深蓝学院
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机器人学中的状态估计

第三次作业讲评

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第一题

$$\underbrace{p(\mathbf{x}_k|\check{\mathbf{x}}_0,\mathbf{v}_{1:k},\mathbf{y}_{0:k})}_{\mathcal{N}(\hat{\mathbf{x}}_k,\hat{\mathbf{P}}_k)} = \eta \underbrace{p(\mathbf{y}_k|\mathbf{x}_k)}_{\mathcal{N}(\check{\mathbf{y}}_k+\mathbf{G}_k(\mathbf{x}_k-\check{\mathbf{x}}_k),\mathbf{R}'_k)}$$

$$\times \underbrace{\int p(\mathbf{x}_k|\mathbf{x}_{k-1},\mathbf{v}_k)p(\mathbf{x}_{k-1}|\check{\mathbf{x}}_0,\mathbf{v}_{1:k-1},\mathbf{y}_{0:k-1})\mathrm{d}\mathbf{x}_{k-1}}_{\mathcal{N}(\check{\mathbf{x}}_k,\mathbf{F}_{k-1}\hat{\mathbf{P}}_{k-1}\mathbf{F}_{k-1}^{\mathrm{T}}+\mathbf{Q}'_k)}$$

推导EKF经典表达式

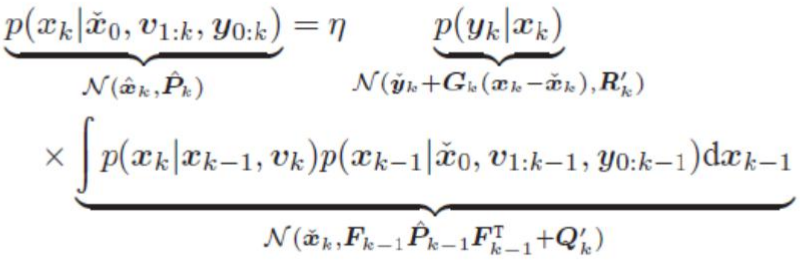
作业概况： 完成情况较好,部分作业未完成本题的证明

证明思路： 可直接按照ppt17 页给出的思路证明

1. 写出归一化积指数部分，它们应该是x(k)的二次型；
2. 比较x(k)二次项系数与一次项系数，得到协方差和均值的表达式；此时你得到信息形式的EKF；
3. 定义卡尔曼增益，推导见第2讲37页，使用SMW得到它两种不同形式；
4. 利用卡尔曼增益，改写第2步中的信息形式，整理成经典形式的EKF

解答：

已知条件如下：



$$\underbrace{p(\boldsymbol{x}_k|\check{\boldsymbol{x}}_0,\boldsymbol{v}_{1:k},\boldsymbol{y}_{0:k})}_{\mathcal{N}(\hat{\boldsymbol{x}}_k,\hat{\boldsymbol{P}}_k)} = \eta \underbrace{p(\boldsymbol{y}_k|\boldsymbol{x}_k)}_{\mathcal{N}(\check{\boldsymbol{y}}_k+\boldsymbol{G}_k(\boldsymbol{x}_k-\check{\boldsymbol{x}}_k),\boldsymbol{R}'_k)}$$

$$\times \underbrace{\int p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1},\boldsymbol{v}_k)\,p(\boldsymbol{x}_{k-1}|\check{\boldsymbol{x}}_0,\boldsymbol{v}_{1:k-1},\boldsymbol{y}_{0:k-1})\,\mathrm{d}\boldsymbol{x}_{k-1}}_{\mathcal{N}(\check{\boldsymbol{x}}_k,\boldsymbol{F}_{k-1}\hat{\boldsymbol{P}}_{k-1}\boldsymbol{F}^{T}_{k-1}+\boldsymbol{Q}'_k)}$$

1、将等式两边展开,分别提取二次项及一次项

左式$= p(x_k|\check{x}_0, v_{1:k}, y_{0:k}) \sim \mathcal{N}(\hat{x}_k, \hat{P}_k)$

$$= \eta \exp\left((x_k-\hat{x}_k)^T \hat{P}_k{}^{-1}(x_k-\hat{x}_k)\right)$$

$$= \eta\exp\left(x_k{}^T\hat{P}_k^{-1}x_k - 2x_k{}^T\hat{P}_k^{-1}\hat{x}_k + \hat{x}_k{}^T\hat{P}_k^{-1}\hat{x}_k\right)$$

得到：二次项：$x_k{}^T\hat{P}_k^{-1}x_k$

一次项：$2x_k{}^T\hat{P}_k^{-1}\hat{x}_k$

解答：

右式=$\eta \underbrace{p(y_k|x_k)}_{\mathcal{N}(\check{y}_k+G_k(x_k-\check{x}_k),R'_k)} \times \underbrace{\int p(x_k|x_{k-1},v_k)p(x_{k-1}|\check{x}_0,v_{1:k},y_{0:k-1})dx_{k-1}}_{\mathcal{N}(\check{x}_k,\underbrace{F_{k-1}\hat{P}_{k-1}F_{k-1}^T+Q'_k}_{\check{P}_k})}$

$$=\eta\exp\left(-\frac{1}{2}(\boldsymbol{y}_k-(\check{\boldsymbol{y}}_k+\boldsymbol{G}_k(\boldsymbol{x}_k-\check{\boldsymbol{x}}_k)))^T\boldsymbol{R}_k'^{-1}(\boldsymbol{y}_k-(\check{\boldsymbol{y}}_k+\boldsymbol{G}_k(\boldsymbol{x}_k-\check{\boldsymbol{x}}_k)))\right)$$

$$\times\exp\left(-\frac{1}{2}(\boldsymbol{x}_k-\check{\boldsymbol{x}}_k)^T\left(\boldsymbol{F}_{k-1}\hat{\boldsymbol{P}}_{k-1}\boldsymbol{F}_{k-1}^T+\boldsymbol{Q}'_k\right)^{-1}(\boldsymbol{x}_k-\check{\boldsymbol{x}}_k)\right)$$

$$=\eta\exp\left(-\frac{1}{2}\left(\boldsymbol{G}_k\boldsymbol{x}_k-(\boldsymbol{y}_k-\check{\boldsymbol{y}}_k+\boldsymbol{G}_k\check{\boldsymbol{x}}_k)\right)^T\boldsymbol{R}_k'^{-1}(\boldsymbol{G}_k\boldsymbol{x}_k-(\boldsymbol{y}_k-\check{\boldsymbol{y}}_k+\boldsymbol{G}_k\check{\boldsymbol{x}}_k))\right)$$

$$\times\exp\left(-\frac{1}{2}(\boldsymbol{x}_k-\check{\boldsymbol{x}}_k)^T\left(\boldsymbol{F}_{k-1}\hat{\boldsymbol{P}}_{k-1}\boldsymbol{F}_{k-1}^T+\boldsymbol{Q}'_k\right)^{-1}(\boldsymbol{x}_k-\check{\boldsymbol{x}}_k)\right)$$

其中二次项：$x_k^T\left[G_k^T{R'_k}^{-1}G_k+{\check{P}_k}^{-1}\right]x_k$

一次项：$2x_k^T\left[G_k^T{R'_k}^{-1}((y_k-\check{y}_k)+G_k\check{x}_k)+{\check{P}_k}^{-1}\check{x}_k\right]$

2、对比左右两式的二次项及一次项

对比二次项：${x_k}^T\hat{P}_k^{-1}x_k$ = $x_k^T\left[G_k^T{R'_k}^{-1}G_k+{\check{P}_k}^{-1}\right]x_k$

$${\hat{P}_k}^{-1}=G_k^T{R'_k}^{-1}G_k+{\check{P}_k}^{-1} \quad (1)$$

对比一次项：

$${\hat{P}_k}^{-1}\hat{x}_k=G_k^T{R'_k}^{-1}(y_k-\check{y}_k+G_k\check{x}_k)+{\check{P}_k}^{-1}\check{x}_k$$

$$\hat{x}_k=\hat{P}_kG_k^T{R'_k}^{-1}(y_k-\check{y}_k+G_k\check{x}_k)+\hat{P}_k{\check{P}_k}^{-1}\check{x}_k \quad (2)$$

解答：

3、定义卡尔曼增益K

定义$K_k = \hat{P}_k G_k^T R_k'^{-1}$　(3)

$$= (\check{P}_k^{-1} + G_k^T R_k'^{-1} G_k)^{-1} G_k^T R_k'^{-1}$$

由 SMW 等式：$(D + CAB)^{-1}CA \equiv D^{-1}C(A^{-1} + BD^{-1}C)^{-1}$

得：$K_k = \check{P}_k G_k^T (R_k' + G_k^T \check{P}_k G_k)^{-1}$　(4)

4、通过卡尔曼增益整理结果

由二次项对比结果(1)式得：

$$\check{P}_k^{-1} = \hat{P}_k^{-1} - G_k^T R_k'^{-1} G_k = \hat{P}_k^{-1}(1 - \underbrace{\hat{P}_k G_k^T R_k'^{-1}}_{K_k\text{的定义}} G_k) = \hat{P}_k^{-1}(1 - K_k G_k)$$

$$\hat{P}_k = (1 - K_k G_k)\,\check{P}_k \quad (5)$$

由一次项对比结果(2)式得：

$$\hat{x}_k = \underbrace{\hat{P}_k G_k^T R_k'^{-1}}_{K_k\text{的定义}}(y_k - \check{y}_k + G_k \check{x}_k) + \underbrace{\hat{P}_k \check{P}_k^{-1}}_{\text{由(5)式}1-K_kG_k} \check{x}_k$$

$$= K_k(y_k - \check{y}_k + G_k \check{x}_k) + (1 - K_k G_k)\check{x}_k$$

$$= K_k(y_k - \check{y}_k) + K_k G_k \check{x}_k + \check{x}_k - K_k G_k \check{x}_k = \check{x}_k + K_k(y_k - \check{y}_k) \quad (6)$$

解答：

整理上述结果，可得EKF 的经典形式：

$$\text{预测：}\begin{cases}\check{\boldsymbol{P}}_k=\boldsymbol{F}_{k-1}\hat{\boldsymbol{P}}_{k-1}\boldsymbol{F}_{k-1}^T+\boldsymbol{Q}_k'\\ \check{\boldsymbol{x}}_k=f\left(\hat{\boldsymbol{x}}_{k-1},\boldsymbol{v}_k,0\right)\end{cases}$$

$$\text{卡尔曼增益：}\quad \boldsymbol{K}_k=\check{\boldsymbol{P}}_k\boldsymbol{G}_k^T\left(\boldsymbol{G}_k\check{\boldsymbol{P}}_k\boldsymbol{G}_k^T+\boldsymbol{R}_k'\right)^{-1}$$

$$\text{更新：}\begin{cases}\hat{\boldsymbol{P}}_k=(1-\boldsymbol{K}_k\boldsymbol{G}_k)\check{\boldsymbol{P}}_k\\ \hat{\boldsymbol{x}}_k=\check{\boldsymbol{x}}_k+\boldsymbol{K}_k\left(y_k-g\left(\check{\boldsymbol{x}}_k,0\right)\right)\end{cases}$$

第二题

$$\mu_x = \sum_{i=0}^{2L} \alpha_i x_i$$

$$\Sigma_{xx} = \sum_{i=0}^{2L} \alpha_i (x_i - \mu_x)(x_i - \mu_x)^{\mathrm{T}}$$

其中:

$$\alpha_i = \begin{cases} \frac{\kappa}{L+\kappa} & i = 0 \\ \frac{1}{2}\frac{1}{L+\kappa} & \text{其他} \end{cases}$$

□ 习题：请验证此式

作业概况： 完成情况很好

证明思路： 直接将 α_i与 x_i 代入左式中验证

解答：

已知：↵

$$x_0 = \mu_x$$

$$x_i = \mu_x + \sqrt{L+\kappa}\,\mathrm{col}_i L \qquad i = 1, \cdots, L$$

$$x_{i+L} = \mu_x - \sqrt{L+\kappa}\,\mathrm{col}_i L$$

$$\alpha_i = \begin{cases} \frac{\kappa}{L+\kappa} & i = 0 \\ \frac{1}{2}\frac{1}{L+\kappa} & \text{其他} \end{cases}$$

验证：

$$\mu_x = \sum_{i=0}^{2L} \alpha_i\, x_i \quad (1)$$

$$\Sigma_{xx} = \sum_{i=0}^{2L} \alpha_i (x_i - \mu_x)(x_i - \mu_x)^T \quad (2)$$

(1)式验证：

$$\begin{aligned}
&\sum_{i=0}^{2L} \alpha_i\, x_i \\
&= \alpha_0 x_0 + \alpha_1 x_1 + \alpha_{L+1} x_{L+1} + \alpha_2 x_2 + \alpha_{L+2} x_{L+2} + \cdots + \alpha_L x_L + \alpha_{2L} x_{2L} \\
&= \frac{\kappa}{L+\kappa}\mu_x + \frac{1}{2(L+\kappa)}\left[\mu_x + \sqrt{L+\kappa}\,col_1 L + \mu_x - \sqrt{L+\kappa}\,col_1 L + \cdots + \mu_x + \sqrt{L+\kappa}\,col_1 L + \mu_x - \sqrt{L+\kappa}\,col_1 L\right] \\
&= \frac{\kappa}{L+\kappa}\mu_x + \frac{1}{2(L+\kappa)} 2L\mu_x \\
&= \mu_x
\end{aligned}$$

解答：

(2)式验证：

$$\sum_{i=0}^{2L} \alpha_i (x_i - \mu_x)(x_i - \mu_x)^T$$

$$= \alpha_0 (\mu_x - \mu_x)(\mu_x - \mu_x)^T + \sum_{i=1}^{L} \alpha_i \left(\mu_x + \sqrt{L+\kappa} col_i L - \mu_x\right)\left(\mu_x + \sqrt{L+\kappa} col_i L - \mu_x\right)^T + \sum_{i=1}^{L} \alpha_{i+L} \left(\mu_x - \sqrt{L+\kappa} col_i L - \mu_x\right)\left(\mu_x - \sqrt{L+\kappa} col_i L - \mu_x\right)^T$$

$$= 2\sum_{i=1}^{L} \frac{1}{2(L+\kappa)} \left(\sqrt{L+\kappa} col_i L\right)\left(\sqrt{L+\kappa} col_i L\right)^T$$

$$= 2\sum_{i=1}^{L} \frac{1}{2(L+\kappa)} \left(\sqrt{L+\kappa}\right)\left(\sqrt{L+\kappa}\right)(col_i L)(col_i L)^T$$

$$= \sum_{i=1}^{L} (col_i L)(col_i L)^T$$

$$= \Sigma_{xx}$$

解答：

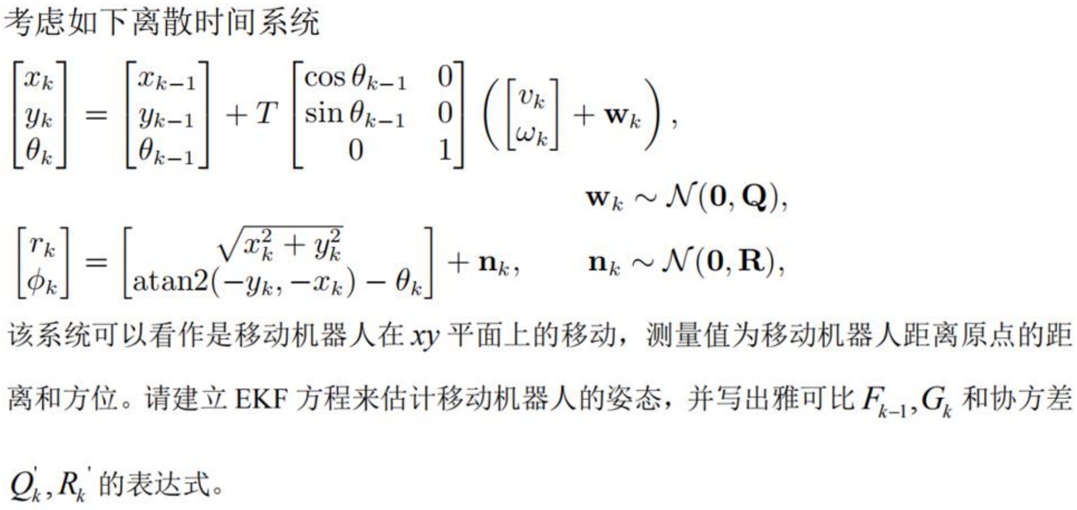
$$\sum_{i=1}^{2L}(col_iL)(col_iL)^T = \Sigma_{xx}$$ 的证明：

$$令\mathbf{L}=\begin{bmatrix} L_{11} & & & \\ L_{21} & L_{22} & & \\ \vdots & \vdots & \ddots & \\ L_{L1} & L_{L2} & \cdots & L_{LL} \end{bmatrix}$$

$$(col_i\mathbf{L})(col_i\mathbf{L})^T = \begin{bmatrix} L_{1i} \\ L_{2i} \\ \vdots \\ L_{Li} \end{bmatrix}\begin{bmatrix} L_{1i} & L_{2i} & \cdots & L_{Li} \end{bmatrix} = \begin{bmatrix} L_{1i}^{\ 2} & L_{1i}L_{2i} & \cdots & L_{1i}L_{Li} \\ L_{2i}L_{1i} & L_{2i}^{\ 2} & \cdots & L_{1i}L_{Li} \\ \vdots & \vdots & \ddots & \vdots \\ L_{Li}L_{1i} & L_{Li}L_{2i} & \cdots & L_{Li}^{\ 2} \end{bmatrix}$$

$$\sum_{i=1}^{L}\begin{bmatrix} L_{1i}^{\ 2} & L_{1i}L_{2i} & \cdots & L_{1i}L_{Li} \\ L_{2i}L_{1i} & L_{2i}^{\ 2} & \cdots & L_{1i}L_{Li} \\ \vdots & \vdots & \ddots & \vdots \\ L_{Li}L_{1i} & L_{Li}L_{2i} & \cdots & L_{Li}^{\ 2} \end{bmatrix} = \mathbf{L}\mathbf{L}^T = \sum{}_{xx}$$

第三题



考虑如下离散时间系统

$$\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + T \begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} v_k \\ \omega_k \end{bmatrix} + \mathbf{w}_k \right),$$

$$\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}),$$

$$\begin{bmatrix} r_k \\ \phi_k \end{bmatrix} = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \operatorname{atan2}(-y_k, -x_k) - \theta_k \end{bmatrix} + \mathbf{n}_k, \qquad \mathbf{n}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}),$$

该系统可以看作是移动机器人在 xy 平面上的移动，测量值为移动机器人距离原点的距离和方位。请建立 EKF 方程来估计移动机器人的姿态，并写出雅可比 F_{k-1}, G_k 和协方差 $Q_k^{'}, R_k^{'}$ 的表达式。

作业概况：完成情况较好，部分作业存在细节性错误

如：1、将噪声项w与输入项ω搞混 2、atan2()求导错误 等等

证明思路：将多个矩阵的运算合并为一个矩阵，再进行后续计算

解答：

$$令X_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix}, V_k = \begin{bmatrix} v_k \\ \omega_k \end{bmatrix}$$

$$\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = f(X_{k-1}, V_k, w_k)$$

$$= \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + T \begin{bmatrix} cos\ \theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} v_k \\ \omega_k \end{bmatrix} + w_k \right)$$

$$= \begin{bmatrix} x_{k-1} + Tcos\ \theta_{k-1}\ v_k \\ y_{k-1+}Tcos\ \theta_{k-1}\ v_k \\ \theta_{k-1} + \omega_k \end{bmatrix} + T \begin{bmatrix} cos\ \theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k$$

$$F_{k-1} = \frac{\partial f(X_{k-1}, V_k, w_k)}{\partial [x_{k-1} \quad y_{k-1} \quad \theta_{k-1}]} \Bigg|_{\hat{X}_{k-1} V_k, 0}$$

$$= \begin{bmatrix} 1 & 0 & -Tsin\theta_{k-1} v_k \\ 0 & 1 & Tcos\theta_{k-1} v_k \\ 0 & 0 & 1 \end{bmatrix} \Bigg|_{\hat{X}_{k-1}, V_k, 0} + \frac{\partial\ T \begin{bmatrix} cos\ \theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix}}{\partial [x_{k-1} \quad y_{k-1} \quad \theta_{k-1}]} w_k \Bigg|_{\hat{X}_{k-1}, V_k, 0}$$

$$= \begin{bmatrix} 1 & 0 & -Tsin\hat{\theta}_{k-1} v_k \\ 0 & 1 & Tcos\hat{\theta}_{k-1} v_k \\ 0 & 0 & 1 \end{bmatrix}$$

解答：

$$w_k' = \frac{\partial f\left(X_{k-1}, V_k, w_k\right)}{\partial {w_k}^T}\Bigg|_{\hat{X}_{k-1}, V_k, 0} w_k$$

$$= \frac{\partial\left(\begin{bmatrix} x_{k-1} + T\cos\theta_{k-1}\, v_k \\ y_{k-1+}T\cos\theta_{k-1}\, v_k \\ \theta_{k-1} + \omega_k \end{bmatrix} + T\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k\right)}{\partial {w_k}^T}\Bigg|_{\hat{X}_{k-1}, V_k, 0} w_k$$

$$= T\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k$$

$$Q_k' = E\left(w_k' {w_k'}^T\right)$$

$$= E\left(T\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k \left(T\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k\right)^T\right)$$

$$= E\left(T\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} w_k {w_k}^T \left(T\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix}\right)^T\right)$$

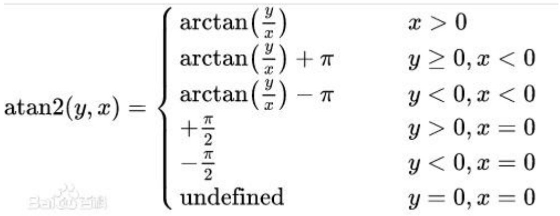
$$= T^2\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} E\left(w_k {w_k}^T\right)\begin{bmatrix} \cos\theta_{k-1} & \sin\theta_{k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= T^2\begin{bmatrix} \cos\theta_{k-1} & 0 \\ \sin\theta_{k-1} & 0 \\ 0 & 1 \end{bmatrix} Q_k \begin{bmatrix} \cos\theta_{k-1} & \sin\theta_{k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

解答：

$$\begin{bmatrix} r_k \\ \phi_k \end{bmatrix} = g(X_k, n_k) = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ atan2(-y_k, -x_k) - \theta_k \end{bmatrix} + n_k$$

$$G_k = \frac{\partial g(X_k, n_k)}{\partial [x_k \quad y_k \quad \theta_k]}\bigg|_{\breve{X}_{k,0}} = \frac{\partial \left(\begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ atan2(-y_k, -x_k) - \theta_k \end{bmatrix} \right)}{\partial [x_k \quad y_k \quad \theta_k]}\Bigg|_{\breve{X}_{k,0}} = \begin{bmatrix} \frac{\breve{x}_k}{\sqrt{\breve{x}_k^2 + \breve{y}_k^2}} & \frac{\breve{y}_k}{\sqrt{\breve{x}_k^2 + \breve{y}_k^2}} & 0 \\ -\frac{\breve{y}_k}{\breve{x}_k^2 + \breve{y}_k^2} & \frac{\breve{x}_k}{\breve{x}_k^2 + \breve{y}_k^2} & -1 \end{bmatrix}$$



$$\operatorname{atan2}(y, x) = \begin{cases} \arctan\left(\frac{y}{x}\right) & x > 0 \\ \arctan\left(\frac{y}{x}\right) + \pi & y \geq 0, x < 0 \\ \arctan\left(\frac{y}{x}\right) - \pi & y < 0, x < 0 \\ +\frac{\pi}{2} & y > 0, x = 0 \\ -\frac{\pi}{2} & y < 0, x = 0 \\ \text{undefined} & y = 0, x = 0 \end{cases}$$

$$\frac{\partial\ atan2(-y_k, -x_k)}{\partial\ x_k} = \frac{\partial\ \left(arctan\left(\frac{y_k}{x_k}\right) + \varphi\right)}{\partial\ x_k} = \frac{1}{1 + \left(\frac{y_k}{x_k}\right)^2}\left(-\frac{y_k}{{x_k}^2}\right) = -\frac{y_k}{{x_k}^2 + {y_k}^2}$$

$$n'_k = \frac{\partial g(X_k, n_k)}{\partial\ {n_k}^T}\bigg|_{\breve{X}_{k,0}} n_k = \frac{\partial(n_k)}{\partial\ {n_k}^T} n_k = n_k$$

$$R'_k = E\left[n'_k {n'_k}^T\right] = E\left[n_k {n_k}^T\right] = R$$

解答：

之后，可对照如下 EKF 的经典递归更新公式，估计移动机器人的姿态:

预测：$$\begin{cases}\check{P}_k = F_{k-1}\hat{P}_{k-1}{F_{k-1}}^T + Q'_k \\ \check{X}_k = f(\hat{X}_{k-1}V_k, 0)\end{cases}$$

卡尔曼增益：$$\boldsymbol{K}_k = \check{\boldsymbol{P}}_k \boldsymbol{G}_k^T \left(\boldsymbol{G}_k \check{\boldsymbol{P}}_k \boldsymbol{G}_k^T + \boldsymbol{R}'_k\right)^{-1}$$

更新：$$\begin{cases}\hat{P}_k = (1 - K_k G_k)\check{P}_k \\ \hat{X}_k = \check{X}_k + K_k(y_k - g(\check{X}_k, 0))\end{cases}$$

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感谢各位聆听！

Thanks for Listening