机器人学中的状态估计 - 作业 3

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1. $y_k|x_k \sim \mathcal{N}(\check{y}_k + G_k(x-\check{x}_k), R'_k)$, $x_k|\check{x}_0, v_{1:k}, y_{0:k} \sim \mathcal{N}(\check{x}_k, F_{k-1}\check{P}_{k-1}F_{k-1}^T + Q'_k)$. 将概率密度相乘得到:

$$\begin{aligned} p(\hat{x}_k|\check{x}_0, v_{1:k}, y_{0:k}) &= \eta p(y_k|x_k)p(x_k|\check{x}_0, v_{1:k}, y_{0:k}) \\ &= \eta \exp\{-\frac{1}{2}[y_k - (\check{y}_k + G_k(x-\check{x}_k))]^T R_k'^{-1}[y_k - (\check{y}_k + G_k(x-\check{x}_k))]\} \\ &\times \exp\{-\frac{1}{2}(x_k-\check{x}_k)^T\check{P}_k^{-1}(x_k-\check{x}_k)\} \end{aligned} \tag{1}$$

显然后验 $\hat{x}_k$ 仍然服从正态分布. 对于指数部分暂时忽略掉常数 $-\frac{1}{2}$，相乘相当于在指数部分相加:

$$((y_k-\check{y}_k) - G_k(x-\check{x}_k))^T R_k'^{-1}((y_k-\check{y}_k) - G_k(x-\check{x}_k)) + (x_k-\check{x}_k)^T\check{P}_k^{-1}(x_k-\check{x}_k) \tag{2}$$

分离出包含 x 的项，得到:

$$(x_k-\check{x}_k)^T G_k^T R_k'^{-1} G_k (x_k-\check{x}_k) - 2(x_k-\check{x}_k)^T G_k^T R_k'^{-1}(y_k-\check{y}_k) + (x_k-\check{x}_k)^T\check{P}_k^{-1}(x_k-\check{x}_k) \tag{3}$$

其中 x_k 的二次项对应 $\hat{x}_k$ 的后验协方差:

$$\hat{P}_k^{-1} = G_k^T R_k'^{-1} G_k + \check{P}_k^{-1} \Rightarrow \hat{P}_k = (G_k^T R_k'^{-1} G_k + \check{P}_k^{-1})^{-1} \tag{4}$$

带入 SMW 公式得到:

$$\begin{aligned} \hat{P}_k &= (G_k^T R_k'^{-1} G_k + \check{P}_k^{-1})^{-1} \\ &= \check{P}_k - \check{P}_k G_k^T (R'_k + G_k\check{P}_k G_k^T)^{-1} G_k \check{P}_k \\ &= (I - \check{P}_k G_k^T (R'_k + G_k\check{P}_k G_k^T)^{-1} G_k)\check{P}_k \\ &= (I - K_k G_k)\check{P}_k \end{aligned} \tag{5}$$

其中 $K_k = \check{P}_k G_k^T (R'_k + G_k\check{P}_k G_k^T)^{-1}$
然后考虑 $\hat{x}_k$ 的均值, 记为 μ, 有:

$$\begin{aligned} \hat{P}_k^{-1}\mu &= G_k^T R_k'^{-1} G_k \check{x}_k + G_k^T R_k'^{-1}(y_k-\check{y}_k) + \check{P}_k^{-1}\check{x}_k \\ &= (G_k^T R_k'^{-1} G_k + \check{P}_k^{-1})\check{x}_k + R_k'^{-1} G_k (y_k-\check{y}_k) \end{aligned} \tag{6}$$

求解 μ 得到

$$\begin{aligned} \mu &= \hat{P}_k[(G_k^T R_k'^{-1} G_k + \check{P}_k^{-1})\check{x}_k + G_k^T R_k'^{-1}(y_k-\check{y}_k)] \\ &= \hat{P}_k(G_k^T R_k'^{-1} G_k + \check{P}_k^{-1})\check{x}_k + \hat{P}_k G_k^T R_k'^{-1}(y_k-\check{y}_k) \\ &= \check{x}_k + (G_k^T R_k'^{-1} G_k + \check{P}_k^{-1})^{-1} G_k^T R_k'^{-1}(y_k-\check{y}_k) \\ &= \check{x}_k + \check{P}_k G_k^T (R'_k + G_k\check{P}_k G_k^T)^{-1}(y_k-\check{y}_k) \\ &= \check{x}_k + K_k(y_k-\check{y}_k) \end{aligned} \tag{7}$$

2.

$$
\begin{aligned}
\sum_{i=0}^{2L} \alpha_i x_i &= \alpha_0 x_0 + \sum_{i=1}^{2L} \alpha_i x_i \\
&= \frac{\kappa}{\kappa+L}\mu_x + \frac{1}{2}\frac{1}{\kappa+L}\sum_{i=1}^{L} x_i \\
&= \frac{\kappa}{\kappa+L}\mu_x + \frac{1}{2}\frac{1}{\kappa+L} 2\mu_x \\
&= \frac{\kappa+L}{\kappa+L}\mu_x \\
&= \mu_x
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\sum_{i=0}^{2L} \alpha_i (x_i - \mu_x)(x_i - \mu_x)^T &= \sum_{i=1}^{2L} \alpha_i (x_i - \mu_x)(x_i - \mu_x)^T \\
&= \frac{1}{2}\frac{1}{\kappa+L}\sum_{i=1}^{2L} (\kappa+L) L_i L_i^T \\
&= \frac{1}{2}\sum_{i=1}^{2L} L_i L_i^T \\
&= \sum_{i=1}^{L} L_i L_i^T \\
&= \begin{bmatrix} L_1 & L_2 & \dots & L_L \end{bmatrix} \begin{bmatrix} L_1^T \\ L_2^T \\ \vdots \\ L_L^T \end{bmatrix} \\
&= \mathbf{L}\mathbf{L}^T \\
&= \Sigma_{xx}
\end{aligned} \tag{9}
$$

3.

$$\begin{aligned}\mathbf{F}_{k-1} &= \left.\frac{\partial f(\mathbf{x}_{k-1}, \mathbf{v}_k, \mathbf{w}_k)}{\partial \mathbf{x}_{k-1}}\right|_{\hat{\mathbf{x}}_{k-1}, \mathbf{v}_k, 0} \\ &= \begin{bmatrix} 1 & 0 & -T\sin\theta_{k-1} v_k \\ 0 & 1 & T\cos\theta_{k-1} v_k \\ 0 & 0 & 1 \end{bmatrix}\end{aligned} \tag{10}$$

$$\begin{aligned}\mathbf{G}_{k-1} &= \left.\frac{\partial g(\mathbf{x}_k, \mathbf{n}_k)}{\partial \mathbf{x}_k}\right|_{\check{\mathbf{x}}_k, 0} \\ &= \begin{bmatrix} \frac{x_k}{\sqrt{x_k^2+y_k^2}} & \frac{y_k}{\sqrt{x_k^2+y_k^2}} & 0 \\ -\frac{y_k}{x_k^2+y_k^2} & \frac{x_k}{x_k^2+y_k^2} & -1 \end{bmatrix}\end{aligned} \tag{11}$$

$$\begin{aligned}\mathbf{w}'_k &= \left.\frac{\partial f(\mathbf{x}_{k-1}, \mathbf{v}_k, \mathbf{w}_k)}{\partial \mathbf{w}_k}\right|_{\hat{\mathbf{x}}_{k-1}, \mathbf{v}_k, 0} \mathbf{w}_k \\ &= \begin{bmatrix} T\cos\theta_{k-1} & 0 \\ T\sin\theta_{k-1} & 0 \\ 0 & T \end{bmatrix} \mathbf{w}_k\end{aligned} \tag{12}$$

$$\begin{aligned}\mathbf{n}'_k &= \left.\frac{\partial g(\mathbf{x}_k, \mathbf{n}_k)}{\partial \mathbf{n}_k}\right|_{\check{\mathbf{x}}_k, 0} \mathbf{n}_k \\ &= \mathbf{n}_k\end{aligned} \tag{13}$$

因此线性化后的协方差矩阵为:

$$\begin{aligned}\mathbf{Q}'_k &= \mathbb{E}[\mathbf{w}'_k \mathbf{w}'^T_k] \\ &= \begin{bmatrix} T\cos\theta_{k-1} & 0 \\ T\sin\theta_{k-1} & 0 \\ 0 & T \end{bmatrix} \mathbf{Q} \begin{bmatrix} T\cos\theta_{k-1} & 0 \\ T\sin\theta_{k-1} & 0 \\ 0 & T \end{bmatrix}^T\end{aligned} \tag{14}$$

$$\mathbf{R}'_k = \mathbb{E}[\mathbf{n}'_k \mathbf{n}'^T_k] = \mathbf{R} \tag{15}$$

将以上式子带入 EKF 迭代公式即可.