Líneas de Transmisión y Antenas

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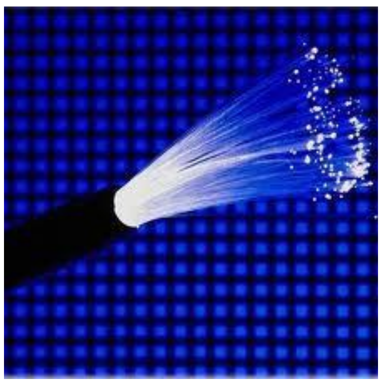
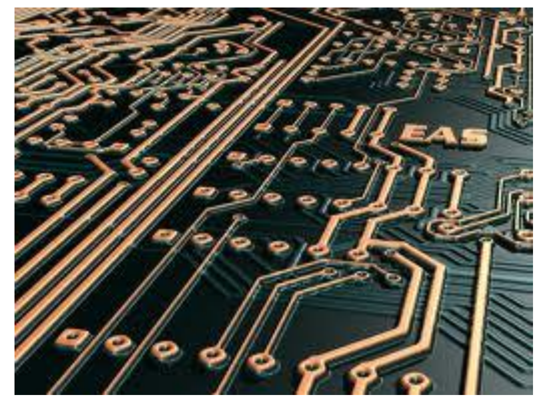
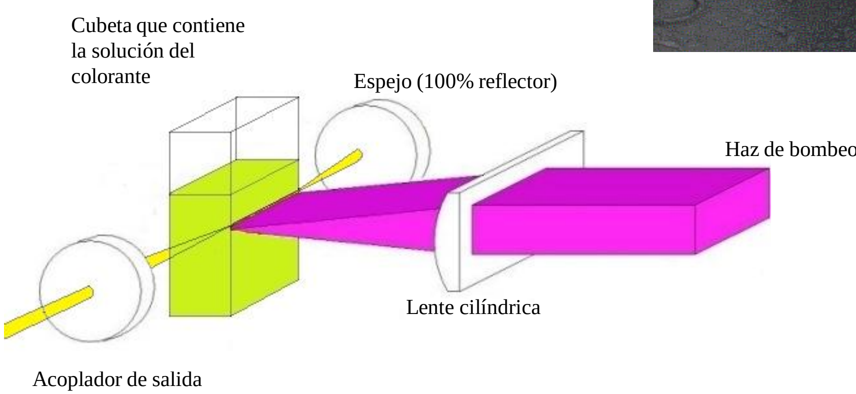
Radio Observatorio de Jicamarca

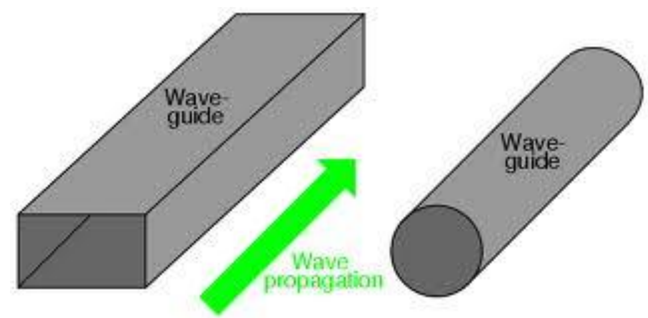
Línea de transmisión (1)

- Definición: Dispositivo para transmitir o guiar energía de un punto a otro









Línea de transmisión. Ecuaciones de Maxwell (1)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

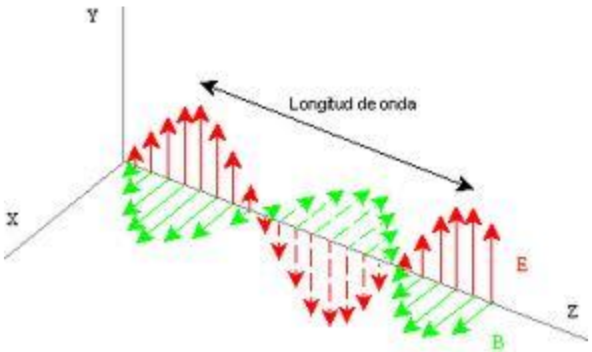
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \cancel{\mu_0 \mathbf{J}} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times H = \frac{dD}{dt}$$ *Ley de Ampere*

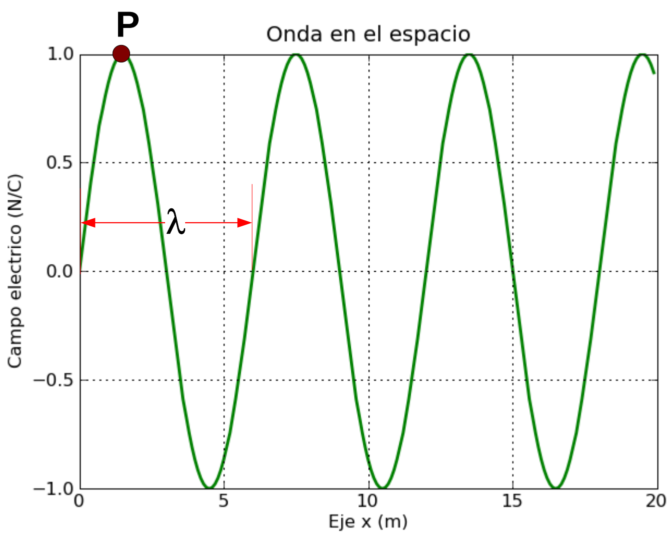
$$\nabla \times E = -\frac{dB}{dt}$$ Ley de Faraday

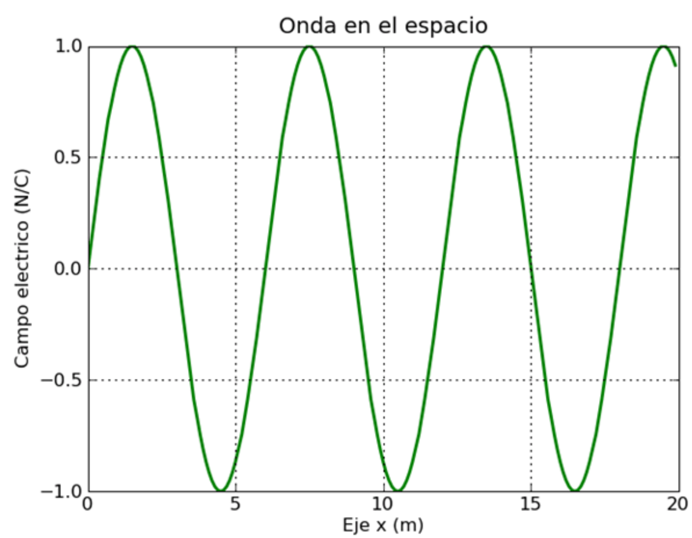
Línea de transmisión. Ecuaciones de Maxwell



$$\frac{\partial^2 Hx}{\partial t^2} = \frac{1}{\mu\epsilon}\frac{\partial^2 Hx}{\partial t^2} \qquad \frac{\partial^2 Ey}{\partial t^2} = \frac{1}{\mu\epsilon}\frac{\partial^2 Ey}{\partial t^2}$$

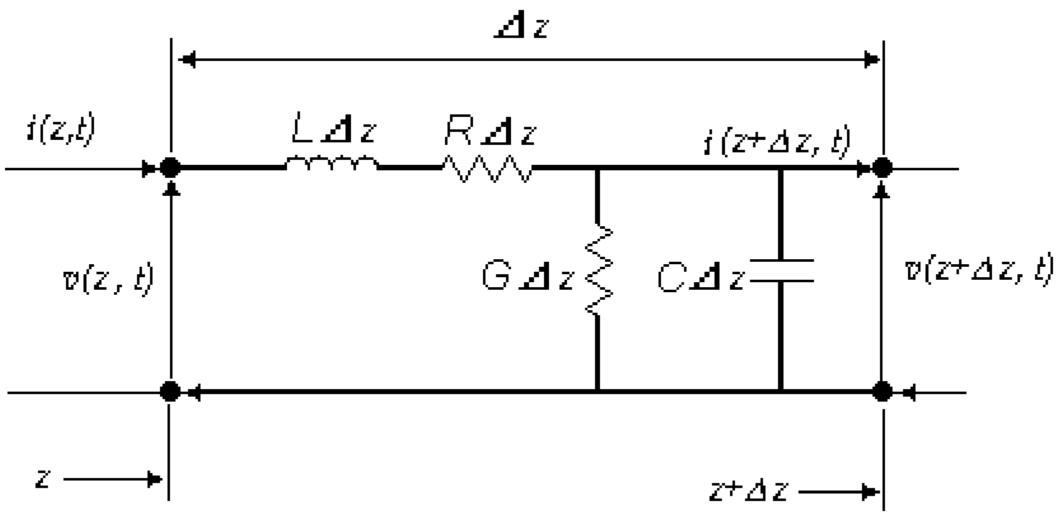
$$E_y = E_0 Sin\left[\frac{2\pi}{\lambda}x + \varpi t\right]$$





$$E(x,t) = Eo * \sin(\frac{2\pi}{\lambda} * x + 2\pi f * t)$$

Línea de transmisión. Solución para cantidades distribuidas de L,C y G



$$\frac{\partial^2 E_y}{\partial x^2} - j\varpi L(G + j\varpi C)E_y = 0$$

$$\frac{\partial^2 V}{\partial x^2} - j\varpi L(G + j\varpi C) * V = 0$$

$$V = E_y * h \Rightarrow E_y = V / h$$

$$\boxed{\frac{\partial^2 V}{\partial x^2} - Y * Z * V = 0}$$

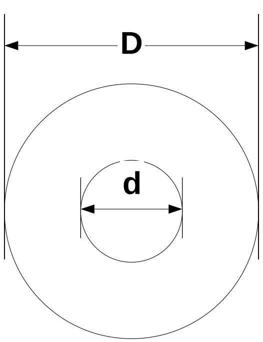
Línea de transmisión, Impedancia Característica

$$\boxed{\frac{\partial^2 V}{\partial x^2} - Y * Z * V = 0}$$

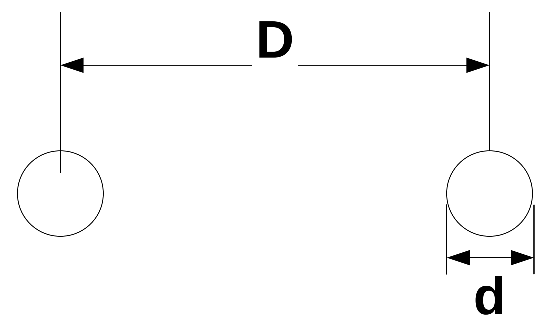
$$V = V_1 e^{j(wt+\beta x)} + V_2 e^{j(wt-\beta x)}$$

$$I = \frac{V_1}{\sqrt{Z/Y}} e^{j(wt+\beta x)} + \frac{V_2}{\sqrt{Z/Y}} e^{j(wt-\beta x)}$$

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

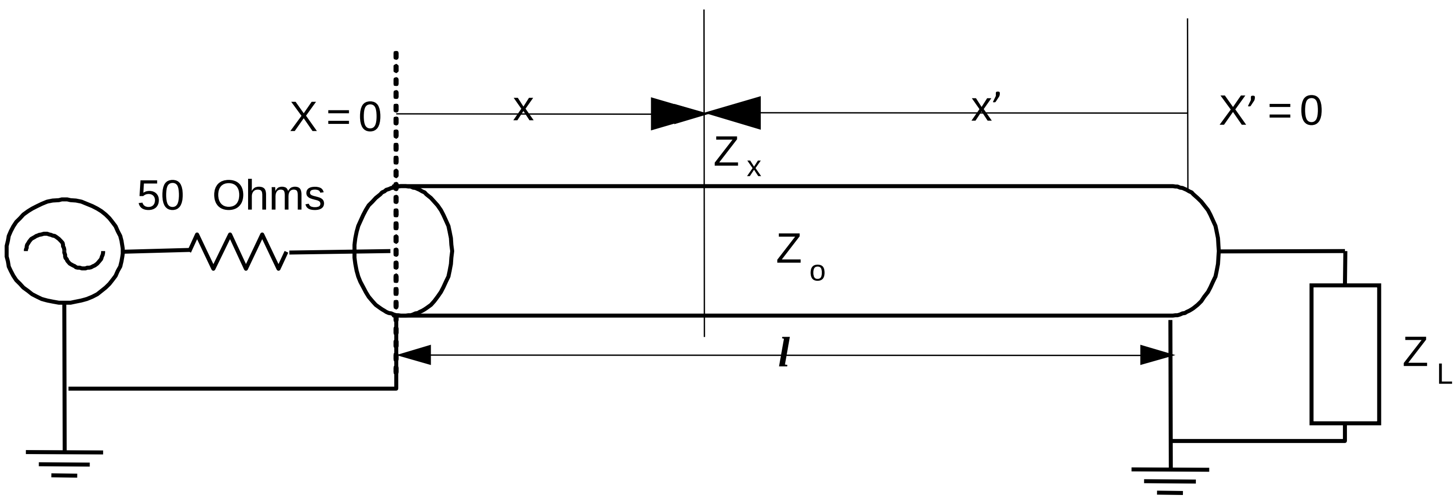


$$Z_o = \frac{138}{\sqrt{\varepsilon_r}} \log(D/a)$$



$$Z_o = 276 \log(D/d)$$

Línea de transmisión. Impedancia de la línea

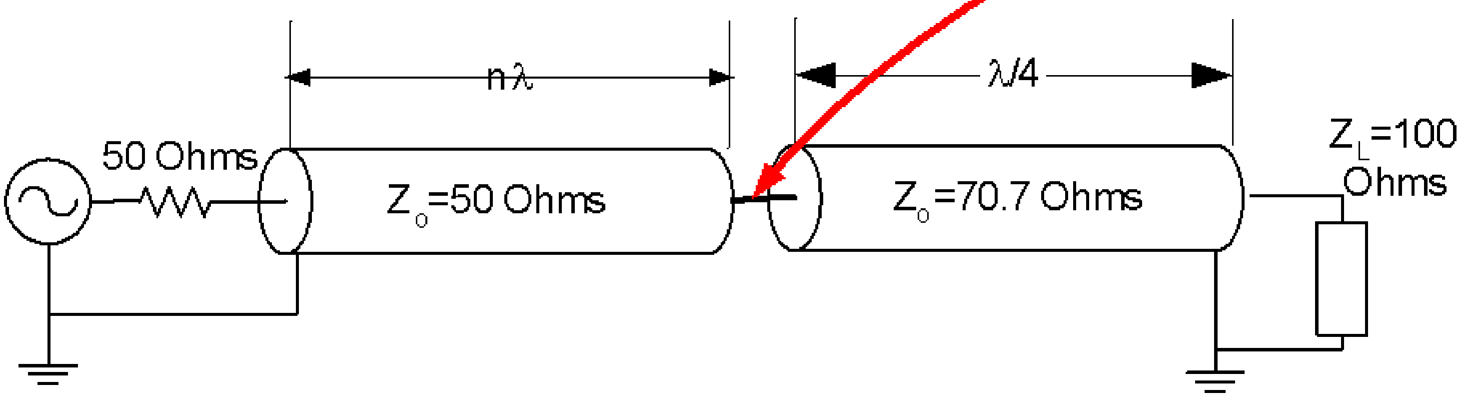


$$Z_x = Z_o * \left(\frac{Z_L + jZ_o \tan\left(\frac{2\pi * (l - x)}{\lambda}\right)}{Z_o + jZ_L \tan\left(\frac{2\pi * (l - x)}{\lambda}\right)} \right)$$

Línea de transmisión. Ecuación del Transformador de lambda/4

$$Z_{\lambda/4} = Z_o * \left(\frac{Z_L + jZ_o \tan\left(\overset{\infty}{\frac{\pi}{2}}\right)}{Z_o + jZ_L \tan\left(\underset{\infty}{\frac{\pi}{2}}\right)} \right) \Rightarrow Z_{\lambda/4} = \frac{{Z_o}^2}{Z_L}$$

$$Z_{\lambda/4} = \frac{70.7^2}{100} = 49.98$$



Línea de transmisión. Ejemplos de Transformador de lambda/4

Transformadores de Lambda cuartos ubicados en el módulo

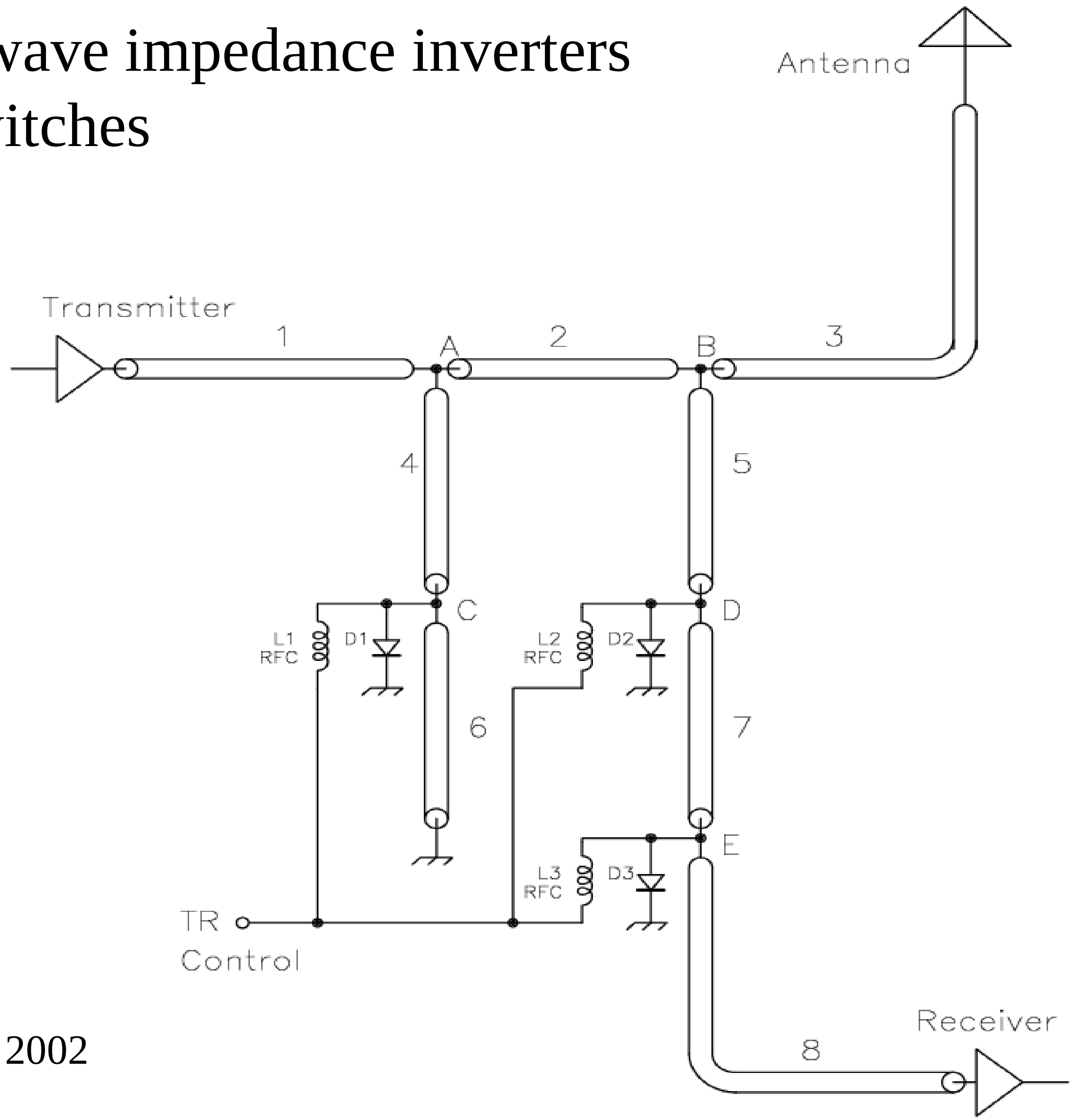
Línea de transmisión. Ecuación del Transformador de lambda/4, caso de corto circuito y circuito abierto

$$Z_{\lambda/4} = \frac{{Z_o}^2}{Z_L}$$

$$Z_{sc} \to \infty \qquad Z_{oc} \to 0$$

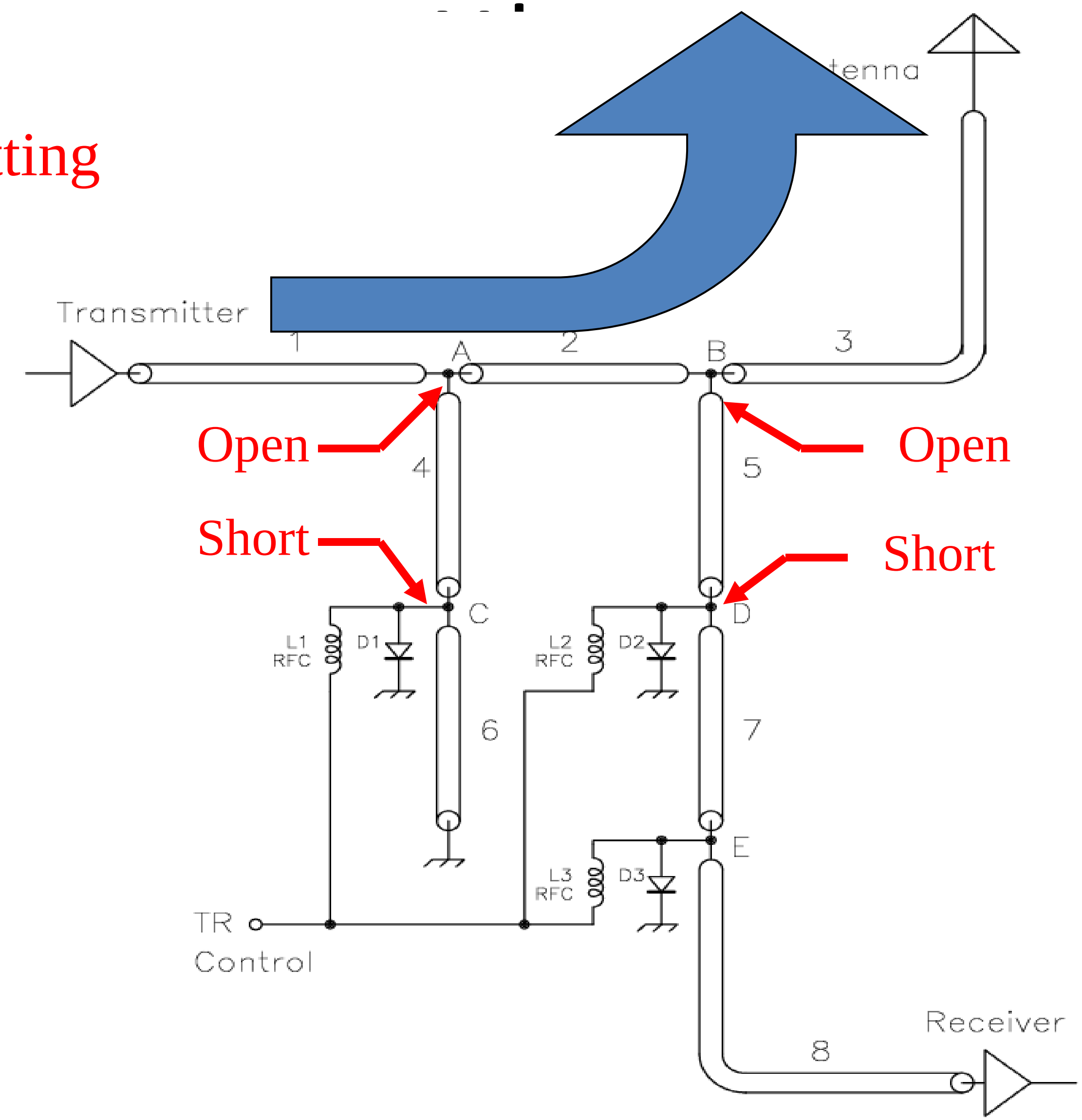
Línea de transmisión. Ejemplos de Transformador de lambda/4

- Quarter-wave impedance inverters
- Diode switches



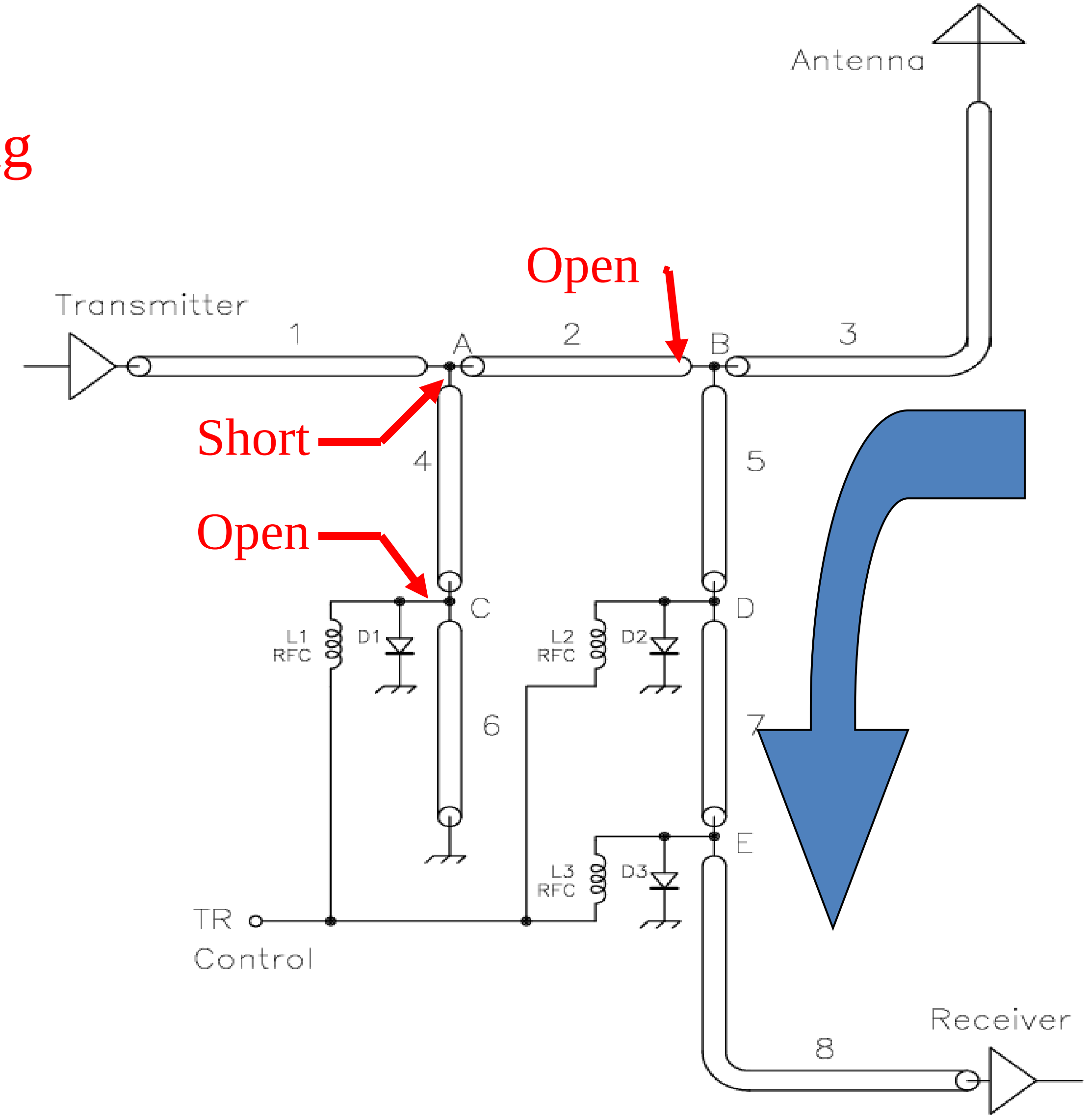
Línea de transmisión. Ejemplos de Transformador de lambda/4

- Transmitting

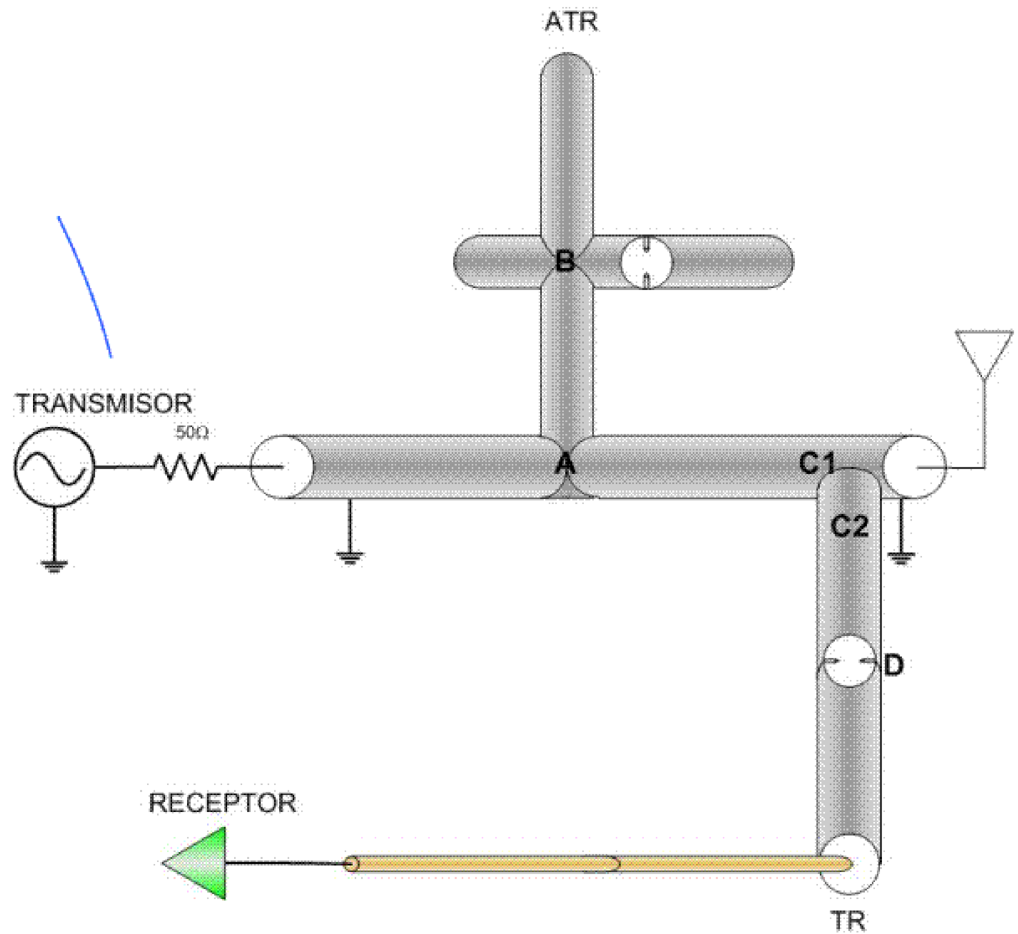


Línea de transmisión. Ejemplos de Transformador de lambda/4

- Receiving



Línea de transmisión. Ejemplos de Transformador de lambda/4



Pregunta 1

Se quiere adaptar 3 líneas de antenas de 50 Ohm a una línea de transmisión de 50 Ohm. ¿Cual debe ser la impedancia característica del adaptador?

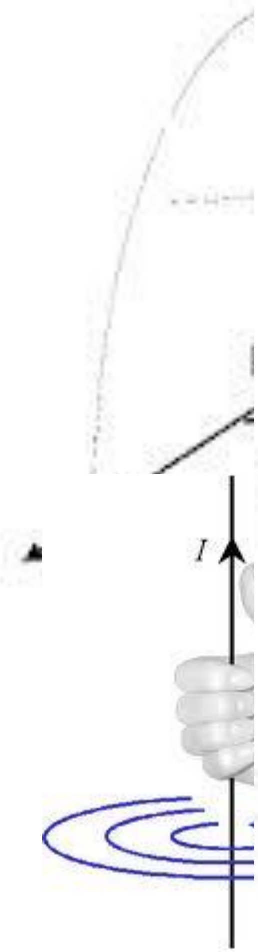
(a) 37 Ohms

(b) 50 Ohms

(c) 28 Ohms

(d) 100 Ohms

Introducción a antenas. Algunos conceptos



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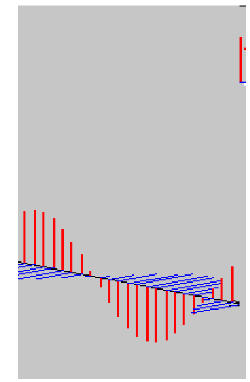
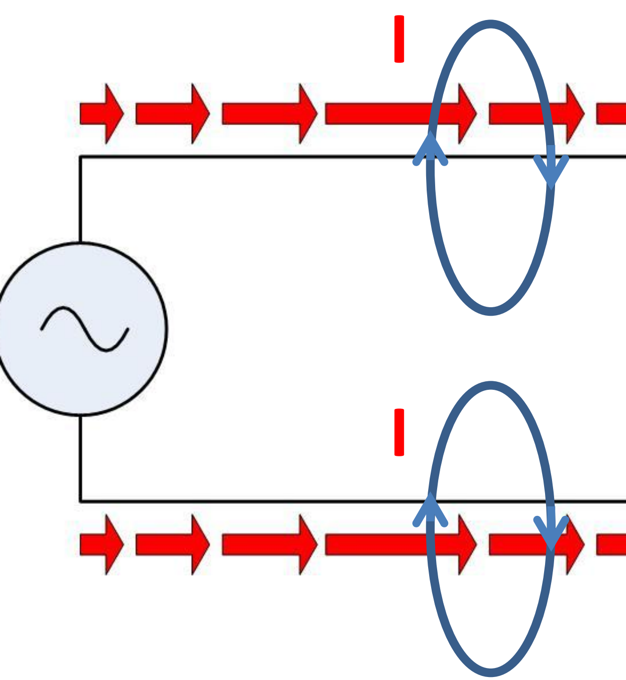
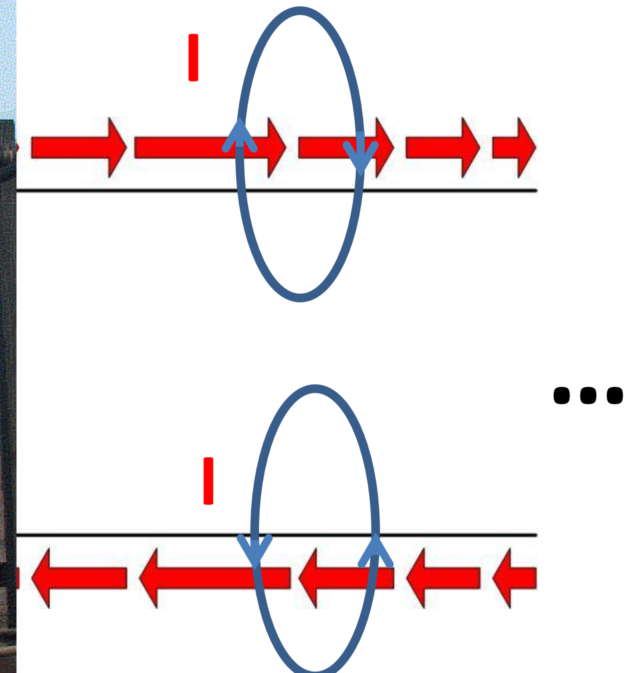


Pregunta 2

Se tiene el circuito (a) y (b). ¿Cual de ellos radiara?



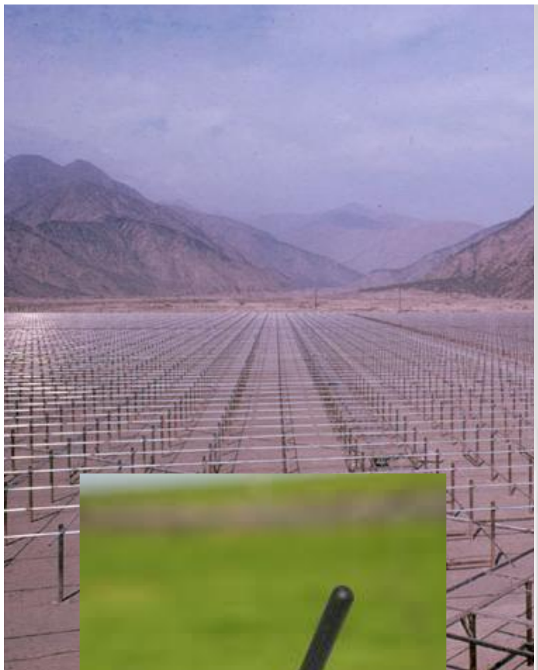
(a)



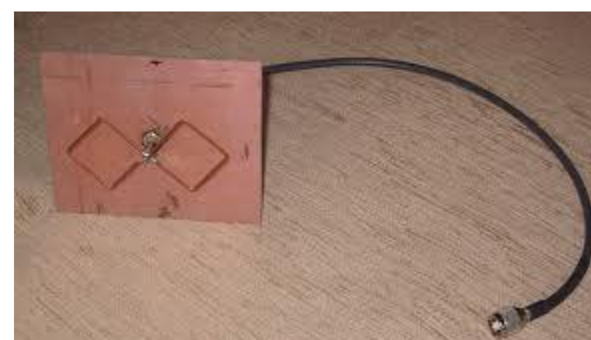
(b)

Antenas . ¿Qué es una antena?

- Cualquier elemento que radie la potencia que se le suministra, con la direccionalidad adecuada







Antenas . Dipolo Elemental, campos lejanos

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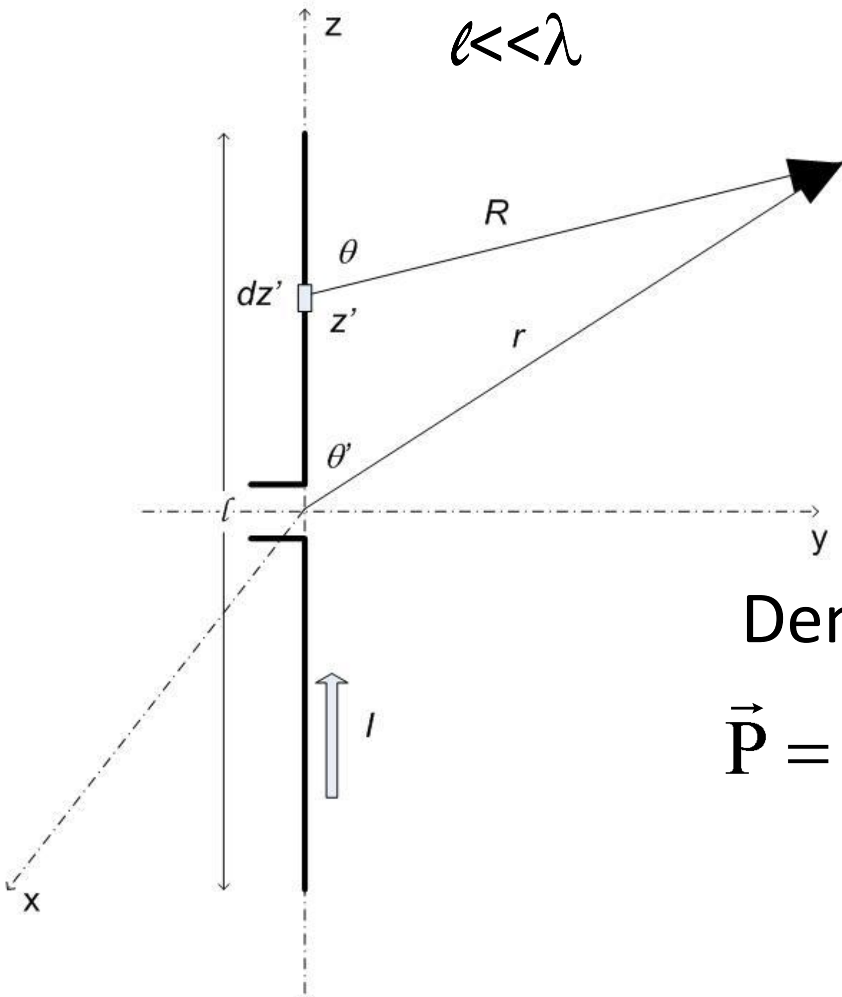


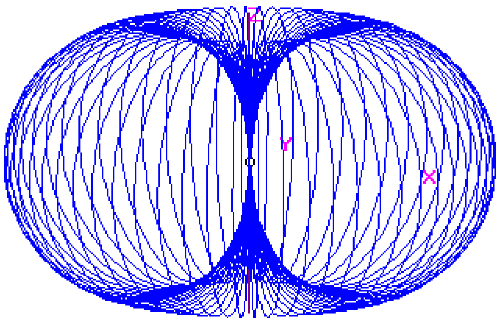
$$\vec{E}_{rad} = j\varpi\mu \frac{Il}{4\pi}\frac{e^{-jkr}}{r}\sin\theta\ \widehat{\theta}$$

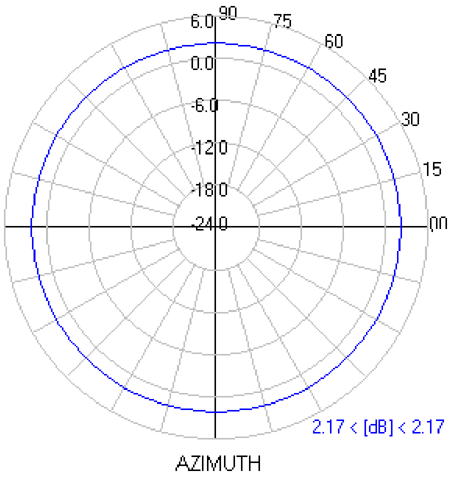
$$\vec{H}_{rad} = jk\frac{Il}{4\pi}\frac{e^{-jkr}}{r}\sin\theta\ \widehat{\phi}$$

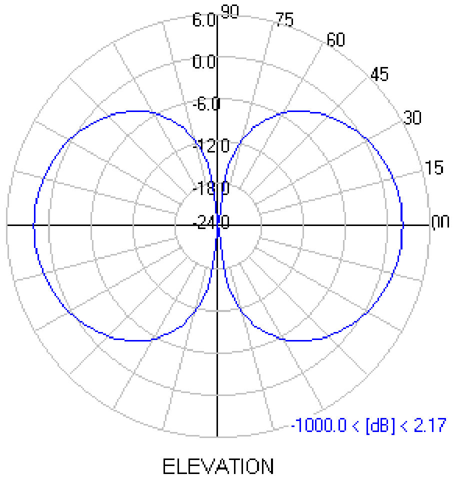
Densidad de potencia radiada

$$\vec{\mathrm{P}} = \mathrm{Re}\left[\vec{E} x \vec{H}^*\right] = jk\frac{(Il)^2}{16\pi^2}\frac{k\varpi\mu}{r^2}\sin\theta^2\hat{r}$$

Antenas . Dipolo Elemental, campos lejanos







Potencia radiada

$$P_r = \iint_S \vec{P}.d\vec{s} = I^2 \pi \frac{\sqrt{\frac{\mu}{\varepsilon}}}{2} \frac{l^2}{\lambda^2} \frac{4}{3}$$

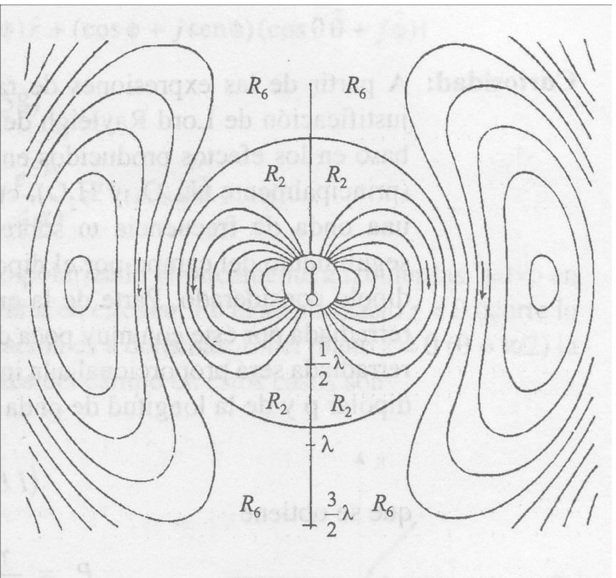
Resistencia de radiación

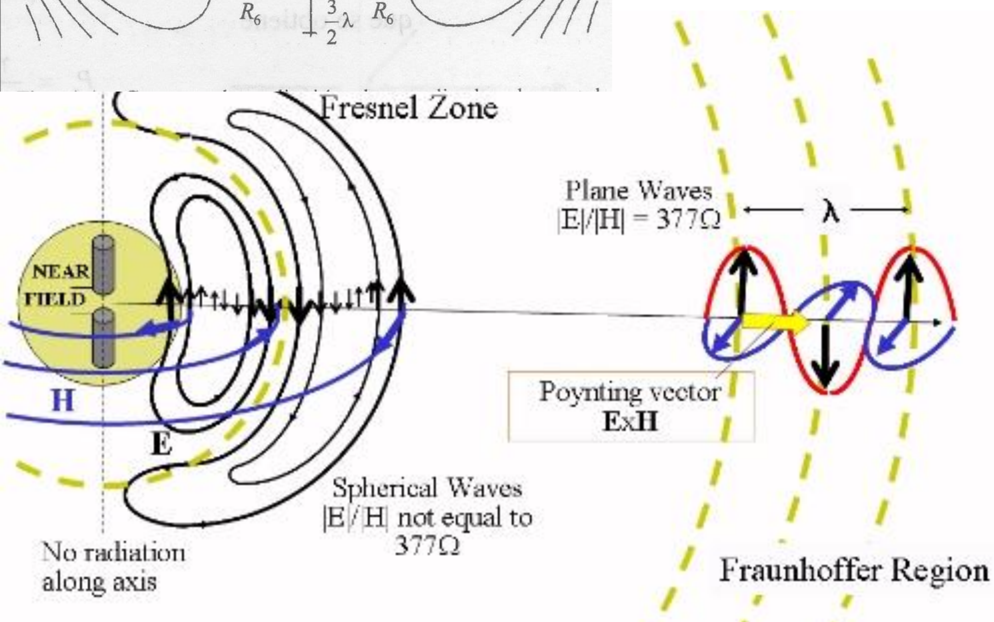
$$R_r = \frac{P_r}{I^2} \approx 790 \frac{l^2}{\lambda^2}$$

Directividad

$$D(\theta,\phi) = \frac{\mathrm{P}}{\mathrm{P}_{med}} = 1.5 \sin \theta^2$$

Antenas . Dipolo Elemental, campos lejanos





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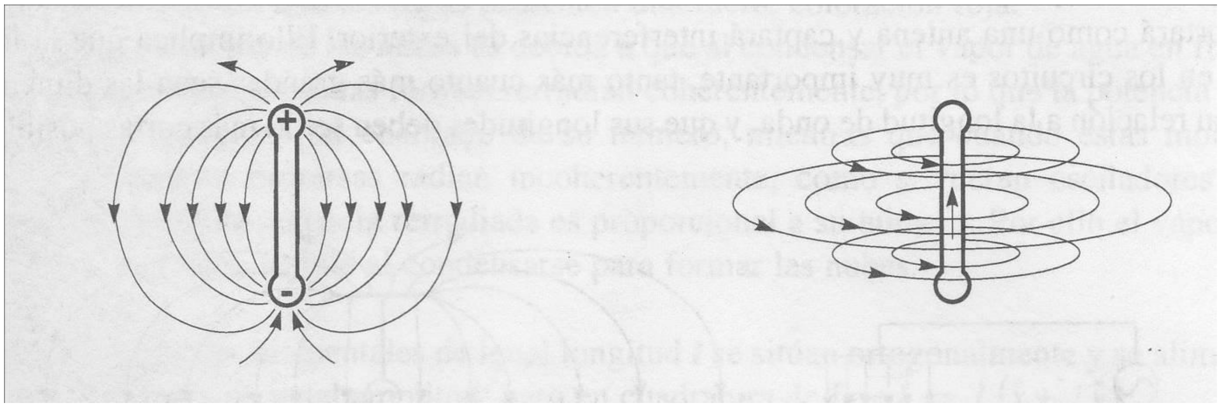
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Antenas . Dipolo Elemental, campos cercanos

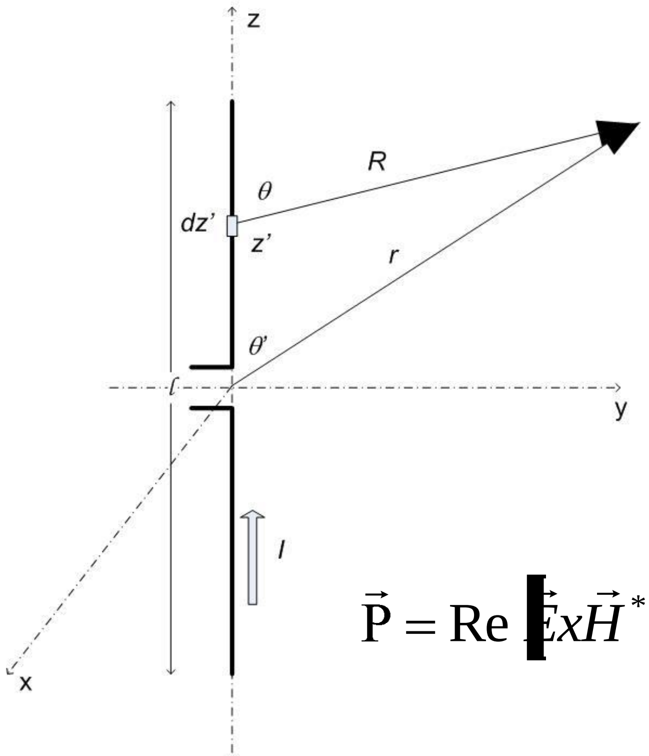


$$H_{\phi}^{i}=\frac{Il}{4\pi}\frac{\sin\theta}{r^{2}}e^{-jkr}$$

$$E_{r}^{i}=\frac{Il}{4\pi}\frac{2\cos\theta}{j\varpi\varepsilon r^{3}}e^{-jkr}=Ql\frac{2\cos\theta}{4\pi\varepsilon r^{3}}e^{-jkr}$$

$$E_{\theta}^{i}=\frac{Il}{4\pi}\frac{\sin\theta}{j\varpi\varepsilon r^{3}}e^{-jkr}=Ql\frac{\sin\theta}{4\pi\varepsilon r^{3}}e^{-jkr}$$

Antenas. Dipolo cilíndrica



$$E_\theta = j\frac{\sqrt{\frac{\mu}{\varepsilon}}}{2\pi r}e^{-jkr}\frac{\cos(\mathrm{kH}\cos\theta)-\cos kH}{\sin\theta}$$

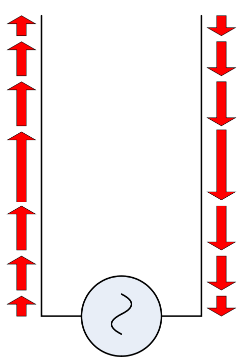
$$E_\phi = 0$$

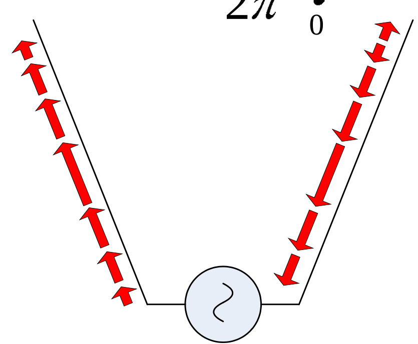
$$H_\theta = 0$$

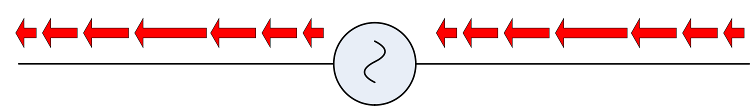
$$H_\phi = \frac{E_\theta}{\sqrt{\frac{\mu}{\varepsilon}}}$$

$$\vec{\mathrm{P}} = \mathrm{Re}\left[\vec{E}x\vec{H}^*\right] = jk\frac{I^2}{4\pi^2 r^2}\left(\frac{\cos(kH\cos\theta - \cos kH)}{\sin\theta}\right)^2$$

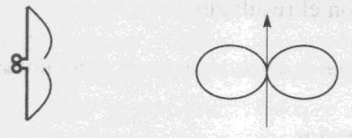
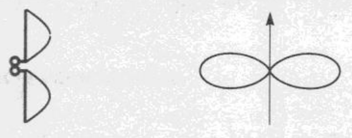
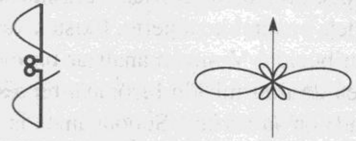
$$P_r = \iint_S \vec{P}.d\vec{s} = I_m{}^2\frac{\sqrt{\frac{\mu}{\varepsilon}}}{2\pi}\int_0^\pi \frac{(\cos(kH\cos\theta - \cos kH))^2}{\sin\theta}\,d\theta$$



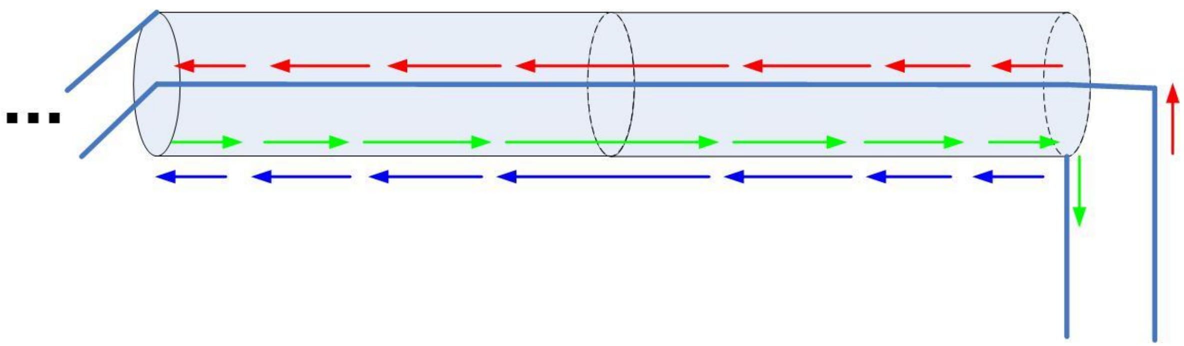


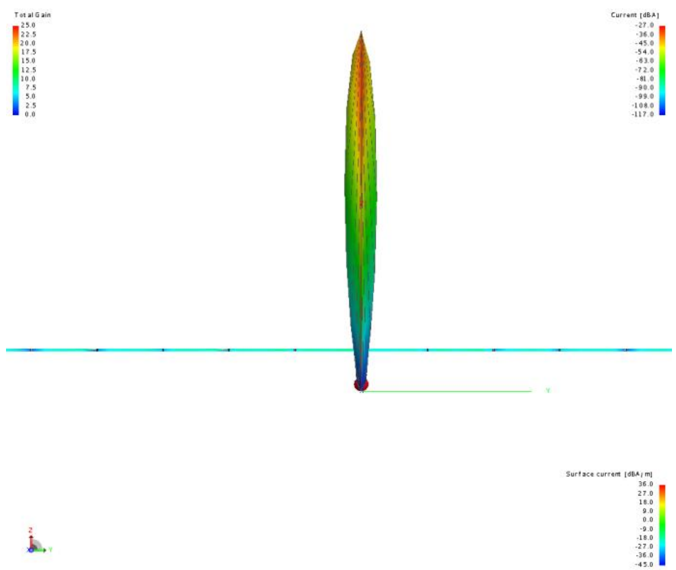


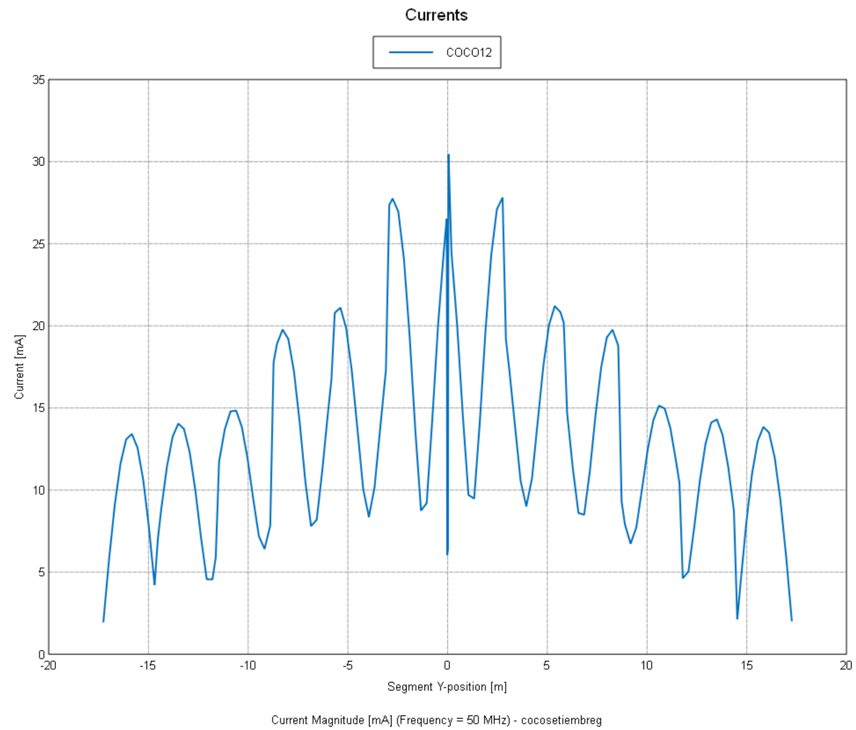
Antenas . Dipolo de varias longitudes

	$H = \lambda/4$ $\Delta\theta_{-3dB} = 78°$	$R_{rm} = 73\ \Omega$ $R_{re} = 73\ \Omega$ $D = 1{,}64$
	$H = 3\lambda/8$ $\Delta\theta_{-3dB} = 64°$	$R_{rm} = 180\ \Omega$ $R_{re} = 360\ \Omega$ $D = 1{,}94$
	$H = \lambda/2$ $\Delta\theta_{-3dB} = 48°$	$R_{rm} = 199\ \Omega$ $R_{re} = \infty\ \Omega$ $D = 2{,}41$
	$H = 5\lambda/8$ $\Delta\theta_{-3dB} = 33°$	$R_{rm} = 105\ \Omega$ $R_{re} = 210\ \Omega$ $D = 3{,}33$
	$H = 3\lambda/4$ $\Delta\theta_{-3dB} = 33°$ $\theta_{max} = 43°$	$R_{rm} = 99{,}5\ \Omega$ $R_{re} = 99{,}5\ \Omega$ $D = 2{,}17$
	$H = \lambda$ $\Delta\theta_{-3dB} = 27°$ $\theta_{max} = 57°$	$R_{rm} = 260\ \Omega$ $R_{re} = \infty\ \Omega$ $D = 2{,}52$

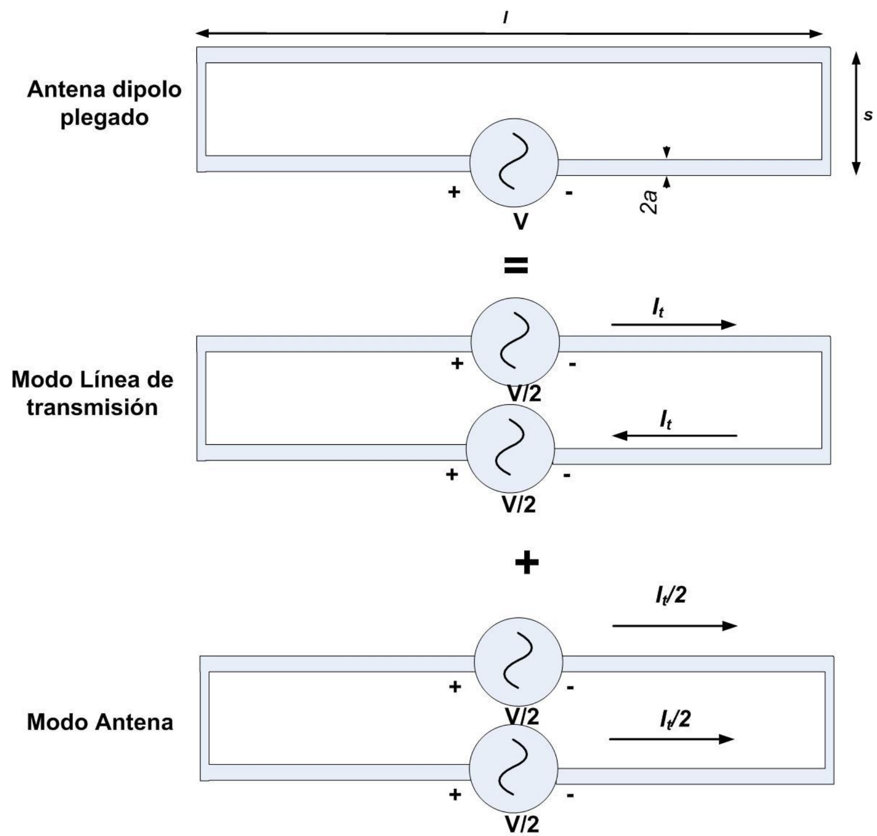
Antenas. COCO



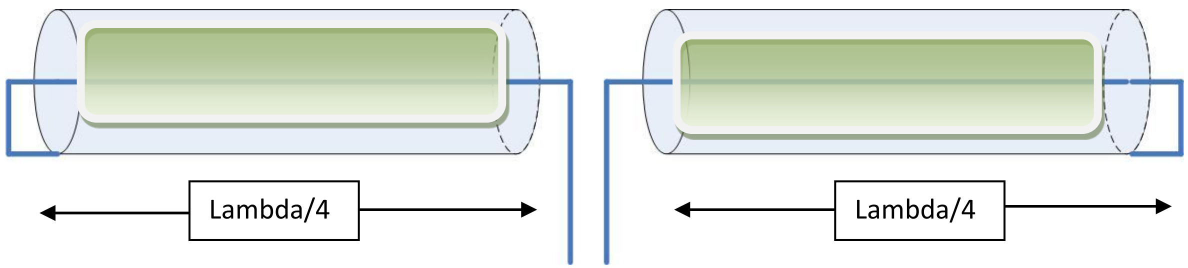


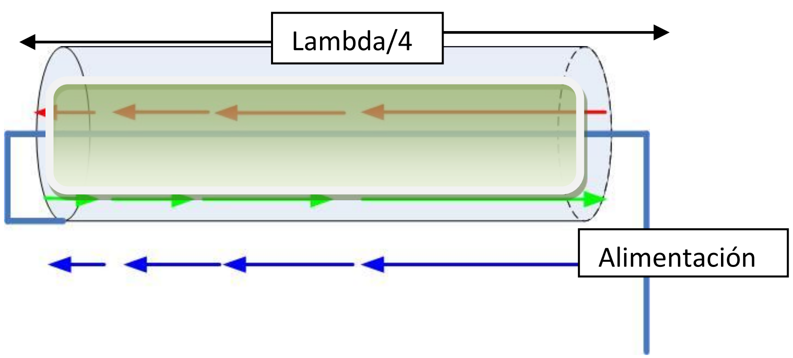


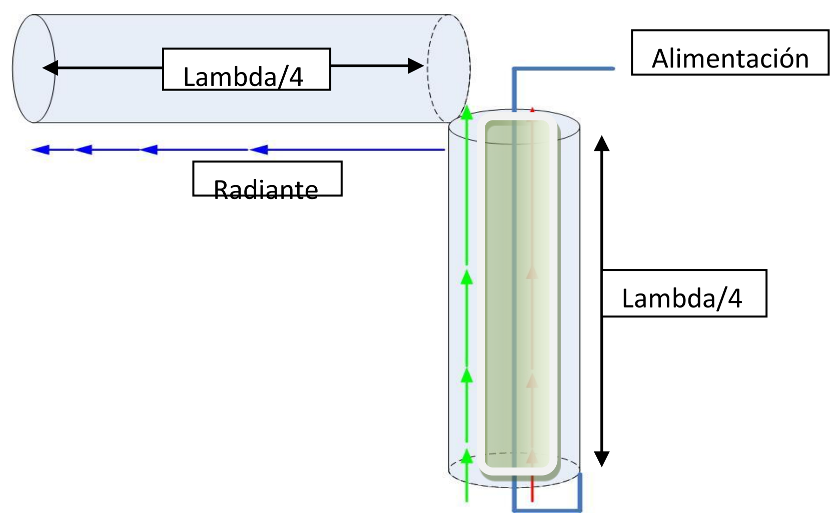
Antenas. Dipolo plegada



Antenas. Dipolo plegada con coaxial







Antenas. Solución utilizando Métodos numéricos

Calcular el vector potencial A(z) en función de V_i

$$A_z(z) = C_1 \cos(kz) + + C_2 \sin(kz) - \frac{jk}{2w\mu_o}\left[\sum_{i=2}^{n} V_i \left[\sin(k|z - z_i|)\right]\right]$$

Resolver I(z) de la ecuación de potencial vector A(z)

$$A_z(\vec{r}) = \int_{V'} \frac{\mu \vec{J}(\vec{r}\,')e - jk|\vec{r} - \vec{r}\,'|}{4\pi|\vec{r} - \vec{r}\,'|} dV'$$

La solución de la integral compleja de A se puede resolver por métodos numéricos, mediante la aproximación de la corriente a funciones conocidas

FIN