分类器组合与集成习题

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1.

(1)

写出拉格朗日函数如下:

$$L = rac{1}{L} \sum_{j=1}^{L} \sum_{i=1}^{K} P(\omega_i | oldsymbol{x}) \ln rac{P(\omega_i | oldsymbol{x})}{P_j(\omega_i | oldsymbol{x})} - lpha \left(\sum_{i=1}^{k} P(\omega_i | oldsymbol{x}) - 1
ight)$$

求偏导:

$$egin{aligned} rac{\partial L}{\partial P(\omega_i|m{x})} &= rac{1}{L} \sum_{j=1}^L \ln P(\omega_i|m{x}) + 1 - \ln P_j(\omega_i|m{x}) - lpha \ & \ln P(\omega_i|m{x}) + 1 - lpha &= rac{1}{L} \sum_{j=1}^L \ln P_j(\omega_i|m{x}) \ & \Rightarrow \ln P(\omega_i|m{x}) &= \ln \left(\prod_{j=1}^L P_j(\omega_i|m{x})
ight)^{1/L} + (lpha - 1) \ & P(\omega_i|m{x}) &= C \left(\prod_{j=1}^L P_j(\omega_i|m{x})
ight)^{1/L} \end{aligned}$$

其中C为归一化因子,由于 $\sum_{i=1}^K P(\omega_i | \boldsymbol{x}) = 1$,应当有:

$$C = rac{1}{\sum_{i=1}^{K} \left(\prod_{j=1}^{L} P_j(\omega_i | oldsymbol{x})
ight)^{1/L}}$$

得证原式。

$$egin{aligned} \min_{P(\omega_i|m{x})} rac{1}{L} \sum_{j=1}^L \sum_{i=1}^K P_j(\omega_i|m{x}) \ln rac{P_j(\omega_i|m{x})}{P(\omega_i|m{x})} \ &\Leftrightarrow \max_{P(\omega_i|m{x})} rac{1}{L} \sum_{j=1}^L \sum_{i=1}^K P_j(\omega_i|m{x}) \ln P(\omega_i|m{x}) \ &\Leftrightarrow \max_{P(\omega_i|m{x})} \sum_{i=1}^K \left(rac{1}{L} \sum_{j=1}^L P_j(\omega_i|m{x})
ight) \ln P(\omega_i|m{x}) \end{aligned}$$

可以知道:

$$\sum_{i=1}^K \left(\frac{1}{L}\sum_{j=1}^L P_j(\omega_i|\boldsymbol{x})\right) \ln P(\omega_i|\boldsymbol{x}) \leq \sum_{i=1}^K \left(\frac{1}{L}\sum_{j=1}^L P_j(\omega_i|\boldsymbol{x})\right) \ln \left(\frac{1}{L}\sum_{j=1}^L P_j(\omega_i|\boldsymbol{x})\right)$$

当:

$$P(\omega_i|oldsymbol{x}) = \left(rac{1}{L}\sum_{j=1}^L P_j(\omega_i|oldsymbol{x})
ight)$$

时候取到极大值。

2.

采用加权投票法,错误率为0.2, 0.3, 0.4, 0.4, 0.4的分类器的权重依次为 $\frac{4}{9}, \frac{2}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{9}$ 。当错误的权重和>正确的权重和的时候,产生错误。

$$P_{err} = 0.3*0.4^3*0.8 + 0.2*(1 - 0.7*0.6^3) = 0.185 < 0.2$$

3.

$$egin{aligned} \min_{lpha,h} \sum_{i=1}^n [y_i - (f_{t-1}(oldsymbol{x}_i) + lpha h(oldsymbol{x}_i))]^2 \ \Leftrightarrow \min_{lpha,h} \sum_{i=1}^n [eta_i^t - lpha h(oldsymbol{x}_i))]^2 \end{aligned}$$

 $\hat{\alpha}\hat{h}(\boldsymbol{x}_i)$ 为当前残差的最优拟合,所以更新过程为:

$$f_t(oldsymbol{x}) = f_{t-1}(oldsymbol{x}) + \hat{lpha}_t \hat{h}_t(oldsymbol{x})$$

这样可以在不影响前面分类器的情况下逐步增加分类器。

算法如下:

- 1. 初始化 $f_0(x) = 0$
- 2. 对m = 1 ... M:
 - 1. 计算 $(\hat{\alpha}_t, \hat{h}_t) = \arg\min_{\alpha, h} \sum_{i=1}^n [\beta_i^t \alpha h(\boldsymbol{x}_i))]^2$
 - 2. 更新 $f_t(oldsymbol{x}) = f_{t-1}(oldsymbol{x}) + \hat{lpha}_t \hat{h}_t(oldsymbol{x})$