特征选择与变换习题

姓名: 甘云冲

学号: 2101213081

1.

(1)

He = 0:

$$He = (I - ee^{T})e = Ie - e(e^{T}e) = e - e = 0$$

HH = H:

$$m{H}m{H} = (m{I} - m{e}m{e}^T)(m{I} - m{e}m{e}^T) = m{I} - m{e}m{e}^T - m{e}m{e}^T + m{e}(m{e}^Tm{e})m{e}^T = m{I} - m{e}m{e}^T = m{H}$$
 $(m{x}_1 - ar{m{x}}, \dots, m{x}_n - ar{m{x}})^T = m{H}m{X}$:

$$egin{aligned} (ar{oldsymbol{x}},ar{oldsymbol{x}},ar{oldsymbol{x}}, ar{oldsymbol{x}}, ar{oldsymbol{x}})^T &= [ar{oldsymbol{x}}(1,1,\ldots,1)]^T = (oldsymbol{X}^Toldsymbol{e}oldsymbol{e}^Toldsymbol{Y} &= oldsymbol{e}oldsymbol{e}^Toldsymbol{X} &= oldsymbol{e}oldsymbol{e}^To$$

 $oldsymbol{\Sigma} = rac{1}{n} oldsymbol{X}^T oldsymbol{H} oldsymbol{X}$:

$$oldsymbol{\Sigma} = rac{1}{n}(oldsymbol{H}oldsymbol{X})^T(oldsymbol{H}oldsymbol{X}) = rac{1}{n}oldsymbol{X}^T(oldsymbol{H}^Toldsymbol{H})oldsymbol{X} = rac{1}{n}oldsymbol{X}^T(oldsymbol{H}oldsymbol{X})$$

(2)

 $oldsymbol{v}_i$ 为 $oldsymbol{X}^Toldsymbol{H}oldsymbol{X}$ 的特征值:

$$egin{aligned} oldsymbol{X}^T oldsymbol{H} oldsymbol{X} oldsymbol{v}_i oldsymbol{H} oldsymbol{X} oldsymbol{X}^T oldsymbol{H} oldsymbol{X} oldsymbol{v}_i & = \lambda_i oldsymbol{H} oldsymbol{X} oldsymbol{v}_i \ oldsymbol{H} oldsymbol{X} oldsymbol{X}^T oldsymbol{H} (oldsymbol{H} oldsymbol{X} oldsymbol{v}_i) & = \lambda_i (oldsymbol{H} oldsymbol{X} oldsymbol{v}_i) \end{aligned}$$

 u_i 为 HXX^TH 的特征值:

$$egin{aligned} oldsymbol{H}oldsymbol{X}^Toldsymbol{H}oldsymbol{u}_i &= l_ioldsymbol{u}_i \ oldsymbol{X}^Toldsymbol{H}oldsymbol{X}^Toldsymbol{H}oldsymbol{X}^Toldsymbol{H}oldsymbol{X}_i &= l_ioldsymbol{X}^Toldsymbol{H}oldsymbol{u}_i \ oldsymbol{X}^Toldsymbol{H}oldsymbol{X}(oldsymbol{X}^Toldsymbol{H}oldsymbol{u}_i) &= l_ioldsymbol{X}^Toldsymbol{H}oldsymbol{u}_i \end{aligned}$$

又由于:

$$egin{aligned} oldsymbol{v}_i^Toldsymbol{X}^Toldsymbol{H}oldsymbol{X}oldsymbol{v}_i & oldsymbol{u}_i^Toldsymbol{u}_i = 1 \ oldsymbol{u}_i^Toldsymbol{H}oldsymbol{X}^Toldsymbol{H}oldsymbol{u}_i = (oldsymbol{X}^Toldsymbol{H}oldsymbol{u}_i)^T(oldsymbol{X}^Toldsymbol{H}oldsymbol{u}_i) = \lambda_i, & oldsymbol{u}_i^Toldsymbol{u}_i = 1 \ oldsymbol{u}_i^Toldsymbol{H}oldsymbol{X}^Toldsymbol{H}oldsymbol{u}_i = 1 \end{aligned}$$

可以知道对应的比例为 $\sqrt{\lambda_i}$ 和 $\sqrt{l_i}$, 即 $m{v}_i = rac{m{X}^T m{H} m{u}_i}{\sqrt{l_i}}, m{u}_i = rac{m{H} m{X} m{v}_i}{\sqrt{\lambda_i}}$ 。

2.

(1)

由于u为B的特征向量:

$$egin{aligned} l_i oldsymbol{u}_i &= oldsymbol{H} oldsymbol{X} oldsymbol{X}^T oldsymbol{H} oldsymbol{u}_i &= oldsymbol{H} oldsymbol{X} oldsymbol{X}^T oldsymbol{H} oldsymbol{u}_i &= oldsymbol{H} oldsymbol{X} oldsymbol{X}^T oldsymbol{H} oldsymbol{u}_i &= oldsymbol{H} oldsymbol{U}_i \end{aligned}$$

于是有:

$$oldsymbol{H}\hat{oldsymbol{X}} = oldsymbol{H}(\sqrt{l_1}oldsymbol{u}_1,\ldots,\sqrt{l_n}oldsymbol{u}_n) = (\sqrt{l_1}oldsymbol{H}oldsymbol{u}_1,\ldots,\sqrt{l_n}oldsymbol{u}_n) = \hat{oldsymbol{X}}$$

(2)

$$\hat{\boldsymbol{B}} = \boldsymbol{H}\hat{\boldsymbol{X}}\hat{\boldsymbol{X}}^T\boldsymbol{H} = \boldsymbol{H}\hat{\boldsymbol{X}}\hat{\boldsymbol{X}}^T\boldsymbol{H}^T = \boldsymbol{H}\hat{\boldsymbol{X}}(\boldsymbol{H}\hat{\boldsymbol{X}})^T = \hat{\boldsymbol{X}}\hat{\boldsymbol{X}}^T = \boldsymbol{B}$$

(3)

$$egin{aligned} s_{ij} &= (oldsymbol{x}_i - oldsymbol{x}_j)^T (oldsymbol{x}_i - oldsymbol{x}_j) \ &= ((oldsymbol{x}_i - ar{oldsymbol{x}}) - (oldsymbol{x}_j - ar{oldsymbol{x}}) - (oldsymbol{x}_j - ar{oldsymbol{x}}) \ &= (oldsymbol{x}_i - ar{oldsymbol{x}})^T (oldsymbol{x}_i - ar{oldsymbol{x}})^T (oldsymbol{x}_j - ar{oldsymbol{x}}) - 2(oldsymbol{x}_i - ar{oldsymbol{x}})^T (oldsymbol{x}_j - ar{oldsymbol{x}}) \ &= b_{ii} + b_{jj} - 2b_{ij} \end{aligned}$$

所以由于 $\hat{\boldsymbol{B}} = \boldsymbol{B}$,即可以直接得到 $\hat{\boldsymbol{S}} = \boldsymbol{S}$ 。

$$oldsymbol{\Sigma}_{arphi} oldsymbol{v} = \lambda oldsymbol{v} \ \Rightarrow rac{1}{n} \sum_{i=1}^n arphi(oldsymbol{x}_i) arphi(oldsymbol{x}_i)^T oldsymbol{v} = \lambda oldsymbol{v} \ \$$

当 $\lambda \neq 0$ 时:

$$oldsymbol{v} = rac{1}{n\lambda} \sum_{i=1}^n arphi(oldsymbol{x}_i) [arphi(oldsymbol{x}_i)^T oldsymbol{v}] = rac{1}{n\lambda} \sum_{i=1}^n [arphi(oldsymbol{x}_i)^T oldsymbol{v}] arphi(oldsymbol{x}_i)$$

其中 $[\varphi(\boldsymbol{x}_i)^T \boldsymbol{v}]$ 为标量,故可以表示为 $\boldsymbol{v} = \sum_{i=1}^n \alpha_i \varphi(\boldsymbol{x}_i)$ 的形式。