Friday IV EN 2752022 - FPV Solutions for week 5

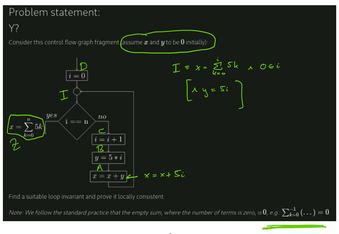
Funktionale Programmierung (Technische Universität München)

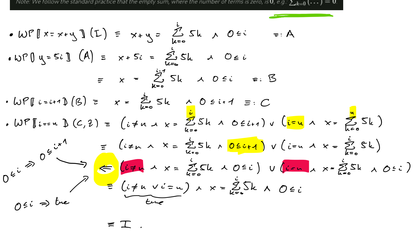
Friday IV - EN - 27/5/2022

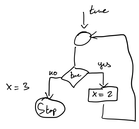
Mittwoch, 17. November 2021 10:59

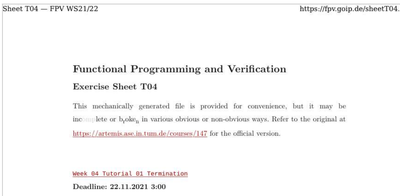
Zulip-Links:

- FPV-Announcements
- FPV-Lecture
- FPV-TechSupport
- FPV-Organization
- FPV-Memes
- FPV_T_IX EN









Sheet T04 — FPV WS21/22 https://fpv.goip.de/sheetT04.html

Functional Programming and Verification

Exercise Sheet T04

This mechanically generated file is provided for convenience, but it may be inc···lete or h·ok·n in various obvious or non-obvious ways. Refer to the original at https://artemis.ase.in.tum.de/courses/147 for the official version.

Week 04 Tutorial 01 Termination

Deadline: 22.11.2021 3:00

Termination

In the lecture, you have learned how to prove termination of a MiniJava program. Discuss these questions:

1. How can you decide whether a termination proof is required at all?
2. What is the basic idea of the termination proof?
3. How is the program to be modified?
4. What has to be proven?
5. How is the loop invariant influenced?

Note: This is a tutorial exercise, you do not need to submit anything for this exercise.

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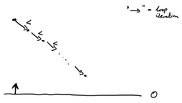
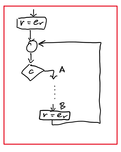
1. Programs without loops always terminate (for $i=0, i<n, i=i+2$ ...)
2. • Goal: Show that the loop is only executed finitely often.
   • Define a variable r which
   - is always positive when the loop is entered [>0]
   - Strictly decreases after every iteration. [r new < r old]



3.



4. We have to find locally consistent assertions A and B and prove:
   • $A \Rightarrow r > 0$ (*)
   • $B \Rightarrow r > e_1$
   "old r" "new r"

5. We follow the same B rules as before and additionally:
   4) I has to include $r = e_0$ and the relations between all variables in e_0.

⚠ We don't write A and B beforehand, but arrive at them through WP. However, we still have to show (*).

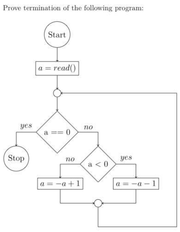
Sheet T04 — FPV WS21/22 https://fpv.goip.de/sheetT04.html

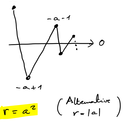
Week 04 Tutorial 02 A Wavy Approach

Deadline: 22.11.2021 3:00

A Wavy Approach

Prove termination of the following program:





$r = a^2$ (Alternative $r = |a|$)

Note: This is a tutorial exercise, you do not need to submit anything for this exercise.

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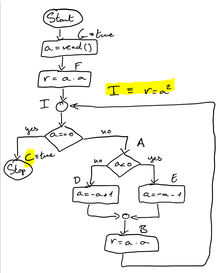


- $WP[\![r = a \cdot a]\!](I) \equiv a^2 = a^2 \equiv true$
  $\Leftarrow r > a^2 \equiv: B$

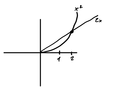
⚠ This check is required in every termination proof. The information about $r \geq e_1$ has to be added.

- $WP[\![a = -a+1]\!](B) \equiv r > (-a+1)^2$
  $\equiv r > a^2 - 2a + 1 \equiv: D$
- $WP[\![a = -a-1]\!](B) \equiv r > (-a-1)^2$
  $\equiv r > a^2 + 2a + 1 \equiv: E$
- $WP[\![a \geq 0]\!](D,E) \equiv (a \geq 0 \wedge r > a^2 - 2a + 1)$
  $\vee (a < 0 \wedge r > a^2 + 2a + 1)$
  $\equiv r > a^2 - 2|a| + 1 \equiv: A$

$A \Rightarrow r > 0$ ✓ ($a^2 - 2|a| + 1 \geq 0$)

- $WP[\![a == 0]\!](A, C) \equiv (a \neq 0 \wedge r > a^2 - 2|a| + 1) \vee a = 0$
  $\Leftarrow I \equiv r < a^2$
- $WP[\![r = a \cdot a]\!](I) \equiv true$
- $WP[\![a = read()]\!](true) \equiv \forall a.\ true \equiv true. \equiv G$

$a^2 \geq 2|a|$ for $a \neq 1$



Recall: local consistency

We say the annotations of a program are locally consistent if for every program point S, postcondition B and precondition A, the following holds:

$A \Rightarrow WP[\![S]\!](B)$

General Method

- For every occurring loop `while (b) s` we introduce a fresh variable `r`.
- Then we transform the loop into:

```
r = e0;
while (b) {
   assert(r>0);
   s
   assert(r > e1);
   r = e1;
}
```

for suitable expressions `e0`, `e1`.

Idea

- Make sure that each loop is executed only finitely often ...
- For each loop, identify an indicator value r, that has two properties
  (1) $r > 0$ whenever the loop is entered;
  (2) r is decreased during every iteration of the loop.
- Transform the program in a way that, alongside ordinary program execution, the indicator value r is computed.
- Verify that properties (1) and (2) hold!