

Module Six Project:
Optimization Problems

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Introduction

The Rockhill Shipping & Transport Company is considering a contract of shipping waste with Chimotoxic, a company that manufactures chemicals for use in industry. Chimotoxic requires Rockhill to receive and transport waste from its six plants to three waste disposal sites. In addition to direct delivery from the six plants to one of the three waste disposal sites, managers are also considering using each plant and waste disposal site as an intermediate transport point. The transport vehicles can drop the goods at the factory or disposal site to be picked up and transported to the final destination by another transport vehicle. Additionally, Rockhill will not incur any operating costs, and Chimotoxic takes care of all handling costs at the plant and where the waste is disposed of. Rockhill only costs the freight, so the manager wanted to consider the possibility of dropping and picking up the product at the intermediate point because it may allow the company to reduce the cost of transportation instead of transporting directly to the destination.

Analysis

In this case, the optimal solution was found through the Solver. Solving linear programming models using slack variables was used to provide the optimal solution to an optimization problem. The decision variable, in this case, is the number of barrels that flow through each plant and waste disposal site. As a manager, Allen is responsible for managing the volume of transport flow from one destination to another to keep transportation costs to a minimum.

Model objective: $Z = \sum C_{ij}F_{ij}$

when C are the costs of transportation per barrel across the plants or waste disposal sites

F are the optimal number of barrels that are transported from one plants or waste disposal sites to another

The requirement for the analysis

An analysis of the transport costs between each plant and waste disposal site must consider the constraint of the volume of chemicals flowing through each plant and the waste disposal site. The volume of barrels that flow through each plant and waste disposal site is

limited to the plant's production and capacity of the disposal site. If the factory and the waste disposal site are used as an intermediate point of transportation, the constraint about the net volume flowing through that point should be provided. The net volume that flows through the plant can be greater than or equal to the plant's capacity. However, the disposal site's capacity is limited to the maximum number that can be stored at one time.

Model constraints: Waste disposal site net outflow \leq Waste disposal site capacity

Plant net outflow \geq Plant capacity

Optimal flow \leq Maximum capacity of Plant or Waste disposal

*Net outflow = (The volume of chemicals transported from the source) – (The volume of chemicals transported to the destination)

A Net outflow formula in excel:

= SUMIF(Start,T11,Optimal_flow)-SUMIF(Destination,T11,Optimal_flow)

For example, T11 = Denver, If the keyword in a “Start” range is equal to Denver, calculates the sum of values in a “Optimal_flow” range. And if the keyword in a “Destination” range is equal to Denver, calculates the sum of values in a “Optimal_flow” range.

Table 1 Model constraints

| Constraints | | | Unit: bbl/week |
|----------------------------|--------------------|--------|-------------------|
| Waste Disposal Site | Net outflow | | |
| Orangeburg | -74 | \leq | 65 |
| Florence | -64 | \leq | 80 |
| Macon | -95 | \leq | 105 |
| Plant | Net outflow | | |
| Denver | 45 | \geq | 45 |
| Morganton | 26 | \geq | 26 |
| Morrisville | 42 | \geq | 42 |
| Pineville | 53 | \geq | 53 |
| Rockhill | 29 | \geq | 29 |
| Statesville | 38 | \geq | 38 |

Table 2 Transportation cost between Plant and Waste Proposal Site

| | <u>Waste Proposal Site</u> | | |
|--------------------------------------------------------------------------------------------|----------------------------|-----------------|--------------|
| <u>Plant:</u> | <i>Orangeburg</i> | <i>Florence</i> | <i>Macon</i> |
| Denver | \$12 | \$15 | \$17 |
| Morganton | \$14 | \$9 | \$10 |
| Morrisville | \$13 | \$20 | \$11 |
| Pineville | \$17 | \$16 | \$19 |
| Rockhill | \$7 | \$14 | \$12 |
| Statesville | \$22 | \$16 | \$18 |
| Table 1: Shipping costs, per barrel of waste from six plants to three waste disposal sites | | | |

Table 3 Transportation cost between Plant and Plant

| | <u>Plant</u> | | | | | |
|--------------------|--------------|-----------|-------------|-----------|----------|-------------|
| <u>Plant:</u> | Denver | Morganton | Morrisville | Pineville | Rockhill | Statesville |
| Denver | \$--- | \$3 | \$4 | \$9 | \$5 | \$4 |
| Morganton | \$6 | \$--- | \$7 | \$6 | \$9 | \$4 |
| Morrisville | \$5 | \$7 | \$--- | \$3 | \$4 | \$9 |
| Pineville | \$5 | \$4 | \$3 | \$--- | \$3 | \$11 |
| Rockhill | \$5 | \$9 | \$5 | \$3 | \$--- | \$14 |
| Statesville | \$4 | \$7 | \$11 | \$12 | \$8 | \$--- |

Table 2: Shipping costs, per barrel of waste from each plant to another plant

Table 4 Transportation cost between waste disposal sites

| | <u>Waste Proposal Site</u> | | |
|-----------------------------|----------------------------|-----------------|--------------|
| <u>Waste Disposal Site:</u> | Orangeburg | Florence | Macon |
| Orangeburg | \$--- | \$12 | \$10 |
| Florence | \$12 | \$--- | \$15 |
| Macon | \$10 | \$15 | \$--- |

Table 5 productivity of plant and capacity of waste disposal site

| Plant: | Waste per Week (bbl) | Waste Site | Maximum (bbl) |
|-------------|----------------------|------------|---------------|
| Denver | 45 | Orangeburg | 65 |
| Morganton | 26 | Florence | 80 |
| Morrisville | 42 | Macon | 105 |
| Pineville | 53 | | |
| Rockhill | 29 | | |
| Statesville | 38 | | |

The total cost of transportation is determined by SUMPRODUCT of unit cost and optimal flow. And then the solver is used to find the best transport route that minimizes the transportation cost and identify all constraints.

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- ☒ Optimal_capacity <= Max_capacity
- ☐ Plant_net_outflow >= Plant_capacity
- ☐ Waste_net_outflow <= Waste_capacity

☒ Make Unconstrained Variables Non-Negative

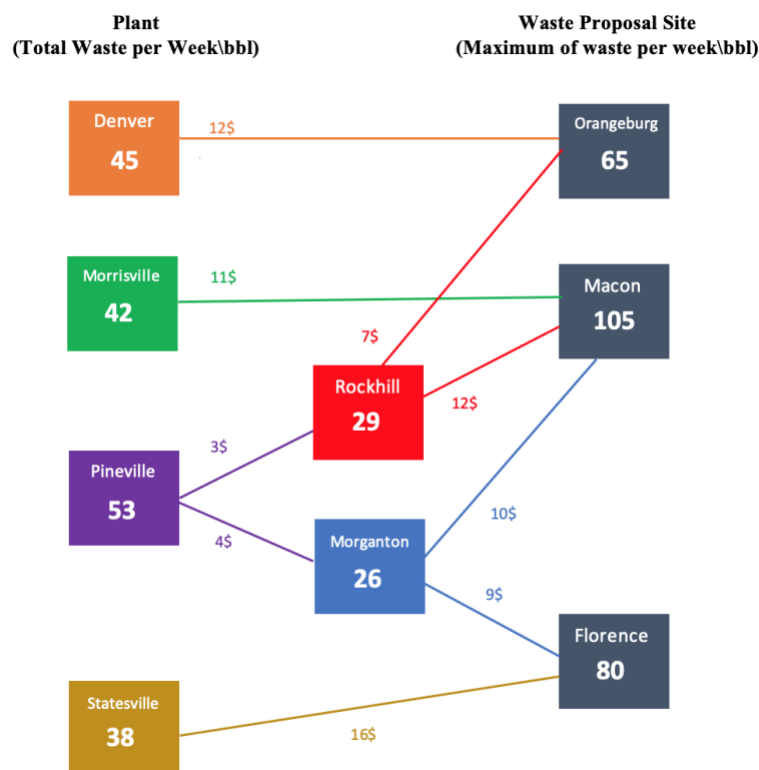
Select a Solving Method:

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

The optimal flow provides the volume of chemicals in the barrel unit transported from the Start to the Destination. The maximum capacity provides the maximum flow of chemicals that can pass through a plant or a waste disposal site in the transportation schedule. By using Solver-Add in, the optimal transportation routes are shown as follows:

| Start | Destination | Unit Cost/bbl | Optimal flow | | Maximum Capacity |
|-------------|-------------|---------------|--------------|----|------------------|
| Denver | Orangeburg | \$12.00 | 45 | <= | 45 |
| Morganton | Florence | \$9.00 | 26 | <= | 26 |
| Morganton | Macon | \$10.00 | 26 | <= | 26 |
| Morrisville | Macon | \$11.00 | 42 | <= | 42 |
| Pineville | Morganton | \$4.00 | 26 | <= | 53 |
| Pineville | Rockhill | \$3.00 | 27 | <= | 53 |
| Rockhill | Orangeburg | \$7.00 | 29 | <= | 29 |
| Rockhill | Macon | \$12.00 | 27 | <= | 29 |
| Statesville | Florence | \$16.00 | 38 | <= | 38 |

The transportation schedule diagram can be summarized as follows:



To conclude, Allen should implement this flow schedule to minimize the company's transportation costs. From the diagram, we can see that 45 barrels of chemicals is transferred from Denver to Orangeburg, and the total cost of this trip is equal to \$540. And then there is a flow of chemicals from Morrisville to Macon with 42 barrels; the trip cost is \$462. Pineville waste is transported to Rockhill with 27 barrels and Morganton with 26 barrels. Both trips generate a cost of \$185 ($27 \cdot 3 + 26 \cdot 4$). From Rockhill, the chemical is transported to two final waste disposal sites: Orangeburg and Macon. The total cost of transportation to these two destinations is \$406. From Morganton, the chemical is delivered to Macon and Florence with the cost of \$494. Finally, there will be chemicals 38 barrels transported from Statesville to Florence. Therefore, the least amount of total cost to be incurred is equal to 2,816 and the chemicals of 286 barrels is carried through all points.