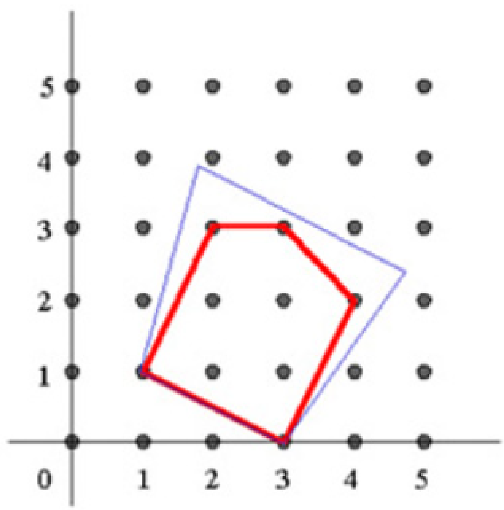
Integer linear programming 整数优化

Programming 是运筹学的说法，Optimizing 是计算机、统计学那边的叫法。
求解整数优化问题(ILP)的两种基本办法是：

- Cutting plane methods
- Enumerative methods

无论是哪一种方法，基本的思路都是**reduce the number of solutions to be searched and eventually arrive at the optimal solution**

关于整数优化，我们可以首先讨论一下它的背景。整数由于其在数域上的离散特点，使得整数优化也被称作"离散优化"(Discrete Optimization). 优化问题我们会讨论可行域的定义.

> 与线性规划连续的**可行域**（可行解组成的集合）不同，整数规划的可行域是离散的。
>
> 
>
>
> 如上图，一条蓝线代表一个线性不等式，但是这里x,y自变量被约束成整数变量，因此可行域变成了红线区域内的9个离散的黑点。（线性规划的可行域是蓝色线段内部所有的区域）
>
> **凸包**（**Convex Hull**）:整数规划所有可行解的凸包围，即图中红线组成的多面体（想象多维的情况）。凸包是未知的，已知的是蓝线的不等式，并且凸包是非常难求解的，或者形成凸包需要指数数量级的线性不等式（图中红线）。如果知道了凸包的所有线性表示，那么整数规划问题就可以**等价**为求解一个凸包表示的线性规划问题。

这里要补充的知识是凸集与凸方程(Convex Set & Convex Function). 在之前的文档中曾简单介绍凸函

数，更完整的介绍将会在看完教材后单开一章讲这个问题。
TODO: Convex def research

Cutting Plane Methods 切割平面法

除课件与教材外，另一篇重要的参考引用资料出自知乎文章“整数规划经典方法——割平面法(Cutting Plane Method)”

The gerneral intent of cutting plane methods for solving ILP

Let's talk about the oldest cutting plane method, it has **several principles**:

- A cut never excludes any integer solution from the new feasible region.
- Each cut will reduce the feasible region of the original LP problem.
- Each cut passes through at least one integer point.
- In a finte number of steps, the ILP problem is solved(can be solved) as an LP problem.

整数规划的精确算法--分支定界法（Branch-and-Bound）

参考资料：【学界】整数规划经典方法--割平面法（Cutting Plane Method）

TODO:先交作业，补完有问题再问

上机作业

编辑器 - D:\Gurobi\win64\matlab\problem1.m

Test_gurobi.m | problem1.m | problem3.m | problem4.m | problem5.m | problem2.m

```
names = {'x' ; 'y'; 'z';'a';'b'};
model.A = sparse([1 0 0 -1/2 5/2; 0 1 0 -1/2 3/2 ; 0 0 1 -1/2 1/2]);
model.obj = [3 5 -1 0 0];
model.rhs = [13/2; 5/2;1/2];
model.sense = '===';
model.vtype = 'C';
model.modelsense = 'min';
model.varnames = names;

result = gurobi(model);
disp(result);
```

命令行窗口

```
Solved in 0 iterations and 0.03 seconds (0.00 work units)
Optimal objective -2.000000000e+00
          status: 'OPTIMAL'
     versioninfo: [1×1 struct]
         runtime: 0.0429
            work: 4.4806e-06
          objval: -2.0000
               x: [5×1 double]
           slack: [3×1 double]
              pi: [3×1 double]
              rc: [5×1 double]
          vbasis: [5×1 double]
          cbasis: [3×1 double]
        objbound: -2.0000
       itercount: 0
    baritercount: 0
       nodecount: 0
          maxvio: 8.8818e-16

x 0
y 0
z 2.000000e+00
a 7.000000e+00
b 4.000000e+00
Obj:-2.000000e+00
>>
```

编辑器 - D:\Gurobi\win64\matlab\problem2.m

Test_gurobi.m | problem1.m | problem3.m | problem4.m | problem5.m

```matlab
names = {'x' ; 'y' ; 'z' ;'a' ;'b'};
model.A = sparse([1 -2 1 0 0; 0 1 -3 1 0 ; 0 1 -1 0 1]);
model.obj = [0 -1 2 0 0];
model.rhs = [2;1;2];
model.sense = '===';
model.vtype = 'C';
model.modelsense = 'min';
model.varnames = names;

result = gurobi(model);
disp(result);
```

命令行窗口

```
Solved in 2 iterations and 0.04 seconds (0.00 work units)
Optimal objective -1.500000000e+00
          status: 'OPTIMAL'
     versioninfo: [1×1 struct]
         runtime: 0.0429
            work: 6.0432e-06
          objval: -1.5000
               x: [5×1 double]
           slack: [3×1 double]
              pi: [3×1 double]
              rc: [5×1 double]
          vbasis: [5×1 double]
          cbasis: [3×1 double]
        objbound: -1.5000
       itercount: 2
    baritercount: 0
       nodecount: 0
          maxvio: 0

x 6.500000e+00
y 2.500000e+00
z 5.000000e-01
a 0
b 0
Obj:-1.500000e+00
fx >>
```

编辑器 - D:\Gurobi\win64\matlab\problem3.m

Test_gurobi.m | problem1.m | problem3.m | problem4.m | problem5.m

```matlab
model.A = sparse([4 2; 2 3; 6 5]);
model.obj = [1 1];
model.rhs = [9;6;15];
model.sense = '<<<';
model.vtype = 'C';
model.modelsense = 'max';
model.varnames = names;

result = gurobi(model);
disp(result);

% print out the result
```

命令行窗口

```
       0    2.0000000e+30   4.125000e+30   2.000000e+00      0s
       2    2.6250000e+00   0.000000e+00   0.000000e+00      0s

Solved in 2 iterations and 0.04 seconds (0.00 work units)
Optimal objective  2.625000000e+00
          status: 'OPTIMAL'
     versioninfo: [1×1 struct]
         runtime: 0.0459
            work: 4.5964e-06
          objval: 2.6250
               x: [2×1 double]
           slack: [3×1 double]
              pi: [3×1 double]
              rc: [2×1 double]
          vbasis: [2×1 double]
          cbasis: [3×1 double]
        objbound: 2.6250
       itercount: 2
    baritercount: 0
       nodecount: 0
          maxvio: 0

x 1.875000e+00
y 7.500000e-01
Obj:2.625000e+00
```

编辑器 - D:\Gurobi\win64\matlab\problem4.m

Test_gurobi.m | problem1.m | problem3.m | problem4.m | problem5.m

```matlab
    model.A = sparse([3 2 1;1 1 1;12 4 1]);
    model.obj = [12 8 5];
    model.rhs = [20;11;48];
    model.sense = '<<<';
    model.vtype = 'C';
    model.modelsense = 'max';
    model.varnames = names;

    result = gurobi(model);
    disp(result);

    % print out the result
```

命令行窗口

```
       3    8.4000000e+01   0.000000e+00   0.000000e+00      0s

Solved in 3 iterations and 0.04 seconds (0.00 work units)
Optimal objective  8.400000000e+01
          status: 'OPTIMAL'
     versioninfo: [1×1 struct]
         runtime: 0.0439
            work: 5.8482e-06
          objval: 84
               x: [3×1 double]
           slack: [3×1 double]
              pi: [3×1 double]
              rc: [3×1 double]
          vbasis: [3×1 double]
          cbasis: [3×1 double]
        objbound: 84
       itercount: 3
    baritercount: 0
       nodecount: 0
          maxvio: 2.7756e-16


x 2
y 5
z 4
Obj:8.400000e+01
```

编辑器 - D:\Gurobi\win64\matlab\problem5.m

Test_gurobi.m | problem1.m | problem3.m | problem4.m | problem5

```
    model.A = sparse([110 160 420 260;4 8 4 14;2 285 22 80]);
    model.obj = [3 9 20 19];
    model.rhs = [2000;55;800];
    model.sense = '>>>';
    model.vtype = 'C';
    model.modelsense = 'min';
    model.varnames = names;

    result = gurobi(model);
    disp(result);

    % print out the result
```

命令行窗口

```
Solved in 2 iterations and 0.05 seconds (0.00 work units)
Optimal objective  6.709635836e+01
          status: 'OPTIMAL'
     versioninfo: [1×1 struct]
         runtime: 0.0598
            work: 6.9904e-06
          objval: 67.0964
               x: [4×1 double]
           slack: [3×1 double]
              pi: [3×1 double]
              rc: [4×1 double]
          vbasis: [4×1 double]
          cbasis: [3×1 double]
        objbound: 67.0964
       itercount: 2
    baritercount: 0
       nodecount: 0
          maxvio: 2.2737e-13

x 1.424428e+01
y 2.707058e+00
z 0
a 0
Obj:6.709636e+01
```