5} \\ f(k) &= \frac{Y}{LE} = \frac{K^{0,5}(LE)^{0,5}}{LE} = \\ &= \frac{K^{0,5}}{(LE)^{0,5}} = k^{0,5} \text{ with } k = \frac{K}{LE} \end{aligned}$$

(b) What is the break-even investment in this economy (the investment needed to keep the capital stock per unit of effective labour constant)?

Mathematically, the change in capital stock per unit of effective labour is as follows:

$$\begin{aligned} \frac{\triangle k}{k} = \frac{\triangle \frac{K}{LE}}{\frac{K}{LE}} &\approx \frac{\triangle K}{K} - \frac{\triangle L}{L} - \frac{\triangle E}{E}, \text{ see below for the exact approach}^1 \\ &= \frac{I - \delta K}{K} - \frac{\triangle L}{L} - \frac{\triangle E}{E} \\ &= \frac{I}{K} - \delta - n - g \end{aligned}$$

[Exact approach:

$$\begin{aligned}
\frac{\frac{\partial k}{\partial t}}{k} &= \frac{\frac{\partial \frac{K}{LE}}{\partial t}}{\frac{K}{LE}} = \frac{\frac{LE\cdot\dot{K}-K(\dot{L}E+L\dot{E})}{(LE)^2}}{\frac{K}{LE}} \\
&= \frac{LE\cdot\dot{K}-K(\dot{L}E+L\dot{E})}{LE\cdot K} \\
&= \frac{\dot{K}}{K} - \frac{\dot{L}E+L\dot{E}}{LE} \\
&= \frac{\dot{K}}{K} - \frac{\dot{L}}{L} - \frac{\dot{E}}{E}.
\end{aligned}$$

For small discrete changes ($\triangle$) of K, L, and E, the equations holds approximately.]

Therefore, if we multiply both sides with $k = \frac{K}{LE}$ this gives:

$$\triangle k = \frac{I}{K}\frac{K}{LE} - (\delta + n + g)k = i - (\delta + n + g)k$$

Hence, the change in capital stock per unit of effective labour per period is

$$\triangle k = i - (\delta + n + g)k \text{ with } i = \frac{I}{LE}$$

The gross investments per unit of effective labour (i) increase the capital stock per unit of effective labour (k). This effect is countered by three other effects: the capital stock per unit of effective labour decreases

- due to depreciation, which reduces the level of capital;
- due to population growth, which leads to the fact that the existing capital is distributed among an increased number of persons;
- due to technological progress, which leads to an increase in worker efficiency.

  The break-even investment is the amount of investment necessary to keep capital stock per unit of effective labour constant:

  $$i = (\delta + n + g)k$$

Since the Solow model assumes that savings and investment are in equilibrium, we can substitute the investments of the equation of motion for k from above with savings:

$$\triangle k = sf(k) - (\delta + n + g)k$$

For $\triangle k = 0$:

$$sf(k) = (\delta + n + g)k$$

(c) Calculate the capital stock per unit of effective labour of the economy in the steady state.

$$\begin{aligned}
\Delta k = 0 \rightarrow sf(k) &= (n + \delta + g)k \\
sk^{0,5} &= (n + \delta + g)k \\
\frac{k^{0,5}}{k} &= \frac{n + \delta + g}{s} \\
k^{0,5} &= \frac{s}{n + \delta + g} \\
k &= (\frac{s}{n + \delta + g})^2 \\
k^* &= (\frac{0,25}{0,1 + 0,3 + 0,1})^2 = \frac{1}{4}
\end{aligned}$$

(d) Calculate the income stock per unit of effective labour of the economy in the steady state.

$$y^* = k^{0,5} = \frac{1}{2}$$

(e) Calculate the consumption per unit of effective labour of the economy in the steady state.

$$c^* = y^* - sy^* = \frac{1}{2} - 0,25 \cdot \frac{1}{2} = \frac{3}{8} = 0,375$$

(f) Calculate the savings rate that maximises the per unit of effective labour consumption of the economy in the steady state.

$$\max_k c = y - sy = f(k) - sf(k) = f(k) - (n + \delta + g)k$$

Golden rule capital stock per unit of effective labour:

$$\begin{aligned} \frac{\partial c}{\partial k} = f'(k) - (n + \delta + g) &\overset{!}{=} 0 \\ f'(k) &= n + \delta + g \\ 0,5k^{-0,5} &= 0,5 \\ k^{-0,5} &= 1 \\ k^{GR} &= 1 \end{aligned}$$

Golden rule savings rate:

$$\begin{aligned} sf(k) &= (n + \delta + g)k \\ s &= \frac{(n + \delta + g)k}{f(k)} \\ s &= \frac{0,5k^{GR}}{f(k^{GR})} = \frac{0,5 \cdot 1}{1^{0,5}} = 0,5 \\ s^{GR} &= 0,5 \end{aligned}$$

Golden rule consumption per unit of effective labour:

$$c^{GR} = f(k^{GR}) - s^{GR}f(^{GR}) = (k^{GR})^{0,5} - s^{GR}(k^{GR})^{0,5} = 1 - 0,5 \cdot 1 = 0,5$$

(g) Sketch the results of (a)–(f) in an appropriate Solow diagramme.

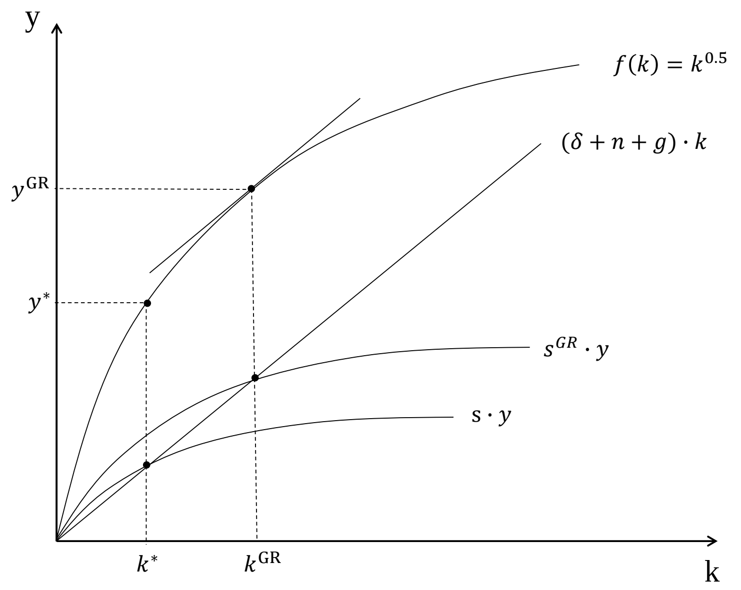

Figure 5: Illustration of the steady-state and the Golden rule steady state.

(h) Determine the growth rates of output per capita, the capital stock per capita, and consumption per capita in the steady state.

In the steady state, the capital stock per unit of effective labour is constant, $\frac{\Delta k}{k} = 0$. Hence $\frac{\Delta(\frac{K}{LE})}{\frac{K}{LE}} = 0$. Analogously, also consumption per unit of effective labour as well as output per unit of effective labour are constant, $\frac{\Delta y}{y} = \frac{\Delta \frac{Y}{LE}}{\frac{Y}{LE}} = 0$ and $\frac{\Delta c}{c} = \frac{\Delta \frac{C}{LE}}{\frac{C}{LE}} = 0$. However, labour the employed force L continues to grow with rate n and the efficiency of the economy E continues to grow with the rate for the technological progress g. Approximately, $\frac{\Delta(\frac{K}{LE})}{\frac{K}{LE}} \approx \frac{\Delta K}{K} - \frac{\Delta L}{L} - \frac{\Delta E}{E}$. Given the growth rates n and g, we can rewrite $\frac{\Delta K}{K} - \frac{\Delta L}{L} - \frac{\Delta E}{E} = \frac{\Delta K}{K} - n - g = 0 \Leftrightarrow \frac{\Delta K}{K} = n + g$. Thus, the capital stock grows with rate $n+g$. We can rewrite $\frac{\Delta K}{K} - n - g = 0$ as $\frac{\Delta K}{K} - \frac{\Delta L}{L} - g \approx \frac{\Delta \frac{K}{L}}{\frac{K}{L}} - g = 0$ with $\frac{K}{L}$ as capital stock per capita. Thus, the capital stock per capita grows with the rate of the technological progress g, $\frac{\Delta \frac{K}{L}}{\frac{K}{L}} = g$. Similarly, output per capita $\frac{\Delta \frac{Y}{L}}{\frac{Y}{L}}$ and consumption per capita $\frac{\Delta \frac{C}{L}}{\frac{C}{L}}$ grow also with rate g.

II. Multiple Choice

Select one answer.

1. **Technological progress and unemployment**
The diagram plots GDP per worker vs capital per worker, both across countries in 1990 (the scatter plots) and the trajectories since 1760 for a few representative countries (the paths).

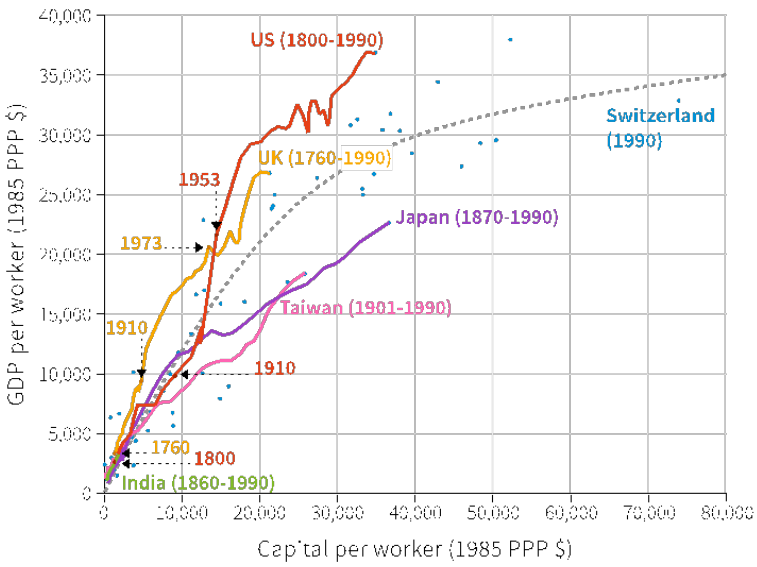

Figure 1

Which of the following statements is correct?

(A) There is no clear evidence of technological progress in the US GDP per worker.

(B) Switzerland has been the most successful country in attaining high GDP per worker by use of its capital.

(C) $\boxed{\text{Taiwan is more capital intensive than the UK in 1990.}}$

(D) The average product of capital has been higher in Japan than in the UK over the years shown.

2. Okun's Law

Figure 2 depicts the relationship between changes in real GDP (in percent) and unemployment rate (in percentage points) in Germany from 1993 to 2016. Each point represents one year.

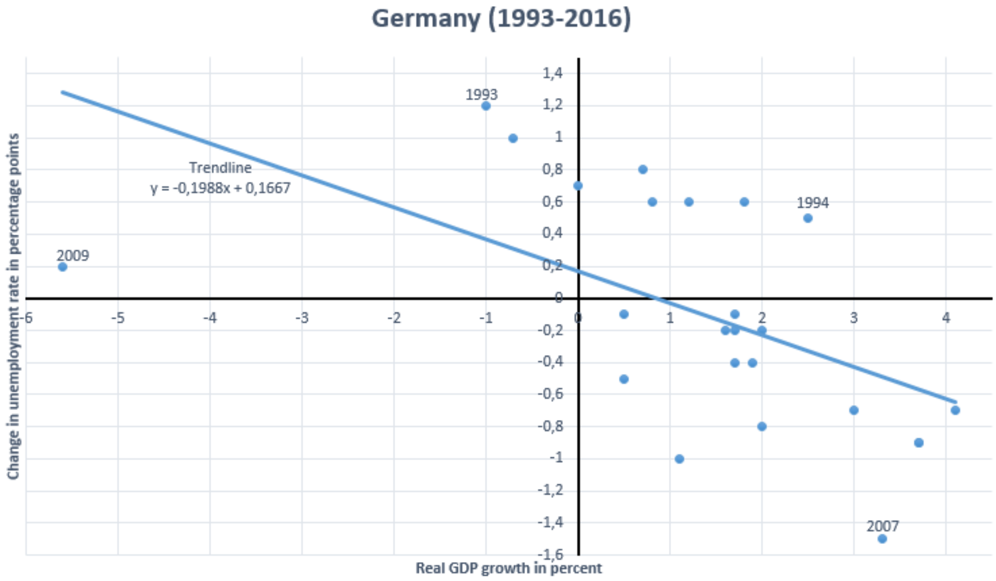

Figure 2

Which of the following statements is true?

(A) A growth in real GDP always results in a decline of the unemployment rate.

(B) $\boxed{\text{A growth in real GDP above 0.9\% on average is accompanied by a decline of the unemployment rate.}}$

(C) In 1994 a growth in real GDP of approximately 2.5% led to a decrease of unemployment by approximately 0.5 percentage points.

(D) An increase of the unemployment rate of 0.1 percentage points on average leads to an increase of real GDP of 0.5%.