机器人学中的状态估计 - 作业 8

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1. 一次项为

$$\epsilon_k^{\wedge} T_{\text{op},k} p_{\text{op},j} + T_{\text{op},k} D\zeta_j = (T_{\text{op},k} p_{\text{op},j})^{\odot} \epsilon_k + T_{\text{op},k} D\zeta_j$$
$$= \begin{bmatrix} (T_{\text{op},k} p_{\text{op},j})^{\odot} & T_{\text{op},k} D \end{bmatrix} \begin{bmatrix} \epsilon_k \\ \zeta_j \end{bmatrix} \tag{1}$$
$$= Z_{jk} \delta x_{jk}$$

二次项为

$$\frac{1}{2} \epsilon_k^{\wedge} \epsilon_k^{\wedge} T_{\text{op},k} p_{\text{op},j} + \epsilon_k^{\wedge} T_{\text{op},k} D\zeta_j = \frac{1}{2} \epsilon_k^{\wedge} (T_{\text{op},k} p_{\text{op},j})^{\odot} \epsilon_k + \epsilon_k^{\wedge} T_{\text{op},k} D\zeta_j \tag{2}$$

其第 i 位为

$$\mathbf{1}_i^T (\frac{1}{2} \epsilon_k^{\wedge} \epsilon_k^{\wedge} T_{\text{op},k} p_{\text{op},j} + \epsilon_k^{\wedge} T_{\text{op},k} D\zeta_j) = \frac{1}{2} \mathbf{1}_i^T \epsilon_k^{\wedge} (T_{\text{op},k} p_{\text{op},j})^{\odot} \epsilon_k + \mathbf{1}_i^T \epsilon_k^{\wedge} T_{\text{op},k} D\zeta_j$$
$$= \frac{1}{2} \epsilon_k^T \mathbf{1}_i^{\circledcirc} (T_{\text{op},k} p_{\text{op},j})^{\odot} \epsilon_k + \epsilon_k^T \mathbf{1}_i^{\circledcirc} T_{\text{op},k} D\zeta_j$$
$$= \frac{1}{2} \begin{bmatrix} \epsilon_k^T & \zeta_j^T \end{bmatrix} \begin{bmatrix} \mathbf{1}_i^{\circledcirc} (T_{\text{op},k} p_{\text{op},j})^{\odot} & \mathbf{1}_i^{\circledcirc} T_{\text{op},k} D \\ (\mathbf{1}_i^{\circledcirc} T_{\text{op},k} D)^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \epsilon_k \\ \zeta_j \end{bmatrix} \tag{3}$$
$$= \delta x_{jk}^T \mathcal{Z}_{ijk} \delta x_{jk}$$

因此

$$\frac{1}{2} \epsilon_k^{\wedge} \epsilon_k^{\wedge} T_{\text{op},k} p_{\text{op},j} + \epsilon_k^{\wedge} T_{\text{op},k} D\zeta_j = \frac{1}{2} \epsilon_k^{\wedge} (T_{\text{op},k} p_{\text{op},j})^{\odot} \epsilon_k + \epsilon_k^{\wedge} T_{\text{op},k} D\zeta_j$$
$$= \sum_i \frac{1}{2} \mathbf{1}_i^T \epsilon_k^{\wedge} (T_{\text{op},k} p_{\text{op},j})^{\odot} \epsilon_k + \mathbf{1}_i^T \epsilon_k^{\wedge} T_{\text{op},k} D\zeta_j \tag{4}$$
$$= \sum_i \delta x_{jk}^T \mathcal{Z}_{ijk} \delta x_{jk}$$