机器人学中的状态估计

第六次作业讲评

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第一题

证明: $(\mathbf{Cu})^\wedge \equiv \mathbf{Cu}^\wedge\mathbf{C}^T$

解答:

对于相应维度的任意向量v有:

$(Cu)^\wedge v$

$= (Cu) \times v$

$= (Cu) \times (CC^Tv)$

由向量叉乘的旋转不变性:

$= C(u \times C^Tv)$

$= Cu^\wedge C^Tv$

故:

$(Cu)^\wedge = Cu^\wedge C^T$

证明思路: 上次作业中原题

证法1:从矩阵关系角度证明(仅介绍此证法)

证法2:将矩阵展开成元素形式证明

第二题

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证明： $(\mathbf{C}\mathbf{u})^\wedge \equiv (2\cos\phi + 1)\mathbf{u}^\wedge - \mathbf{u}^\wedge\mathbf{C} - \mathbf{C}^T\mathbf{u}^\wedge.$

解答： 由书表格 7-2 有： $(\mathbf{W}\mathbf{u})^\wedge \equiv \mathbf{u}^\wedge\left(\operatorname{tr}(\mathbf{W})\mathbf{1} - \mathbf{W}\right) - \mathbf{W}^T\mathbf{u}^\wedge$

故：

$$\begin{aligned}(\mathbf{C}\mathbf{u})^\wedge &\equiv \mathbf{u}^\wedge\left(\operatorname{tr}(\mathbf{C})\mathbf{1} - \mathbf{C}\right) - \mathbf{C}^T\mathbf{u}^\wedge \\ &\equiv \operatorname{tr}(\mathbf{C})\mathbf{u}^\wedge - \mathbf{u}^\wedge\mathbf{C} - \mathbf{C}^T\mathbf{u}^\wedge \\ &\equiv (2\cos\phi + 1)\mathbf{u}^\wedge - \mathbf{u}^\wedge\mathbf{C} - \mathbf{C}^T\mathbf{u}^\wedge\end{aligned}$$

其中：

$$\begin{aligned}\operatorname{tr}(\mathbf{C}) &= \operatorname{tr}\left(\cos\phi\mathbf{1} + (1-\cos\phi)\mathbf{a}\mathbf{a}^T + \sin\phi\mathbf{a}^\wedge\right) \\ &= 3\cos\phi + (1-\cos\phi)\underbrace{\operatorname{tr}(\mathbf{a}\mathbf{a}^T)}_{1} + \sin\phi\underbrace{\operatorname{tr}(\mathbf{a}^\wedge)}_{0} \\ &= 2\cos\phi + 1\end{aligned}$$

证明思路：

证法1：使用书中表格表格 **7-2**公式 （仅介绍证法1，另外两种方法思路不难，直接推导就可以了，可能推起来稍微复杂一些）

证法2：使用罗德里格斯公式展开旋转矩阵

证法3：将矩阵写成元素形式，展开化简

第三题

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证明：$\exp\left((\mathbf{C}\mathbf{u})^{\wedge}\right) \equiv \mathbf{C}\exp\left(\mathbf{u}^{\wedge}\right)\mathbf{C}^{T}$

证明思路： 将exp泰勒展开然后化简

解答： 根据公式（7.19），可知：

$$\exp\left((\boldsymbol{C}\boldsymbol{u})^{\wedge}\right) \equiv \sum_{n=0}^{\infty} \frac{1}{n!}\left((\boldsymbol{C}\boldsymbol{u})^{\wedge}\right)^{n}$$

第一问中证明$(\boldsymbol{C}\boldsymbol{u})^{\wedge} \equiv \boldsymbol{C}\boldsymbol{u}^{\wedge}\boldsymbol{C}^{\mathrm{T}}$，故：

$$\exp\left((\boldsymbol{C}\boldsymbol{u})^{\wedge}\right) \equiv \sum_{n=0}^{\infty} \frac{1}{n!}\left((\boldsymbol{C}\boldsymbol{u})^{\wedge}\right)^{n} \equiv \sum_{n=0}^{\infty} \frac{1}{n!}\left(\boldsymbol{C}\boldsymbol{u}^{\wedge}\boldsymbol{C}^{\mathrm{T}}\right)^{n}$$

$$\equiv \sum_{n=0}^{\infty} \frac{1}{n!}\left(\boldsymbol{C}\boldsymbol{u}^{\wedge}\underbrace{\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}}_{\mathbf{1}}\boldsymbol{u}^{\wedge}\underbrace{\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C}}_{\mathbf{1}}\boldsymbol{u}^{\wedge}\boldsymbol{C}^{\mathrm{T}}\cdots\boldsymbol{C}\boldsymbol{u}^{\wedge}\boldsymbol{C}^{\mathrm{T}}\right)$$

$$\equiv \sum_{n=0}^{\infty} \frac{1}{n!}\boldsymbol{C}\left(\boldsymbol{u}^{\wedge}\right)^{n}\boldsymbol{C}^{\mathrm{T}} = \boldsymbol{C}\underbrace{\sum_{n=0}^{\infty} \frac{1}{n!}\left(\boldsymbol{u}^{\wedge}\right)^{n}}_{\exp(\boldsymbol{u}^{\wedge})}\boldsymbol{C}^{\mathrm{T}}$$

$$\equiv \boldsymbol{C}\exp\left(\boldsymbol{u}^{\wedge}\right)\boldsymbol{C}^{\mathrm{T}}$$

第四题

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证明： $(\mathcal{T}\mathbf{x})^\wedge \equiv \mathbf{T}\mathbf{x}^\wedge\mathbf{T}^{-1}$

证明思路： 展开证明

解答：

$$\mathcal{T}\mathbf{x} = \begin{bmatrix} \mathbf{C} & \mathbf{r}^\wedge\mathbf{C} \\ \mathbf{0} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{bmatrix} = \begin{bmatrix} \mathbf{C}\boldsymbol{\rho} + \mathbf{r}^\wedge\mathbf{C}\boldsymbol{\phi} \\ \mathbf{C}\boldsymbol{\phi} \end{bmatrix}$$

$$(\mathcal{T}\mathbf{x})^\wedge = \begin{bmatrix} (\mathbf{C}\boldsymbol{\phi})^\wedge & \mathbf{C}\boldsymbol{\rho} + \mathbf{r}^\wedge\mathbf{C}\boldsymbol{\phi} \\ \mathbf{0}^T & 0 \end{bmatrix}$$

$$\begin{aligned}
\mathbf{T}\mathbf{x}^\wedge\mathbf{T}^{-1} &= \begin{bmatrix} \mathbf{C} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi}^\wedge & \boldsymbol{\rho} \\ \mathbf{0}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{C} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} \mathbf{C} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\phi}^\wedge & \boldsymbol{\rho} \\ \mathbf{0}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{C}^T & -\mathbf{C}^T\mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{C}\boldsymbol{\phi}^\wedge\mathbf{C}^T & -\mathbf{C}\boldsymbol{\phi}^\wedge\mathbf{C}^T\mathbf{r} + \mathbf{C}\boldsymbol{\rho} \\ \mathbf{0}^T & 0 \end{bmatrix}
\end{aligned}$$

第四题

解答：

由第一题可知：$(\mathbf{C}\phi)^\wedge = \mathbf{C}\phi^\wedge\mathbf{C}^T$

$$\mathbf{r}^\wedge\mathbf{C}\phi = \mathbf{r}\times(\mathbf{C}\phi) = -(\mathbf{C}\phi)\times\mathbf{r} = -(\mathbf{C}\phi)^\wedge\mathbf{r} = -\mathbf{C}\phi^\wedge\mathbf{C}^T\mathbf{r}$$

$$(\mathcal{T}\mathbf{x})^\wedge = \begin{bmatrix} (\mathbf{C}\phi)^\wedge & \mathbf{C}\rho + \mathbf{r}^\wedge\mathbf{C}\phi \\ \mathbf{0}^T & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{C}\phi^\wedge\mathbf{C}^T & -\mathbf{C}\phi^\wedge\mathbf{C}^T\mathbf{r} + \mathbf{C}\rho \\ \mathbf{0}^T & 0 \end{bmatrix} = \mathbf{T}\mathbf{x}^\wedge\mathbf{T}^{-1}$$

第五题

证明： $\exp\left((\mathcal{T}\mathbf{x})^\wedge\right) \equiv \mathbf{T}\exp\left(\mathbf{x}^\wedge\right)\mathbf{T}^{-1}$

证明思路：将exp展开然后再化简

解答：

$$\exp\left((\mathcal{T}\mathbf{x})^\wedge\right) = \sum_{n=0}^{\infty}\frac{1}{n!}\left[(\mathcal{T}\mathbf{x})^\wedge\right]^n$$

$$= \sum_{n=0}^{\infty}\frac{1}{n!}\left[\mathbf{T}\mathbf{x}^\wedge\mathbf{T}^{-1}\right]^n$$

（使用7.5.4题结论）

$$= \mathbf{1} + \mathbf{T}\mathbf{x}^\wedge\mathbf{T}^{-1} + \frac{1}{2!}\mathbf{T}\mathbf{x}^\wedge\underbrace{\mathbf{T}^{-1}\mathbf{T}}_{\mathbf{1}}\mathbf{x}^\wedge\mathbf{T}^{-1} + \frac{1}{3!}\mathbf{T}\mathbf{x}^\wedge\underbrace{\mathbf{T}^{-1}\mathbf{T}}_{\mathbf{1}}\mathbf{x}^\wedge\underbrace{\mathbf{T}^{-1}\mathbf{T}}_{\mathbf{1}}\mathbf{x}^\wedge\mathbf{T}^{-1} + \cdots$$

$$= \mathbf{1} + \mathbf{T}\mathbf{x}^\wedge\mathbf{T}^{-1} + \frac{1}{2!}\mathbf{T}\left(\mathbf{x}^\wedge\right)^2\mathbf{T}^{-1} + \frac{1}{3!}\mathbf{T}\left(\mathbf{x}^\wedge\right)^3\mathbf{T}^{-1} + \cdots$$

$$= \mathbf{T}\left[\sum_{n=0}^{\infty}\frac{1}{n!}\left(\mathbf{x}^\wedge\right)^n\right]\mathbf{T}^{-1}$$

$$= \mathbf{T}\exp\left(\mathbf{x}^\wedge\right)\mathbf{T}^{-1}.$$

第六题

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证明： $\mathbf{x}^{\wedge}\mathbf{p} \equiv \mathbf{p}^{\odot}\mathbf{x}$

证明思路： 展开化简

解答：

$$\boldsymbol{x}^{\wedge} = \begin{bmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{bmatrix}^{\wedge} = \begin{bmatrix} \boldsymbol{\phi}^{\wedge} & \boldsymbol{\rho} \\ \mathbf{0}^{\mathrm{T}} & 0 \end{bmatrix}, \quad \boldsymbol{p}^{\odot} = \begin{bmatrix} \boldsymbol{\varepsilon} \\ \eta \end{bmatrix}^{\odot} = \begin{bmatrix} \eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge} \\ \mathbf{0}^{\mathrm{T}} & \mathbf{0}^{\mathrm{T}} \end{bmatrix}$$

$$\boldsymbol{x}^{\wedge}\boldsymbol{p} \equiv \begin{bmatrix} \boldsymbol{\phi}^{\wedge} & \boldsymbol{\rho} \\ \mathbf{0}^{\mathrm{T}} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \eta \end{bmatrix} \equiv \begin{bmatrix} \boldsymbol{\phi}^{\wedge}\boldsymbol{\varepsilon} + \boldsymbol{\rho}\eta \\ 0 \end{bmatrix}$$

$$\boldsymbol{p}^{\odot}\boldsymbol{x} \equiv \begin{bmatrix} \eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge} \\ \mathbf{0}^{\mathrm{T}} & \mathbf{0}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{bmatrix} = \begin{bmatrix} \eta\boldsymbol{\rho} - \boldsymbol{\varepsilon}^{\wedge}\boldsymbol{\phi} \\ 0 \end{bmatrix} = \begin{bmatrix} \eta\boldsymbol{\rho} + \boldsymbol{\phi}^{\wedge}\boldsymbol{\varepsilon} \\ 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\phi}^{\wedge}\boldsymbol{\varepsilon} + \boldsymbol{\rho}\eta \\ 0 \end{bmatrix}$$

故， $\boldsymbol{x}^{\wedge}\boldsymbol{p} \equiv \boldsymbol{p}^{\odot}\boldsymbol{x}$

第七题

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证明： $\mathbf{p}^T\mathbf{x}^\wedge \equiv \mathbf{x}^T\mathbf{p}^\circledcirc$

证明思路： 展开化简

解答：

$$\boldsymbol{p}^\circledcirc = \begin{bmatrix} \boldsymbol{\varepsilon} \\ \eta \end{bmatrix}^\circledcirc = \begin{bmatrix} \mathbf{0} & \boldsymbol{\varepsilon} \\ -\boldsymbol{\varepsilon}^\wedge & \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{p}^{\mathrm{T}}\boldsymbol{x}^\wedge \equiv [\boldsymbol{\varepsilon}^{\mathrm{T}} \quad \eta] \begin{bmatrix} \boldsymbol{\phi}^\wedge & \boldsymbol{\rho} \\ \mathbf{0}^{\mathrm{T}} & 0 \end{bmatrix} \equiv [\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\phi}^\wedge \quad \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\rho}]$$

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{p}^\circledcirc \equiv [\boldsymbol{\rho}^{\mathrm{T}} \quad \boldsymbol{\phi}^{\mathrm{T}}] \begin{bmatrix} \mathbf{0} & \boldsymbol{\varepsilon} \\ -\boldsymbol{\varepsilon}^\wedge & \mathbf{0} \end{bmatrix} \equiv [-\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{\varepsilon}^\wedge \quad \boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\varepsilon}]$$

其中，

$$-\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{\varepsilon}^\wedge = (\boldsymbol{\varepsilon}^\wedge\boldsymbol{\phi})^{\mathrm{T}} = (-\boldsymbol{\phi}^\wedge\boldsymbol{\varepsilon})^{\mathrm{T}} = \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\phi}^\wedge$$

$\boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\varepsilon}$ 为标量，即 $\boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\varepsilon} = (\boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\varepsilon})^{\mathrm{T}} = \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\rho}$

故，

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{p}^\circledcirc \equiv [-\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{\varepsilon}^\wedge \quad \boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\varepsilon}] \equiv [\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\phi}^\wedge \quad \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\rho}] \equiv \boldsymbol{p}^{\mathrm{T}}\boldsymbol{x}^\wedge$$

第八题

证明： $(\mathbf{Tp})^{\odot} \equiv \mathbf{Tp}^{\odot}\mathcal{T}^{-1}$

证明思路：展开化简

解答： $\mathbf{Tp} = \begin{bmatrix} \mathbf{C} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix} = \begin{bmatrix} \mathbf{C}\varepsilon + \eta\mathbf{r} \\ \eta \end{bmatrix}$ $\quad (\mathbf{Tp})^{\odot} = \begin{bmatrix} \eta\mathbf{1} & -(\mathbf{C}\varepsilon + \eta\mathbf{r})^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix}$

$$\mathbf{Tp}^{\odot}\mathcal{T}^{-1} = \begin{bmatrix} \mathbf{C} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \eta\mathbf{1} & -\varepsilon^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} \begin{bmatrix} \mathbf{C} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \mathbf{C} & \mathbf{r} \\ \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \eta\mathbf{1} & -\varepsilon^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} \begin{bmatrix} \mathbf{C}^T & (-\mathbf{C}^T\mathbf{r})^{\wedge}\mathbf{C}^T \\ \mathbf{0} & \mathbf{C}^T \end{bmatrix}$$

$$= \begin{bmatrix} \eta\mathbf{C} & -\mathbf{C}\varepsilon^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} \begin{bmatrix} \mathbf{C}^T & (-\mathbf{C}^T\mathbf{r})^{\wedge}\mathbf{C}^T \\ \mathbf{0} & \mathbf{C}^T \end{bmatrix}$$

第八题

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解答：

$$
=\begin{bmatrix} \eta \mathbf{C}\mathbf{C}^T & \eta \mathbf{C}\left(-\mathbf{C}^T\mathbf{r}\right)^\wedge \mathbf{C}^T - \mathbf{C}\boldsymbol{\varepsilon}^\wedge \mathbf{C}^T \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix}
$$

$$
=\begin{bmatrix} \eta \mathbf{1} & \eta \mathbf{C}\left(-\mathbf{C}^T\mathbf{r}\right)^\wedge \mathbf{C}^T - \mathbf{C}\boldsymbol{\varepsilon}^\wedge \mathbf{C}^T \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix}
$$

$$
\begin{aligned}
\eta \mathbf{C}\left(-\mathbf{C}^T\mathbf{r}\right)^\wedge \mathbf{C}^T - \mathbf{C}\boldsymbol{\varepsilon}^\wedge \mathbf{C}^T &= \mathbf{C}\left[\eta\left(-\mathbf{C}^T\mathbf{r}\right)^\wedge - \boldsymbol{\varepsilon}^\wedge\right]\mathbf{C}^T \\
&= -\mathbf{C}\left[\eta \mathbf{C}^T\mathbf{r} + \boldsymbol{\varepsilon}\right]^\wedge \mathbf{C}^T \\
&= -\left[\mathbf{C}\left(\eta \mathbf{C}^T\mathbf{r} + \boldsymbol{\varepsilon}\right)\right]^\wedge \\
&= -\left(\eta \mathbf{C}\mathbf{C}^T\mathbf{r} + \mathbf{C}\boldsymbol{\varepsilon}\right)^\wedge \\
&= -\left(\eta \mathbf{r} + \mathbf{C}\boldsymbol{\varepsilon}\right)^\wedge
\end{aligned}
$$

所以

$$
(\mathbf{T}\mathbf{p})^\odot = \begin{bmatrix} \eta \mathbf{1} & -\left(\mathbf{C}\boldsymbol{\varepsilon} + \eta \mathbf{r}\right)^\wedge \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} = \begin{bmatrix} \eta \mathbf{1} & \eta \mathbf{C}\left(-\mathbf{C}^T\mathbf{r}\right)^\wedge \mathbf{C}^T - \mathbf{C}\boldsymbol{\varepsilon}^\wedge \mathbf{C}^T \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} = \mathbf{T}\mathbf{p}^\odot \boldsymbol{\mathcal{T}}^{-1}
$$

第九题

证明：

$$(\mathbf{Tp})^{\odot^T}(\mathbf{Tp})^{\odot} \equiv \boldsymbol{\mathcal{T}}^{-T}\mathbf{p}^{\odot^T}\mathbf{p}^{\odot}\boldsymbol{\mathcal{T}}^{-1}$$

解答：

由上一问的结果 $(\mathbf{Tp})^{\odot} \equiv \mathbf{Tp}^{\odot}\boldsymbol{\mathcal{T}}^{-1}$

$$\begin{aligned}(\mathbf{Tp})^{\odot^T}(\mathbf{Tp})^{\odot} &= \left(\mathbf{Tp}^{\odot}\boldsymbol{\mathcal{T}}^{-1}\right)^T\left(\mathbf{Tp}^{\odot}\boldsymbol{\mathcal{T}}^{-1}\right)\\ &= \boldsymbol{\mathcal{T}}^{-T}\mathbf{p}^{\odot^T}\boldsymbol{T}^T\boldsymbol{T}\mathbf{p}^{\odot}\boldsymbol{\mathcal{T}}^{-1}\end{aligned}$$

两侧相同，只需证明： $\mathbf{p}^{\odot^T}\mathbf{T}^T\mathbf{T}\mathbf{p}^{\odot} = \mathbf{p}^{\odot^T}\mathbf{p}^{\odot}$

$$\begin{aligned}\mathbf{p}^{\odot^T}\mathbf{T}^T\mathbf{T}\mathbf{p}^{\odot} &= \begin{bmatrix}\eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge}\\ \mathbf{0}^T & \mathbf{0}^T\end{bmatrix}^T\begin{bmatrix}\mathbf{C} & \mathbf{r}\\ \mathbf{0}^T & 1\end{bmatrix}^T\begin{bmatrix}\mathbf{C} & \mathbf{r}\\ \mathbf{0}^T & 1\end{bmatrix}\begin{bmatrix}\eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge}\\ \mathbf{0}^T & \mathbf{0}^T\end{bmatrix}\\ &= \begin{bmatrix}\eta\mathbf{1} & \mathbf{0}\\ -(\boldsymbol{\varepsilon}^{\wedge})^T & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{C}^T & \mathbf{0}\\ \mathbf{r}^T & 1\end{bmatrix}\begin{bmatrix}\mathbf{C} & \mathbf{r}\\ \mathbf{0}^T & 1\end{bmatrix}\begin{bmatrix}\eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge}\\ \mathbf{0}^T & \mathbf{0}^T\end{bmatrix}\end{aligned}$$

第九题

$$
\begin{aligned}
&= \begin{bmatrix} \eta\mathbf{1} & \mathbf{0} \\ -(\boldsymbol{\varepsilon}^{\wedge})^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{C}^T\mathbf{C} & \mathbf{C}^T\mathbf{r} \\ \mathbf{r}^T\mathbf{C} & \mathbf{r}^T\mathbf{r}+1 \end{bmatrix} \begin{bmatrix} \eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} \\
&= \begin{bmatrix} \eta\mathbf{1} & \mathbf{0} \\ -(\boldsymbol{\varepsilon}^{\wedge})^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{1} & \mathbf{C}^T\mathbf{r} \\ \mathbf{r}^T\mathbf{C} & \mathbf{r}^T\mathbf{r}+1 \end{bmatrix} \begin{bmatrix} \eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} \\
&= \begin{bmatrix} \eta\mathbf{1} & \eta\mathbf{C}^T\mathbf{r} \\ -(\boldsymbol{\varepsilon}^{\wedge})^T & -(\boldsymbol{\varepsilon}^{\wedge})^T\mathbf{C}^T\mathbf{r} \end{bmatrix} \begin{bmatrix} \eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} \\
&= \begin{bmatrix} \eta^2\mathbf{1} & -\eta\boldsymbol{\varepsilon}^{\wedge} \\ -\eta(\boldsymbol{\varepsilon}^{\wedge})^T & (\boldsymbol{\varepsilon}^{\wedge})^T\boldsymbol{\varepsilon}^{\wedge} \end{bmatrix} \\
&= \begin{bmatrix} \eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix}^T \begin{bmatrix} \eta\mathbf{1} & -\boldsymbol{\varepsilon}^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{bmatrix} \\
&= \mathbf{p}^{\odot^T}\mathbf{p}^{\odot}
\end{aligned}
$$

所以：$(\mathbf{Tp})^{\odot^T}(\mathbf{Tp})^{\odot} = \boldsymbol{\mathcal{T}}^{-T}\left(\mathbf{p}^{\odot^T}\mathbf{T}^T\mathbf{T}\mathbf{p}^{\odot}\right)\boldsymbol{\mathcal{T}}^{-1}$

$$= \boldsymbol{\mathcal{T}}^{-T}\mathbf{p}^{\odot^T}\mathbf{p}^{\odot}\boldsymbol{\mathcal{T}}^{-1}$$

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感谢各位聆听！

Thanks for Listening