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Cover design: Mariusz Banachowicz

ISSN 0869-8286 (print format)
e-ISSN 2451-0602 (electronic format)

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Publisher: Copernicus Center Press Sp. z o.o.
Cholerny 501, 32-060 Łuski POLAND
tel. (+48) 12 448 14 12
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www.ccpress.pl



Michał Heller	
<i>At the interface of theory and experience</i>	7
Adrian Heathcote	
<i>Realism, irrationality, and spinor spaces</i>	9
Paweł Polak	
<i>Philosophy in technology: A research program</i>	11
Roman Krzanowski	
<i>From philosophy in science to information in nature: Michael Heller's ideas</i>	13
Sławomir Grzegorz Leciejewski	
<i>New experimentalism and computer-aided experiments</i>	15
Timothy Tambassi	
<i>For the sake of simplicity: Applying software design parsimony to the content of information system ontologies</i>	17
Andrzej Bielecki, Ryszard Stócki	
<i>The concept of structural information and possible applications</i>	19
Wojciech P. Grygiel	
<i>The applicability of the concept of the field of rationality in the explanation of the fundamental role of symmetries in physics</i>	23
Tadeusz Sierotowicz	
<i>Theology of science: Its collocation and critical role for understanding of limits of theological and scientific investigations</i>	25
Kamil Trombik	
<i>Andrzej Fullński as a representative of the concept of philosophy in science</i>	27
Andrzej Anderwald	
<i>The interdisciplinary profile of theology—fashion or necessity?</i>	29
Roman Krzanowski	
<i>Introduction to topo-philosophy</i>	31

The founding motto of philosophy in science is “tracking down big philosophical problems in contemporary science.” Knowing the basic history of philosophy and the history of science, we more or less know what “big philosophical topics” mean. The most representative topics of this kind include: time, space, causality, matter, life, consciousness, thinking... The tables of contents of philosophy textbooks could be copied to continue this list. These topics are big not only when they remain at a high level of generality, but also when they get down to special cases and particular sub-problems. Sometimes it is only then that they fully reveal their big format.

But where in science should we pursue these topics? As usual, when struggling with a difficult question, it is worth limiting ourselves to an easier case. Such a “methodologically easier” case is, of course, physics; this is where we will focus our attention in this short essay.

But where exactly in physics should we look for these philosophical topics? To be sure, in the core of modern physics, that is, at the interface of theory and experience. The final instance for physical theories is experience, but experience without theory would be reduced to crude sensory perceptions, which have little to do with science and are completely powerless against more advanced physical theories. Not only should we look for traces of great philosophical problems in the interface between the theories of physics and experiment, but this interface itself creates a great philosophical problem which could only be vaguely intuited in the old problems of philosophical epistemology.

For obvious reasons, the problem of the relationship between the mathematical formalism of theory and empirical data is also one of the main, if not simply the main, problem in contemporary philosophy of science. Moreover, this problem is becoming more and more urgent. Some theories of modern physics seem to reach domains in which experiment is impossible, either for financial reasons (theories of extremely high energies) or for even more fundamental reasons (theories of multiverses). Is physics without the possibility to confront its hypotheses with experimental data still physics? The question of the relationship between formalism and experience becomes the question of the identity of physics as a science.

Undoubtedly, the identity of modern physics was determined by its empirical character. Rapid progress in physics occurred precisely when experience became the main criterion for the acceptability of its theories. The turning point in the emergence of modern science was the departure from the belief, cultivated throughout antiquity and the Middle Ages, that the universe can be reconstructed basing on rigorous deduction from “first principles” and the understanding that such a deduction must—as Whitehead elegantly put it—face “irreducible and stubborn facts”, and if the facts stubbornly persist despite the results of the deduction, then the whole deduction, together with its conclusions, must be abandoned.

As physical theories became more and more sophisticated, the understanding of their empirical character (that is supposed to constitute the identity of physics) became less and less obvious. In fact, the entire history and philosophy of science of the last two centuries has revolved around this concept.

Empiricism achieved its maximum in the views of logical empiricism, which postulated the reduction of the entire theoretical “superstructure” of modern physics to direct empirical data. Although logical empiricism did not survive into the 21st century, it left a strong mark on contemporary philosophy of science. One of the clearest features of this heritage are empiricist tendencies. Of course, there is no return to the idea of direct reports of experimental results (the so-called elementary propositions), to which all physical theories should be reduced. No one denies that mathematical formalism is an important element of physical theories, but in many so-called case studies, i.e. in methodological analyzes of specific theories or models of contemporary physics, we find attempts to distinguish as clearly as possible those elements of formalism that can be directly associated with measurement procedures. What is evident in these attempts is the idea that a given physical theory will be more empirical the more precisely it can be done.

This is not how it works in the scientific practice of physicists. The practice of physics is much more monolithic. When you enter a modern physics laboratory, you take a closer look at all this complicated equipment (if it is possible at all, because it may have dimensions far beyond what you can see) and look at the diagrams in which the results of the experiment are encoded, you can really have the impression that you are touching a nerve of reality. But you only need to look a little more carefully into what is actually happening here to understand that it is impossible to draw even a relatively sharp line separating what is theoretical from what is empirical.

It would seem that at least what is theoretical can be clearly distinguished from what is empirical. After all, “theoretical” is simply the mathematical formalism of a theory. But that is not entirely true. Because the mathematical formalism of the theory can virtually contain the results of future measurements. This is eloquently evidenced by the history of the field equations of general relativity, which “knew” about future empirical discoveries (microwave background radiation, gravitational radiation and many others) much earlier than they could be made.

It is often said, somewhat metaphorically, that theoretical and empirical elements in physical theories are nonlinearly coupled with each other. This is an apt metaphor. Just as the solution of a nonlinear differential equation cannot be decomposed into the sum of two solutions to that equation, a physical theory cannot be decomposed into the sum of a theoretical component and an empirical component.

According to aesthetic criteria, that go back to the shadows of logical empiricism, this would be an argument on behalf of the thesis that the theories of modern physics do not meet the criterion of being an empirical science. I think that it is just the opposite: physics is an empirical science precisely because the empiricism runs so deep into its theoretical body that it cannot be separated from it.

This coupling of mathematical formalism and empirical results, the element of rationalism and the element of empiricism, constitutes a Big Philosophical Problem. We have here not only a case for *philosophy in science*, but also a beautiful example of what physics contributes to Big Philosophical Problems.

Bibliography

Mathematics, as Eugene Wigner noted, is unreasonably effective in physics. The argument of this paper is that the disproportionate attention that philosophers have paid to discrete structures such as the natural numbers, for which a nominalist construction may be possible, has deprived us of the best argument for platonism, which lies in continuous structures—in fields and their derived algebras, such as Clifford algebras. The argument that Wigner was making is best made with respect to such structures—in a loose sense, with respect to geometry rather than arithmetic. The purpose of the present paper is to make this connection between mathematical realism and geometrical entities. It thus constitutes an argument against formalism, for which mathematics is merely a game with humnaly set rules; and nominalism, in which whatever mathematics is used is eliminable in the final analysis, by often insufficiently specified means. The hope is that light may be cast on the stubborn mysteries of the nature of quantum mechanics and its mathematical formulation, with particular reference to spinor representations—as they have been developed by Andrej Trautman. Thus, according to our argument, QM may appear more natural, as we have better reasons to take spinor structures as irreducibly real, a view consonant with the work of Trautman and Penrose in particular.

indispensibility, nominalism, spinors, complex numbers, incommensurability.

Many who have more than a passing interest in mathematical physics have been impressed by the intimate connection that exists between quite advanced mathematics and the elucidation of our best physical theories, and being so impressed have taken this as an argument for a form of mathematical platonism. Yet, in the wider philosophical community, and certainly in the culture at large, nominalism *seems* (perhaps only to a jaundiced eye) to dominate. Thus we have a rather stark opposition between philosophy and science in which the two sides appear to be largely talking past one another, and little that is said advances the debate in a successful manner.

The present paper is an attempt to get beyond this impasse by offering a way of reasting the issues, so that 1) a central part of the nominalist intuition can be seen to have some plausibility; and 2) that nevertheless the platoinist can be seen to be correct in that mathematical physics does in fact offer an argument for the reality of mathematical entities. Indeed, my suggestion will be that there is a straight line between the motivation for platonism among the ancient Greeks and platonism today. Thus the main claim of the present work is that there is a mechanism for the expansion of our mathematical ontology that is directly tied to our progress in mathematical physics, a connection that is unlikely to be accidental. In brief: the taking of roots is often *ontologically ampliative*.

We may begin by noting that perhaps the most important way that the discussion has gone astray is through the historical focus on the arithmetic of the natural numbers, a focus that was present in Kant as well as Frege, and that flowed naturally through the reductive programmes of the 20th Century. The natural numbers were seen to have first place in the *ordo cognoscendi*: they were our original mathematics—account for these and all else will somehow surely fall into place. In due course philosophical discussion became bound to the twin poles of arithmetic and set theory—the latter having first place in the *ordo essendi*. Though nominalists and realists disagreed on what should be in our ontology, they were at least disposed to agree on what mathematics we should be considering.

The implicit thought here seems to be that whatever we can say about the natural numbers we will be able to say about any other mathematical structure. However I want to suggest that this is false: that the natural numbers are a special case that lend themselves to a very special nominalist explanation, an explanation that does not extend to other mathematical entities in which we might be interested.

1. A nominalism for arithmetic

Let us begin by giving the Peano axioms in their second-order form. We modify them in a way that is now customary by taking the first number as 0. Since 0 is the additive unit it means that much more of what would ordinarily be considered elementary arithmetic is derivable. However it also means that we would have to be careful in the statement of divisibility. Peano's own statement would lead to problems unless modified, for it would allow division by zero.¹

Where Peano speaks, in the first axiom (and then throughout) of the *n* being a member of a set *N*, I will be explicit that this set is to be the set of natural numbers *N*.

AXIOMS FOR PEANO ARITHMETIC

PI : 0 is a natural number;
PII : For every natural number *n*, *n* + 1 is a natural number
PIII : For every natural number *n*, *n* + 1 ≠ 0;
PIV : For all natural numbers *n* and *m*, *n* + 1 = *m* + 1 if and only if *n* = *m*;
PV : If *φ* is a property of numbers such that: 0 is *φ*, and for every natural number *n*, if *n* is *φ*, then *n* + 1 is *φ*, then all natural numbers *n* are *φ*;
PVI : *n* + 0 = *n*;
PVII : *n* + (*m* + 1) = (*n* + *m*) + 1;
PVIII : *n*.0 = 0;
PIX : *n*.(*m* + 1) = (*n*.*m*) + *n*;
PX : *n*.(*m* + *p*) = (*n*.*m*) + (*n*.*p*).

These axioms, as is well known, are derived from Dedekind's *Was Sind und was sollen die Zahlen?* ([dedekind_was_1888](#)), and Dedekind had there shown that his axiom-set is categorical. His method, as outlined in his letter to Hans Keferstein in 1890, is not to appeal to known features of the natural numbers—this, he says, would result in a vicious circularity—but to give axioms that ought to determine any infinite, well-ordered set ([van_heijenoort_frege_1967](#)).

But now we come to the crucial point. Not only are these axioms such that they characterise the natural numbers, they *also characterise the numerals that name the natural numbers*. For the numerals are also a well-ordered infinite set and begin with a first numeral '0'. To achieve this isomorphism we must understand that numerals are not identical with inscriptions of numerals: there are numerals that no one *will ever, or could ever*, write down. But no matter, these numerals exist and there are many that cannot be written down but can be characterised by a definite description—thus the name "Graham's Number" is an abbreviation of a definite description where the numeral itself could not be written down without a secondary abbreviated notation.

Of course, there will be some nominalists for whom an infinite set of numerals is already going too far in the direction of platonism: it must be understood that the way out of the problem that I am offering here will not be a way that is open to them. But a rigid Inscriptionism is, I believe, a most difficult position to extract explanatory content from, and so we must await someone who is prepared to try to make it work. At any rate I say no more about such a view here.

Allowing ourselves an infinite set of numerals we can check the Peano axioms to see what they mean when applied to numerals. As already noted neither Peano nor Dedekind mention numbers, for their purpose in providing an axiomatisation is to characterise numbers without circular descriptions. So, adapting Peano, we have simply:

P*PI : 0 ∈ *N*;
P*PII : If *n* ∈ *N* then *n* + 1 ∈ *N*;
P*PIII : If *n* ∈ *N* then *n* + 1 ≠ 0;
P*PIV : For *n* and *m* ∈ *N*, *n* + 1 = *m* + 1 if and only if *n* = *m*;
P*PV : If *φ* is a property of the members of *N* such that: '0' is *φ*, and
P*PVI : etc.

Since addition is simply an operation that takes a member of *N* to

Now the philosophical point should be clear: since there is an is down a numeral but I cannot write down a number. By analogy, to

Now if we take a Medieval conception of nominalism, we may be and Kronecker, though undoubtedly many others followed in the 2nd

Some credence is given to this position if we ask ourselves, sym manipulate are *numerals*. Likewise with associativity: the order in

Now I will say that I think we have here the beginning of an int contrast I am not sure that sense can be made of saying that *num* Confusion between name and referent is rife in this area, and of for

But I cut this discussion short to say that, ultimately, I do not b next two sections.

However, I think that something like the above reasoning was i

Mathematics began as an abstract discipline, I suggest—as opposed Nor can it be certain that the Pythagoreans were the first to const

If were a fraction then there would be a set of natural numbers *S* – *k*. We now have

This shows that *m* is a member of *S* since 2*k* – *k* is obviously a wh

and thus *m* = *k*(–1) is less than *k*). So we have found an *m* < *k*, ' The beauty of this proof, besides its great simplicity, is that it Fundamental Theorem of Arithmetic.

The Pythagoreans of the 6th Century no probably did not have left to the mathematician Theodorus to extend the proof up to 17 c

A lost book of Apollonius is meant to have gone further and consid The mathematical significance of this discovery has been thoro

The best way to approach this question is to ask what could not worth in a marketplace. I don't say that such a nominalising strate

Secondly, it might be thought that some geometrical magnitudes of the triangle will each be $\frac{1}{2}k$, which is irrational. The same can b phenomenon revealed by the Pythagorean proof was sometimes ref

The third form of Nominalism is the one that I regard as initial their discovery: that they had discovered numbers that were *unsay*

In fact as late as Euclid, Heath reminds us that the term that i

In saying that irrational numbers are unsayable we do not of cou expressible in *numbers exact*, and even God could not find such an *c* there is no finite expression in numerals, or *numbers exact*: it is in t

To say that is unsayable in numerals must also be to wonder w remained agnostic as to whether it was number in a new sense of t

geometria, as the study that encompasses these entities, and *arith*

If there are entities without numerical names then those entities natural numbers there come algorithms for the common arithmetic

It seems that we have in this reconstruction a quite solid argument

- There exist mathematical entities for which there is no places
- These entities figure in the measurement of space and time in

In a sense we have here an indispensability argument. But this 'indi invoking the existence of any items not already available to the reali is restricted just to explaining positive metrical facts, not all facts.

And yet though this gives us a realism of the real numbers—it is

But the way we go beyond this beginning point is exactly the s Diophantus, or generally in real numbers. But whether mathematic

The stability of mathematical ontology up to the 15th Century The tale has been told often enough of the discovery of the me

and the interpretation of this, geometrically, as a mean proportion The striking thing about the way complex numbers arise in the solving this equation from Scipione Dal Ferro, with *b* = 15, and *c*

This will give us, on substitution:

or

where each cube root has three solutions. One of these, 2 + *i* along

The philosophical puzzle that Bombelli faced was this: the root Bombelli the puzzle must have verged upon paradox: for he did no

This kind of root has in its calculation different operations th

This geometric proof of Dal Ferro's equation appears late in Bombel a representation in the Euclidean plane, the nice-named *Argand pla*

En passant this helps to solve another puzzle. It has sometimes And once we have an instantiation for Euclidean space then we get

Our realism, or platonism, has taken us as far as complex numb than had been realised—in both Pythagoras and Cardano-Tartaglia

Complex numbers are used routinely in quantum mechanics—bu about complex numbers. This situation may have changed recentl y transferred to one between Alice and Charlie, even though they hav protocol that could test this difference. If it were to come out

If this is so, what we have is a mathematical discovery that is c A very similar case is provided by the *quaternions*. Hamilton's o may feel quite differently. W. K. Clifford's use of them in what we

We can find an even more significant discovery that affords a bette in his ([cartan_les_1913](#)) Elie Cartan discovered an entirely i 1927 as a way of describing electron spin (followed, independently, by Clifford's usage of it. In Weyl's *Classical Groups* ([weyl_classical](#)

The normalization requires the possibility of *extracting sq* essential. The orthogonal transformations are the automorph

¹ An extra condition stating that in all cases of *m/n*, *n* ≠ 0 would be su ² See Burton and Walsh ([button_philosophy_2018](#)) for a discussion

³ No direct evidence of Rosellini's position survives, only the ruins of hi ⁴ One can find something of this view expressed in Whitehead's *Univers*

⁵ Man-Keung Siu ([siu_esternernn_1998](#)). See also P. Shin ([shiu_2002](#))

⁶ The Plank's length might be thought to be a candidate for such a find ⁷ Additional evidence is provided by the title of a lost work of Democrit

⁸ Euclid in Heath translation ([euclid_thirteen_1956](#)). ⁹ That Plato came at some time in his adulthood to be imbued with Pyt investigated with zeal. In this way, accordingly, this was the first time th

¹⁰ The further, in [optics] and mechanics [...] Philodemus *History of the i* ¹¹ His notation for $\sqrt{-1}$ was R (0 : *m* - 1) which translates directly to $\sqrt{-1}$

¹² Thus I am here resisting the idea that the indispensability of matheus ¹³ See for example Liang Shin-Hahn ([hahn_complex_1994](#)); also Kai ¹⁴ For the fraught history of quaternions see Simon Ahlmann's ([ahlman](#)

¹⁵ Reiser and Weyl ([brauer_spinors_1935](#)). For the English translat

of *N* it is also well-defined on numerals: it is simply counting forward. Likewise for multiplication. Thus the remaining Peano axioms will also have a clear meaning.

'n the two models of the Peano axioms, and since we use the numerals to speak of numbers, there is always a danger that we will confuse the two—and, the nominalist may say, we *have* confused them, and confused them throughout history. Thus we are, whether we are nominalists or realists, simply creating confusion if we say that 'numbers can be written down'. I can write lear, I cannot write down Mary but I can write down Mary's name, 'Mary'. So when we speak of writing down numbers we are already confusing a name with the referent of the name. Thus in Peano's axiomatization what is written down and axiomatised are numerals.²

nothing but numerals, that these do not refer to numbers, as a name refers to a thing, but that they are *all there is* to what we think of as number. Thus numerals are a *flatus vocis*, in Rosellini's phrase, an empty wind, and mathematics is simply a game with rules for the manipulation of these numerals. In the 19th Century there is evidence that this was the view of Heilmholtz ly the Formalists.³

ominalism, what the law of the commutativity for multiplication means: if we multiply together two numbers *a* and *b* then the order of the multiplication does not matter. But, says my imaginary nominalist, surely the *order of an operation* suggests something that we *do*, some way of manipulating objects, in a particular sequence, and the only objects available for us to *n* is performed suggests an action with consequences. After all, *to add* and *to multiply* are verbs and require objects on which the action is to be performed.⁴

' not about nominalism that could be developed further, and one that would be helpful in clearing our minds of long standing confusion. In particular it may help us understand what we mean when we make a distinction between the *potential infinite* and the *actual infinite*, for there is a clear sense in which there are a potential infinity of numerals that we may write down. By ' potentially infinite: either they are finite or they are infinite, and there is nothing in between these two cardinalities. Nor, if it is numbers themselves that are being thought of as potentially infinite, is it all clear what would be *releasing* or *realising* this potential. For *whom* is this potential realised? *When* is it being realised? Can these numbers *return* to be being unrealised?

be correct for anything more than the natural numbers (and in the light of an argument to come in §5, not even there). It depends on our having numerals which can stand in proxy for natural numbers and thinks of numerical operations as manipulations of those numerals. But, as Hilbert realised, this cannot be extended to the real or complex numbers—a point I come to in the

hagoreans and Plato: as long as we had to think *only* of the natural numbers we were able to be lulled into a state of Nominalism about numbers. But when irrational magnitudes were discovered there was no longer a way to avoid realism. The argument for this, with some historical evidence, is given in the next section.

2. Plato and incommensurability

I to accounting—with the Pythagorean discovery that the square root of two cannot be either a whole number or a ratio of whole numbers. There are now many proofs of this, but here is a beautiful, little-known one by Theodor Estermann ([estermann_irrationality_1975](#)). (It isn't known what proof the Pythagoreans actually used, though there has been much speculation.

) when multiplied by said fraction would yield a natural number. And if there is such a set then by well-ordering there is a least member of that set: call it *k*. So *k* is a natural number and by definition the smallest such number. But on the hypothesis that is a fraction we can find a number *m* in that is smaller than *k* for which *m* is also a whole number. Thus consider *m* = *k*(–1) = *k*

$$m\sqrt{2} = (k\sqrt{2} - k)\sqrt{2} = 2k - k\sqrt{2}.$$

this *m* is also less than *k* (the number 1 was chosen specifically so that we would have

$$0 < \sqrt{2} - 1 < 1$$

ary to the hypothesis that *k* was the least member in *S*. Repeating the proof will produce an infinitely descending set of natural numbers, which is impossible.

properties of natural numbers and ratios of same. As Man-Keung Siu has pointed out there is an interpretation of this proof in the geometry of triangles, but the proof itself is free of any geometric assumptions.⁵ The proof can also be generalised to the square root of any number that is not a perfect square, as Estermann noted, while requiring no heavy theorems like the

'f (if they had, the generalisations to other non-square numbers would have been evident to them) but no matter—they had some other that proved the same fact: cannot be either a whole number or a ratio of whole numbers. And it is a simple application of the Pythagorean Theorem that the diagonal of a unit square has a length that is *and so, such a length must exist*. It is for this reason that, by Euclid's time, we can generalise the discovery to the square roots of all numbers that are not perfect squares (and again, we cannot be sure what proof was used). By the time of Euclid this discovery was well-developed as the theory of *incommensurable* magnitudes, and developed in books V, IX and X of the *Elements*. In Book X Euclid extends the theory of irrationals to all that have the form

$$\sqrt{a \pm \sqrt{b}}.$$

ere unordered—possibly including π .

by Knorr ([knorr_evolution_1975](#)) and Fowler ([fowler_mathematics_1999](#)). But what about the metaphysical significance? In metaphysical terms, what can be, and what can it not be?

g is that, given the above proof and others like it, we cannot automatically think that a nominalising strategy that might look promising for the natural numbers or the rationals will work for . Thus it might be thought that we could regard number as an abstraction *for our purposes* from aggregates of individuals, as in, *five sheep, three goats*—that this is a *social fact*, like their

ad volumes—might be this number . But this cannot be right either. The hypotenuse of a right-angled isosceles (RAI) triangle with catheti of unit length then the hypotenuse will have the length . But if we had chosen instead to make the hypotenuse of unit length then the catheti *utandis*, for areas and volumes. Whereas it might be plausible to think of *things* as having natural units—one goat, one sheep, one neutron,—this cannot be carried across to geometrical magnitudes. And if there are no natural units for geometrical magnitudes then no other such magnitude is intrinsically irrational either.⁶ It is for this reason that, by Euclid's time, the

measurability. This is a pair-wise relation. The catheti and the hypotenuse of a RAI triangle cannot *both* be whole numbers or ratios of whole numbers: one must fail, but it is an arbitrary choice which one is made to fail. The consequence is that cannot be identified with geometrical magnitudes in an absolute sense.

ble, and the one that was outlined in the first section, above. The trouble is that this view will not work either for . This is because there is *no numerical* expression—I must emphasise '*numerical*' to forestall the irrelevant objection that " is itself such an expression—for this or any other irrational number. In fact this seems to be *how* the Pythagoreans themselves understood this can be found in Plato's statement in *The Republic*: such numbers (or magnitudes) were *arritelon* (unspeakable or unsayable).⁷

ated as 'rational' was *rieta*, meaning *sayable*, and the obvious root of *arritelon*. By contrast the word in Euclid that we translate as 'irrational' was *alogos* which can have as many meanings as that very loaded word *logos*—but will certainly include *beyond words*.⁸

' did the Greeks mean) that there is no form of words which will describe such numbers, for the expression 'the square root of two' is obviously such an expression. The point is that there is no finite expression in *numerals* that will do so. As Leibniz put it, in his *Dialogue on Human Freedom and the Origin of Evil*, of 1695 ([leibniz_dialogue_1980](#)), such magnitudes as are not *log* *infinite* forms of expression then we can think of these numbers as limits, for example by the approximation method known as *anthyphairisis*, which was known in Plato's time. And this in itself leads to a continued fraction representation of these numbers, as discovered by Pietro Catadi, Brouncker, Wallis, and Euler. But all of these means of expression are essentially infinite.

' are unsayable. Every schoolchild learns at least one manifestation of this profound fact: the decimal representation of would be an infinite, non-recurring string of numerals. Cutting it off after any finite length will give a rational number that is *not equal to* . So is something beyond what we can express in numerals. The habitual confusion between numerals and numbers that

er at all. This is the important ontological issue to which we have become numb, but which was still very much a live issue in the 19th Century. It is a familiar point that 'numbes' for the Greeks meant *natural* numbers, though they also understood ratios of these natural numbers. So it is possible that Plato could have said, cautiously, that there was *something* that was but it was some *other* kind of entity whose square was a number! And yet there was at least one good argument for thinking of these unsayable entities as numbers in a new sense: the square root of 4 is a number, namely 2; so the square root of 2 surely ought to be something of the same kind, despite being 'unsayable'. They marked their caution by distinguishing between

' evidence in the later dialogues that Plato was prepared to take the step of expanding the concept of number to including these new entities, at *Epinomis* 990d, for example.⁹

ed into such names—and the proof that there are an uncountable number of real numbers means that not every real number *can* receive a name of any kind. Thus even if we allow ourselves to make use of countable *ad hoc* names—as we do with 's' or e—or disguised definite descriptions—as we do with '—we still have a significant problem. For with the numerals expressing the

' there is no such natural extension of these algorithms for these *ad hoc* names. How would Plato (or any mathematician before the 19th Century) go about adding and π ? Can we be sure that $\times = \sqrt{6}$? In fact it was Dedekind who noted, as late as 1858, that it had never been proven but only assumed that for real numbers

$$\sqrt{a}.\sqrt{b} = \sqrt{ab}.$$

se_square_2012). So a formalist or fictionalist conception of Nominalism—in which mathematics is *just* the manipulation of symbols according to set rules—has to confront the fact that here we have entities for which there can be no systematic naming procedure. Moreover this must have been evident even in Plato's day, for there is a complete absence of discussion of adding

thematical realism.
 nstrual.
 ture, but also in particular physical problems, including those that require the use of calculus. Moreover their properties explain certain things that are impossible: namely the Delian cube problem, squaring the circle, etc.

to targeted in this case, for it is not simply an indispensability to modern science, but has a more general cast: an indispensability to nature herself. For if the nature of irrational numbers is able to explain the *impossibility* of carrying out particular acts, how does a nominalist or a fictionalist strategy have anything that can equally explain that impossibility? After all, neither are ng for less, and so have fewer resources. As far as I'm aware there is no answer to this in the existent literature. The only nominalist strategy of which I'm aware that might have something to say here is that of Harry Field in his ([field_science_1980](#)). Field helps himself to a particular space-time manifold model to argue that real numbers are unnecessary, but his argument eat fails in general (I take it up in section 5) and if it fails there is nothing to replace it.

provide us with a reason to be realist about other mathematical entities.
 athematics itself evolved beyond this beginning point. Euclid is the germ from which mathematics grew, by demonstration from axioms which are self-evident. For 1700 years mathematics consisted of furthering the work of Euclid by enlarging on the subjects of geometry, arithmetic and analysis. Abstraction led to algebra, whether in whole number solutions, as in that of y solving equations or giving proofs, the method by which mathematical knowledge was gained was hardly a mystical intuition. Mathematical truths are known by proof and calculation.

f Platonism and the re-establishment of the Academy in Florence under Marsilio Ficino and Cosimo de Medici, laid the ground for the next expansion: the discovery of the complex numbers.

ble equations by Tartaglia and its theft and publishing by Cardano in 1545. The interpretation of the root of –1 as a geometric mean of 1 and –1 obtained by solving

$$\frac{1}{x} = \frac{x}{-1}$$

o the ordinary number line gives us 'two-dimensional numbers', removing linearity as an essential condition of what it is to be a number. Again, the mathematical aspect of the discovery of complex numbers has been well-described elsewhere, but what of the philosophical significance?

o is that they seem to force themselves upon us. We are looking for real solutions to a cubic equation, which itself has only real coefficients, and yet complex numbers arise naturally on the way to the real number solutions. Thus consider this example, from Bombelli's *L' Algebra*: $x^3 = 15\,x + 4$. The three roots of this equation are 4, $-2 - \sqrt{3}$, $-2 + \sqrt{3}$. They can be found by

$$x = \sqrt[3]{\frac{c}{2} + \sqrt{\frac{c^2}{4} - \frac{b^3}{27}}} + \sqrt[3]{\frac{c}{2} - \sqrt{\frac{c^2}{4} - \frac{b^3}{27}}}$$

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$$

$$x = \sqrt[3]{2 + 11i} + \sqrt[3]{2 - 11i}$$

te 2 – *i*. Bombelli must have found, since it yields the root 4, which he gives as a solution to the equation. (Bombelli would have been inclined to discard the negative roots.)

are acceptable numbers, or at the very least, one of them is; but the method by which we reach them involves taking the cube roots of numbers that appear unbel or 'sophistical'. And the cube roots themselves are also unreal or sophistical. But it is only by adding together these unreal numbers (in conjugate pairs) that we reach the roots, that we *must* take seriously. For numbers as proper—by contrast he had no problem with irrational numbers—and yet he was taking the square root of negative numbers, and then taking the cube root of the complex radicals that resulted—and then adding them pairwise.¹⁰ He declared this discovery as the discovery of a new kind of cubic radical and said that he had a geometrical proof of it. He says:

I has a different name...[I]t will seem to most people more sophistic than real. This was the opinion I held too until I found its geometrical proof (translated in Federica La Nave and Barry Mazur's ([la_nave_reading_2002](#))).

makes the geometric proofs of the existence of irrationals: in a sense complex numbers stand to irrationals as Dal Ferro's equation stands to Pythagoras's Theorem—they both emerge as surprising solutions given well-recognised inputs.¹¹ However it was not for another 100 years, when Wallis and then De Moivre showed that -1 could be not just be proven to exist but also given tance was assured. But they—i.e. complex numbers—come to us as a natural extension of our previous ontological commitments—they were not 'posited' for the purposes of doing physics, or whatever, they were instead a discovery that emerged naturally from pursuing ordinary mathematics. And it is *this* that gives one the confidence that they exist.¹²

' discovery that our physical space is not Euclidean but instead has a Riemannian curvature shows that Euclidean geometry is 'wrong'. This, I think, is a mis-saying. The geometrical representation of the complex numbers shows that the axioms for two-dimensional Euclidean geometry are instantiated after all. They are just not instantiated in the way one might have thought.

d operators all as part of the machinery for the description of that space. The rich connections between Euclidean geometry and the real and complex numbers have been thoroughly explored, and need no further comment. Again, this is an issue we come back to.¹³

metry with no reliance on the usefulness of mathematics to physics—and Bombelli died 60 years before the appearance of even Galileo's *Dialogue Concerning the Two Chief World Systems*. Most curiously, the expansion of the mathematical ontology—or, to put it more accurately, the realisation that there was more ontology implicit in the initial commitment to whole numbers led taking roots. Once again: taking roots has been ontologically ampliative. In fact had the ancients been prepared from the outset to countenance negative numbers then the process of taking roots might have led directly to the complex numbers two millennia earlier.

evidence that their use is unavoidable? Until recently the answer would have seemed to be 'no', for it always looked possible to translate standard quantum mechanics on the complex field (CQM) into a more cumbersome real number form (which we will abbreviate to RQM). This is hardly any form of nominalism, but it has been a standard suggestion made against being realists

'gues that there are situations in CQM that cannot be explained in RQM ([renou_quantum_2021](#)). The gist of the argument is that if we take three individuals, Alice, Bob and Charlie, and have two entangled photons shared between Alice and Bob, and another two shared between Bob and Charlie: when Bob measures the two particles he has received the entanglement is ticles from a common source. The claim of Renou et al. ([renou_quantum_2021](#)) is that this transfer of entanglement can't be explained in RQM, though it can be explained in CQM. They calculate an entanglement coefficient, based on the Clauser-Horne-Shimony-Holt inequality, of 6, which is higher than the maximum attainable by RQM. There is also an experimental elieve then complex numbers would not after all be eliminable in favour of real numbers.

What might Weyl have meant by this enigmatic final remark? We find it echoed by Michael Atiyah. ‘No one fully understands spinors. Their algebra is formally understood but their general significance is mysterious. In some sense they describe the ‘square root’ of geometry and, just as understanding the square root of -1 took centuries, the same might be true of spinors.’ (quoted in Farnelo (**farnelo_strangest_2009**)).¹⁶ What is the ‘square root of geometry’? Isotropic vectors are those whose ‘length’—as given by the square of the modulus—is zero. So let $\mathbf{x} = (x_1, x_2, x_3)$ be an isotropic vector in a three-dimensional space. In fact we will specify that the space is \mathbb{C}^3 to make the connection with the physics more apparent—thus each of the components is a complex number. The isotropic vectors form a two-dimensional surface in \mathbb{C}^3 , and for each we will have

$$x_1^2 + x_2^2 + x_3^2 = 0.$$

Each such isotropic vector has associated with it two numbers ξ_0 and ξ_1 given as solutions to the following three equations:

$$\begin{aligned}x_1 &= \xi_0^2 - \xi_1^2, \\x_2 &= i(\xi_0^2 + \xi_1^2), \\x_3 &= -2\xi_0\xi_1,\end{aligned}$$

where these are of the form

$$\xi_0 = \pm\sqrt{\frac{x_1 - ix_2}{2}} \quad \text{and} \quad \xi_1 = \pm\sqrt{\frac{-x_1 - ix_2}{2}}.$$

These two numbers parameterize the two-dimensional surface of isotropic vectors. The vector

$$\begin{pmatrix} \xi_0 \\ \xi_1 \end{pmatrix}$$

is a spinor. But as with Bombelli’s solution to Dal Ferro’s formula there are two choices, depending on the sign, as the solutions come in yoked pairs (again the cross terms are discarded). So we also have

$$\begin{pmatrix} -\xi_0 \\ -\xi_1 \end{pmatrix}$$

as a second solution, analogous to the partnering of -1 and -1 .

Though Atiyah spoke of spinors as being ‘square roots’ of (isotropic) vectors, Cartan himself refers to them as ‘polarisations’—‘en quelque sorte un vecteur isotrope *orienté* ou *polarisé*’, where a rotation of this vector through 2π changes this polarisation of the isotropic vector (**cartan_theory_1966**). They are of course now ubiquitous in physics since fermion states are spinors. These are not unknown in relativity theory either—the light cone is represented by isotropic vectors and has associated with it spinors (with real components) which are time-like. This was the point of view emphasised by Cartan in his 1937 lectures, with particular emphasis on Minkowskian geometry. Since Brauer and Weyl in (**brauer_spinors_1935**) had given an algebraic view, Cartan wanted to emphasise their relation to space-time geometry. Thus he presented

[...] a purely geometrical definition of these mathematical entities: because of this geometrical origin, the matrices used by physicists in Quantum Mechanics appear of their own accord, and we can grasp the profound origin of the property, possessed by Clifford algebras, of representing rotations in space having any number of dimensions. (Cartan (**cartan_theory_1966**) from his Introduction.)

But, with respect to his conception of spinors, he also pointed to the impossibility of using the usual coordinate transformation techniques in Riemannian geometry (a remark that was sometimes mistakenly construed as an impossibility proof of introducing spinors into general relativity).

Spinors are closely related to the Atiyah-Singer Index theorem and K-Theory, the Seiberg-Witten theory and Alain Connes’ non-commutative geometry. Roger Penrose has made them the centrepiece of his proposed unification scheme for relativity and quantum mechanics in Twistor theory (**penrose_road_2004_ah**). Their fundamental character would be hard to overestimate—and yet they emerged, firstly, from pure mathematics, only to (independently) come, some 13 years later, to represent a property that had no macroscopic visualisation: a hitherto unsuspected property of matter that arose first from the abstract study of Lie groups—from the Lie group $\mathrm{SO}(3)$ and its double cover $\mathrm{SU}(2)$. This is surely one of the most dramatic and least heralded examples of the uncovering of mathematical structures in nature. And here the mathematics seems very close to being directly physically detectable in the form of spin eigenvalues. And due to the character of the double cover $\mathrm{SU}(2)$ spinors have the remarkable property that if we pick an isotropic vector and rotate it through 2π it returns to its original position but the spinor is only rotated through π and its sign is reversed. It takes a rotation of 4π to bring it back to its original state. It is argued in Christian (2014) that this is also measurable.¹⁷ Moreover it is remarkable that the spin values of the fermions and bosons arise directly from the dimension of the irreducible representations of the Lie algebra $\mathfrak{sl}(3)$, which is the Lie algebra of the groups $\mathrm{SO}(3)$ and $\mathrm{SU}(2)$ —the former giving the spin values for bosons and the latter for fermions.

The non-classical nature of a spinor’s double-rotational invariance is surprising and constitutes a challenge to the idea that particles can be seen as physical objects in the classical manner. Despite this, and acknowledging that it represents only a partial solution to the geometrical problem, Penrose has ingeniously utilised the properties of the Riemann Sphere $\mathbb{P}(1)$ to give a graphical representation of the pure states of spin. It is when one moves to higher fermionic spin states that this picture—the *Majarana picture*—becomes highly non-classical and defies ready visualisation. Penrose pointed out that as we aggregate matter to form higher spin values that there is no convergence to the classical picture, rather the opposite.

[...] we see that a randomly chosen quantum system with a large angular momentum (large j value) has a state defined by a Majorana description consisting of $2j$ points more-or-less randomly peppered about the sphere S^2 . This bears no resemblance to the classical angular momentum state of a system of large angular momentum, despite the common impression that a quantum system with large values for its quantum numbers should approximate a classical system! [...] The answer is that almost all ‘large’ quantum states do not resemble classical ones (**penrose_road_2004_ahpenrose_spinors_1987penrose_spinors_1988**).

But despite defying ready geometric visualisation, spinors are required in quantum theory. Since the work of Cartan, Weyl, and then Chevalley in the 1950’s it has become clear that the natural home for a discussion of spinors is Clifford Algebra. And within the Clifford Algebra in which the simplest expression of quantum mechanical spin is representable, the 8-dimensional algebra usually denoted \mathcal{Cl}_3 , the real numbers and the complex numbers are naturally represented as sub-algebras. Thus, spinors represent a *culmination of algebraic structure* within the structures applicable in physics, that includes the real and complex numbers, and also the quaternions. And it is the unit quaternions that are the spinors as defined by Pauli. Thus Clifford Algebras encapsulate and relate together these seemingly different mathematical structures—all of which are intimately related to our most successful physical theories and in the case of the real and complex numbers, spinors, and quaternions, actually preceded them.

We can close the circle on the progression that we have been noting here: from right angled triangles to the Pythagorean understanding of irrationality and the real numbers, to complex numbers, to spinors, by mentioning a remarkable fact: Pythagorean triples can be understood as generating spinors defined on the null vectors of \mathbb{Z}^3 . This is due to the mapping induced by the Euclidean parameters (p, q) , with $p > q$, to the Pythagorean triples (x, y, z) by

$$x = p^2 - q^2, \quad y = 2pq, \quad z = p^2 + q^2.$$

At least one of the numbers (x, y) must be even. The *primitive* Pythagorean triples are those that are mutually prime with z positive and y even, or $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ is primitive and $\frac{1}{2}$ is odd. Thus the triple $(3, 4, 5)$ is standard, whereas $(4, 3, 5)$ is not. Then, it is provable that for every standard Pythagorean triple there is a pair of Euclidean parameters that are relatively prime which generate the Pythagorean triple. This is then a one-to-one correspondence (bijection) between the directions in \mathbb{Z}^2 and the null directions in \mathbb{Z}^3 .¹⁸

Euclid’s discovery of the parameterisation of Pythagorean triples may be viewed then as the first recorded use of a spinor space.

This in turn is related to complex numbers: $c = p + qi$, since the norm is equal to cc^* , the complex number multiplied by its conjugate, which is

$$p^2 + q^2.$$

And the square of the complex number is

$$(p^2 - q^2) + 2pqi.$$

Thus the squares of certain integer complex numbers generate Pythagorean triples. Or, to put it another way, Pythagorean triples have square roots that are integer complex numbers. A comparison with the immediately preceding discussion of isotropic vectors shows that Euclid’s three equations for Pythagorean triples are analogous to the equations that define a spinor in Cartan’s formulation. Pythagorean triples *are* spinors in \mathbb{Z}^3 ! As Kocik (**kocik_clifford_2007**) puts it: ‘Euclid’s discovery of the parameterisation of Pythagorean triples may be viewed then as the first recorded use of a spinor space.’

This appears to vindicate Weyl’s mysterious remark.¹⁹ But it also emphasises that there is a connection between the metric on the space and the definition of spinors—so that the latter actually *requires* the former. This dependence is further discussed in Bär *et al.* (**bar_generalized_2005**) and Bourguignon *et al.* (**bourguignon_spinorial_2015**).

Let us return briefly to Penrose’s idea of the centrality of the Riemann sphere. As noted, he pointed out that a spin- $\frac{1}{2}$ particle can have the possible directions in which its spin can be measured mapped to the Riemann sphere. But he then said:

Although quantum amplitudes seem to be very abstract things, having this strange ‘square root’ relation to a probability, they actually have close associations with space-time geometry (**arnold_mathematical_2000**).

To make this connection he noted that being situated at a point in space the light cone at that point can also be represented by a Riemann sphere. This sphere represents all of the light like rays that pass through the observer’s point in space. This Riemann sphere is then conformally deformed if we pass to another observer passing through that same point with a different velocity. Thus the non-reflective Lorentz transformations can be represented by complex conformal transformations of the Riemann sphere. It would be interesting to consider that these different ussages of the Riemann sphere could be unified by Cartan’s geometrical picture of spinors as square roots of null vectors.²⁰

5. Realism defended

The enlargement of mathematical ontology from Pythagoras through to Cartan and Weyl is properly the uncovering of structure already present, and uncovered through the process of doing ordinary mathematics—solving equations, constructing proofs, analysing existing mathematical structures. And through this process mathematicians have given us an understanding of real numbers and analysis, including differential geometry: complex numbers and their associated structures in geometry and algebra; and spinors and their structures. In these three cases the mathematical structures preceded, sometimes by centuries, their application in physics.

We can thus see the danger in an over-reliance on the indispensability thesis. There is a strongly pragmatist construal of this thesis that would have it that the *only* reason we should believe in mathematical entities is their usefulness in physical explanation—with the implication that if they had *not* found an application in physical explanation we would not have reason to believe in them. This does an injustice to the very thing that makes mathematical epistemology unique: *proof*. A far more compelling fact about the use of mathematics in physics is that the mathematical discoveries made by entirely different methods often precede the discovery that they can be found also in the natural world. It is this that should keep the Nominalist awake at night. But we should accept a more modest role for indispensability: that physics is capable of providing a layer of additional confirmation that mathematical structures and entities exist, and moreover that this existence should not be regarded as an *abstract* matter, for they are part of the fabric of the Universe.

Thus let us consider the most well-developed nominalist view: that proposed by Hartry Field in his (**field_science_1980**). The central idea is to take congruences on a Newtonian space as giving one all the ‘numbers’ that we need. And yet I think it misses the mark. As suggested earlier, if the nominalist is permitted to help himself to space-time as a flat 4-dimensional differentiable manifold *with a metric structure* then he has thereby helped himself to the real numbers already, both in the metric and also in the differentiable structure. For an n -dimensional differentiable manifold is locally isomorphic to \mathbb{R}^n .²¹ In fact a 4-dimensional, not necessarily flat, differentiable manifold proves to be unlucky for Field and nominalists generally as it is the only dimension for which there can exist an infinite number of *distinct* quasi-conformal structures—and thus there can be more than one way to determine the local mappings to \mathbb{R}^4 that are conformally inequivalent (**donaldson_quasiconformal_1989**). These are not simply many different metrics definable on a differentiable manifold—which would be a trivial point and would not distinguish 4-manifolds. Rather the quasi-conformal structures are distinct in that no finite amount of stretching or shrinking of the metric will deform one quasi-conformal manifold into another, despite being topologically identical. (Guided as we are by 2- and 3-dimensional topology this seems impossible to visualise.) The problem for Field is that these infinite possibilities are precisely the kind of *abstracta* that his nominalism cannot countenance.

If the space-time is Newtonian (as it is for Field) then the metric is globally singular though well-defined on the time-like fibrations—this alone creates complications since then his congruence relations are only defined on the fibrations. If it is Minkowskian then it is globally well-defined everywhere.²² Field is of necessity a substantialist about space-time geometry but I cannot see how comparatives will allow him to give a Nominalist construal of the light cone structure, since all points that are light-like separated *have 0 distance from one another by the Lorentz-signature metric, even when they are collinear!* This by itself refutes the idea that congruence can be a nominalist substitute for the role that the metric structure plays! Since Minkowski space-time is a more realistic space-time structure than Field’s preferred euclidean space this seems definitive.

But I’d like to sketch an auxiliary argument of a different kind, which suggests that Field’s strategy does *not* do away with numbers in the way he suggests, even on Euclidean space. Suppose that there were two four-dimensional manifolds, one with its metrical structure determined by a mapping to \mathbb{R}^4 and the other with the mapping to the quaternions \mathbb{H} or even to $\mathbb{C} \times \mathbb{C}$. According to Field’s nominalism these spaces are acceptable because they can be construed substantively, though the metrical structures are not taken to be substantive, because they involve numbers. So his plan is to eliminate these metrical structures using his reformulation thesis in favour of segment-congruence. But this presents him with a dilemma: either the results of this elimination gives us the same ‘space-time’, or they are different. If they are the same then the reformulation has eliminated crucial information—because multiplication (and therefore segment length) acts differently in these cases—but if they are different then the metrical structures are still present in an implicit form: we have simply stopped using useful numerical world. We could run this same argument with a comparison of \mathbb{R}^4 with Minkowski space-time, or with a different signature metric entirely, such as $(-+++)$, or, most significantly, with a Kähler 4-manifold in which there is more than one metric-like structure.

If real numbers are smuggled in in the form of geometric structure then the nominalist, though helping himself to a lot, has still not got enough even for the simplest cases of quantum mechanics. If we consider the Hilbert space of the spin- $\frac{1}{2}$ particle there is simply nothing available at all. The problems are then only compounded from this point on. Once we begin to consider quantum field theory we must consider spaces of operators that are defined on each of the space-time points. So let us consider the noncommuting operators of the electro-magnetic field: then the algebra will be an infinite-dimensional noncommutative algebra. Disposibilist Instrumentalism has no hope in this case, nor has it ever been attempted.²³

The *Indispensibilist* Instrumentalist might accept all of this as evidence of the indispensability of said mathematics but insist that we can think of the mathematics as merely ‘indexing’ the physical facts. The term ‘indexing’ comes from Melia (**melia_wesseling_2000**) and is meant to cover the use of real numbers for distances as well as other cases of measured magnitudes. However it is not at all clear what else it is meant to cover and without a very clear recipe for applying the term the charge of question begging will be hard to avoid (see Daly and Langford (**daly_mathematical_2009**) for a defence of this way of understanding Indispensibilist Instrumentalism).²⁴

Thus it is hard to see how we can account for *dimensionless* physical constants—such as Summerfeld’s *fine structure constant* $\alpha = 0.0072973525693\dots$, first introduced in 1917. The constant has the (or one) meaning:

$$\alpha = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c}.$$

Here ϵ_0 is the electric constant and e^2 is the square of the elementary charge of an electron. The value of the constant has been measured very accurately—and the accuracy is always improving, but it does not seem to be related to any known mathematical constants, and note that it isn’t clear, and may never be clear, whether it is rational or irrational.²⁵ And yet, it has been argued that if α were different by even a small amount then the Universe would not exist: matter as we know it would not exist. However it is not its precise value that is our concern here, but simply the fact that it is a *dimensionless number*. For the nominalist view is that numbers don’t exist, and thus that α does not exist either. But if that is the case then, never mind its exact value, *no* such value exists—and so matter can’t exist. No form of nominalism of which I’m aware has made an attempt to deal with this problem of dimensionless constants such as α —and no strategy suggests itself. That is the real argument in its starkest form, and indeed may summarise the point of this paper: *either numbers exist or nothing exists*.²⁶

But, as hinted at earlier, I believe we can find a simpler case, with ancient and venerable Platonic credentials, that seems rather clearly to not be a case of mere indexing. And it is one that is equally as hard for any form of nominalism that is currently espoused.²⁷

The argument is as follows. Premise 1: Whether an action *can* be performed, or a task completed, has a determinate truth value: one either can or can’t. Premise 2: Whatever facts the ability to perform the act or complete the task depends upon must likewise be determinate. But consider the task set by the Delphic oracle to the Delians: they were required to double the size of their altar—which was cubic shaped. And let us suppose, as Plato apparently did, when the Delians approached him on the matter, that this doubling of the cube must be done only within constructive geometry, that is with straightedge and compass, anything else being merely approximate.²⁸ The doubling of the cube requires finding $\sqrt[3]{2}$ which is irrational (the proof is an easy generalisation of some of the proofs of the irrationality of $\sqrt{2}$). But it is also a non-constructible number—as proven by Wantzel in 1837. And this means that there is no way to perform the action required by the oracle. So it is false that $\sqrt[3]{2}$ is constructible and so false that the Delians task can be performed. The same argument can be run using *squaring the circle* as the example, where the impossibility depends upon the transcendental character of π , which implies that it too is non-constructible.²⁹ The point is that mathematics is not just, as a discipline, indispensable to science, it is that mathematical facts *constrain and determine physical facts, and cannot easily be distinguished from them*. Thus, as another example, it is the topology of the 2-sphere that determines that there must be some point on Earth where the wind does not blow. It is impossible to partition explanation into the physical versus the mathematical in a way that leaves Nominalism with any clear content. Once we have let in what is needed for physical explanation then mathematics has been let in as well. This is particularly the case with the structures chosen here: the division algebras and the spinor structures. Mathematics and physics seem to have converged.

6. The royal road to ontology

It is time to take stock.

In the process of taking roots we have jumped from a discrete structure to continuous structures, in other words to geometry. In the first instance this led us to the real numbers, via incommensurable magnitudes and irrational numbers. Then in a second step we were led to the complex numbers and their richer geometry. And then, through complex numbers, Clifford algebras, and quaternions, we arrived at spinors. I’ve argued that there is no plausible nominalist strategy that can account for these structures: Field’s nominalist strategy won’t work and—even if its problems could be set aside (as I believe they cannot)—we would confront the problem of the dimensionless constants. This latter problem defeats even a putative structuralist solution. Nor is a narrow indispensibilist explanation plausible. My suggestion in this paper has been that the major steps of this progress would warrant a realistic attitude to these entities even if one could lay aside the application of this mathematics to physics.

So the process of taking roots turns out to be ontologically ampliative, and resists nominalistic reduction. Should we find this surprising? One might suggest, aphoristically, that platonism manifests itself in its most irresistible form as geometry. In support we may quote Shing-Tung Yau, the inventor of Calabi-Yau manifolds, on the importance of geometry:

Since the time of the Greek mathematicians, geometry has always been at the centre of science. Scientists cannot resist explaining natural phenomena in terms of the language of geometry. Indeed, it is reasonable to consider geometric objects as part of nature. Practically all elegant theorems in geometry have found applications in classical or modern physics (**arnold_review_2000**).

This of course is not to seek to take anything away from algebra, or to suggest that arguments for realism do not extend to algebras. How could they not when there is such a close relationship between algebra and geometry? If geometry may be likened to the face, then algebra is the mind behind the face. As Kähler said (in a philosophical essay): ‘[...] one must interpret the development of algebra as the revelation of the realm of ideas postulated by *Plato*’ (**kähler_II_2003**).

Thus this argument for mathematical realism gives precedence to the reals over the integers, and to the complex numbers over the reals. This is not to say that nominalism can easily deal with the integers—I believe that even here it must fail. But in mathematical terms the integers are now just one example of a commutative ring, one among an infinite number of others—and quantum mechanics has directed our attention to the *non*-commutative rings as possibly equally or more fundamental. The primacy of the three associative division algebras in mathematical explanation—the reals, the complex numbers and the quaternions—is what I mean by saying that these ‘almost geometrical structures’ are the primary basis for mathematical realism, a meaning that is in accord with Plato’s own emphasis on the importance of geometry. These division algebras and their associated higher structures, such as Clifford Algebras, or spin representations, are structures about which we must be realist.³⁰ It is here that the evidence is most irresistible. Indeed, if we turn the matter around, we could say this: the *only* plausible explanation for physics continually using the seemingly abstract mathematical structures uncovered by mathematics is that our universe contains those mathematical facts as generalised, non-local, parts of itself. In short: as ‘geometry’. My historical conjecture is that this was itself Plato’s original insight, inscribed on the entrance to the Academy: *Let No-one Unskilled in Geometry Enter Here*.

Acknowledgement: I wish to express my warm thanks to the readers for the journal who offered useful suggestions.

Declaration: The author declares that there are no conflicts of interest, no funding issues, and no ethics issues involved with this paper.

Bibliography

¹⁶ In a direct reference, Atiyah, in his 2013 conference lecture ‘What is a Spinor?’ quoted Weyl’s line verbatim.

¹⁷ See Penrose and Rindler (**penrose_spinors_1987penrose_spinors_1988**). Also Claude Chevalley (**chevalley_algebraic_1997**), particularly the afterword by J.-P. Bourguignon; also Louesto (**louesto_clifford_2001**).

¹⁸ See Trautman (**trautman_pythagorean_1998**) Proposition 1. These ideas are developed in greater detail in Kocik (**kocik_clifford_2007**), though without acknowledging Trautman’s prior work. Kocik links this with quasi-quaternions and the Apollonian Gasket.

¹⁹ Of course we might also add, for further evidence, that the square root of the classical Laplacian is the Dirac operator of relativistic quantum mechanics—and this takes us to the Lorentz invariant spinors of Cartan. There is thus a sense, not entirely figurative, in which quantum mechanics is the square root of classical mechanics, as suggested by Penrose.

²⁰ Penrose (**penrose_road_2004_ah**) does not reference Cartan in this context.

²¹ This style of criticism of Field was signalled early on by Michael D. Resnik in his review of Field’s book (**resnik_hartry_1983resnik_how_1985steinser_applicability_1998**).

²² This leads David Malament in his review (**malament_science_1982**) to shift Field’s case to a Klein-Gordon scalar field theory.

²³ Of course, we can accept, with Field, that there is no canonical natural isomorphic mapping of an n -manifold to \mathbb{R}^4 . But that is less than Field needs, for we can allow that the metric is defined only up to a scale factor without abandoning the idea that the metric is a part of the space. The metric simply becomes an equivalence class of numerical assignments, equivalent up to a scalar factor. But by only allowing congruence classes of intervals Field ends up with less than this—and here we come back to one of the main themes of this paper—for he cannot capture the facts about incommensurable magnitudes that so impressed the Greeks. Thus consider again the 1:1-triangle. Congruence classes will allow Field to say that the two catheti are the same length, but that no integer that is assigned to that class will allow an integer to be assigned to the hypotenuse, or vice versa. Incommensurability is a pairwise, *metrical* relation, and it is entirely *intrinsic* to the space. So it holds for every coordinatization in the equivalence class that defines the metric.

²⁴ I say nothing in this paper about structuralism, as I’ve discussed it elsewhere—see Heathcote (**heathcote_exhaustion_2014**). In its anti-realist form structuralism is unable to address the objections made here.

²⁵ The fine structure constant is often given in the reciprocal form $\alpha^{-1} = 137.035999206\dots$.

²⁶ As Wolfgang Pauli is alleged to have said: ‘When I die my first question to the Devil will be: “What is the meaning of the fine structure constant?” Of course there are other dimensionless physical constants besides α that could make the same point.

²⁷ Jody Azoumi has recently resurrected, in his (**azoumi_deflating_2006**) and (**azoumi_talking_2010**), a form of pure Fictionalism about mathematics—mixed with a form of social constructionism—that seems particularly vulnerable to this challenge, as it makes no attempt to deal with the mathematics that occurs in physics and is content to discuss the counting and computation of natural numbers(**batterman_explanatory_2010**).

²⁸ Thus note Plutarch’s comment on this: ‘And therefore Plato himself dislikes Eudoxus, Archytas, and Menaechmus for endeavouring to bring down the doubling of the cube to mechanical operations; for by this means all that was good in geometry would be lost and corrupted, it falling back again to sensible things, and not rising upward and considering immaterial and immortal images [...]’ Platonic Questions, Quest. 2, *Moralia*.

²⁹ As proven by Lindemann in 1882. The impossibility of squaring the circle was probably known by the time Plato was writing: Aristophanes ridicules circle-squarers in *The Birds*.

³⁰ In this context, the importance of group representation theory in quantum physics is worth emphasising. For here we take an often complicated non-linear algebraic object like a group and we consider it under the aspect of a geometric object by considering a homomorphism to a vector space. That this is especially fruitful has been argued often, as far back as Weyl (**weyl_theory_1931**) or Wigner (**wigner_unitary_1939**). For additional comments on this see Heathcote (**heathcote_multiplicity_2021**).

Philosophy in technology is a research program that studies the philosophical roots of engineering and technology. By virtue of their education, technologists believe that the limits, goals, possibilities, and effects of technology on society and humankind are exclusively technological problems, so their solutions must lie exclusively in technology. In contrast, philosophy in technology asserts that the resolutions to these problems need to be rooted in an understanding of their philosophical origins. In this program paper, we define the objectives of philosophy in technology together with the kinds of questions it explores, the methods it uses, and its differences to the philosophy of technology.

philosophy in technology, philosophy of technology, engineering perspective, semantic gap, philosophy in science, theology.

The man who has no tincture of philosophy goes through life imprisoned in the prejudices derived from common sense, from the habitual beliefs of his age or his nation, and from convictions which have grown up in his mind without the co-operation or consent of his deliberate reason. To such a man the world tends to become definite, finite, obvious; common objects rouse no questions, and unfamiliar possibilities are contemptuously rejected. (russell_problems_1912).

[Philosophy] removes the somewhat arrogant dogmatism of those who have never travelled into the region of liberating doubt (russell_problems_1912).

1. Introduction: The Need for a new approach for reflecting on technology¹

This paper discusses the concept of information that was formulated by Michael (Michał) Heller. Heller—a philosopher, theoretical physicist, cosmologist, and theologian—provided a complex image of information and its role in nature, one that is rarely found in studies of information. Heller posited that the laws of nature may be interpreted as information, or as providing information, with this being a complementary view to scientific structuralism (not discussed in this paper). According to Heller, the informational content of a structure (in nature) is inversely proportional to that structure's degree of freedom. The more constrained or complex, while also being less likely to exist, a structure is, the more information it contains. In Heller's view, the concept of information presented in the Shannon's Theory of Communication (ToC) is inadequate for expressing the notion of information beyond the concept of a numerical measure of a signal structure. Information in Heller's research comes very close to the concepts of Jacquette's and Perzanowski's combinatorial ontology (the concepts not discussed in this paper) and the general theory of information (GTI) of Mark Burgin, although Heller himself did not make these connections.

1. Introduction

A modern concept of information (and its quantification) in science and technology was introduced (not created) into the scientific and technical discourse in the mid-20th century by Shannon (shannon_mathematical_1948), Shannon and Weaver (ShannonWeaver1949ShannonWeaver1998), and Weaver (weaver_mathematics_1949), and a flood of research publications on information followed (carvalho_60_2009). Nevertheless, after decades of continuous efforts, we still only have a rather vague understanding of what information is.¹ Instead of one definition, we have many (adrians_information_2020krzanowski_ontological_2022). Most discussions of information are limited to a specific context, such as biological information, information in communication, pragmatic information, semantic information, symbolic information, physical information, quantum information, natural information, environmental information, or structural information with variants in each of the classes, although this list is not exhaustive.

On a few occasions, in an attempt to express information more comprehensively as a fundamental aspect of reality (an intuition shared with our pre-Socratics, religious colleagues, and some physicist with the bent for metaphysics),² researchers have formulated enigmatic *koens* like “everything is Information” (jones_everything_2018), “information is the difference that makes a difference” (sloman_what_2018), or “It from Bit” (wheeler_information_1989). Different versions of these have become entrenched in popular culture, yet these sayings do not explain much.³ They serve as useful quips in TED talks or alike, but they are without much impact beyond (tetlow_phil_2017).⁴ Heller as a philosopher, theoretical physicist, cosmologist, and theologian provided a complex image of information and illustrated its role in cosmology, something that is rarely found in studies of information. In what follows, we discuss Heller's account of information, present his extant claims and views about Shannon's information entropy, and present the enigmatic idea of harmony between abstract mathematical structures and nature. We also discuss how Heller's concepts of information fit into the wider modern discussion about information, including the GTI and the idea of the latent order of nature.⁵

A word of caution: Heller's ideas on information do not form a comprehensive theory like Shannon's TOC, Floridi's *General Definition of Information* (GDI) (floridi_philosophy_2011) or Burgin's *General Theory of Information* (GTI) (burgin_information_2003). They are dispersed throughout his papers on cosmology and philosophy of science and are more akin to Heraclitus or pre-Socratic fragments than to Shannon's, Floridi's, or Burgin's comprehensive theories. Thus, they have to be in some way weaved out of the larger context. Interpretation of such dispersed fragments is riddled with dangers. On one hand, we want to understand what Heller is telling us about information. On the other hand, we do not want to over-interpret his ideas, as it has been done (sometimes) with pre-Socratics. Therefore, the following presentation of Heller's thoughts on information may be seen by some as incomplete. But, we prefer the presentation to be incomplete in this sense, rather than incorrect, stating what Heller said but not what Heller might have said. Thus, the reader will often find our comments on Heller's fragments ending with the pose, “Heller does not clarify this intuition further,” and so we don't do it either.

2. Heller and Information

Michael Heller's views on information resulted from his studies of the fundamental structures of the cosmos (i.e., the Universe), its mathematical models, and the properties of nature (i.e., physical phenomena).⁶ In a series of observations, Heller outlines his vision of information in nature. The first fragment (1) comes from Heller's book *The Introduction to the Philosophy of Science*:

(1) The informational interpretation of the laws of nature may be seen as a complement rather than a competing option to scientific structuralism (heller_filozofia_2009).

Heller posits that the laws of nature may be interpreted as information, or as providing information, in a view that complements scientific structuralism.⁷ What “complements” means here is unclear, but it may be interpreted as saying that there is no dichotomy between the structural and information views of nature, with structure and information both being characteristic of nature. In fragment (2), Heller interprets Shannon's theory of communication and claims that the increase in the information content of a structure is inversely proportional to the structure's degrees of freedom.⁸

(2) According to the modern theory of information, the increase in informational content arises in transition from a set with a larger number of degrees of freedom to a more limited set. For example, the informational content of a set of all letters will increase for a set of letters that expresses some sentence (heller_filozofia_2009).

Heller observes how the laws of nature impose constraints on nature's structures, so they control, in a way, what can and what cannot be (i.e., not everything is possible in physics).⁹ What is possible is limited to the very large number of combinations of fundamental elements, so it is constrained by physical laws. The presence of quantum or discrete building blocks then makes the universe possible. This view is also reflected in the models of the universe in combinatorial ontologies, or ancient atomism.

For Heller, the laws of nature act like information (fragments (3) and (4)) in determining and constraining what is possible.

(3) Thus, information increases when the number of degrees of freedom decreases. (4) Limited sets (i.e., sets with constraints imposed on them) are nothing but certain structures, and every structure has certain information. The more restrictions that a given structure possesses, the more information it contains (heller_filozofia_2009).

The more constrained or complex structures are, and therefore less likely to exist, the more information they contain, based on Shannon's law. Thus, do the structures (in nature) code information (fragment (5)) or express information?

(5) As the world is a certain structure, it contains information, because this structure-world encodes information. This information is decoded by science and formulated as the laws of nature (heller_filozofia_2009).

Would Heller suggest here that the laws of nature are informat

This interpretation of nature, information, structures, and natu

(6) Modern theoretical physics suggests that the world does i

This information, as natural laws, is partially decoded and exp

In fragments (9) and (10), Heller states that while the laws of i

(9) The decoded fragments of information are denoted as scien

(heller_nauka_1995).

In other words, the laws of nature are the causes, or the results

According to Heller, the mathematical models of nature are highly s

strange harmony. This would be the position of modern Platonism

Further explanations for the concepts of nature, structures, inf

(11) As one must have some image of the world, the image of

(heller_ewolucja_1987).

In particular, modern physics does model the universe as math

In fragments (13) and (14), Heller suggests that even if there is

(13) Even if the real world contains something beyond the fo

This statement approaches the position of epistemic structurali

In fragments (15 and 16), Heller posits that this concept of info

(15) The same concept can be expressed as follows: If we defi

than to what this structure is filled with. In this view, the st

This theory perceives structure as something for encoding somet

information entropy), with the elementary unit of information bei

definition of a kilogram defines what mass is. It is instead simply q

ion merely their expression? Alternatively, maybe it is a chicken-and-the-egg problem. Nevertheless, we do not get an answer to this question in Heller's writings.

discussed in Heller's article titled “Nauka i wyobraźnia” (Science and Imagination) (heller_nauka_1995). In fragments (6,7, and 8), Heller positions structures and laws of nature as information.

ure but is a structure. (7) This structure contains encoded information or is information. (8) Science decodes its fragments by fitting mathematical structures to the structures of the cosmos (heller_nauka_1995).

laws. While scientific laws do represent a fragment, or an aspect, of cosmic structures, even though they are obviously much less complex than the natural structures, Heller does not explain in what sense the laws of nature are natural structures.

res are not isomorphic, they act in concert with nature, which perhaps refers to a sort of codependency.

odels of nature. (10) The mathematical structures of our theories and the structure of the cosmos are not isomorphic, but there is a strange resonance, a harmony between them. Because of this, resonance-harmony theories are grossly simplified in comparison to the structures of the cosmos, but they harmonize with the world, reproducing some of its [structural] properties

ries to some degree, although to what degree we are not sure. The point behind this remark is that laws and natural structures are not the same but somewhat codependent. Heller refers to the similarity between nature and abstract mathematical structures as harmony. Harmony, as proposed by Heller, is an intriguing (strange) property of the abstract models of the cosmos.

spect to the complexity of nature, and formalized. In other words, they have a high level of abstraction. They are not of the same “nature” as physical entities, so how are they able to reflect some of nature's properties quite accurately? In these abstractions, one may be tempted to see Platonic forms and nature as their realization, and such a view would certainly explain this

Platonism, which by the way has little to do with the ontology of Plato (linnebo_platonism_2018).

n can be found in Heller's paper titled “Evolution of the concept of mass” (heller_ewolucja_1987). In fragments (11) and (12), Heller posits that information can be thought of as a foundational element of nature instead of matter.

ional “stuff” must be substituted with another one. The image of the world not as a material composite but as a pure form would correspond much better with the findings of modern physics. (12) All models of the cosmos constructed by modern physics are abstract mathematical models. They do not have anything else but shape and structure (i.e., purely formal schema)

through shapes/structures without content. In this view, information is expressed in, or by, the “empty” mathematical structures, or these structures are information. Nevertheless, Heller does not clarify this intuition further.

d these “Platonic” structures, modern science is unable to detect it.

ethods of physics cannot detect it; this something slips through the net of mathematical-empirical methods. (14) In this sense, the world of physics is a pure form (heller_ewolucja_1987).

ie mathematical version of structuralism) in claiming that the structures of nature are mathematical structures of which nothing else (i.e., ontology) cannot be known.

on the concept of information arising in the Shannon-Weaver-Hartley theory of communication (ToC).

the constraint on degrees of freedom (possibilities), each law of physics is information as it limits the possibilities of nature. (16) One may think that the “stuff” of the universe is nothing else but information. But our current understanding of information is purely formal (e.g., Shannon-Hartley theory of information). Thus, information is reduced here to structure rather

dd is an information code, or encoded information, and the role of science is to decipher this code (heller_ewolucja_1987).

is the “stuff of the universe.” Thus, the concept of information in the ToC is inadequate for expressing the notion of information beyond the concept of a numerical value. In fact, the ToC does not define information, as some have mistakenly concluded, but rather measures it. To put it more precisely, the function defined by Shannon is referred to as a measure of information (i.e., 1). Indeed, Shannon's measure of information (i.e., the entropy of information) does not define information, just as the definition of a kilogram does not define what mass is. The entropy of information merely quantifies a specific property of a modulated physical phenomenon (i.e., a signal) under certain assumptions of syntax. Thus, it no more defines information than the in

property of a certain physical phenomenon (a signal) under certain assumptions. Thinking of the ToC this way is less prone to misinterpretations and may be closer to Shannon's original intention.

3. Heller on Information in Perspective

If we were to consider the most insightful ideas from Heller, who

(shannon_bandwagon_1956), foresaw this profusion of concept

information entropy, in Heller's view, is a metric for certain observ

The next insight from Heller's work would be the laws of nature but

such as knowledge or data, as many do (loose_discipline_1997).

In Heller's view, with “information expressed in or by” on

(burgin_information_2003burgin_theory_2010burgin_structural_2017).

In the GTI, information is stratified according to the global

(burgin_theory_2010burgin_structural_2017). The Physics

information.

A more detailed explanation of the GTI can be gained from Bur

from its import; the veracity of scientific theories is not voted in or

’ The statement that “the concept of information in the ToC is inadequate for expressing the notion of information beyond the concept of a number” would certainly count as one. Most studies of information in any domain base their concepts of information on Shannon's information metrics (i.e., information entropy). Few people, including Shannon himself

his idea and warned against this. Indeed, these “Shannon's extensions” are often over-interpretations (of the original intent) or to put it more bluntly, misinterpretations of the original idea and purpose. Shannon developed his ToC as a theory of communication for measuring the efficiency of a communication channel in the presence of noise and little more than this. Shannon's

at depending on what information is, may or may not contain information. As it happens, if we ask in what sense is this information, we generally get lost in explanations, or mathematics.¹⁰

what expressed as the natural (and by extension any) structures and laws of nature while being neither of these. According to Heller, the structures only encode or express information. Information lies beyond the visible and is expressed in, or by, “empty” mathematical structures, or these structures are information. Interestingly, Heller never associates information with meaning.

998sagegrande_information_1999dretske_knowledge_1999floridi_philosophy_2011floridi_semantic_2010leński_information_2010vernon_artificial_2014). However, Heller's information in the physical world is just form or form behind form, with meaning as in knowledge coming from, and with, us.

al structures,” information comes close to Platonic or platonic forms,¹¹ a metaphysical position that has a ring of truth to it, but this does not go down well with headline physicalists. Nevertheless, the fact is that Burgin's theory of information (GTI) is arguably the most comprehensive conceptualization of information proposed so far

urgin_structural_2017burgin_is_2022), and it includes Heller's metaphysical aspect of information in some form, thereby granting Heller's intuitions legitimacy of sorts.

world, as represented by the Existential Triad, which comprises the world's top-level components as a unified whole that reflects the unity of the world. This triadic structure is rooted in the long-standing traditions of Plato and Aristotle, and it comprises three components: the Physical (i.e., material) World, the Mental World, and the World of Structures

s the physical reality that is studied by natural and technological sciences, while the Mental World encompasses different forms and levels of mentality. Finally, the World of Structures comprises various kinds of ideal structures. The Existential Triad involves differentiating information into two fundamental classes: ontological information (i.e., information in nature) and mental

sd above. Due to its metaphysical import, the GTI may not be to everyone's liking, but it does not make the theory itself any less comprehensive or wrong; philosophy is not a beauty contest, even if it seems to be so from time to time. Further, the fact that the GTI is not known outside of the narrow circle of experts in the philosophy of information does not take away anything

atic process or won in a popularity contest (a point that some people may miss). Moreover, in the authors view, we do not have anything better than the GTI theory, at least for now.

4. Beyond Heller's Information

Out of Heller's fragment (11) and the works of other cosmologists' (

forms (the external form or shape of an object) should be better r

or low-entropy structures (krzanowski_inquiry_2023)¹⁴ or information as a latent order in nature.¹⁵ The concept of information as the potential of nature to create low-entropy (thermodynamic entropy) complexes (structures) appears to resemble the concept of Aristotelian potency, but the precise nature of this apparent similarity needs further research. Several recent

1 to as self-organization, to create forms or complexes (eigen_laws_1993). The self-organization property of nature is observable in everything from snowflake structures to organic life and the cosmos (reeves_heure_1986schrodinger_what_2012).¹⁶ Nevertheless, we should add that potentiality in its modern form does not attribute Aristotelian *telos* to nature.

ould therefore be the subject of a separate study. (See the discussion about nature's potencies in the work of Bird (bird_natures_2007) or Austin and Marmodoro (simpson_structural_2017).)

ves),¹² which envision information as a fundamental element of nature, grew the idea that information cannot be identical to, or identified with, the external form or shape of an object, structure as such, syntax, or even semantics because these “things” are temporal and ephemeral, whereas a fundamental element of nature should have a more stable existence.¹³ These material

dium through which information discloses itself to us. Heller's position, rather than information itself. To address this insight, Heller proposed that information is “an abstract form” or “something beyond the form,” which verges on the Platonic realm.

or low-entropy structures (krzanowski_inquiry_2023)¹⁴ or information as a latent order in nature.¹⁵ The concept of information as the potential of nature to create low-entropy (thermodynamic entropy) complexes (structures) appears to resemble the concept of Aristotelian potency, but the precise nature of this apparent similarity needs further research. Several recent

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5. Conclusion

Heller's intuitions about information in nature are not part of the m

delight of exploring the deep unknown. Then again, is this not wh

Being a hard-core scientist, Heller never abandoned the philoso

“in what sense are laws of nature natural structures,” and “informat

explicitated. Why we did not try to interpret Heller's ideas on inform

The connection between Heller and the GTI, the most complex

comprehensive as it is, it is the best we currently have, having bee

The possible role of information in nature has been discussed in

Burgin and Mikikiniemi (burgin_is_2022), Turk (turek_filozof

Hidalgo (hidalgo_why_2015), Wilczek (wilczek_beautiful_20

(krzanowski_ontological_2022). This list is certainly not exha

Heller's writings about information should be seen on a par wit

ation research, fortunately, otherwise we would have few reasons to talk about his work. Heller's intuitions belong to studies into the deep foundations of reality and border (for some) on metaphysics. It is certainly a path less travelled, one reserved rather for a minority of more open minds. With this comes the (sort of) penalty of not being frequently referred to, albeit with the

res of science and philosophy reside?

(heller_how_2019polak_philosophy_2019polak_beyond_2022) however at the cost of introducing metaphysical ambiguities. We may argue that Heller did not clarify his ideas about information and that some of his claims are enigmatic (e.g., “laws of nature are information, or information is their expression only,” “structures code information or express information,” or by “empty” mathematical structures or these structures are information”). This leaves the reader feeling somewhat uneasy. Yet the concepts Heller was grappling with are not well understood, and even now, nobody has proposed any better elucidations for them. At least with Heller, our ignorance and ambiguities about information and the foundations of reality have been

s we have said in the introduction, we try to report what Heller said, not what his claims might have implied.

for the nature of information we have, adds some importance to Heller's perspectives (it shows that Heller's ideas on information fits well into a larger comprehensive theory), but it also legitimizes the GTI itself. This is because Heller's perspective is built upon a deep understanding of the foundation of nature and physics.¹⁷ The GTI, meanwhile, is a complex construct, and

isite philosopher and mathematician extraordinaire, not through a deep study of nature, as was the case with Heller's ideas,¹⁸ but rather through the deep conceptual analysis.

re researchers who have conceptualized information as something more fundamental in nature (like Heller proposed) rather than just an idea or knowledge over the past 50 years includes von Weizsäcker (weizsacker_einheit_1971), Burgin (burgin_information_2003burgin_theory_2010burgin_structural_2017), Burgin and Feistel (burgin_structural_2017),

ak_rozwazania_1981), Collier (hanson_intrinsic_1990), Reeves (reeves_heure_1986), Stonier (stonier_information_1990), Devlin (devlin_logic_1991), De Mul (mul_informatization_1999), Polkinghorne (Polkinghorne2000), von Baeyer (baeyer_information_2005), Seife (seife_decoding_2006), Dodig-Crnkovic (dodig-crnkovic_alan_2012),

oll_big_2017), Rovelli (rovelli_reality_2016), Davies (davies_demon_2019), Sole and Elena (sole_viruses_2019), Schroeder (schroeder_philosophical_2005schroeder_structural_2017), Wheeler (wheeler_information_1989), Landauer (landauer_irreversibility_1961landauer_information_1991landauer_physical_1996), and Krzanowski

s a comprehensive overview of the recent (going back to the1960s) seminal discussions on this topic.

se authors, and should enter the canon of works on this topic, because his insights and intuitions not only confirm their studies but offer a perspective about the role of information in nature that is grounded in cosmology and physics rather than just in conceptualizations and philosophy, as is often the case with works on information.

Bibliography

¹ For historical, pre-Shannon, notes on the concept of information, see V

² The papers were published in an edited volume by Davies and Gregors

³ The fact that these phrases have been entrenched in popular culture do

⁴ Not surprisingly, visions of information as a fundamental element of a

⁵ A part of this paper has been published as D. Phil. thesis (krzanowski

⁶ All Heller's quoted writings here have been translated from Polish into

⁷ Heller's views about laws of nature and structuralism may be found in

⁸ The degrees of freedom is the number of independent variables (dimen

⁹ An interesting interpretation of the relation between the laws of nature

¹⁰ Any measure of information based on shape/form does not measure it

¹¹ The term “Platonic” refers to the original teachings of Plato himself, v

¹² Not surprisingly, visions of information as a fundamental element of a

¹³ The stability in time of physical objects, which is denoted as persistence

¹⁴ See B. 14.

¹⁵ The term “latent order” should always be interpreted as the “latent or

¹⁶ We regard snowflakes as low-entropy complexes that epitomize the pers

¹⁷ For more popular publications by Heller on the cosmos, science, and i

¹⁸ This point is important. Philosophy, mathematics, cosmology, and sci

history_2005), Adrians (adrians_information_2020), or Glick (glick_information_2011).

AVIAT-5).

ruer. It makes them what they are—a staple of popular culture. Further, one of these koans, a well-known “It from Bit” (wheeler_information_1989) implying human effect on QM has been proven wrong in the Delayed Choice Quantum Eraser experiment, the point which, of course, popular publications miss to the detriment of the scientific truth. As other ‘koans’ do not pretend to express scientific

tool as in https://onbcommons.org/).

¹² In the author's view, enthusiasm about the apparent deep meaning of i

dispersed in popular publications, has not been reflected in advanced discussions on philosophy of information.

(2022).

hor.

2009).

tem may be characterized by or exist within.

on of natural world is suggested by Laughlin. He writes that “At the most fundamental level, the laws of physics are laid out in plain sight for everyone to see. Yet you cannot generally predict things with these equations [...] [however, there are] collective principles of organization encrypted into these equations” (laughlin_crime_2008). Thus, you may say that the laws of physics define principles of

er its effect in nature. In addition, any measure of information based on shape/form/morphology actually contains/conceals a time variable, as pointed out by Burgin (burgin_theory_2010), so such measures should be indexed by time. For example, Shannon's information entropy ‘IE’ should be rewritten as ‘IE’.

onic” refers to modern versions of Plato's metaphysics.

ate from computer or data scientists or communication and networking engineers but rather people working intimately with information and nature.

something to exist through time simpliciter. All physical things, including the Universe itself, persist in that they come into existence, exist for a certain time (possibly changing forms on the way), and disappear (as in Heraclitian flux), at least this is the view of The Standard Model of Cosmology. (See the discussion about the SMC in, for example, the work of Smeenk and Ellis (smeenk_philosophy_2017).

l of nature to create complex morphologies.” Wheeler denotes this latent order, it seems, as a principle of organization (wheeler_information_1989).

bjects or naturally organized complexes (reeves_heure_1986). Forming complexes (i.e., ice crystals) that later disintegrate exemplifies nature's flux and the transition from low-high-low organizational states. Under specific conditions, nature forms low-entropy systems in local violation of the second law of entropy. Complex, highly organized natural systems are characterized by low entropy, while chaotic

ntropy systems can go on for as long as the required conditions are satisfied. For an extended discussion of low-entropy complexes and information, see the work of Krzanowski (krzanowski_inquiry_2023).

¹⁷ For more popular publications by Heller on the cosmos, science, and i

¹⁸ attempt to address fundamental questions using their own different methodologies, and often they diverge in their conclusions. Nevertheless, when their conclusions agree in some cases, it significantly strengthens the results of their inquiries.

Although many information system ontologies [ISOs] claim to be parsimonious, the notion of parsimony seems to influence the debate on ISOs only at the level of vague and uncritical assumption. To challenge this trend, the paper aims to clarify what it means for ISOs to be parsimonious. Specifically, section 2 shows that parsimony in computer science generally concerns software design and, together with elegance, is one of the two aspects of the broader notion of simplicity. Section 3 transforms the main claims of parsimony in software design into claims about the content of ISOs, the combination of which is hereafter called “parsimony of content”—where “content” refers only to the content of ISOs. Sects. 4-7 discuss the application of this parsimony to the design of ISOs, and outline different kinds (and combinations) of parsimony of content. Finally, section 8 considers whether parsimony of content could provide some criteria both for selecting and/or classifying the contents of ISOs and for choosing between different and equally consistent ISOs.

information system ontologies, ontological aims, parsimony, representation primitives, simplicity.

There are two ways of constructing a software design: one way is to make it so simple that there are obviously no deficiencies, and the other way is to make it so complicated that there are no obvious deficiencies.
Tony Hoare (1980)

1. Introduction

According to Turner (**turner_computational_2018**), there are two methodological advantages to adopting parsimony in software design:

- diminishing the amount of work,
- reducing the risk of error.

“This is in line with Quine, who, in the case of theories, argues that parsimony carries with it pragmatic advantages, and that pragmatic considerations themselves provide rational grounds for discriminating between competing theories” (**turner_computational_2018**). Acknowledging such advantages, however, does not imply that the adoption of parsimony is mandatory. Indeed, in speaking of information system ontologies [ISOs], Smith (**Horidi_ontology_2004**) and Grenon (**mum_primer_2008**) remark that nothing prevents ISOs from:

- [1] endorsing/rejecting different assumptions,
- [2] including parsimony among those assumptions,
- [3] considering the possibility of multiple forms of parsimony, and then repeating [1-2].

Despite [1-3], the adoption of parsimony is so common for ISOs that many ISOs implicitly and uncritically assume this notion. To prevent parsimony from influencing the debate on ISOs at the level of an implicit and uncritical assumption, this paper aims to clarify what it means for ISOs to be parsimonious. Sect. 2 shows that parsimony in computer science generally concerns software design and, together with elegance, is one of the two aspects of the broader notion of simplicity. Sect. 3 transforms the main claims of parsimony in software design into two claims about the contents of ISOs, the combination of which is hereafter called “parsimony of content”—where “contents” refers only to the contents of ISOs. Sects. 4-7 discuss the application of this parsimony to the design of ISOs, and outline different kinds (and combinations) of parsimony of content. Finally, Sect. 8 considers whether parsimony of content could provide some criteria both for selecting and/or classifying the contents of ISOs and for choosing between different and equally consistent ISOs.

2. Parsimony in software design

One of the main reasons why computer scientists place simplicity at the core of good and/or successful software design¹ is that simplicity contributes to the transparency and reliability of the design.² According to Turner (**turner_computational_2018**), simplicity does not have a single meaning in this context; rather, it refers to two distinct and related notions: elegance (or syntactic simplicity) and parsimony (or ontological simplicity).³ Elegance generally concerns the graspability, clarity, transparency, correctness, efficiency, consistency, generality, uniformity, and explanatory power of software.⁴ Parsimony links software design with its specification⁵, and insists that

- [4] software solutions do not go beyond *what is required*.

While Turner further specifies the meaning of “what is required” in [4] by claiming that

- [5] software should solve the problem it aims to solve, but no more,

Pawson (**pawson_minimum_1998**) takes one step further. First, he considers

- [6] parsimony to have been achieved when it is no longer possible to improve software by subtraction.

Then, he adds that

- [7] parsimony is the quality that software applications have when their details, and junctions have been reduced to the essential.

[7] in turn means that

- [8] the link between the design and the aims of software (see [4-6]) is reduced to the components, details, and junctions of the software.

[4-8] (together) imply that

- [9] parsimony concerns the [9.1] aims of software and [9.2] its components, details, and junctions.

3. Parsimony in information system ontologies

Section 2 has shown that:

- [10] simplicity is at the core of good and/or successful software design,
- [11] simplicity can be divided into elegance and parsimony.

Turner (**turner_computational_2018**) adds that

- [12] design is everywhere in computer science.

This means that, if [10-12] hold, parsimony also applies to the Gruber (**lu_ontology_2009**) defines ISOs as follows:

- [13] ISOs are sets of representational primitives (henceforth, primitives) to model a domain (of knowledge).⁶ Primitives are primarily instances, classes, properties, and relations.⁷

Therefore, based on [4-9], applying parsimony (of software design) to ISOs means:

- [14] ISOs should not go beyond the problem(s) they aim to solve;
- [15] the components, details and junctions of ISOs, that is the primitives, should be reduced to the essential (see [6-7] and [9.2]).

Henceforth, by “parsimony of content” (where “content” refers to the content of ISOs) I will mean the application of [4-9] to [13], namely [14-15]. There are two main reasons for this emphasis on “content”—rather than on “parsimony of ISOs” in the broader sense. The first reason is that, within the debate on ISOs, the notion of parsimony is chiefly associated with the content of primitives.⁸ Therefore, to speak of “content” in “parsimony of content” and this relation. The second reason is that parsimony of content does not (aspire to) exhaust the debate on the parsimony of ISOs. In other words, there may *in principle* be other parsimonies involved in the ISOs debate, as well as other ways of applying [4-9] to ISOs. And this is also in line with [10-12], which do not rule out that parsimony could be “everywhere” in ISOs, and thus **2018**). Moreover, although it would transitively follow from [8-9] that

- [16] parsimony of content deals with both [14-15],

we should also consider the possibility of

- [17] following [14-15] separately.

Indeed, if adopting parsimony of content means following both [14-15],

nothing prevents us from adopting parsimony of content partially, that is, from adopting either [14] or [15] by itself.

4. On the rivers of the UK

To specify what it means to adopt parsimony of content in practice, let us consider the ISO ISO_I aimed at $[A_I]$ listing and $[A_2]$ classifying all the rivers of the UK. Unless A_I and A_2 are further specified, A_I is fulfilled if and only if

- [18] no river in the UK is excluded from ISO_I ,

whereas achieving A_2 means

- [19] providing any classification of such rivers.

- [18] generally refers to the notion of completeness (of ISOs),⁹ and

- [20] the contents of an ISO should be exhaustive¹⁰ with respect to the domain that the ISO aims to model.

For ISO_I , [20] means that the nearly 1,500 rivers crossing the UK should find their place among the contents of ISO_I , which ultimately fall within (one of) the primitives of ISO_I (see also [13]), no matter which primitive. And

- [21] classify the rivers according to their biotic and/or topographic features;
- [22] systematize the rivers according to the geographical region(s);
- [23] catalogue the rivers according to some (arbitrary) length intervals;
- [24] consider [21-23] together;
- [25] provide any arbitrary classification.

The reason why there can be many ways to achieve A_2 is that

there are many criteria for classifying the UK’s rivers. Therefore, to the extent that each of [21-25] classifies the UK’s rivers, there is no way to prefer one among [21-25] over the others, at least according to A_2 .

5. On the aims of information system ontologies

According to [14-15], applying parsimony of content to ISO_I entails

- [26] ISO_I should not go beyond its aims;
- [27] (and) the primitives of ISO_I should be reduced to the essential.

As for [26], ISO_I has two aims: A_I and A_2 . In accordance with [26],

ISO_I is expected to

- [28] list all the UK’s rivers (see A_I),

- [29] classify those rivers (see A_2),

- [30] do nothing more than what [28-29] specify.

[28] implies [18], [29] leads to [21-23], and thus assumes that the

only ways of fulfilling [26], or that ISO_I could not go beyond its aims in different ways. [30] limits ISO_I ’s tasks to [28] (or A_I) and to [29] (or A_2). This means that, according to [30], ISO_I should not, for example,

- [31] list the UK’s lakes,

- [32] (or) classify Germany’s rivers,

because [31-32] would go beyond A_I and A_2 , and hence contradict [26].

[29]. All this also implies that

- [33] (all) ISO_I ’s contents should be consistent with and functionally adequate to the aims of ISO_I , but also that

- [34] no content of ISO_I should go beyond the aims of ISO_I .

However, things can get complicated in cases like the following: let us consider the ISO ISO_I aimed at $[A_I]$ listing and $[A_2]$ classifying all the rivers of the UK. Unless A_I and A_2 are further specified, A_I is fulfilled if and only if no river in the UK is excluded from ISO_I , whereas achieving A_2 means providing any classification of such rivers. [18] generally refers to the notion of completeness (of ISOs), and [20] the contents of an ISO should be exhaustive¹⁰ with respect to the domain that the ISO aims to model. For ISO_I , [20] means that the nearly 1,500 rivers crossing the UK should find their place among the contents of ISO_I , which ultimately fall within (one of) the primitives of ISO_I (see also [13]), no matter which primitive. And [21] classify the rivers according to their biotic and/or topographic features; [22] systematize the rivers according to the geographical region(s); [23] catalogue the rivers according to some (arbitrary) length intervals; [24] consider [21-23] together; [25] provide any arbitrary classification. The reason why there can be many ways to achieve A_2 is that there are many criteria for classifying the UK’s rivers. Therefore, to the extent that each of [21-25] classifies the UK’s rivers, there is no way to prefer one among [21-25] over the others, at least according to A_2 .

6. Completeness and parsimony of content

The principle of completeness (of ISOs) states that the contents of an ISO should be exhaustive for the domain that the ISO aims to model (see [20]). Applying completeness to (ISO_I ’s) A_I implies [14], but does not exclude that:

- [35] the same river appears twice (or several times) in ISO_I ,
- [36] ISO_I also includes the UK’s lakes and/or Germany’s rivers.

Conversely, applying parsimony of content to A_I implies [18], i.e., no river in the UK is excluded from ISO_I .

Because of [33-34]—which are ultimately inferred from [26]. From [36], however, it does not follow that completeness and parsimony of content are mutually contradictory, since:

- [37] ISOs may consistently follow both completeness and parsimony of content.

[1].

To justify [37], we could consider the negation of [36] to be only:

completeness, as well as a necessity for parsimony of content. The same, we may add, can be said for the negation of [35]. If so,

- [38] how does the negation of [35] follow from parsimony of content?

It is reduced to the essential. If [27], an (easy) solution might be to avoid repetitions, so that

- [39] each content of an ISO should appear only once in the same ISO.

Now, [39] is based on [27], which follows from [15], which in turn follows from [14-15]. Moreover, maintaining [39] means affirming the negation of [35], which is a necessity for parsimony of content and a possibility for completeness. But if so, [37] can also be justified by [35].

¹ On software design, see Allen (**allen_formal_1997**); Baljon (**baljon_formal_1997**); Wirth (**wirth_design_1979**).
² On simplicity in software design, see also Wirth (**wirth_design_1979**).
³ See also Baker (**baker_simplicity_2016**), who analyzes the distinctness of elegance and parsimony within the philosophy of science debate.
⁴ On elegance in software design, see Bentley and McRoy (**bentley_enough_1998**).
⁵ One referee rightly pointed out that there are other ways of relating elegance and parsimony: e.g., Soler (**soler_what_2002**); Turner (**turner_ontology_2018**).
⁶ For further (and competing) definitions of ISO, see Neches et al. (**neches_1991**); Gruber (**gruber_translation_1993**); Guarino and Giaretta (**guarino_ontologies_1995**); Bernaras et al. (**bernaras_building_1996**); Borst (**borst_construction_1997**); Swartout et al. (**swartout_toward_1997**); Studer et al. (**studer_knowledge_1998**); Guarino (**guarino_formal_1998**); Uschold and Jasper (**uschold_framework_1999**); Sowa (**sowa_guided_2005**); Gelernter (**gelernter_machine_1998**).
⁷ Instances are the lowest-level components, the basic units, of ISOs (**lu_ontology_2009**).
⁸ See Tambassi (**tambassi_philosophy_2021**).
⁹ See Burgin et al. (**burgin_sharing_1999**); Yao et al. (**yao_benchmark_2008**).
¹⁰ See Tambassi (2021b). “Exhaustive” in [20] also refers to the debate on

Parsons (**parsons_philosophy_2015**).
Jon (**humble_1979**).
e and parsimony within the philosophy of science debate.
Gelernter (**gelernter_machine_1998**). Oram and Wilson (**oram_beautiful_2007**); Hill (**de_mol_elegance_2018**); Turner (**turner_computational_2018**).
sion. The example they give is simplicity in understanding the code (i.e. “semantic simplicity”), including self-commenting code, which is simple in terms of understanding the code. I fully agree with them. I can only note here that this paper is not intended to exhaust the debate on the relationship between simplicity and parsimony. For more details on semantic simplicity, see Gelernter (**gelernter_machine_1998**).
991); Gruber (**gruber_translation_1993**); Guarino and Giaretta (**guarino_ontologies_1995**); Bernaras et al. (**bernaras_building_1996**); Borst (**borst_construction_1997**); Swartout et al. (**swartout_toward_1997**); Studer et al. (**studer_knowledge_1998**); Guarino (**guarino_formal_1998**); Uschold and Jasper (**uschold_framework_1999**); Sowa (**sowa_guided_2005**); Gelernter (**gelernter_machine_1998**).
10 See Burgin et al. (**burgin_sharing_1999**); Yao et al. (**yao_benchmark_2008**).
11 See Bittner and Smith (2008).
12 See Tambassi (2021b). “Exhaustive” in [20] also refers to the debate on

7. Parsimony of content and (representational) primitives

While [39] follows from [27], this is not all. Indeed, “ ISO_I ’s primitives should be reduced to the essential” seems to be open to different interpretations, such as:

- [40] reducing the types of the primitives we use (to the essential);
- [41] reducing the tokens of the primitives we use (to the essential);¹¹
- [42] combining [40] and [41].

To explain [40–42], let us imagine that ISO_I follows [23] and thus classifies all the UK’s rivers according to some (arbitrary) length intervals. $ISO_I \wedge [23]$ therefore has two aims: (A_S) to list all the UK’s rivers and (A_L) to classify them according to [23]. Now, [40] suggests reducing the types of primitives: using fewer primitive types to model a domain (see [13] and [20]) is preferable to modelling the same domain using more primitive types. This means that placing $[S_L]$ the UK’s rivers among the instances of $ISO_I \wedge [23]$ and the intervals of length among the classes of $ISO_I \wedge [23]$ (respectively) would be preferable to $[S_d]$ placing these rivers and length intervals among the instances, classes, and properties of $ISO_I \wedge [23]$. Indeed, S_I uses fewer primitive types than S_S . [41] is instead ambiguous. It may refer to

[41.1] an ISO’s overall amount of tokens,

meaning that the tokens of $ISO_I \wedge [23]$ should be reduced to the essential. Now, while A_S (simply) requires that all of the nearly 1,500 rivers crossing the UK find their place among the contents of $ISO_I \wedge [23]$ (for example, among the instances of $ISO_I \wedge [23]$), A_L might be fulfilled in different ways. Supposing, for example, that each length interval corresponds to a class of $ISO_I \wedge [23]$, [41.1] suggests that $[S_L]$ classifying the UK’s rivers by means of two length intervals (e.g., “longer than 100 miles” and “shorter than 100 miles”) is preferable to $[S_L]$ classifying those rivers by means of five length intervals (e.g., “between 0–30 miles”, “between 40–80 miles”, “between 80–120 miles”, and so forth). Why so? Because S_S requires (almost) 1,500 instances and 2 classes, 1,502 tokens in total, whereas S_L requires (almost) 1,500 instances and 5 classes, 1,505 tokens in total. (This also means that, insofar as S_S and S_L are both consistent with the aims of $ISO_I \wedge [23]$, it is irrelevant to [41.1] whether S_L is more detailed than S_S). But [41.1] also represents a way of balancing the overall tokens of $ISO_I \wedge [23]$ within the various primitives. For example, if achieving A_S and A_L required that S_S also include 10 properties and S_L includes 2 properties, then S_L would be preferable to S_S . In other words, the reduction of tokens within one primitive should not be at the expense of a proliferation of tokens within the whole ISO.

[41.1], however, is not the only way to interpret [41], which might also refer to

[41.2] the tokens of each primitive.

In turn, [41.2] could have two interpretations: [41.2.1] and [41.2.2]. [41.2.1] indicates that modelling $ISO_I \wedge [23]$ by means of, for example, $[S_S]$ 10 classes, 2 relations and 1,500 instances is preferable to modelling $ISO_I \wedge [23]$ by means of $[S_d]$ 2 classes, 3 relations, 2 properties and 1,500 instances. For although there is no difference between S_S and S_d in terms of the tokens of instances, and S_d is preferable to S_S in terms of the tokens of classes, S_S is preferable to S_d in terms of the tokens of relations and properties. This means that, according to [41.2.1], we should prefer S_S over S_d , insofar as S_S is preferable with regard to both relations and properties, and S_d is preferable only with regard to classes. [41.2.2] instead focuses on the tokens of each primitive, the reduction of which is independent from one primitive to another, and does not directly concern [41.1] or [41.2.1]. In other words, [41.2.2] offers the chance to apply [41] (and more generally [15] or [27]) to one and only one primitive. Consequently, we could have a [41] based on tokens of classes when the tokens of classes are reduced to the essential, a [41] based on the tokens of relations when the tokens of relations are reduced to the essential, and so on for each primitive. All this does not imply that those applications of [41] to one and only one primitive cannot be combined to improve [15], [27] and/or [40–41], nor that the list of primitives will never change, and with it the varieties of applications of [41] to which the primitives refer.

However, there are also ambiguities surrounding [42]. Firstly, it is unclear whether

[43] the combination refers to [40] and [41.1], or [40] and [41.2.1], or [40] and [41.2.2], or [40], [41.1], and [41.2.1], and so on.

Secondly,

[44] once [43] is clarified, we should also define the order of priority of the combination.

To clarify [44], let us suppose that the combination refers to [40] and [41.1]. Giving priority to [40] means that reducing the types of primitives is more important than reducing the total number of tokens: that is, both primitive types and tokens should be reduced to the essential but the reduction of the tokens comes after that of the types of primitives. Giving priority to [41.1] means the opposite.

8. Parsimony (of content) as a set of criteria

According to [14], ISOs should not go beyond their aims, whatever these may be. As regards the contents of an ISO, [14] means that they should all be consistent with the ISO’s aims (see [33–34]). According to [15], for any ISO, we should reduce the types of primitives (see [40]), the total number of tokens (see [41.1]), or the tokens of each primitive (see [41.2.1] and [41.2.2]) to the essential. Alternatively (see [42]), we could adopt [40] and one or more of [41.1], [41.2.1], and [41.2.2] by defining their priority. According to [16], we should adopt both [14] and [15], or better [14] and at least one of [40], [41.1], [41.2.1], [41.2.2], or [42].

On this basis, let us focus on ISO_I ’s A_S , according to which ISO_I should provide a classification of the UK’s rivers. Now, insofar as A_S does not specify any criteria to classify the UK’s rivers and [21–25] are (all) consistent with A_S , there is no reason why we should not regard

[45] [21–25] as *equally consistent* with A_S .

But, if [45], how are we to choose among [21–25]? The fact that the criteria, if any, are not deducible from A_S does not imply or guarantee that [14–16] provide any criteria. In other words,

[46] choosing among [21–25] may both [46.1] (at least partially) depend *and* [46.2] not depend on (some of) [14–16].

In turn, [46] does not imply or guarantee that

[47] once we choose among [21–25], [14–16] provide criteria for selecting and/or classifying the contents of ISOs.

All this means that parsimony of content (in general) can provide:

- [48] some criteria for choosing among different and equally consistent classifications/ISOs;
- [49] some criteria for selecting and/or classifying the content of ISOs;
- [50] both [48] and [49];
- [51] neither [48] nor [49].

9. Concluding remarks

Since some ISOs adopt parsimony as an implicit and uncritical assumption, and/or without explaining what parsimony specifically consists of (or refers to), these pages sought to clarify the point. In this regard, I introduced the notion of parsimony of content, showing that

[52] this parsimony concerns two main claims, [14–15], as well as their connection, [16], from which [33–34], [37], [39–40], [41.1], [41.2.1], [41.2.2], [43–44] and [48–51] follow.

[52] broadly suggests that the adoption of parsimony of content has to do with

[53] the interpretation and combination of claims about parsimony of content,

[54] specifying whether parsimony of content provides some criteria for choosing among different classifications/ISOs and/or for selecting and/or classifying the contents of ISOs.¹²

All this means that

- [55] the notion (and application) of parsimony of content is multifaceted;
- [56] an informed adoption of parsimony of content requires [53–54].

It does not follow from [55–56] that parsimony of content exhausts the debate on the parsimony of ISOs, nor that ISOs are bound to adopt parsimony of content. In other words, [55–56] are consistent with [1–3], thus ensuring the plurality of the methodological approaches shaping the debate on ISOs.

Funding. This paper has received funding from the European Research Council under the European Union Horizon Europe Research and Innovation Programme (GA no. 101041596 ERC—PolyphonicPhilosophy).

Disclaimer. Funded by the European Union. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union. Neither the European Union nor the granting authority can be held responsible for them.

Bibliography

¹¹ The distinction between [40] and [41] is largely based on Fiddaman and Rodrigues-Pereyra’s (fiddaman_razor_2018) distinction between two different forms of ontological economy. According to the authors, the principle of qualitative economy requires us to avoid multiplying types of entities when not necessary, while that of quantitative economy requires us to avoid multiplying token entities when not necessary. For further reading on ontological economy, see Sober (sober_simplicity_1975), Lewis (lewis_counterfactuals_1973), van Inwagen (van_inwagen_ontology_2001), Lando (lando_ontologia_2010), and Schaffer (schaffer_what_2015).

¹² Unlike some computer scientists (floyd_assigning_1967), I have not considered the possibility of combining parsimony of content with modularization: indeed, breaking down complex ISOs into (in)dependent modules would simply defer the question of adopting parsimony of content to both complex ISOs and their (in)dependent modules.

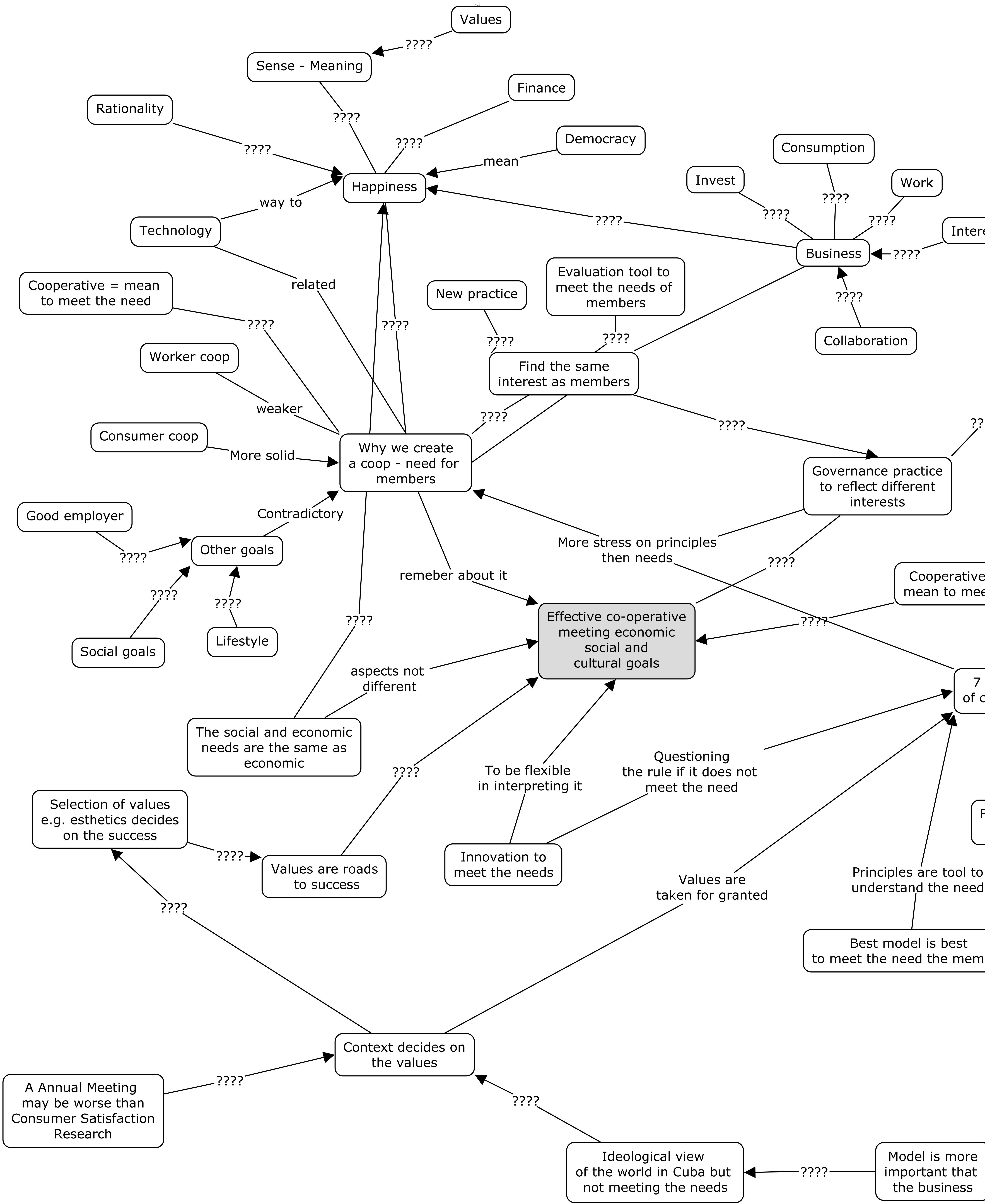


Figure 4: An expert map of an effective cooperative (stocki_tacit_nodate).

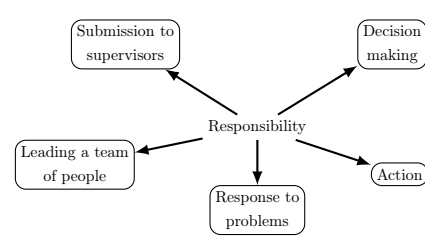
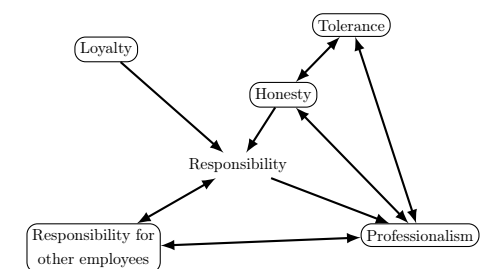


Figure 5: Examples of two cognitive maps obtained during investigations described in subsection 4.1.



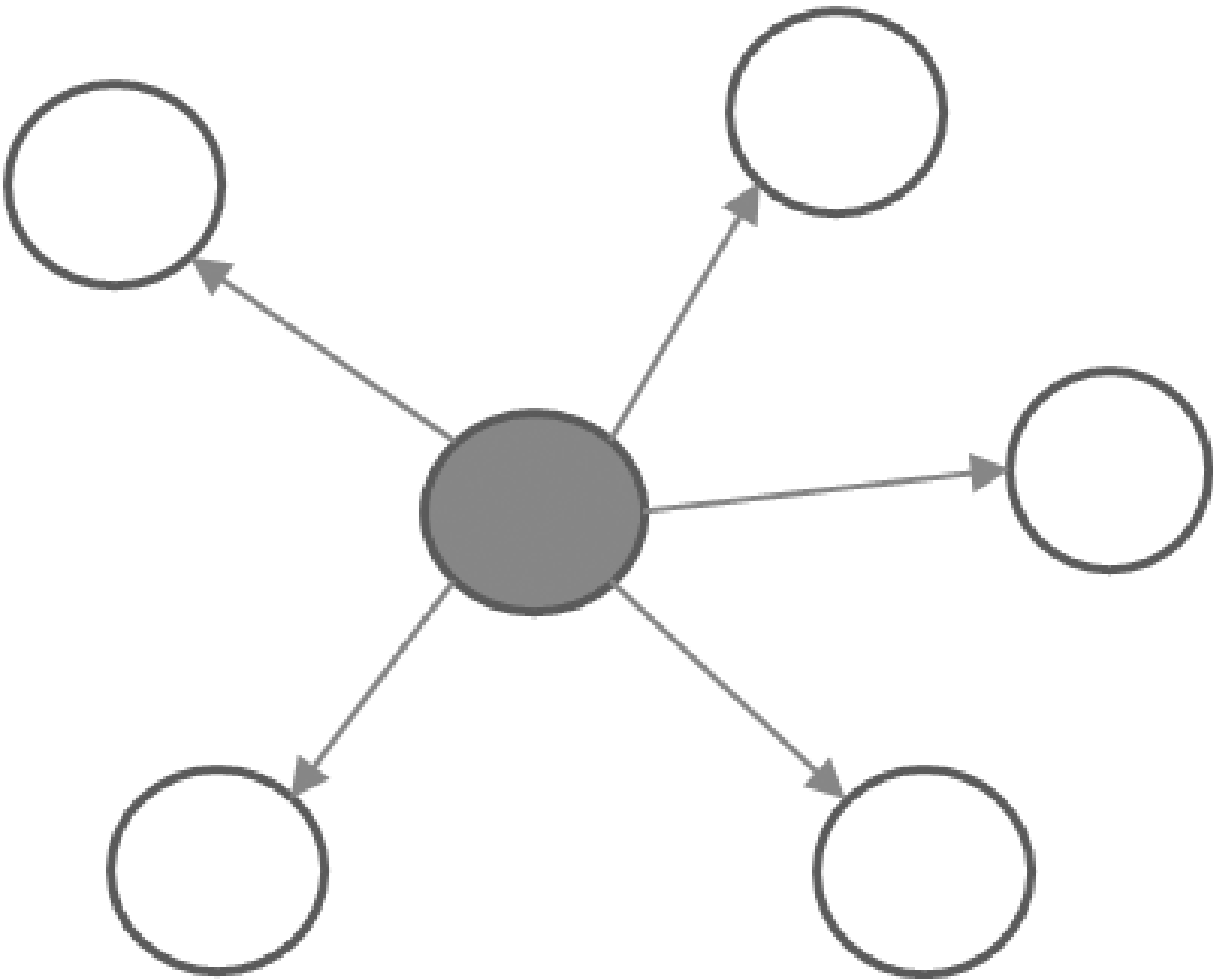
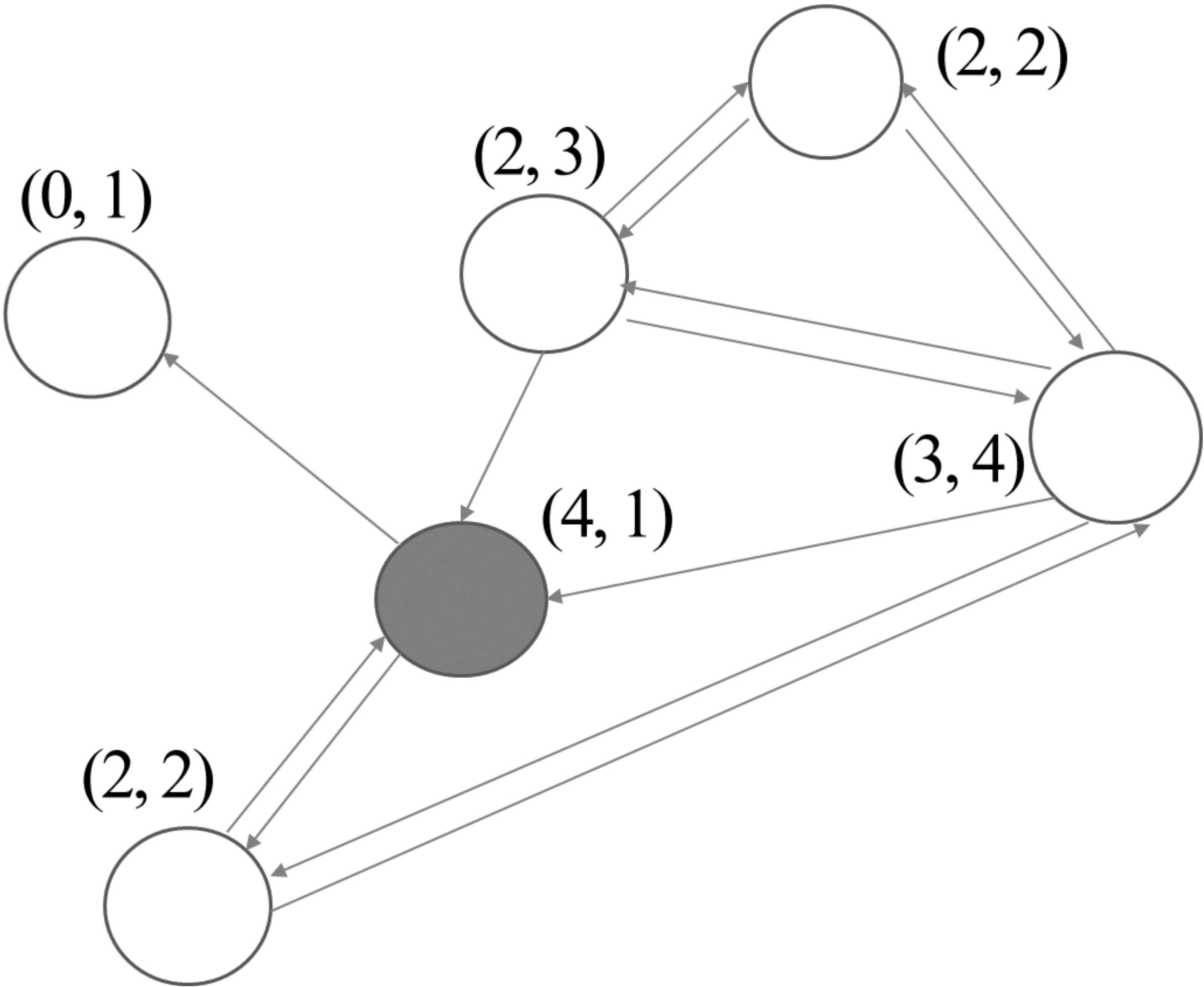


Figure 6: The structure of cognitive maps presented in Fig.5. The numbers in parentheses in graph G_2 denote indegree and outdegree of the node.



5. Concluding remarks

Let us summarize the proposed concept of information and the presented example of its application. The concept of information at the current stage of the studies is formulated in purely mathematical way. The definitions of various types of information have been put forward and their basic properties have been specified. This was done with care for formal correctness and completeness. Although the concept was originally dedicated to applications for analysis of biological structures and processes ([bielecki_general_2015](#)) and was preliminary tested by using to analysis of molecular cybernetics ([bielecki_information_2022](#)) it turned out that the concept is also applicable to analysis of cognitive maps. As for the analyzed example of applications, formal analysis of such maps with the assistance of structural information concept allows the researchers to go beyond from the simple analysis of the map content to the analysis of the maps structure and complexity. As is visible in the example above, we can, for instance, analyze the role of the central concept of responsibility in detail showing that in the first map if removed, it disintegrates the structure of the whole graph, whereas the same removing of the central concept from the second graph causes only partial disintegration of the structure of the graph. This means that in critical moments in the decision making process, when some important aspect is undermined, the first manager may have no indication how to behave whereas the second one may reconstruct the decision with the help of a substitute. Such formal analysis of the concepts make visible properties of our thinking that were, so far, treated as tacit. Furthermore, the proposed approach allow the researcher to calculate precisely amount of information in the cognitive maps.

Bibliography

The introduction of the concept of the field of rationality and its correlates (the field of potentiality and the formal field) by Józef Życiński and Michał (Michael) Heller opened up space for the philosophical explanation of the unreasonable effectiveness of mathematics in capturing regularities built into the physical reality. The presented study is a response to the clear incentive of these authors towards the development of the understanding and applicability of these concepts. It is argued that identifying symmetries within the field of rationality not only helps to articulate the fundamental role of symmetries in physics but it provides a better grasp on the issue of potentialities for the emergence of complexity in the Universe. Also, some global properties of this field can be more deeply comprehended. By indicating the drawbacks and limitations of this approach, perspectives for further inquiry into the meaning and usefulness are suggested.

symmetry, ontology, potentiality, emergence, field of rationality, field of potentiality.

Introduction

The concept of the *field of rationality* has been proposed independently by Józef Życiński and Michał (Michael) Heller in order to address two fundamental questions within the philosophical reflection on the nature and method of mathematics and physics: (1) how mathematical objects and structures exist and (2) why mathematics is so effective in the physical sciences.¹ The development of the contemporary physics has revealed that the formalisms of the fundamental physical theories rely on symmetry manifested by the appropriate symmetry groups. Also, symmetry is a principal tool by which the unification of physics has become possible thereby making the Universe intelligible at an unprecedented scale (**gross_role_1996**). This outcome has found its vocal expression in a phrase coined by Wolfgang Pauli who referred to the ubiquity of symmetry in physics as *Gruppenpest* (the plague of symmetry). The importance of deepened philosophical analysis of why the type of symmetries known as gauge symmetries is so effective in physics has been emphasized by Michael Redhead (**brading_interpretation_2003**) in the following assertion: “The gauge principle is generally regarded as the most fundamental cornerstone of modern theoretical physics. In my view its elucidation is the most pressing problem in current philosophy of physics”. The philosophical concerns regarding symmetries in physical theories continue to spark interest and discussions from a wide range of perspectives (**dardashti_editorial_2021**).

The aim of this study is to show how the understanding of the internal structure and the global properties of the field of rationality can be deepened by taking into account that symmetries play such an extremely important role in physics. By identifying symmetries within the field of rationality a metaphysical argument for this state of affairs will become available. The need for this deepening has been clearly expressed by Heller (**heller_field_2014**) in his assertion that “the idea never went beyond its seminal stage” and still remains “fuzzy”. The additional advantage of identifying symmetries within the field of rationality is that one can better explicate the nature of potentialities for the emergence of physical structures in the course of the Universe’s history commencing at the moment of the Big Bang.

The objective of this study will be carried out in four steps. Firstly, an introduction to the origins and the meaning of the field of rationality as well as its derivatives referred to as the formal field and the field of potentialities will be offered. A special emphasis will be made on how Życiński attempted to capture the process of the emergence of the physical structures in the Universe as the actualization of potentialities latent in the field and what are the possible shortcomings of this attempt. Secondly, the specificity of the formalisms of the symmetry based physical theories will serve as a premise to propose a relation between the formal field and the field of rationality and to introduce the concept of the *field of symmetries*. Thirdly, the formal field as well as the connection between symmetry and structure will be utilized in advancing better understanding of potentialities and the ensuing dynamics leading to the emergence of structures in the Universe. Some useful references to the contemporary discussions on potentialities will be made. Fourthly and lastly, keeping in mind that the inquiry is intended more as an exploration of the possible interpretative perspectives of the field formal field and the field of rationality, some suggestions concerning further investigative efforts will be offered. Since identifying symmetry groups within the field of rationality implies a decidedly realist position in regards to the status of symmetries within the fundamental fabric of the Universe, this study explores a new dimension of metaphysical issues that arise in the context of contemporary science.

The field

The concept of the *field of rationality* was originally proposed by Józef Życiński and introduced with detailed justification in (**zycinski_filozoficzne_1987**). In a nutshell, this field comprises all possible mathematical structures as well as all possible relations of inference between them and some section of this field provides a matrix for the physical functioning of the Universe. This clearly reflects the fact that only a small portion of mathematics turns out to be relevant from the point of view of physical applications. As long as this field is considered from purely formal point of view only, Heller prefers to call it the *formal field* and to link the field of rationality with the ontological claim positing it as an existing entity that justifies the possibility mathematics as the activity of the human mind (**heller_uchwycie_1997**). This, of course, reveals Platonic preferences of Heller and Życiński to which they openly subscribe (**zycinski_swiat_2013**). Unfortunately, both these authors remain somewhat ambiguous whether the field of rationality should refer to the world of mathematics as a whole or to its portion that is physically relevant only. Since it is the ontological interpretation of the field of mathematical structures that shows the desired explanative power in regards to the possibility of mathematics and its applicability in physics, for the purpose of the conceptual clarity the Platonic world of all possible mathematical structures will be referred to as the *formal field* and its physically applicable portion as the *field of rationality*. The distinct ontological character of these two fields finds its natural environment in the Platonic ontology of the three worlds of *math*, *physics* and *mind* proposed by British mathematician and theoretical physicist, Roger Penrose (**penrose_road_2004**). In this ontology, the physical world emerges in its entirety from the objectively existing Platonic world of mathematical structures. Undoubtedly, the Platonic interpretation of the formal field of mathematical structures reinforces a strong metaphysical claim but, at the same time, it does justice to the preferred standpoint of mathematicians treating the object of their study as a objectively existing reality which they do not construct but discover (**penrose_road_2004**).

While the above paragraph shows only a general statement of what the field of rationality is, Życiński took up the challenge to delve deeper into its nature. In his view, the key role of the field of rationality is to capture the fact that “the fundamental level of reality is constituted by an abstract network of formal relations and the reality of the observed physical substrate is secondary with respect to the formal relations whose existence we discover in the physical processes which are concrete exemplifications of these structures” (**zycinski_status_1995**).² In order to provide a suitable illustration of this assertion, Życiński resorted to quantum field theory and, in particular, to the metaphor based on the process of formation of particles as a result of the excitation of the lowest energy field, that is, the vacuum. Following the suggestion of American particle physicist, Heinz Pagels (**pagels_cosmic_1983**), Życiński treated the vacuum as a reservoir of potentialities out of which physical structures could emerge in the evolution of the Universe and, ultimately, find their exemplification in concrete physical systems. And this is the very reason why he proposed to regard the field of rationality as the *field of potentiality*.

His favorite examples of the emergent structures were the Kepler laws of the planetary motions and the Mendeleev’s periodic table which—in his opinion—should have both already existed in the early Universe prior to the appearance of planets and chemical elements. He maintains that although these laws must have been somehow coded in the structure of the Universe so their actualization in concrete objects occurred strictly by natural powers, there must remain a “radical separation” between these two domains of existence (**zycinski_pole_2006**). In other words, on one hand he wished to secure the workings of the physical causality in effecting this actualization and yet to preserve some form of otherness of the field of rationality to sensibly articulate the idea of potentiality.

It is not difficult to see that this ambiguity makes Życiński’s argumentation inconclusive and that he never came up with a satisfactory way out of it. Initially, he opted for the Platonic metaphysical view of the field of rationality based on the atemporal character of the abstract structures comprising the field of rationality. While this dualist stance allowed for a clearer articulation of their potentiality with respect to the domain of physicality, it effectively prevented their causal activity in this domain. Życiński (**zycinski_pole_2006**) has eventually abandoned the Platonic view of the field of rationality in favor of its ontological interpretation by naming the field of rationality the *nomie structure* of the Universe (from Greek *nomos* = law) which reflects much closer relationship of this field with the laws that govern the Universe. In his introduction to Życiński’s *Świat matematyki i jej materialnych cieni* Heller parallels this conceptual change with the transformation of the philosophical school of Plato in which the ostensibly dual metaphysics has been converted into ontology by Plato’s successors: Speusippus and Xenocrates (**zycinski_wstep_2013**).³ Heller opines that this is precisely where the final ontological stance of Życiński qualifies and where the idea of the mathematicity of the Universe has its roots.

Życiński’s ontological turn finds its corroboration in the approach to quantum gravity pursued by Heller and his collaborators with the use of the non-commutative geometries (**heller_emergence_1998heller_poczetek_2002**). This approach leads to the elimination of the notion of space and time on the fundamental level of the physical reality thereby offsetting the dichotomy between the atemporal and the temporal as means of delineating what is abstract and ideal and what is concrete. Consequently, potentiality ceases to be the attribute of abstract Platonic world but shifts over to the domain of the physical and can enter into the causal interactions with the concrete. Contrary to the Platonic stance, this situation neutralizes the barrier for the physical causation in actualizing potentialities but, by this very fact, it makes the articulation of potentiality more difficult.

By bringing up only a handful of examples illustrating the usefulness of the concept of the field of rationality Życiński *de facto* provides only some local characteristics of this field without much attention to its more fundamental global properties. However, intimations of this kind of description appear in his insistence that the field of rationality as a whole imposes constraints on the ontology of the Universe rendering some phenomena and processes impossible (**zycinski_filozoficzne_1987**). According to Życiński, the existence of the field as a constraint manifests itself in the unchangeability of the physical constants, stability of the physical processes and—most importantly—symmetries and their invariants. In order to substantiate this claim he recalls Pagels’ observation that the majority of the history of modern physics are the discoveries of new symmetries (**pagels_cosmic_1983**). Engaging the field of rationality to explain the role of symmetries as the cornerstone of c

It turns out that Życiński’s incentive to investigate the global pr algebra as separate categories whereby it provides an overview “from is the Universe categorial” suggests that the field of rationality is r the ubiquity of symmetry in physical theories.

The first indication that there may exist connections between the fi measure and which precedes the group theoretical account of symme resulting symmetry and in what exactly it consists” (**brading_int**

The next important piece of information on how to locate symm to the order of real physical existence” (**heller_telihards_2003**). / which the Poincaré group belongs, are additionally equipped with d spaces thereby generating distinct meaningful physical situations.

The simplest and quite illustrative examples in that regard are t have several unitary irreducible representations which refer to differe Consequently, considering that the abstract groups may have repre

In order gain further insight into the relations between the form the symplectic structure of a Hamiltonian, for instance. This state c empirically but not mathematically equivalent: Hilbert spaces, Feyn domain of the artifact of description. Interestingly enough, such inf

[Nature’s] fundamental laws control a substratum of which w

The third general feature of a physical theory with symmetries i oscillation can be effectively described as a 3-level system of a dynamical systems (the **12k_collaboration_constraint_2020**)—ne

Although by taking into account the ubiquity of symmetries in p naming the field of rationality “the field” has yet another advantage ubiquity of symmetries in physics a conjecture can be put forward t relevant symmetries present in the field of rationality seem to form

A close corollary of identifying symmetries within the field of rationa rationality containing abstract structures was placed in the Platonic reservation of how such abstract groups can exert their causal influ

Keeping in mind that symmetries impose restrictions on the pr concretes is fulfilled. In a way, the number of physically relevant re

The second way of associating potentiality with symmetries has based on the geometric symmetries of the geometrical objects. In his the one who arranged them. Kosso concluded that the messy room l a chair constructed out of atoms where each atom is rotationally s

As Debs and Redhead (**debs_objectivity_2007**) point out in sides of the same coin”. Debs and Redhead do not pursue any rigoro the symmetry group gets smaller, the number of invariants grows at purpose of this study.

A good example of the relationship between the operations of y whose invariant is the Euler number and any transformation is als

It is commonly known that the structuring and diversification of crystal is less symmetrical than perfect homogeneity”. The nature o approaches a critical value resulting in a new asymmetric but stable of undergoing a phase transition could be regarded as having pote

The presence of the groups of symmetries in the field of rationa a symmetry group that has been spontaneously broken. From a no lowering of a symmetry present in this field where the original larg

Also, the identification of symmetries in the field of rationality s asymmetric but it does not mean that the fundamental laws are nc symmetries that are being broken. The extent to which this mecha

The identification of symmetries within the field of potentialiti

Everything points to the fact that at the beginning there wa

An attentive reader will quickly notice that in this quote Heller potentiality in light of which a unified theory would encode potent

It turns out the the issue of potentiality is one of the central on effect—that is, that a certain outcome will occur. For instance, a neq based physical theories presents a considerable challenge. In this reg detailed study will need to follow.

In the conclusion of the presented inquiry it is worth to bring out th posed within their mathematical frameworks. Nevertheless, it is cruc of rationality has been obtained. Moreover, the identification of sym

The identification of symmetries within the field of rationality a geometry in the pursuit of the theory of quantum gravity that has

One can rightly expect that the development of Heller’s idea of f relate to the field of symmetries in a yet unknown way (**heunen_1** invariance will reveal its full mathematical meaning suggesting tha

Acknowledgments: The author wishes to express his thanks to

nd symmetry can be found in the philosophical understanding of the term *rationality*. The term itself has diverse meanings deriving from the Latin term *ratio* and it may stand for reason, relation as well mathematical proportion. This coincides with the original understanding of symmetry developed in the ancient Greece which reflects the etymology of the term as the *common*

d by Brading and Castellani, symmetry remains closely linked with unity which in the ancient meaning is derived by proportion and in the modern by the symmetry operations belonging to a precisely defined transformation group. They assert that “the way which this unity is realized on one hand, and how the equal and different elements are chosen on the other, determines the **3**”. This, in turn, correlates with the *normative* character of symmetry, namely, that the invariance with respect to a group of transformations imports significant restrictions on the theory’s form as well as on the form of its equations (**brading_introduction_2003**).

of rationality comes from Heller’s attempt to compare the process of the formation of a physically meaningful representation of an abstract group with the commencement of its existence in the philosophical sense of the term. He grounds this inference in the analogy to St. Anselm’s proof of the existence of God on the premise that there occurs a transition from the formal order Heller offers the example of the irreducible unitary representations of the Poincaré group which describe properties of all existing elementary particles and fields. Considered in themselves, groups are but sets of abstract objects defined by the group operation satisfying the group axioms. The Lie groups, are continuous groups playing key role in physical applications and to folds (**schwichtenberg_physics_2018**). However, these abstract objects begin to “do physics” once they are represented as group structure preserving operations on a uniquely selected mathematical space most frequently considered as linear transformations of a vector space. By using representation theory, one can study how a given group operates on a variety of vector

(1,1) symmetries. Since both these symmetries offer powerful tools in advancing our understanding of the properties of quantum systems, they are undoubtedly important elements of the field of rationality. While there is only one unitary and finite dimensional representation of the SU(2) compact group, the SU(1,1) group, which is probably the simplest non-compact Lie group, erent states and serve to study physically distinct systems (**yourdas_analytic_2006**). Since the abstract structure of the SU(1,1) group leads to several distinct physical realizations, it seems rational to locate the abstract groups within the formal field while their physically pertinent representations, which are symmetries, should find their place in the field of rationality.

e not physical (e.g., non-unitary representations), one can postulate the existence of the *field of symmetries* that constitutes the subfield of the formal field which contains all possible abstract groups and symmetries regardless of their physical relevance. f symmetries and the field of rationality, one needs to take into account three general features of physical theories that rely on symmetries. Firstly, the formalisms of these theories feature mathematical structures other than symmetry groups such as topology, manifolds or differential geometry. Secondly, physical theories contain symmetries that are physically irrelevant such as ⁴ of its source in the fact that physical theories put forward by physicists are but approximations of the structure of the physical reality and as such they may contain structural elements that do not pertain to reality but they are artifacts of the workings of the human mind. A good example in this regard is given by the four possible formulations of quantum mechanics that are ⁵, C*-algebras and density matrices. As Heller points out, these formulations are different representations of the quantum reality taken in an informal sense that they encode some of the structural features of this reality and only structural invariants of these representations refer to the fabric of the microworld (Heller 2011, pp.144-145). Whatever remains variant is relegated to the es given as the defining feature of symmetry whereby symmetries constitute mathematical tools which discriminate between what pertains to reality and what is a surplus structure, that is, an artifact of a theory. Paul Dirac (**dirac_principles_1930**) asserts the following:

mental picture without introducing irrelevancies. The formulation of these laws requires the use of the mathematics of transformations.

a fact that although symmetries provide important constraints for the dynamical equations, they don’t determine them uniquely and other factors need to be taken into account in their derivation. For instance, neutrino oscillation is a phenomenon where an impact of symmetry on dynamic properties (equations of motion) becomes particularly visible. The three-flavor neutrino a highly non-trivial Hamiltonian directly related to Pontecorvo-Maki-Nakagawa-Sakata mixing matrix relating mass and flavor states (**banerjee_quantum-information_2015bilenky_neutrino_2016**). A form of this matrix depends on the CP symmetry constraining neutrino properties. If the CP symmetry is violated—as it seems to be the case according to the recent particle become distinguishable and evolve in time with different Hamiltonians generating their evolution. One can identify measurable properties of the neutrino by indicating particular form of the time evolution and its symmetry (**richter_logett-garg_2017**).

Initially tempted to match the field of rationality with the field of symmetries, considerations presented above show that the situation is more complex and a more nuanced approach needs to be adopted. It has been already suggested that the abstract groups and symmetries belong to the formal field and that this field contains all possible mathematical structures. It turns out sep mathematical definition of a field associates a certain quantity with each of its points. By way of analogy, a particular instance of rationality such as those indicated by Życiński can be linked with a corresponding point of the field. Such a point stands for a section of the fundamental ontic structure of the Universe represented by physical theories. Taking into account the oup is located in the neighborhood of the points of the field of rationality and it may constrain structures proper to a given point. As a result, a symmetry group will turn up in the physical theory that describes reality’s structure at this point and it will exert influence on the properties of the systems subject to the regime of this theory and its equations. Ultimately, physically s section of the field of symmetries, that is a part of the formal field, with the field of rationality. Unfortunately, at this stage of analysis it is not possible to explain why this cross section contains the symmetries that it does and not any other. One may also legitimately doubt whether, beyond a mere statement, such an explanation is even possible.

Exploring potentiality

ity of clarifying Życiński’s ambiguity in regards to the nature of potentialities latent in the field of rationality. It turns out that one can think of these potentialities in two different ways based on how the “radical separation” between the abstract and the concrete comes about. The first way arises in some accordance with Życiński’s original metaphysical outlook where the field of eraly generating the much desired “radical separation” between the abstract and the concrete. It is not hard too see that the proposed placing of the abstract groups such as SU(2) and SU(1,1) in the formal field and not in the field of rationality does justice to this radical separation when the formal field corresponds to the Platonic universe of mathematics. With the obvious al domain, this separation has to serve as the only reason for now why these groups should be regarded as potencies that become actualized in the form of the properties of fields and particles when unitarily represented in concrete linear spaces.

s/dical objects they describe, it is worthwhile to point an important difference between the two abstract groups. In contradistinction to SU(2), the SU(1,1) has several physically meaningless representations suggesting that its abstract structure is refracted in a number concrete physical realizations whereby Życiński’s demand of one abstract structure underpinning a number of

ccesses of symmetry breaking. Let us start with the difference between symmetry and design. The opinion that symmetry is a key element of the design of the Universe has been expressed by American physicist Anthony Zee (**zee_fearful_2007**). It has been critically analyzed by American philosopher of science, Peter Kosso, who suggested an intuitive origin of this assertion le this intuition, Kosso (**brading_symmetry_2003**) gave a simple but telling example of juxtaposing a messy and ordered room. While in a messy room one can quite easily shift items around without upsetting its invariant structure and frustrating its owner, an ordered room does not admit of practically any displacements of its furnishings that would escape the attention of y and less design while the ordered less messy and more design. Consequently, design means not symmetry but the breaking of symmetry suggesting that producing a design connotes rather having intentional control over the choice of the desired symmetries than being subjected to a constraint. As a confirmation of his conclusions Kosso recalls Steven Weinberg’s example of chair itself is not. In other words, the building of a chair by its designer has led to the decrease of symmetry.

l and intuitive way, symmetry and invariance are complementary ideas bound by the relation of *duality*. In mathematics duality is known to be a broad concept and its precise definition is given when duality is applied to specific cases, for just that context. The main idea contained in duality is that it points to a deeper structure that manifests itself in twofold manner as “two f an underlying structure that symmetry and invariance may represent but they wish to articulate the *interchangeability* of these concepts with special emphasis on their *reciprocity*. In particular, they refer to the fact that the higher the symmetry group of a structure, the more changes it can endure indicating that it is less constrained because it contains less invariants. So if comes richer (more rigid). In other words, the decrease of the size of the symmetry group, that is, the symmetry breaking, leads to the emergence of more complex structures resulting in the growth of complexity. Manchak and Barrett (**manchak_hierarchy_2023**) demonstrate that this relation bears more nuanced character but its informal treatment should suffice for the

invariant structures are the different geometries with the Euclidean being the most rigid that is having the greatest number of invariants and the smallest symmetry group, through affine geometry where the requirement of constant length is loosened and only the parallel lines are preserved. Yet less structure comes with the projective geometry. The “softest” structure is topology s continuity, that is, the structure of the neighborhoods of points. Ripping the structure apart would mean changing topology and breaking the structure’s symmetry.

ty occurs by means of the processes of symmetry breaking. Peter W. Anderson (**anderson_more_1972**) offers an example the formation of a crystal which leads to the lowering of the symmetry: “the general rule, however, even in the case of a crystal, is that the large system is less symmetrical than the underlying structure would suggest: symmetrical as it is, a symmetrical crystal has received an extensive treatment in physics leading to the identification of two basic mechanisms through which symmetry might be broken: *explicit* and *spontaneous* (**brading_meaning_2003**). The mechanism of the spontaneous symmetry breaking occurs when the lowest energy symmetrical solution becomes unstable under small perturbations as some parameter (e. Inasmuch as Życiński’s illustration of the actualization of the potentialities in the field of rationality by means of the excitation of a vacuum could with some reservations reflect the mechanism of the spontaneous symmetry breaking (e.g., the excitation of the quantum harmonic oscillator), phase transitions yield a much better example in this regard. A system that is capable posal to assume a more ordered state due to symmetry breaking as a certain external parameter is changed (i.e., decrease of temperature).

Other straightforward understanding of what it means that a physical structure is contained in this field. Since following the explanation provided in a previous section symmetries relate to the corresponding invariant structures via the relation of duality, a concrete structure may be considered as encoded within the field of rationality by means of an appropriate subgroup of y view, duality stands for a mathematically precise relation between these two different structures suggesting that Życiński’s postulate of the “radical separation” between the abstract and concrete finds its expression in this reciprocity. In summary, the actualization of a physical structure that emerges from the field of rationality could be then understood as a process of the p

ommentates the potentiality to bring forth a diversity of concrete structures which commence their physical existence as accessible for the scrutiny of the scientific method. of better insight into Życiński’s claim that concrete physical systems are instantiations of the general physical laws that govern their dynamics (e.g., the Kepler laws). When symmetry is spontaneously broken, the solutions of the equations of motion are no longer invariant under the action of the equation’s symmetries. Phrased differently, the world around us appears to us very ough the new lowest energy solutions are asymmetric, they are related through the action of symmetry transformations and the whole set maintains the symmetry of of a given theory and its laws. Thus the lower symmetry solutions do not violate the symmetry properties of these laws. And conversely, the patterns exhibited by the behavior of nature provide clues to the

to such instances as the planetary systems fulfilling the Kepler laws of motion would need much more detailed analysis that remains beyond the confines of this study.

and justification in a path that is in some sense reverse to that of symmetry breaking, namely, a path that hypothetically leads back to a structure that has the potency of producing every possible complexity in the Universe. In addressing this issue Heller (**heller_uchwycie_1997**) asserts the following:

—an extremely rich and geometrically simple mathematical structure. The subsequent symmetry breakings (the separation of each of the four interactions) gave rise to increasing diversity. The dream of the theory of everything is the dream of discovering of the mathematical structure from which everything has its origin. ⁴

procal relation between symmetry and invariance as applied to the early stages of the Universe. In order to unify bosons and fermions, supersymmetry requires a sufficiently large symmetry group which should in turn yield relatively few invariants thereby making the corresponding geometry simple. This observation signals an interesting connection between unification and oger number of possible concretizations. For instance, such increased potentiality could manifest itself in a theory unifying gravity with the three other interactions because, as Heller (**heller_poczetek_2002**) admits: “it is very difficult to find a symmetry rich enough to combine the spatiotemporal symmetry of gravitation with the dynamic symmetry of other interactions”. y metaphysics and it concerns the ongoing discussion on the nature of powers and dispositions and these concepts are used into the explanation of what the laws of nature are (**friend_dispositions_2023**). In most general terms, to attribute a disposition to a thing means that if certain conditions are fulfilled, then that thing will behave in a certain way, or produce a certain article is an entity that, if brought together with another negatively charged particle, it will experience a repulsive force. As French (**french_doing_2020**) clearly shows, while the articulation of dispositions and powers in regards to objects of everyday experience is a fairly straightforward task, the shift to the domain of the abstract mathematical formalisms of the symmetry mately ask what is the metaphysical significance of the fact that, for example, the spinor representation of the Poincaré group encodes the properties of electrons and quarks. Chances are that the application of the concepts of the formal field and the field of rationality may turn out instrumental in sorting out these difficulties. In order to accomplish that, however, a separate

Conclusions

n of symmetries within the field of rationality—much the same as the postulate of the field itself—are philosophical interpretations. This means that they cannot influence the progress of physics but they provide answers to why this progress is possible. In other words, they do not modify or oppose the formalisms of the physical theories but they address questions which cannot be he efficacy of the proposed interpretation relies on an *a posteriori* observation derived from the practical aspects of theoretical physics, revealing that symmetry serves as a fundamental underpinning in all physical theories. The major contribution of the inquiry consists in that, by relying on this observation, a novel insight into the global structure of the formal field and the field se domains fortifies a robust realist standpoint concerning their ontological status, thereby opening up avenues for exploring their metaphysical significance. What might escape even the most sophisticated metaphysical consideration is why the cross section of the field of symmetries with the field of rationality contains these and not other symmetries that are physically relevant.

utilization carry a number of shortcomings and are in need of further development to address their full philosophical import. For instance, no reference was made to the different kinds of symmetries that enter into the theoretical frameworks (external (i.e., spatio-temporal), internal, gauge). Moreover, in light of the works of Heller on the application of the non-commutative tioned of this study, some promising results can be obtained when the concept of a group is generalized with that of a *grasoid* (**heller_evolution_2006**). This indicates that identifying symmetry groups within the field of rationality may bear an approximate character only.

ld of rationality as the field of categories will provide further support for the meaningfulness of the field of rationality but at some point it might face its conceptual limitations as well. What appears promising from the point of view of this study is that some deep connections have been identified between categories and symmetry suggesting that that the field of categories may . Consequently, the process of “unfuzzing” of the field of rationality remains a challenge as one needs to constantly re-represent it with the use of more abstract conceptual frameworks allowing for the gradual unveiling of its nature. Ultimately, however, one cannot exclude that the intuitively understood duality expressing the relation of reciprocity between symmetry and

their incisive comments and to Professor Jerzy Dajka from The Department of Physics of the University of Silesia in Poland for his help in sorting out complex formal technicalities.

Bibliography

¹ An extensive overview of the origin and the development of the concep

² Translated from Polish by Wojciech P. Grygiel.

³ For an in-depth analysis of the transformation of the Platonic School s

⁴ Translated from Polish by Wojciech P. Grygiel.

1. Introduction

The paper presents a brief outline of the Michal Heller's programme of theology of science, with a specific attention to its collocation and critical role with respect to both theology and science. The former consideration is based on a third domain of truths (Hans Urs von Balthasar), while the latter is inspired by Józef Tischner's presentation of religious thinking. Theology of science as such will be described with the reference to Larry Laudan's approach, considered here as a very useful and pragmatic tool for the description of basic concepts of this theology.

theology of science, third domain of the truths, research tradition, Michal Heller, Hans Urs von Balthasar, Józef Tischner, Larry Laudan.

In the vast area of study designated "faith and reason" the theology of science occupies a special position. While considered a branch of theology, the theology of science has a specific topic of study, namely science, which do not belong to theology proper. This situation raises a number of uncertainties, including questions regarding its methodology and locus of enquiry. In this essay, I will focus my attention on two issues: the collocation of theology of science in the realm of theological investigation, and the purpose it serves for both theology and science. In fact, the theology of science serves in communicating faith to a secular world and in developing a reasonable and informed faith. But not only, as it will be explained later.

I will follow the approach of Donald Lococo developed in his *Life in One Breath: Meditations on Science and Christology* (**lococo_life_2021**). As he writes, modern reflections on theology and science evince a "large lacunae, owing to the near ablation from consideration" of some of the "most significant twentieth-century Catholic theologians, namely Balthasar and Karl Rahner".¹ "[O]ne can hardly conceive of building on the theology of any denomination without paying attention to its most deeply influential thinkers" (**lococo_life_2021**).

Lococo mentions these two names only. Of course, in order to give a fuller account of the development of theology in the context that interests me, the list of names should be considerably longer, including theologians as Ratzinger, Guardini or Teilhard de Chardin. As this text is not intended to be a review paper, but aims to formulate a working hypothesis inspired by Balthasar's approach, his works will provide the basis for further considerations. As far as the role of science in religious thinking and the limits of the theology of science are concerned, I will be guided by the thought of Józef Tischner (Polish priest and an eminent philosopher), whose ideas on religious thinking were developed in the Kraków academic milieu, not without a dialogue with the thought of Michael (Michał) Heller. At the same time it is worthwhile to remember the possible correlation between Balthasar's and Tischner's thinking (**wolowski_problem_2019**).

With these thoughts in mind, I will state my point of view as follows. As to the understanding of theology of science itself, I will follow Michael Heller's approach, briefly outlined in the first section.² Then, I will enquire into the theology of Hans Urs von Balthasar, a pre-eminent theologian in the Catholic tradition, looking for the answer to the question about the collocation of theology of science in the domain of theological research (section 2). Next, Michael Heller's programme will be further examined, and framed, in the broader context of Larry Laudan's research tradition (section 3).³ In the conclusion, Józef Tischner view of religious thinking will be questioned in order to describe the role of theology of science in developing a reasonable faith and in understanding of limits of both science and theology (section 4).

2. Michael Heller's theology of science programme

This essay has its *raison d'être* in the faith of the Church. Before proceeding, an important clarification must be made. The main participants in the conversation reported in this essay belong to the circle of Catholic Church. Coherently, the views expressed by the Christian Catholic theology represent what can be considered "the First Truth Discourse" on God and His Revelation.⁴ Thus, it presupposes the existence of God, who reveals Himself, and the legitimacy of science which "begins with the self-revelation of the trine God in the Incarnation of the divine Logos, the Word, the Son, and the expositor [Auscloer] of the Father" (**balthasar_theo-logic_2004**). Stated otherwise, the essay has its locus in theology, which refers to talk about God and God's Word of Revelation in the Catholic Church (**barth_church_2010**).

The purpose of Revelation is not to communicate truths about the natural world that satisfy the innate curiosity of the human being, but above all to show the path leading to salvation.⁵ Revelation is not informative in the way that ordinary knowledge is informative. It does not add to our list of facts about the world or the universe in which we live. Instead, it is existential in the sense that it concerns the deepest dimension of human existence and gives direction and meaning to human life. For this reason, the knowledge gained through revelation cannot be combined with the findings of science, which focus on the material universe and do not touch the deeper question of meaning. As John Paul II puts it, addressing a group of scientists and researchers, "Divine Revelation, of which the Church is the guarantor and witness, does not in itself entail any scientific theory of the universe, and the assistance of the Holy Spirit does not guarantee the explanations we propose regarding the physical constitution of reality" (**john_paul_ii_address_1983**).

Nevertheless, within the framework of theology, it is possible to reflect critically on those truths of Revelation which allow for a deeper understanding of science as a specifically human activity dedicated to the world created by God. This critical reflection is the very purpose of that branch of theology that might be called "theology of science".

According to Michael Heller, the theology of science is a branch of theology that engages the experimental sciences, their existence, foundations, methods and results, with the understanding that the experimental sciences study the world created by God. As a branch of theology, the theology of science has all the characteristics of theology as a discipline. Its context for reflection is the life of the believer, the Church, and its methods and sources are not extraneous to those used in other theological disciplines. Consequently, a theology of science can be thought as an authentic research tradition within Catholic theology.⁶

The basic premise of the theology of science is thus one that has already been put forward: the statement that the universe was created by God. It should be specified here that, for theologians, the concept of the universe encompasses all that has been created by God. Of course, the universe of science and the universe of theology are not identical. The former pertains to the material world while the universe of theology goes beyond the material or visible world. However, while the two realms are separate, theology cannot bring forward theses that contradict those advocated by the sciences. It cannot, therefore, enter arbitrarily into the specific domain of the experimental sciences.

The thesis that the world, and indeed the universe, came into existence through God's special design has to be completed by the thesis which affirms the absolute dependence of everything that exists on the Creator. Traditional theology, following in the footsteps of traditional philosophy, thus used to speak of the "contingency of the world". The thesis that the world is utterly dependent on God not only for its creation but also for its continued existence is one of the essential elements of Christian doctrine concerning creation; however, the way God interacts with world is not a question that will be addressed in this essay.

Rather, I will focus on the rationality and comprehensibility of the world, a primary focus within the theology of science. As Heller writes, "with the theology of creation is connected another problem, the problem of the rationality of the world. [...] by the rationality of the world I mean that property of the world by which it can be studied rationally. This investigation of the world belongs to the domain of science and the accomplishments within the sciences are the best attestation to the rationality of the world. From a theological perspective, the rationality of the world is the mark of the Creator's rationality" (**maczka_wstep_2015**). This theme is frequently highlighted in the theology of science. In the Christian doctrine of creation, it belongs to a study of the Logos-Word. Olaf Pedersen writes that "the identification of the divine logos with Christ [...] make it possible to connect in a fundamental way faith in Christ with the quest for understanding the inherent rationality of nature, or even to see this rationality as a sign of God's immanence in the world" (**pedersen_historical_1990**).

Finally, the question of values needs to be mentioned in connection with the theology of science. It is well known that the method of the experimental sciences is insensitive to values: normative and value statements do not belong to the language of the experimental sciences. This thesis has been put forward since at least the time of the Vienna Circle formed in the 1920s. It does not mean, however, that the material world has nothing to do with values. On the contrary, from the standpoint of theology, the creation of the world is essential for the realization of God's project of love and salvation. This project takes into account not only everything that the experimental sciences seek to discover and investigate, but also what is called a "value system", that is an axiology. Hence, reflection on the experimental sciences from an axiological point of view is also one of the tasks of the theology of science.

3. A Third domain of truths

Clear from what has been so far written is that the theology of scien what is the precise meaning of that statement? What is the specific, short, the question is about the specific domain (the material object theology and sciences.⁷

One of the possible solutions to the problem suggested by this li can be further elaborated and slightly changed since it identifies the stimulating remarks which outline a possible, more profound, I eve According to Balthasar, "the world as it concretely exists is one author of *Theo-Logic* emphasizes that the natural fundamental struc increasing amount of theological data. This is so, because "the sup supernatural truth from philosophical research; the supernatural h

Balthasar proceeds to describe three ways in which theologial d given our knowledge of the incarnation, and the second way entails According to this statement, between the two domains of the natur This third domain of truths is constituted by truths "visible" on object of theology of science. As a matter of fact, it satisfies all fou

discipline of theology and shares with science an interest in the natural world, albeit from a particular perspective, which is different from that of the experimental sciences. Michal Heller and his commentators emphasise that it is a perspective which considers the world as created by God. Therefore, the theology of creation is considered a pillar of the theology of science. But f the theology of science, which, while guaranteeing its belonging to the field of theological enquiry as such, nevertheless distinguishes it from other theological disciplines and from the sciences as well? The question pertains, on the one hand, to the place of the theology of science within theology and, on the other, to the relation of theology of science to the natural sciences. In *4 science*. To properly belong to theology and science this domain must fulfil the following conditions: (1) it must belong to the domain of theology as such; (2) it must also belong to the domain of the sciences; (3) it must allow theology of science to be considered a distinct theological discipline, distinct from the sciences; and, last but not least (4) it must ensure the autonomy of

ven suggested by Szczurek (**maczka_teologia_2015-1**). In his essay on the structure of theology of science, he advocates that theology of science is an authentically theological discipline working with scientific results as interpreted by the philosophy of science in the light of Revelation and the Ultimate Aim of the man. Interesting as this thesis may be, Szczurek's suggestion theology of science with science as seen by philosophy of science. Consequently, it presents the theology of science in the guise of philosophy of science, that is, as another way of mediating science and its achievements. Whether a more radical interpretation of theology of science that does not collapse the discipline into one that already exists remains to be investigated. Some *ological* – insight, can be found in the works of Hans Urs von Balthasar, mainly in the first volume of his *Theo-Logic* (**balthasar_theo-logic_2000**). Let us follow his train of thoughts. *adly* related positively or negatively to the God of grace and supernatural revelation". Consequently, "the world, considered as an object of knowledge, is always already embedded in this supernatural sphere, and, in the same way, man's cognitive powers operate either under the positive sign of faith or under the negative sign of unbelief" (**balthasar_theo-logic_2000**). The and knowledge are by no means eliminated or altered in their essence by their inclusion in the supernatural sphere. Therefore, philosophical thought, in its capacity for abstraction, can probe them apart from conscious reflection on their supernatural imbuement. However, as philosophical thought probes the concrete object of enquiry deeper and deeper, it begins to encounter an ot in the deepest structures of being, leaves them through and through, and permeates them like a breath of an omnipresent aroma". For that reason, Balthasar asserts that it is impossible not to include theological data in thinking about the nature of things: "it is not only impossible, it would be sheer folly to attempt at all costs to banish and uproot this fragrance of pregated nature so deeply that there is simply no way to reconstruct it in its pure state" (**balthasar_theo-logic_2000**).

a concrete philosophical thought. There is, of course, the unc conscious assimilation of such data in philosophical enquiry (Balthasar gives the example of Plato). Then there is a kind of secularization of theological data, whereby the data is given the status of rational, properly human truths (e.g. modern rationalism and existentialism). The first way, however, is no longer accessible st divine Revelation, which can hardly be justified theologically and is therefore unsuitable for a theology of science. There remains a third way: "to describe the truth of the world in its prevalently worldly character, without, however, ruling out the possibility that the truth we are describing in fact includes elements that are immediately of divine, supernatural provenance". 1984 (**heller_stwozrodzenie_1982**cyrcinski_w_1984)polak_teologia_2015rodzen_teologia_2021). M. Heller's writings on theology of science goes back to 1992 (**oleksowicz_w_2020**). As to the main bibliography, see: (**heller_new_1996**maczka_wstep_2015maczek_teologia_2014maczka_teologia_2015maziarka_w_2016)polak_teologia_2016oleksowicz_w_2020rodzen_teologia_2021).

ditions, that is only when illuminated by "a supernatural ray". Which truths belong to this domain? Balthasar indicates, as an example, the First Vatican Council teaching that natural reason suffices "to know with certainty the one true God as our Creator and Lord through creatures" (**balthasar_theo-logic_2000**). This truth could be the foundation for a specific, material t the beginning of this science. Indeed, it is the supernatural light (theology) that illuminates the natural world (of science). What is so illuminated by that supernatural light is what theology of science explores. Given this approach, it follows as a matter of course that theology and science remain effectively autonomous in their specific fields.⁸

4. Theology of science as research tradition

Having described the specific object of theology of science, I'll rest research traditions, interprets science as intellectual activity of solv methodological do's and don'ts" (**laudan_progress_1977**). A given research tradition consists of various theories (which are a particular tradition and distinguish it from other traditions". One

in which the individual symbols stand for, respectively: *I* – metaphysical and methodological commitments, *O&R* – basic objects&relationships, *M* – methodology accepted in the particular research tradition, *{T}* – the set of theories proposed in the framework of the reseac *{p}* – problems occurring in the given field of reflection (at the Laudan believed that his approach could be applied, after mak *I* – the existence of God as described in Christian Tradition (s *O&R* – a third domain of truths, *M* – overall methodology of theology in the Christian Tradition *{T}* – e. g. evolution and creation as presented in (**heller_new** *{p}* – *first order problems*: contingency, comprehensibility of the sciences?

But, after all, who needs such a research tradition? Doesn't it pr a few passages from the Message to give an example of what is at s

the Church and the scientific community will inevitably inter challenge theology far more deeply than did the introduction c of human knowing, may invigorate and inform those parts of An example of a "potentially important resource" could be be (**lococo_life_2021**). Again, significant for what? Significant and The second reason is the critical role played by the theology of scien approach to religious thinking, as it provides a very profound insig/ absolutizing both the scene and scientific rationality") can be descri

For Tischner religious thinking is the thinking of "the man whose res "someone's thinking, thinking with *someone* and thinking about *son* ground under one's feet. Man would not be a dramatic existence b

For the people involved in living the drama, writes Tischner (**jagiello_jozef_2020**).

However, in the context of religious thinking, is the objectified a (**tischner_myslenie_2011-1**). Religious thinking is in opposition only scene of the human drama, also manifests itself as a metaphor On the other hand, the rapid development of the sciences and tl the contingency of the world itself (**tischner_myslenie_2011-1**) These considerations permit to sum up the train of thought of t science preserves the rational character of both theology and science unique metaphorization of the scene. The issues outlined above op

h, and whose faith is seeking reason thinks in a religious manner. His faith becomes manifest in his thinking, and his thinking becomes manifest in his faith" (**jagiello_jozef_2020**). The religious thinking makes possible different, sometimes contradictory, theologies. But each theology exists because of religious thinking, not vice versa. Religious thinking, as with all thinking, is aking has three dimensions: a subjective dimension (I think), a dialogic dimension (I think with you), an objective dimension (we think about it)" (**jagiello_jozef_2020**). Roughly speaking these dimensions correspond to Tischner's description of a human being as a dramatic existence: "to be a dramatic being is to: live in the present time, with other people around and the factors: opening up to another man, opening up to a scene of drama and to the passage of time" (**jagiello_jozef_2020**). Religious thinking in its objective dimension turns to the stage of human drama:

is above all a plane of meetings and partings, a sphere of freedom, in which man searches for a home, bread and God, and where he finds a graveyard. The stage is at man's *feet*. [...] Man experiences the stage by objectifying it, turning it into a space filled with 'objects', which he then arranges in a variety of wholes that serve him looking for its essential design

5. Conclusions: On the role of theology of science

to teach us how to go to heaven, and not how the heavens go" (**mcnullen_augustine_1999**). As first, the theology of science appears to be an authentic theological discipline, having as its basic objects and relationships of study the domain called by Hans Urs von Balthasar "a third domain of truths". The second point to be stressed is that the bond between science and theology within the theology of science appears both critical and bilateral. Besides, the theology of science is never more reasonable than when it recognizes the limits of its methods, and never less so than when it presumes to be adequate to the full reality of the human and the divine.¹² Rephrasing these words, one might say that theology is never more reasonable than when it recognizes the limits of its metaphors of the stage, and never less so than when it presumes to offer the arch perspectives. To give just one example: a systematic presentation of the science-faith/theology relationship in the works of Hans Urs von Balthasar. This topic seems urgent, as so far it has been almost completely ignored by researchers studying the Swiss theologian thought.

Bibliography

¹ It is worth to note, that Karl Rahner thought is also present in Michael Heller's approach.

² Donald J. Lococo has observed that "over the last quarter-century and it appears in the next section. For an understanding of the difference between the two approaches, see the introduction of the book.

³ The first paper on that topic was published by Michael Heller in 1982 and I will follow his approach, which is very useful and pragmatic.

⁴ As to the science and religion dialogue in the Kraków School, see: (**broz_2021**).

⁵ I use this expression following (**tyson_christian_2022**).

⁶ Cardinal Baronio has expressed this idea very clearly: "The intention of the theology of science is to bring the science of the universe into the sphere of the faith of the Church".

⁷ The fourth condition may appear not obvious. Some scholars consider it as a methodological problem, but I think it is not.

⁸ Paul Tyson, in his remarkable book on theology of science, tries to rethink the methodology of theology of science, and he introduces the concept of "Integrative Zone of Knowledge and Understanding".

⁹ The reference to Larry Laudan's approach is purely pragmatic as it is the most widely used in the field.

¹⁰ For a critical appraisal of M. Heller's research tradition, see: (**polak_2021**).

¹¹ For a critical evaluation of these interpretations of the scene, see: (**hu_2021**).

¹² See J. McGrath in his introduction to (**lococo_life_2021**).

The interdisciplinary approach to issues on the border between science and philosophy has become a permanent part of Krakow's intellectual landscape, with an important element of this local tradition being the phenomenon of the so-called "philosophizing scientists," who are researchers and thinkers who address problems specific to philosophy, especially the philosophy of science and the philosophy of nature, based on scientific investigations. Over the past century, these philosophizing scientists have included Marian Smoluchowski, Tadeusz Garbowski, Zygmunt Zawirski, and many others (**heller_krakowska_2007****polak_u_2011****polak_19th_2011****polak_tradycja_2018**). Following World War II, cooperation between philosophers and scientists developed in Krakow, mainly among the friends of Karol Wojtyła (**heller_poczatki_2006****trombik_koncepcje_2021****trombik_stworzyc_2022**). At the time, physicists associated with the Jagiellonian University were widely influential in this milieu, alongside others including Jerzy Janik and Andrzej Fuliński.

Following the election of Cardinal Wojtyła as pope, Janik and Fuliński remained active participants in local interdisciplinary projects, which were initiated from that point on by Michał (Michael) Heller and Józef Życiński. After 1978, both these scientists from the Jagiellonian University became involved in organizing seminars at Castel Gandolfo, which provided an opportunity for meetings and discussions between scientists, philosophers, and theologians throughout the pontificate of John Paul II. These meetings continued previous interdisciplinary conferences organized by Wojtyła during his time in Krakow in the 1960s and 1970s (**trombik_stworzyc_2022**).

Janik's work in philosophy has already had its initial reception within the Polish academic community (**fulinski_profesor_2015**), but the case is completely different with Fuliński's work. Nevertheless, this scientist's activity seems noteworthy for at least two reasons: First, it fits into the tradition of having a dialogue between science and philosophy, something that was successfully achieved in the circle of the Polish Pope's associates. It therefore provides important evidence about the crossing of boundaries between natural sciences and philosophy that took place in Polish culture over the past several decades. Second, although Fuliński's academic achievements lie primarily in the area of statistical physics, he is not limited to the area of pure science. Taking various issues that are present in contemporary science as a starting point, Fuliński has often expressed his philosophical competencies, as evidenced by the numerous, valued articles in which this physicist discussed various issues in the field of the philosophy of science and philosophy of nature.¹ It is worth mentioning that the term "philosophical activity" in Fuliński's case is not confined to a short period—it accompanied him continuously for several decades. Moreover, in his articles, this physicist often returned to previously discussed philosophical issues, trying to philosophize within the context of the natural sciences at various stages on his scientific path.

Fuliński's ties to the interdisciplinary milieu centered around Heller and Życiński, and this makes it possible to consider his activities within the context of the broader phenomenon known as the Krakow School of Philosophy in Science (**trombik_koncepcje_2021****polak_krakow_2022**). In this paper, I propose that Fuliński's publications fit well with the style of practicing the philosophy of nature that was initiated by Heller. Moreover, I believe that Fuliński himself, due to his cooperation with the local interdisciplinary milieu and the specificity of his philosophical works, deserves to be regarded as a representative of the Krakow School of Philosophy in Science (**polak_krakow_2022**).

In the remainder of this article, I will present Fuliński's profile and discuss a selection of his philosophical views, with the focus being especially on those aspects of his philosophical activity that fit with the trend of philosophy in science (**polak_philosophy_2019****trombik_koncepcje_2021**).

Between Kraków and Castel Gandolfo: Fuliński as a philosophizing scientist

Andrzej Fuliński began his academic career in Krakow. In 1955, he was awarded a master's degree in theoretical chemistry at the Jagiellonian University before obtaining his doctorate five years later at the Polish Academy of Sciences in Warsaw. He later obtained his habilitation in 1966 at his *alma mater*. Fuliński's scientific activity was highly appreciated by the academic community. In 1975, he became the head of the newly established Department of Statistical Physics at the Jagiellonian University. Fuliński, together with his colleagues, dealt primarily with describing the phenomena that occur in complex systems using broadly understood statistical physics methods. His research achievements resulted in, among other things, being awarded a full professorship in 1980 and becoming director of the Institute of Physics at the Jagiellonian University.

A period of increased scientific activity for Fuliński coincided with the initiatives of Michał Heller and Józef Życiński, who in Krakow had developed, on behalf of the Pontifical Academy of Theology, some large-scale interdisciplinary activities that had been previously initiated by Wojtyła (**trombik_stworzyc_2022**). Their areas of interest included, among other things, issues on the border between philosophy and physics, as well as the general methodology of science. They took up philosophical issues in the sciences and not just in their own publications. They also promoted and developed the idea of philosophy in science within the Center for Interdisciplinary Studies [in Polish: Ośrodek Badań Interdyscyplinarnych (OBI)], which since the 1980s has been an important, although informal, institution aimed at deepening the dialogue between science and philosophy. This goal was achieved thanks to interdisciplinary meetings, conferences, and publications mainly appearing in the periodical *Zagadnienia Filozoficzne w Nauce* (**heller_poczatki_2006****polak_philosophy_2019****trombik_origin_2019**).

From the very beginning, Fuliński engaged in various interdisciplinary initiatives that were undertaken first by Wojtyła and then by Heller and Życiński. He participated in seminars, panel discussions, and conferences organized by the OBI (**liana_z_1999**), and he also took part in the "Krakow Methodological Conferences" that have replaced the earlier interdisciplinary meetings since the 1990s. Fuliński also regularly appeared at the Castel Gandolfo Seminars, which were held from 1980 at the summer residence of John Paul II. The Pope wanted these meetings to be a continuation of the discussions on the border between science, religion, and philosophy that he had started with Krakow's scholars as early as the 1950s (**janik_nauka_1981****nowina_konopka_kontakty_2020**). Among physicists, Fuliński was, along with Janik, the most frequent participant in these seminars. During his stay in Castel Gandolfo, he had the opportunity to deliver a number of papers, and his speeches were later published in the form of articles in special issues of *Nauka-Religia-Dzieje* [Science-Religion-History], which has been in circulation since 1981.

Fuliński also published in the already mentioned *Zagadnienia Filozoficzne w Nauce* (**fulinski_profesor_2015****fulinski_fluktuacyjny_2017**). His philosophical papers have also been published in magazines such as *Znak* (**fulinski_o_1993**), *Studia Philosophiae Christianae* (**fulinski_maszyzna_1989**), and *Prace Komisji Filozofii Nauk Przyrodniczych* (**fulinski_czy_2010**), as well as in post-conference materials published by the OBI (**janik_glos_1990****heller_co_1991****heller_jednosc_2003-2**). The topics of his work fit well with the issues raised by Heller, Życiński, and their students. Fuliński dealt with issues like the relationship between science and philosophy (together with an analysis of the "two cultures" phenomenon), the ontological aspects of physics, the problem of the mathematical universe, the issue of reductionism in science and philosophy, the issue of time, the issue of determinism, and the concept of chance. At the same time, these issues were vigorously discussed by the representatives of "philosophy in science" (**trombik_koncepcje_2021**), and Fuliński himself regularly referred to the publications of Heller and Życiński in his works.

In the 1980s, Fuliński's cooperation with the OBI community deepened. The Krakow physicist even became one of the reviewers for Włodzisław Skoczny's doctoral dissertation, which was titled "Filozoficzne aspekty Zasady Antropicznej" ["Philosophical Aspects of the Anthropic Principle"], written under the supervision of Życiński and defended at the Pontifical Academy of Theology in 1986. Fuliński was also keenly interested in the publications of Heller and Życiński. A good example of this is his review of their book *Wszczętna—maszyzna czy myśł?* [*The Universe—a machine or a thought?*] that was published in the periodical *Studia Philosophiae Christianae* (**fulinski_maszyzna_1989**). It should also be noted that between 1988 and 1991—at the request of John Paul II and together with Heller, Życiński, and Zygmunt Kolenda—Fuliński prepared the work "Reports on the socio-political situation in Poland" (**heller_jan_2020**).² This proves not only the enormous trust that the pope had in Fuliński but also the spirit of understanding and cooperation that existed between the Krakow physicist and the creators of philosophy in science, a cooperation that continued into later years. Even over the last decade, Fuliński has repeatedly participated in various scientific initiatives of the Copernicus Center for Interdisciplinary Studies, an institution that was established by Heller after receiving the prestigious Templeton Prize in 2008, with this being a 21st century continuation of the former OBI.

The indicated connections between Fuliński and the Krakow interdisciplinary milieu seem so important and so large scale that they provoke questions about the mutual dependencies that existed, including philosophical ones. When reconstructing Fuliński's views, it is worth noting their references to the concept of philosophy in science. Due to the limited length of this article, I will limit myself here to discussing just some selected philosophical ideas in Fuliński's works, ones that will illustrate the mutual connections and dependencies, namely the issue of the relationship between science and philosophy (and also the relationship between science and religion), the problem of reductionism, and the dispute over the mathematical nature of the universe.

Toward interdisciplinary research: Selected philosophical issues in Fuliński's works

The "philosophy in science" project, as initiated by Heller and Życiński during their academic careers, they quickly developed a style of *praesyntheses* in the form of Thomism, Hegelianism, and so on (**heller_philosophy_2011**). Similar thoughts were echoed by Heller's students and colleagues,³ who despite their strictly metaphysical interests, usually rejected the products of philosophical systems as being unsuitable for interdisciplinary research (**polak_krakow_2022**). elian-Thomistic trend. In this aspect, his thoughts corresponded well with those of Heller and his colleagues. According to Fuliński, Thomistic philosophy was not only outdated, especially in the context of issues bordering science and philosophy, but also harmful in light of the social mission of the Church, which wanted to establish contact with contemporary intellectual culture.

Fuliński shared the view that the discrepancy between science a culture has over time become one of the causes of the gap both between the right to treat in advance all those who do not want to recognize methods for analyzing the results of natural sciences.

His approach was not intended to discredit the intellectual heritu worth noting here that Fuliński clearly pointed to the historical im-

Both Heller and Życiński were convinced of the need to develop methodological reflection. A similar approach can be seen in the w

One of the basic goals behind Heller's and Życiński's efforts was

Such interactions can be seen, for example, in the transition a and common culture. The most obvious example is the conce processing process is the concept of time (**janik_fizyka_16**

The interpenetration of the precise language of science with the therefore the need to integrate the various disciplines that describ unity of the world, and consequently the postulated unity of know (**zycinski_trzy_1990****heller_czy_1998**).

Fuliński believed that at the root of the growing autogonism lies opening article for the first seminar at Castel Gandolfo, where he s

For the Krakow physicist, reductionism is "an attempt to reach understanding of reductionism he proposes expresses not just a spe- cto Fuliński, reductionism understood like this would be a real three-

I see the dangers of today's reflection on the world not in red viewing the world in terms of purpose, causality, blind fate, c

In his papers, Fuliński suggested distinguishing between method metaphysics. According to Fuliński, stating that the properties of i

At one point, Fuliński even wrote that "there is no contradiction equivalent of Popper's World 1), with him excluding the sphere of th

Another issue to which Fuliński devoted considerable attention i is not a product of the mind but refers to a reality that exists inde

Firstly, it was obvious to Fuliński that nature exhibits important captured in a broader context, with this also taking into account o

When confronted with the question of whether a scientific theo [...] the statements that theoretical physics discovers objectiv when we talk about physics, we tend to emphasize the more

Fuliński therefore distanced himself from the question of whethe context of the dispute, as well as to the fact that "the problem of t

It is worth emphasizing, however, that Fuliński's analyses in the a more nuanced attitude to the idea of the mathematical universe,

During his scientific career, Fuliński became well known not just as Fuliński's publications clearly bear the mark of "philosophy in : philosophical problems that are entangled in empirical theories; an

(A): methodological analyses of science-culture relations, includin (B): problems of time, determinism, the question of chaos, and s (C): the question of the mathematicality of the world and the pre

It is noteworthy that Fuliński's approach to analyzing philosop representatives of the School, such as Życiński, who willingly refer- Significantlly, the activities of the Krakow physicist fell into, ano interdisciplinary undertakings. I think this thread should be develo

Thinking about the research perspectives related to the School's about the historical development of the School but could also bring

milieu, was a proposal to practice philosophy within the context of the results of contemporary mathematical and natural sciences (**heller_how_2019****polak_philosophy_2019**). As scholars coming from a Catholic background, Heller and Życiński were formed during their studies in seminary by the spirit of the Thomistic philosophy of nature (**heller_wierze_2016**), but that was far removed from the Aristotelian and Thomistic trend. There were several reasons for this: According to Heller, Thomism as a metaphysical system was not capable of creatively addressing key problems on the border between science and philosophy (**janik_nowa_1990**). Moreover, Heller was skeptical of all philosophical systems and critical of the so-called great **heller_philosophy_2011**). Similar thoughts were echoed by Heller's students and colleagues,³ who despite their strictly metaphysical interests, usually rejected the products of philosophical systems as being unsuitable for interdisciplinary research (**polak_krakow_2022**).

elian-Thomistic trend. In this aspect, his thoughts corresponded well with those of Heller and his colleagues. According to Fuliński, Thomistic philosophy was not only outdated, especially in the context of issues bordering science and philosophy, but also harmful in light of the social mission of the Church, which wanted to establish contact with contemporary intellectual culture.

specially neo-Thomism), suspecting, probably not without reason, that it is today one of the causes of mutual distrust, between the community of people of science and the Church" (**fulinski_maszyzna_1989**).

ay have its origins in the overly strong connection between the Church's teachings and neo-Thomistic philosophy, a type of philosophy that is inadequate for addressing problems that have emerged in the context of the modern natural sciences, so it is unattractive for the scientific community. Elsewhere, Fuliński even suggested that the historical rooting of Thomism in Western idy and, from a broader perspective, between humanistic culture and scientific culture, thus contributing to the emergence of the so-called "two cultures" phenomenon (**snow_two_1959**). Fuliński wrote: "It is possible that the roots [of this phenomenon] could be looked for in the Thomistic doctrine. The Thomist assumes that he possesses the Absolute Truth, which gives him mentality was then taken over by both amateur philosophers⁴ and scientists" (**fulinski_o_1993**). It was obvious to Fuliński that the Thomistic philosophy of nature, even in a modified form like Louvain Thomism,⁵ could not be reconciled with contemporary scientific knowledge, so it should be abandoned altogether before looking for a more adequate system with better

, however. On the contrary, the development of a different type of reflection was intended to establish a new platform of understanding between science and faith.⁶ Together with his colleagues and students, Heller made a similar assumption when developing the concept of philosophy in science (**polak_science-religion_2021****PolakRodzen2023****polak_krakow_2022**). It is ianity in the emergence of modern science (**janik_fizyka_1981**), which over time also became the main view of, among others, Życiński, who devoted a book to this issue (**zycinski_inspiracje_2000**). It was in close contact with science and the latest logic and methodologies. The prerequisites for practicing this kind of philosophy include anti-separationism (i.e., a rejection of the thesis that there is a radical epistemological rift between the sciences and philosophy) and an openness to the changes and modifications being dictated by the development of the sciences and which serve as a good example of the practical application of the assumptions of the "philosophy in science" project.

pen the dialogue between philosophy and natural sciences. The search for contacts between broadly understood humanities and the mathematical-empirical sciences is also noticeable in Fuliński's work. Taking part in the discussion of the "two cultures," he emphasized how numerous interactions between science and culture exist that are visible, for example, at the level of language:

oncepts, in the cycle: philosophy and common parlance, science, and common parlance and general culture. Philosophy or common parlance introduces some concepts. Science takes them over, when it is prepared to do so, and on examining them carefully, processes them in its own way. Eventually, this concept is returned, albeit in a processed form, into everyday language [...] An example of a concept that is currently being refined by detailed science, and at the same time, in a purified form, is beginning to pass into general culture, is the notion of heredity, which originates in common parlance and the related more technical notion of the gene, innate traits, and so on. Finally, an example of a concept that is just beginning to enter this

age of culture is a key, although not the only, area of possible interaction between natural sciences and the humanities. Fuliński also noticed other examples of mutual influences, paying attention, for example, to the importance of various cultural creations in the context of scientific discovery (**janik_fizyka_1981**). The view shared by Fuliński about the unity of the world and (despite them coming from different perspectives) is the main reason for him rejecting the separation paradigm. He also expresses hope "that the understanding of the unity of the world and the unity of culture will return to our way of thinking in a purified and processed form in the specific sciences, including the humanities" (**janik_fizyka_1981**). The idea of the edy the existing rupture in culture, as manifested by the gap between humanists and representatives of empirical sciences.⁸ The concept of "philosophy in science" also sought to counteract this discrepancy: Heller and Życiński emphasizing in their works the need to break down the walls between science and culture and justifying it in a manner similar to Fuliński

ngs, a simplified, colloquial image of science that is deeply rooted in culture. According to Fuliński, various areas of misunderstanding exist between the humanities and science, and one of the key ones is the dispute over evaluating the reductionist method. The issue of reductionism in physics appears in many of Fuliński's works. He already devoted attention to this issue in his reductionist attitude specific to science is sometimes treated by humanists with a great deal of suspicion, but how does Fuliński himself respond to this type of allegations?

sies to the basic laws of nature and, if possible, to one basic law of nature" (**heller_watpliwosci_1990**). Nevertheless, according to Fuliński, the proposed concept of reductionism is significantly different from the reductionisms of the past that were grounded in mechanistic or scientific philosophies (**fulinski_o_1993****heller_jednosc_2003-2**). Fuliński is aware that the out "it is sometimes actually the adoption of a certain ontology, the belief that there is a some unifying principle, some central order of things and phenomena" (**janik_czesc_1990**). Nevertheless, the reductionism of physics, as Fuliński puts it, does not mean the belief that everything can be reduced to one simple "world-machine" model that explains all phenomena. According

e, biology to chemistry or physics, emphasizing the role of chance in evolution, or such like. The pitfalls today lie in the fact that the tendency to think in simple models is strongly established among very wide circles of thinking people: the struggle for existence, the selfish gene, the class struggle, agent activity, and so on. The class of such simplifications also includes sity. The danger is that belief in simple models leads to belief in simple recipes for understanding the world, tanning it, and even worse, repairing all its sins and imperfections (**fulinski_maszyzna_1989**).

ism and the ontological version of reductionism, but he also defined the relationship between them fluently. Drawing attention to the benefits of using reductionist procedures in science, he emphasized that the ontological equivalent of reductionism, as long as it is applied to the scope of the physical world, does not have to necessarily lead to a monistic, extremely physicalistic

'levels of the world are reducible to some basic law is not the same as asserting that it is possible to model the entirety of reality according to one pattern and based on one language.

etionism of physics, the search for a unified description of the natural world, and the existence of a transcending world of freedom, the products of which are not fully determined by the laws of nature, with them containing an element of human creation" (**fulinski_o_1993**). This suggests that Fuliński applied the reductionist theory to the world of physical objects (i.e., the l the results of its activity, such as the issue of self-awareness, the problem of free will⁹, the issue of values, and so on. This approach to the problem was not so distant from the methodological and ontological views expressed in the OBI community, such as what can be seen, for example, in the works of Życiński related to the concept of emergence (**zycinski_wszeczwiat_2009**).

l works is the problem of the mathematical nature of the world. The question of "Is the world mathematical?" was one of the most important and frequently discussed issues by Heller and Życiński. Many representatives of the OBI formulated an affirmative answer to this question, and their views often moved towards mathematical Platonism (the subject of mathematics research itive entities). Fuliński was slightly more cautious in this context (**janik_glos_1990**), with him clearly not taking sides in the philosophical dispute.

ng, so we can model it mathematically, but he also believed that the fact that the world can be described mathematically does not mean that reality is mathematical in the ontological sense (**janik_glos_1988****janik_glos_1990****fulinski_o_1993**). Thus, he postulated that the relations between the description of the world (i.e., physical theory) and the world itself should be

tively existing laws or just constructs a description of the world, Fuliński answered that the problem was apparent and that the two claims should not be considered to be contradictory. A scientific theory can be a reflection of reality as well as its reconstruction, structuring, and even a kind of "creation." A good illustration of this view is given in his following words:

or that theoretical physics constitutes the description of the world, are probably not contradictory. Like a work of art, like an artistic creation, theoretical physics is both a reconstruction (in a different order) of the world and, to some extent, the creation of this world, except that when we talk about art, we tend to emphasize, to some extent, the moment of creation, but **janik_o_1988**).

a kind of ontology of the world, as has been assumed, for example, by Życiński (**zycinski_swiat_2013**). Although he did not question this possibility, he demanded greater caution when examining this dispute, pointing to, among other things, the linguistic difficulties that philosophers and scientists encounter here. He pointed to terminological ambiguities that appear in the undary nature of language in relation to perception is directly related to the understanding of the mathematical nature of the world and the ontological status of theoretical physics" (**janik_glos_1988****heller_co_1991**) and how these make the metaphysical question about the nature of reality require very subtle analyses and caution when formulating an answer.

blem of the mathematical nature of the world were positively received in the OBI community (**zycinski_teizm_1988**). On analyzing the works of other representatives of the Krakow interdisciplinary community, it can be discerned that they took Fuliński's critical remarks into account. Such critical positions, which also came from other authors, could consequently influence y noticeable in the works of the younger generation of philosophers from Heller's milieu, such as Ł. Łamża and M. Holoń.

An attempt to summarize

Isa as a scholar who was sensitive to philosophical issues. For many years, he has been involved in the dialogue between science and philosophy and participated in various interdisciplinary projects, with him publishing a number of works primarily in the area of the philosophy of nature and the methodology of science.

cts, the Krakow physicist has addressed issues that fit into the project of philosophy that was outlined by Heller (**heller_jak_1986****heller_how_2019**). In his programmatic paper, Heller indicated that the subjects of interest for philosophy in science include (A) the influence of philosophical ideas on the development and evolution of scientific theories; (B) traditional l reflections on the assumptions of empirical science. The issues discussed by Fuliński correspond to each of the three areas of "philosophy in science", e.g.:

tion; this group could also include, among other things, works on the history of science and philosophy, devoted, for example, to the achievements of the "philosophical physicist" Marian Smoluchowski (**fulinski_fluktuacyjny_2017**);

• **1993****fulinski_profesor_2015**;

• narity and unity of nature (including the issue of reductionism).

o turned out to be close to the style of Heller. The works of the Krakow physicist show that he rejected the radical isolationism of science and philosophy, and he was also very critical of systemic philosophical concepts like Thomism. He placed his reflections within a scientific context while remaining open to traditional metaphysical problems.¹⁰ This was appreciated by some ablications (see footnote 1).

he early formative period for the concept of "philosophy in science" and the milieu of Heller and Życiński (**trombik_koncepcje_2021**). It is therefore possible to speculate that Fuliński was not just part of the Krakow School of Philosophy in Science current but also a creative influence within this school, both philosophically and organizationally, having participated in various l in a future, larger dissertation that would more comprehensively study the life and work of Fuliński.

re that it would be worth undertaking detailed research to indicate the possible scope of the impact on Heller and Życiński's milieu from other philosophizing scientists, such as Jerzy Janik, Andrzej Staraszkiewicz, Zygmunt Chyliński, Małgorzata Głódz, Jerzy Rayski, Leszek Sokolowski, Alicja Michalik, or Marek Szydlowski.¹¹ Such research would not only enrich our knowledge vesting and often still-current philosophical views that are part of native interdisciplinary traditions.

Bibliography

¹ Fuliński's papers were cited in various philosophical works, including a

² The pope read these reports carefully. Fuliński recognized fragments of

³ In the case of Życiński, the problem is somewhat more complex, becau

⁴ Fuliński used this term to refer to those philosophers who, as he used t

⁵ The Louvain type of Thomism was an attempt at the turn of the 19th

⁶ Remarks related to this can also be found in, *inter alia*, the transcript

⁷ It is worth noting here that in Fuliński's works, one can find numerous

⁸ In this context, Fuliński referred to the "mirror metaphor" from Profes

This metaphor can be extended, in particular, to philosophy and physics

⁹ Particularly interesting in this respect are Fuliński's analyses about the

¹⁰ However, on various occasions, Fuliński himself has expressed a distance

¹¹ How much harm is associated with the implementation of some results of

¹² It is worth adding that some of the mentioned representatives of philo

Studies of the complexity of reality, which are carried out at the intersection of different areas of knowledge, are nowadays gaining more and more supporters among researchers. Three directions of these studies dominate today: multidisciplinary, intradisciplinary and interdisciplinary. The last and the most frequently used means taking up common research projects by scientists coming from different disciplines. This occurs at each stage of the project beginning with the formulation of a research problem, then proposing appropriate hypotheses and ultimately interpreting data that were obtained (**kasper_interdisziplinarnitat_1996**). Since the Second Vatican Council, theology has also become more open to cooperation with other sciences, including not only the humanities, but various disciplines of empirical sciences such as physics, chemistry, biology, psychology, sociology and cognitive science. The need of this kind of cooperation is signaled at present not only by the representatives of theology but by those involved in empirical sciences as well. Some researchers even go so far as to make the future of theology and its presence at universities dependent on the interdisciplinary direction of research. However, the interdisciplinary openness of theology raises a number of questions: Does opening theology to other disciplines not threaten to break the internal identity of its content? Why should a theologian listen to the voices of representatives of other sciences since many of them are not interested in his research discipline? Does theology have something to say to other sciences in an interdisciplinary dialogue? Is theology itself interested in opening up to new *loci theologici alieni*? Can the interdisciplinary nature of theology help in creating an integral concept of man and the world? Can interdisciplinary theological research contribute to the clarification and transmission of faith among people with scientific and technical mentality?

An unequivocally positive answer to the questions posed is provided by the authors of the book *Evolutionary Theology*: Wojciech Grygiel, a natural philosopher, chemist, theologian, and Damian Wąsek, a theologian. As representatives of various disciplines through their joint work they give a concrete example of the interdisciplinary cooperation.¹ Their project is methodological in nature: it is to “show how the development of science can entail the development of theology, and what assumptions must be met to result in a constantly deepening insight into the divine essence” (**grygiel_teologia_2022**). The book consists of two main parts: *Assumptions* (pp. 15–151) and *Problems and Hypotheses* (pp. 152–236).

The first part, consisting of five chapters, discusses the methodological assumptions of evolutionary theology. Chapter 1 under the title *Revelation: between formulas and relation* (**grygiel_teologia_2022**), considered by the authors to be the most important for capturing the essence of evolutionary theology, presents a dynamic concept of theology of revelation in the perspective of its historical development. This approach shows, with all its internal dynamics, the importance of the concept of revelation in the construction of evolutionary theology. The category linking the conducted considerations is the relationship between man and the self-giving God taking place against the background of changing images of the world. Chapter 2 entitled *Truths of Faith: Between Immutability and Evolution* (**grygiel_teologia_2022**) presents the tension between what is essential, unchangeable and changeable in the interpretation of the truths of the Christian faith. The latter conditioned by the time context, especially the specific image of the world related to it, may be subject to change, reinterpretation or even correction. In reference to theological thought, e.g., John Henry Newman (**grygiel_teologia_2022**) and Karl Rahner (**grygiel_teologia_2022**) the authors clearly point to the evolution of the Christian doctrine taking place in compliance with certain rules. “The doctrine—as they write—is not about accepting historical formulations, but their inner essence. At the same time, one should be aware that this “explication” cannot be expressed in an ahistorical way” (**grygiel_teologia_2022**). Unlike the descriptive titles of the previous chapters, the third chapter with a metaphorical title *Theology as a Work for Orchestra* (**grygiel_teologia_2022**) deals with the classification of the theological places. Theology distinguishes two types of its sources: proper places (*loci proprii*) and auxiliary ones (*loci alieni*). It is the latter, based solely on human authority, that is the subject of the analysis conducted in this chapter. The authors are particularly interested in defining more precisely the criteria for interdisciplinary dialogue involving theology (**grygiel_teologia_2022**), so strongly related to the topic of *loci alieni*. This issue seems crucial in developing a methodology for evolutionary theology. The fourth chapter, *From a static to a dynamic image of the world* (**grygiel_teologia_2022**), deals with the analysis of various ways of interaction between the language of theological statements and the current image of the world. The adoption of the hermeneutic category of the image of the world, as proposed here, makes it possible to better capture and understand the complex contextuality of not only scientific, but also religious and theological beliefs (**grygiel_teologia_2022**). In a broader historical perspective, as indicated by the title of the chapter, the authors successfully indicate both the ongoing transformations of the image of the world effected by the new discoveries in physics and biology, as well as the related reinterpretations at the level of the philosophy of science (**grygiel_teologia_2022**). In the fifth chapter, *In the flow of logos* (**grygiel_teologia_2022**), one more argument for the need to develop evolutionary theology is presented. This way of practicing theology in the evolving Universe strives not only to protect against its marginalization by taking into account the current image of the world in theological reflection, but “is primarily aimed at a much deeper insight into the mysteries of who God is in essence” (**grygiel_teologia_2022**). This chapter is a supplement to the methodological assumptions of evolutionary theology and it is particularly important in setting the epistemic boundaries that prevent from making unjustified extrapolations between natural and theological cognition. “The existence of such a border—as we read—is necessary for revelation to make sense, that is, for there needs to be room for the transcendent voice of God who speaks from beyond the immanence of the Logos to its interior” (**grygiel_teologia_2022**).

The second part of the book, which consists of three chapters, deals with the application of the discussed assumptions of evolutionary theology in practice. The authors present “several selected reinterpretation problems [...], aptly illustrating how these assumptions ‘work’ in specific cases” (**grygiel_teologia_2022**). And so, in chapter six, *Adam, where are you? Evil and The Original Sin* (**grygiel_teologia_2022**) they synthetically discuss the problem of evil and suffering as well as the classical doctrine of the original sin in the perspective of the evolving image of the world. The presented reinterpretation of the doctrine of original sin is a strong argument for the necessity of practicing theology in the perspective of evolutionary theology (**grygiel_teologia_2022**). Chapter Seven: *Soul: Between Nature and Divine Intervention* (**grygiel_teologia_2022**) deals with the analysis of the current topic of the relationship between theology and neuroscience: “between the biological reality shaped in the process of evolutionary development, and the one that is associated in traditional theology with direct divine intervention” (**grygiel_teologia_2022**). The emphasis is placed here on showing the consequences of this type of relationship both for the reinterpretation of the theological concept of the soul and the very contribution of neuroscience to the theological discourse. In chapter eight, *In the Footsteps of Agency: Cognitive Religious Studies* (**grygiel_teologia_2022**), the authors take up a new research concerning the phenomenon of religion within the cognitive sciences. In their considerations, they strive to capture and show the impact of the scientific knowledge on the formation of religious beliefs. Specific issues are analyzed in turn, such as: the origin of religious beliefs, the doctrine of intelligent design and miraculous events. Some doubts may be raised by the presented analysis of a miracle which links its recognition as God’s way of acting in the world with interpretations inspired by the thought of St. Augustine and St. Thomas Aquinas (**grygiel_teologia_2022**). It seems that reference to the modern semantic concept of a miracle, especially in the layer of its cognition including scientific and religious knowledge, gives the possibility of the interpretation of a miracle better harmonizing with the evolving image of the world.

The second part of the discussed book is not only an example of a practical application of the methodological assumptions of evolutionary theology presented in the first part but it also provides specific arguments against the anti-Christian theses of Richard Dawkins. They have been quite widespread recently mainly through his *The God Delusion* (**Dawkins2006TheGodDelusion**) as well as through the naturalistic ideas propagated by the supporters of the new atheism.²

In conclusion, the book *Evolutionary Theology* is a successful study that shows how interdisciplinarity in theological research leads to a departure from the one-dimensional scientific paradigm and gives the opportunity to develop a holistic view of nature and man. Especially from the perspective of the fundamental theology interdisciplinarity is not a fashion but rather a necessity and an expression of the understanding the complexity of reality which no single science is able to grasp integrally. The research project of the book, its structure in which it combines issues in the field of systematic theology and philosophy of science as well as science itself shows the importance and necessity of the interdisciplinary research for the development of modern theology. The interest of a modern man in the scientific knowledge is an expression of a certain sign of the times in which his expectations, needs and requirements are revealed. Also, the reviewed book is a positive example in search of the new forms to integrate faith and reason as part of the dialogue between theology and other sciences. It shows how to defend the rationality of the Christian faith as it confronts the claims of the contemporary science. The book adds its voice to the attempts of providing this kind of defense which are present in the Anglosaxon literature by such authors as John Polkinghorne, Alister McGrath, Gary Kroege, Andrew Pinsent, Markus Holden, and in German by Jürgen Moltmann, Christian Link, Dieter Hattrup, Ulrich Lüke and Alexander Loidinger.

A reliable interdisciplinary exchange may not only lead to discovering the boundaries of one’s own scientific discipline, but also to an increase in methodological awareness. Such discoveries result in the mutual cleansing of past errors and guard against unwarranted extrapolations so that theology does not turn into pseudoscience and science into unconscious theology. The reliance on the contribution of the authors’ own scientific community to interdisciplinary research, which fits into the research perspective of evolutionary theology, should also be positively assessed. Grygiel and Wąsek not only refer to the achievements of the great precursors and initiators of the Copernicus Center of the Interdisciplinary Studies, Michał Heller and Józef Zychliński or their own publications, but they also incorporate the achievements of philosophers and theologians associated with the Pontifical University of John Paul II, such as: S. Wsiolek, J. Bremer, J. Mączka, Z. Liana, R.J. Woźniak, L. Kamykowski or T. Dziadek.

Based on the above considerations, I recommend the book *Evolutionary Theology* not only to anyone who is interested in the interdisciplinary dialogue and who wishes to do theology within the changing image of the world but to anyone who is looking for the justification of a personal Christian creed against the claims of mentality dominated by the scientific thinking.

Abstract

This review pertains to the book *Evolutionary Theology (Teologia ewolucyjna)* written by Wojciech P. Grygiel and Damian Wąsek. The book presents a distinct and modern viewpoint on theology by offering a comprehensive analysis of the characteristics of theological language and utilizing it to reevaluate certain theological beliefs, such as the concept of original sin, within the framework of the ever-changing understanding of the Universe. This approach contributes significantly to the restoration of theology’s credibility in modern culture by bridging the gap between science and theology.

Keywords

evolutionary theology, interdisciplinarity, science and theology.

Bibliography

¹ It is worthy of note that this is not the first interdisciplinary work by t
² E.g., Sam Harris (**harris_end_2004; harris_better_2006**), Chris

ample, see (**wasek_teologia_2018Wasek2021Grygiel2019IntelligentDesigngrygiel_cognitive_2021**).
chens_god_2007).

In philosophy, it is always refreshing to introduce unconventional ideas. It requires a certain audacity from the author; she/he may face the wall of silence or be slammed by academia, both treatments being undesirable. But still these are more rewarding than gathering laurels for beating the dead philosophical cats like Humes, Leibnizts, Wittgensteins, Whiteheads and others, the practice that for many philosophers is their life opus. Bartłomiej Skowron¹’s book *Part and Whole: Towards Topo-Ontology* published by Oficyna Wydawnicza Politechniki Warszawskiej in 2022² is certainly not in this last category.

The book is quite rich in content and topics. It may be seen, as the author suggests, as a review of mereological and topological perspectives (**skowron_czesc_2021**) or a topological vision of what exists (**skowron_czesc_2021**). The book may also be regarded as an advanced introduction to topology, mereology, and mereo-topology as well as their historic roots, beginning with Plato and ending with Brentano and Ingarden.³ The concept of an advanced introduction is clearly an oxymoron, yet it still seems to reflect the book’s content. For example, the definitions and formalism in the book certainly go beyond an introductory level but the chapters are relatively short, hence introduction. However, the unique contribution of this book lies, it seems, somewhere other than the essays on these topics.

We believe that the center of gravity for this book lies in its discussion of topo-philosophy,⁴ so we expect Skowron to introduce us to topo-philosophy and explain what topo-philosophy is, who has engaged with it, and where it may go in future.

Now, why might topo-philosophy be interesting and worthy of attention? The answer to this question is rather long but rewarding. We are told the following: “Philosophy, in particular its theoretical part, is too difficult to be apprehended with common sense and everyday reasoning” (**skowron_czesc_2021**). So, what is needed to address this? The author states that a deep understanding of philosophical ideas requires a deep understanding of the fundamental philosophical structures for concepts like that of the whole and parts, of unity, of foundation, of place, and of autonomy. Topo-philosophy—as a fusion of topology, topo-ontology, mereology, and philosophy—offers tools for analyzing these complex philosophical structures by juxtaposing them with concepts from topology, such as topological spaces, connectedness, borders, subspaces, density, dimensions, and metrics. Now, let us attempt a simpler explanation.

Philosophy is about ideas and their structures, while topology is about the properties of a geometric object that are preserved under continuous deformations, mereology is the study of parts and the wholes they form and topo-ontology—fusion of topology, mereology and ontology—is about topological-like structures of ontological concepts. This means that topo-philosophy is about topological representations of philosophical ideas that go beyond mere ontology. In the author’s own words, “philosophy using spatio-topological concepts is denoted as topo-philosophy” (**skowron_czesc_2021**).

A two other explanations of topo-philosophy can be found in the book: (1) “[...] topo-philosophy belongs to mathematical philosophy or some philosophy that uses the language of mathematics to express philosophical concepts, with the proviso that topo-philosophy uses the language and concepts of topology” (**skowron_czesc_2021**). Alternatively; (2) “Topo-philosophy is based on the judicious application of ideas of geometry [esprit de géométrie is Skowron’s suggestion]” to philosophy (p.169). Geometry always involves an ordering of things, and topo-philosophy is simply doing the same in ordering the conceptual space of philosophy (p.171). “Judicious application” may be the key phrase here, because “topologization” must be done with “esprit de finesse” (again Skowron’s suggestion), otherwise it may lose its power to give insight into non-topological ideas and morph into a barren abstract discourse.

Thus, how to “topologize” philosophy can be learned by studying applications of topo-philosophy, for which Skowron provides ample examples. Possibly the most accessible one, thanks to it having already enjoyed wider recognition, is the catastrophe theory of Rene Thom. Nevertheless, Skowron discusses applications of topo-philosophy in epistemology, physics, robotics, data analysis, and models of the mind and the central nervous system. Indeed, topo-philosophy is really coming out into the open.

We see the emerging applications of topo-philosophy in research for AI, information, and deep neural networks (DNN), which are topics not covered in this book. Quantified theories of information have a topological side in terms of topological information and information geometry. Information geometry was defined by its founder Shim-ichi Amari (**amari_information_2016**) as “[...] a method of exploring the world of information by means of modern geometry.” Information geometry studies information science—which is an umbrella term grouping statistics, information theory, signal processing, machine learning, and AI (**nielsen_elementary_2020**)—through geometry. Information geometry provides a context-free, pure method for studying relations like the distance between, for example, probability distributions. Information science can be viewed as the science of deriving models from data, which is often presented as the geometry of decision making, such as through curve fitting and classification (**nielsen_elementary_2020****nielsen_many_2022**). Topological information views information geometry as being topological. Thus, information is topological in the sense that the relations between systems that manipulate and exchange information can be captured through topological relations.

A topological representation of information and computing allows for Turing machines and computing to be generalized to information manipulation on tangle machines.⁵ (For more about information topology, see the works (**moskovich_tangle_2014****carmi_tangle_2015**)). The advantages of information geometry and topological information lie in their power to capture various forms of information processing (e.g., information science, decision science) in context-free formal systems based on geometry or topology, thus allowing for results to be generalized from a specific domain.

Of course, if the topological perspective is so revealing, we may wonder why we did not realize this before. Indeed, Skowron’s book is an eye opener to some extent.

However, focusing on topo-philosophy may not do Skowron’s work justice, because it is only a small part of his book. Substantial parts are devoted to reviewing topological research, mereological concepts, mereo-topology, and historical notes. How then should we view these sections? One way is to regard them as a sort of background introduction to topo-philosophy, but why? Well, if you want to learn about topo-philosophy, you need to understand some basic tenets of mereology, topology, mereo-topology, and topo-ontology, so these sections are helpful as a reference. It is certainly useful to have them in one place.

One could also forget the notion that the book is about topo-philosophy (the subtitle of the book suggests an introduction to topo-ontology) and treat it as a series of detailed essays on topological and mereological concepts, with them being connected by the overall theme of topo-ontology, of which a discussion of topo-philosophy is an integral part. From this viewpoint, Chapter 3 being about topo-philosophy is not central, as we previously presumed. Instead, all the chapters are equally important, and the message is the entire book, which elaborates on the main title and subtitle.

Thus, one may think of the book as a review of the main tenets of topo-philosophy (unfortunately quite short) together with a background discussion of topology, mereology, mereo-topology, and so on. Alternatively, one may introduce the book as a review of the main tenets of topology, mereology, mereo-topology, and so on together with a side discussion of topo-philosophy(appropriately quite short).

The problem with this second option, however, is that it takes the punch away from the book in terms of its novelty, because topology, mereology, and mereo-topology are rather well-known, well-studied topics.⁶ In contrast, topo-philosophy seems fairly novel,⁷ despite its deep historical roots, so as something rather unique, topo-philosophy would be a good choice to serve as the fulcrum for the book, as we originally suggested.

There are a few more impressions from reading the book. The book is certainly not an easy read, and the presentation of topology, mereology, and mereo-topology is relatively advanced. For an expert, the book offers a fairly comprehensive review of these topics. In contrast, if one wants to learn about topology, mereology, or mereo-topology, these sections in the book are not the place to start. As we said earlier, it is a rather advanced introduction. In other words, the book provides a formal introduction to the topics and is rather shy on conceptual or intuitive perspectives. (For an easier ride into topology, see, for example, the work of Earl (**earl_topology_2019**) and the philosophy of mereology by Laudo (**laudo_mereology_2017**).) Skowron is well aware of this, however, and from time to time, he shows a lighter side (Socrates’ sting). Overall, though, the thorough, formal approach makes the book a hard nut to crack. Every author has to make choices, and this book was certainly not intended for display on airport bookstands.

There are also a few minor things that catch the eye: (1) The claims for the “entropy of philosophical systems” (**skowron_czesc_2021**) and “entropy as a measure of unpredictability” (after Hutchins (**hutchins_concepts_2012**)), seem to be a misadventure, albeit one that is quite popular in philosophy. Thermodynamic entropy is a well-understood physical phenomenon that has little to do with the state of philosophical systems. Any application of thermodynamic entropy concept outside of its proper context, while quite common (**muller_history_2007**), are misleading.⁸ (2) The book would benefit from a more extended synthesis of the discussed ideas. We have a short synthetic view of what the topo-philosophical method may be but more would benefit the book. (3) The connection between English sources (many quoted key works are in English) and the Polish text would be greatly facilitated if the author provided a lexicon of English technical terms rendered in Polish. (4) The book may also benefit by focusing on topo-philosophy and its main actors, objectives, and applications from a historical perspective. As we have said, topo-philosophy is where the novelty of this book appears to dwell, so why not dedicate the book to it at the expense of thinning out the contextual parts? In addition, a more focused book would be more amenable to being published in English, which I think would be worth doing. (5) Moreover, an English edition of the entire book, or selected parts thereof, would bring some interesting works from Polish philosophers to a wider audience, so it is certainly worth considering.

Overall, the book is a well-executed foray into topo-ontology or topo-philosophy, depending on whichever lens you prefer to use. More specifically, whatever perspective you may adopt, Skowron offers a much-needed review of the main discussions, players, applications, and perspectives related to topo-philosophy, something that is hard to find collected in one place, so this is certainly a plus. What the reader may wish to see, however, is more of the author’s synthesis for the presented ideas, expanded beyond “Towards general topology of object and its parts” in section 7. One may also follow up on Skowron’s ideas in his recently published paper ‘A metaphysical foundation for mathematical philosophy’ (**wojtowicz_metaphysical_2022**).

Abstract

In philosophy, it is always refreshing to introduce unconventional ideas. It requires a certain audacity from the author; she/he may face the wall of silence or be slammed by academia, both treatments being undesirable. But still these are more rewarding than gathering laurels for beating the dead philosophical cats like Humes, Leibnizts, Wittgensteins, Whiteheads and others, the practice that for many philosophers is their life opus. Skowron’s book is certainly not in this category. Bartłomiej Skowron undertakes such a discovery trip into an unknown land in his book *Part and Whole: Towards Topo-Ontology*, which was published by Oficyna Wydawnicza Politechniki Warszawskiej in 2021.

Keywords

topology, topoontology, mereology, topo-philosophy.

Bibliography

¹ Bartłomiej Skowron is an adjunct professor at the Faculty of Administra

ontology (**skowron_contemporary_2020****krzanowski_contemporary**

² The book was published in Polish with the title *Część i całość. W stronę*

³ Taking a historical perspective in philosophy is always a rewarding exe

⁴ This view is also hinted at by the author, with him saying that “the th

⁵ Tangle machines are topologically inspired diagrammatic models. The

⁶ Substantial resources on these topics are available. In fact, Skowron on

⁷ A Google search for topo-philosophy directed us to a Wiki page on EI

mereo-topology is well established. This also means that LLMs like GPT

⁸ For level-headed physicists, entropy—or order and disorder—is nothing

English alphabet, count how often they occur in Hamlet and calculate I

nces at Warsaw University of Technology. B. Skowron research interests include formal ontology (part-whole theory, mereotopology), phenomenology, philosophy of morality, axiology, philosophical anthropology, the basis and philosophy of mathematics, applied logic and applied topology. He recently edited a special issue of ZFN on category theory (**eckstein_is_2020**) and the book on contemporary Polish

as book is also available in in an open access model as a PDF file at <https://philarchive.org/archive/SKOCIC-2>.

ch us humility. This goes contrary to the deep-seated conviction in Anglo-Saxon philosophy that philosophy began with Hume and Locke et al., and everything that came before—with the exception of Plato, Aristotle, and a few others—were musings about ultimate questions that were of no importance, both in terms of the questions and the musings.

after discussing topo-philosophy] is essential for the book” (**skowron_czesc_2021**).

her natural notion of equivalence. Equivalent tangle machines may differ locally, but globally they share the same information content. The goal of tangle machine equivalence is to provide a context-independent method to select, from among many ways to perform a task, the ‘best’ way to perform the task.” (**carmi_tangle_2015**).

ness for all the presented ideas, both historical and modern, so his book is a self-contained, comprehensive source of knowledge for the discussed topics.

1970 Mexican acid Western art film based on “symbolism and Eastern philosophy,” a topic certainly outside the scope of Skowron’s book (accessed at https://en.wikipedia.org/wiki/EI_Topology). In addition, topo-philosophy does not register in the Google Ngram Viewer, so this concept has been banished into the Internet’s conceptual never-never land. In contrast, the presence of topology, mereology and ing essays on topo-philosophy anytime soon, but it may do so for topology or mereology (*signa temporum*).

⁸ For level-headed physicists, entropy—or order and disorder—is nothing to be seen and discussed in conjunction with temperature and heat, and energy and work. And, if there is to be an extrapolation of entropy to a foreign field, it must be accompanied by the appropriate extrapolations of temperature and heat and work. If we wish, we can now assign an entropy to the message which Shakespeare sent us when he wrote Hamlet: We look up the probability of each letter of the nation entropy). People do that and we may suppose that they know why. Ingenious as this joke may be, it provides no more than amusement.” (**muller_history_2007**).