Physics 382 Superconducting Tunnel Junction Lab

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1   Introduction

In this lab, you will recreate the experiments that won Ivar Giaever a share of the 1973 Nobel Prize in Physics, for the experimental demonstration of quantum mechanical tunneling in superconductors. Giaever used electron tunneling to directly measure the superconducting energy gap Δ, thus testing the recently-published BCS theory of superconductivity in a remarkably simple way. Tunneling experiments have since become a standard tool for studying electron energy level structure in superconductors. Finally, the superconducting "tunnel junctions" on which these experiments were preformed have since become important areas of research in their own right, as the basic building blocks of some of the most sensitive detectors and amplifiers in existence. In recreating Giaever's experiments you will gain experience in thin film deposition, cryogenics, automated data acquisition (DAQ) in MATLAB, and χ^2 analysis, all important tools in experimental physics today.

Resources are available on classes*v2 to help you carry out these experiments. This document is supposed to motivate the experiment and serve as an introduction to the equipment and techniques you will use. You should read through section 4 in advance of your first lab session, section 5 before your third lab session, and sections 6 and 7 before you start your analysis. Several questions (italicized) are posed to the reader throughout – you are not expected to formally answer these in your lab writeup; their purpose is simply to ensure that you're following the argument. Consult your TF if any of this material is confusing.

This document is not a step-by-step guide to the complex cryogenics and evaporation procedures – your TF will walk you through these sections, but if you're interested, "Junction Lab Instructions for TFs" on classes*v2 contains step-by-step instructions by a past TF on how to run this portion of the lab. Another useful resource on classes*v2 is "Discovery of Electron Tunneling into Superconductors" [1], a collection of documents relevant to the original experiments, including Giaever's Nobel acceptance lecture and a few of his (very readable) early papers on the subject.

2   Theory

(a)   Relevant Stat Mech and Solid State Physics Background

Depending on your background in statistical mechanics and solid state physics, you may already be familiar with the material in this section, but it is presented here for those who have not taken courses in these subjects.

The discrete electronic energy levels of any solid-state system are so closely spaced together that for all intents and purposes they form a continuous **band**. Degeneracy at each discrete energy level generalizes in the continuum limit to a function $N(E)$ called the **density of states**, which tells you how many electron states are available as a function of energy. An energy range in which $N(E) = 0$ is called a **gap** in the spectrum. We will generally be interested in the question of which levels are occupied and which are not, the answer to which will depend on temperature.

At absolute zero, every electron goes to the lowest-energy state it is allowed to occupy, but electrons are fermions, and therefore cannot occupy the same state. Thus, the ground state configuration is to fill up all $N(E)$ states with energy between E and $E + dE$, moving up in energy from ground state until all electrons have been accounted for. The threshold between occupied and unoccupied states is called the **Fermi level**. More precisely, the Fermi level is defined as the energy for which the occupation probability is 1/2; at 0 Kelvin, where there are no random thermal fluctuations, all states with lower energy have occupation probability 1 and all states with higher energy have occupation probability 0.

Since the highest-energy electrons are responsible for electrical conduction, it is probably more intuitive to think of the Fermi level

as the typical energy of current-carrying electrons; the voltage between two conductors is the difference between their Fermi levels divided by the electron charge e. It is conventional to set $E = 0$ at the Fermi level.

At finite temperatures T, electrons slightly below the Fermi level may be excited above it by thermal fluctuations; the probability that a state at energy E is occupied is given by the **Fermi-Dirac distribution**

$$f(E) = \frac{1}{e^{E/k_B T} + 1}$$

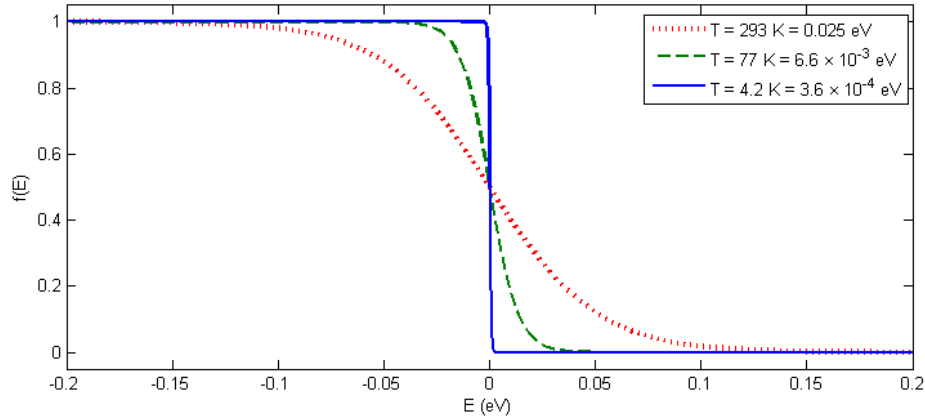where k_B is the Boltzmann constant. The Fermi-Dirac distribution at several temperatures is plotted below:

Figure 1: The Fermi-Dirac distribution at several temperatures.

We can see that this reduces to a step function as $T \to 0$, but there's always a little bit of "smearing" within a few $k_B T$ of the Fermi level: in general, there will be some empty states below the Fermi level, and some filled states above.

Lastly, we consider what happens if the band structure of a material is such that all the states below a gap, and none of the states above it, are filled at $T = 0$. The convention is then to define the Fermi level at the middle of the gap; in this case, the effects of these thermal fluctuations only "turn on" for $k_B T$ comparable to the width of the gap. To facilitate comparison of thermal energy with the energy differences that appear on band diagrams, a useful unit conversion to bear in mind is

$$1 \text{ kelvin} \approx 10^{-4} \text{ electron-volts} \tag{1}$$

(b) A Very Qualitative Introduction to Superconductivity

The most striking feature of the behavior of superconductors is the rapid disappearance of electrical resistance below a certain **critical temperature** T_c, often in the range 1-10 K. Superconductivity was first discovered in 1911, but it wasn't until the publication of the Bardeen-Cooper-Schrieffer (BCS) theory in 1957 that the microscopic basis for this dramatic behavior was finally understood. The BCS theory is beyond the scope of this lab; a good reference if you're interested is the book by Tinkham [2].

Qualitatively, BCS theory showed how electron-phonon interactions could produce an effective attractive interaction between electrons, causing electrons to "pair up" in momentum space even though they avoid each other in real space (to minimize the effects of their repulsive Coulomb interactions). Each of the resulting **Cooper pairs** is a bosonic quasiparticle, so arbitrarily many Cooper pairs can occupy the same state.

One of the main predictions of BCS theory is that the energy spectrum of a superconductor has a gap with half-width Δ between the ground state (in which all electrons are paired up) and its continuum of excited states, in which some electrons are unpaired, and behave more or less like they do in a normal metal.

The chief source of electrical resistance in a normal metal is electron-ion scattering, and the typical energy transferred in such a scattering event at temperature T is $\sim k_B T$. For $k_B T < \Delta$, most of the electrons are paired up in the ground state, and they simply don't scatter off ions: there are no empty states for them to scatter into! This accounts for the rapid disappearance of

resistivity and suggests that the critical temperature should be intimately related to the energy gap; a more careful treatment yields a temperature-dependent gap of the form

$$\Delta(T) \approx 3.06 k_B T_c \sqrt{1 - \frac{T}{T_c}} \tag{2}$$

This is an approximate expression which only holds for T sufficiently close to T_c; at sufficiently low temperatures a better approximation is $\Delta(T) \approx \Delta(0) = 1.76 k_B T_c$. BCS theory also predicts a definite functional form for the density of states above the gap:

$$\frac{N_s(E)}{N_n(0)} = \frac{E}{\sqrt{E^2 - \Delta^2}} \text{ for } E > \Delta \tag{3}$$

where $N_n(0)$ is the density of states of the same material in its normal state ($T > T_c$), at the Fermi level. Qualitatively speaking, the density of states is huge for energies barely above the gap, and then drops off rapidly at higher energies.

(c) Overview of Tunneling

Tunneling should be familiar to you as a basic problem from the wavefunction formulation of quantum mechanics: when a free particle with energy E is incident on a thin potential barrier of height $V > E$, its wavefunction on the other side of the barrier has the same frequency and reduced (but nonzero) amplitude, corresponding to a finite probability of finding the particle with the same energy in the region it could not access classically:
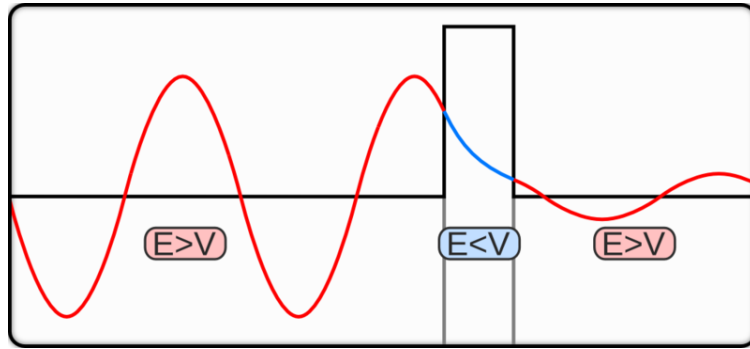

Figure 2: Simple illustration of quantum tunneling. Image from Wikipedia.

The above diagram only allows us to represent the tunneling of a single particle; to discuss tunneling in macroscopic, many-body systems, a slightly different picture is appropriate:
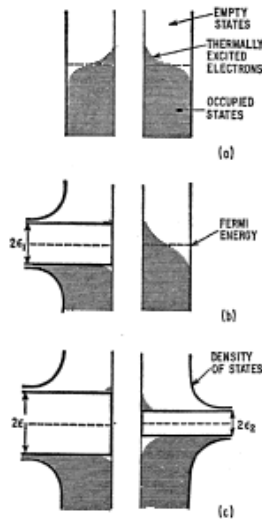

Figure 3: Band diagrams for (a) NIN, (b) SIN, and (c) SIS tunneling. Gap half-width denoted by ϵ. Image from [1].

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The vertical axis is once again energy, and the blank space in the middle represents the potential barrier, which in practice is a thin insulating layer (hence "I;" the materials on either side are normal metals ("N") or superconductors ("S")). Occupied and empty states, along with the Fermi level and the functional form of the density of states, are indicated in the diagram. The effects of thermal fluctuations as depicted in each diagram should make sense in light of the discussion in part **(a)**.

One thing that may seem strange about the diagrams in figure 3 is that the spectrum of the superconductors below the gap doesn't look anything like BCS theory as described above, with a single ground state occupied by a large number of Cooper pairs. Instead, the BCS density of states is simply reflected about the Fermi level. This abstraction, which has the virtue of avoiding much more complicated questions about the tunneling of the highly correlated Cooper pairs, is called the **semiconductor model**; it fits experimental data surprisingly well, and we will use it to make quantitative predictions about tunneling.

As in the simpler wavefunction picture above, tunneling in the semiconductor model occurs at constant energy. Intuitively we expect the tunneling rate at a certain energy to be proportional to the difference between the excess of electrons over unoccupied states on the two sides of the barrier. We should thus expect no net tunneling in the any of the diagrams in figure 3. Applying a voltage difference V between the two metals shifts the Fermi level on one side up by eV; this may induce electron tunneling, which will manifest as a current due to the charge of the electrons. It should also be clear from figure 3 that the tunneling current will be symmetric with respect to the sign of the voltage difference.

Two normal or superconducting metals separated by a thin insulating layer collectively form a **tunnel junction**. Giaever's brilliant idea was to measure Δ by looking for the sharp rise of the current through an SIN junction when the voltage across the junction hits Δ/e. We will now work out this result in greater detail.

(d) Comparison of NIN, SIN, and SIS tunneling

To derive a quantitative expression for the current through a tunnel junction, let's first consider the current flowing in one direction (say, left to right). The current due to left-to-right tunneling at a given energy must be proportional to both the number of occupied states on the left and the number of unoccupied states on the right. The net left-to-right tunneling current will be the sum of the currents due to tunneling at different energies. Mathematically, this amounts to the statement

$$I_{L \to R} = A \left|\mathcal{T}\right|^2 \int_{-\infty}^{+\infty} N_L(E)f(E)N_R(E + eV)[1 - f(E + eV)]\mathrm{d}E$$

where $\left|\mathcal{T}\right|^2$ is the tunneling probability and A is an uninteresting proportionality constant related to the junction geometry. Without loss of generality I have assumed that the Fermi level on the right has been lowered by eV so that this describes a constant-energy transition. We can write down a similar expression for the tunneling current in the opposite direction:

$$I_{R \to L} = A \left|\mathcal{T}\right|^2 \int_{-\infty}^{+\infty} N_R(E + eV)f(E + eV)N_L(E)[1 - f(E)]\mathrm{d}E$$

The observed tunneling current will be the difference $I_{L \to R} - I_{R \to L}$. Simple algebra then yields

$$I(V) = A \left|\mathcal{T}\right|^2 \int_{-\infty}^{+\infty} N_L(E)N_R(E + eV)[f(E) - f(E + eV)]\mathrm{d}E \tag{4}$$

Let's pause to consider the simplifying assumptions implicit in this result. First, we've fudged things a little bit in taking the bounds of the integral to $\pm\infty$, since there are actually no states infinitely far below the Fermi level. However, it's easy to see that for $|E| \gg |eV|$, the integrand becomes vanishingly small, so extreme energies contribute very little to the integral, and it doesn't much matter where we put the limits.

Second, it is physically reasonable to expect that the tunneling probability could depend on energy, but this makes things very complicated – Giaever simply assumed $\left|\mathcal{T}\right|^2$ = constant, and it was later shown that this approximation is reasonable over the range of energies that contribute meaningfully to the integral. Lastly, we've treated this as a 1D problem when in reality electrons will have components of their momentum parallel to the barrier surface – this turns out not to affect our result provided the junction materials are isotropic.

All the above has applied equally to NIN, SIN, and SIS junctions; note also that a single physical junction whose components are superconductors with different transition temperatures can exhibit NIN, SIN, or SIS behavior depending on its temperature. To specify the problem further, we need to plug in a specific form for the density of states on either side of the barrier. As the

diagrams in Figure 3 suggest, we assume the density of states for a normal metal is constant; this is empirically true sufficiently close to the Fermi level, and once again the behavior of the integrand far from the Fermi level is unimportant.

Thus, in **NIN tunneling**, we can pull both N_L and N_R out of the integral:

$$I_{\text{NIN}} = A\,|\mathcal{T}|^2\,N_{Ln}(0)N_{Rn}(0) \int_{-\infty}^{+\infty} [f(E) - f(E + eV)]\mathrm{d}E$$

$$= A\,|\mathcal{T}|^2\,N_{Ln}(0)N_{Rn}(0) \cdot eV \tag{5}$$

$$= \frac{V}{R_0} \tag{6}$$

where

$$R_0 = \frac{1}{eA\,|\mathcal{T}|^2\,N_{Ln}(0)N_{Rn}(0)}$$

is a temperature-independent constant with dimension of resistance; thus the NIN $I(V)$ curve is linear, or **ohmic**. You should be able to compute the integral explicitly at $T = 0$; though it's not a priori obvious that the answer should be temperature-independent, numerical evaluation of the finite-temperature integral yields the same result. I want to emphasize that although the $I(V)$ curve for tunneling through a small insulating layer looks like that of an ordinary resistor, the physical mechanism involved is quite different. The fact that the tunneling resistance is temperature-independent highlights this difference dramatically.

Next, we consider the same junction below T_c of the metal on the left. Then we have **SIN tunneling**:

$$I_{\text{SIN}} = A\,|\mathcal{T}|^2\,N_{Rn}(0) \int_{-\infty}^{+\infty} N_{Ls}(E)[f(E) - f(E + eV)]\mathrm{d}E$$

$$= \frac{1}{eR_0} \int_{-\infty}^{+\infty} \frac{N_{Ls}(E)}{N_{Ln}(0)}[f(E) - f(E + eV)]\mathrm{d}E \tag{7}$$

with $N_{Ls}(E)/N_{Ln}(0)$ given by equation (3). Note that the constant that appears out front is the same R_0 as in the normal case, but this time there is implicit temperature-dependence buried in the integral. Once again, the integrals must be computed numerically for $T \neq 0$; you will need to do this as part of your analysis, but to get an intuitive sense of the behavior of $I(V)$, it is easiest to look at the band diagram:
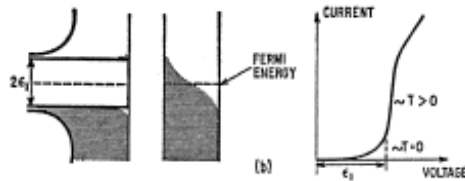

Figure 4: Band diagram and $I(V)$ curve for SIN tunneling. Gap half-width denoted by ϵ. Image from [1].

At $T = 0$, there will be no tunneling whatsoever until the the energy difference between the two Fermi levels is $eV = \Delta$. At this point, regardless of the sign of eV, there is suddenly an enormous number of occupied states on one side at the same energy as empty states on the other, so the current will shoot up. Increasing the voltage difference enables tunneling at higher energies, so the current will continue to increase. For $V \gg \Delta/e$, the superconducting density of states asymptotes to that of a normal metal, so the $I(V)$ curve will approach the linear form seen in the NIN case above.

At finite temperatures, electrons thermally excited above the Fermi level can tunnel for $eV < \Delta$. The number of such electrons drops exponentially with energy, so instead of a sharp cutoff, the $I(V)$ curve should have an exponential tail with characteristic width $k_B T/e$, as illustrated above.

Lastly, we consider **SIS tunneling** in the same junction with both metals superconducting:

$$I_{\text{SIS}} = A\,|\mathcal{T}|^2 \int_{-\infty}^{+\infty} N_{Ls}(E)N_{Rs}(E + eV)[f(E) - f(E + eV)]\mathrm{d}E$$

$$= \frac{1}{eR_0} \int_{-\infty}^{+\infty} \frac{N_{Ls}(E)}{N_{Ln}(0)} \frac{N_{Rs}(E + eV)}{N_{Rn}(0)}[f(E) - f(E + eV)]\mathrm{d}E \tag{8}$$

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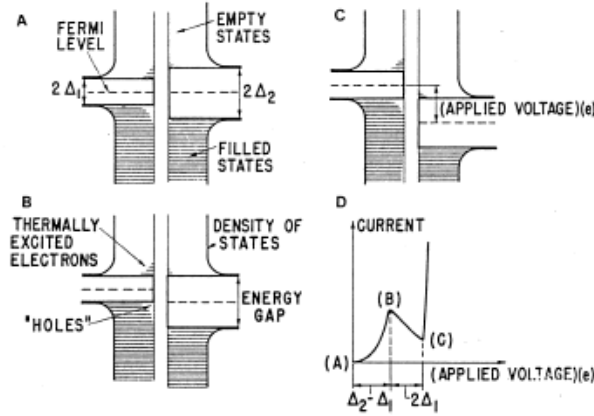Again, we will get a qualitative sense of the behavior of the integral by looking at the band diagram:

Figure 5: Band diagrams at several different voltages and $I(V)$ curve for SIS tunneling at $T \neq 0$. Image from [1].

To make sense of the unusual $T \neq 0$ $I(V)$ curve in diagram D, let's first consider the simpler case of $T = 0$. In diagram A there is no voltage across the junction. In diagram B, at voltage $|\Delta_1 - \Delta_2|/e$, the top of the gaps on both sides are level, but in the absence of thermal fluctuations, there are no occupied states above the gap, so nothing happens. It should be clear that there will be no tunneling until $V = (\Delta_1 + \Delta_2)/e$ in Diagram C, at which point the bottom of one gap is level with the top of the other; the current will rise sharply when the voltage difference passes this threshold. In other words, a $T = 0$ SIS junction behaves exactly like a $T = 0$ SIN junction with $\Delta = \Delta_1 + \Delta_2$.

However, things are qualitatively different for $T > 0$, the case illustrated in the diagrams above. There is now a peak at $V = |\Delta_1 - \Delta_2|/e$, the lowest voltage at which all of the thermally excited electrons can participate in tunneling. As in the SIN case, the tunneling has an exponential tail of width $k_B T$ for lower voltages, when only some of the excited electrons are able to tunnel. What's more unusual is that the current actually decreases for increasing voltage in the regime $|\Delta_1 - \Delta_2| < eV < \Delta_1 + \Delta_2$: this kind of behavior is called **negative resistance**, because the differential resistance $dV/dI < 0$.

The existence of the negative resistance region may be understood as a consequence of the BCS density of states. No additional electrons participate in tunneling as the voltage is increased above $|\Delta_1 - \Delta_2|/e$, and the existing electrons on the left are further above the gap on the right, where the density of states is smaller. This behavior ends abruptly at $eV = \Delta_1 + \Delta_2$, when tunneling below the gap becomes possible. Once again, for $eV \gg \Delta_1 + \Delta_2$, the $I(V)$ curve asymptotes to that of an NIN junction.

(e)   Conclusion and Further Comments on SIS Junctions

We have seen that the $I(V)$ curves of superconducting tunnel junctions have distinctive features that allow us to directly measure the gap (in the SIN case), or both gaps (in the SIS case). Moreover, the precise functional form of the approach to the Ohmic limit and the negative resistance region are both consequences of the BCS density of states (3), and thus provide a way to test this particular prediction of the theory.

Any discussion of tunneling in superconductors would not be complete without a cursory mention of the **Josephson effect**, for which Brian Josephson received another share of the 1973 Nobel. The Josephson effect is an example of a macroscopic quantum phenomenon: a single wavefunction describes the collective behavior of all the Cooper pairs in a superconductor. Josephson correctly predicted that when two superconductors are separated by a thin potential barrier, Cooper pairs will tunnel in response to a phase difference between their wavefunctions: this tunneling manifests as a sharp spike in current at 0 voltage. That is, not only does resistanceless "supercurrent" flow within one superconductor, it can flow between two superconductors provided the potential barrier separating them is sufficiently small.

The Joesphson effect is the basis of the exquisitely sensitive SQUID amplifiers used in fields as diverse as quantum computing and dark matter detection. Roughly speaking, it is a macroscopic manifestation of the "probability current" you likely discussed in your quantum mechanics class when recasting Schroedinger's equation as a continuity equation for the wavefunction. A quantitative treatment requires a more thorough understanding of superconductivity, and in particular the semiconductor

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model we have used thus far completely fails to account for it. You may observe the Josephson supercurrent in the course of this lab.

The remainder of this document will focus on the equipment and techniques you will use to make these measurements. But first, I will briefly digress on one of the more remarkable properties of SIS junctions: the extremely sharp nonlinearity of their $I(V)$ curves near $(\Delta_1 + \Delta_2)/e$. The rest of this section is not directly relevant to your measurements; you can skip it if you're not interested.

As those of you with electronics or engineering background know, one of the most fundamental properties of linear systems is that an input at frequency ω always produces an output at the same frequency; nonlinear devices are needed for frequency conversion. For such a device the amplitude of the output at the new frequency is usually proportional to the second derivative of the $I(V)$ curve (more generally, the transfer function) evaluated at the operating point. For an ideal SIS junction, the $I(V)$ curve at $(\Delta_1 + \Delta_2)/e$ is vertical at any temperature; the second derivative is infinite!

Of course, an infinite output is not physical for any device, but before we examine where the idealization that led to this result breaks down, it's worth examining more closely why we get such a remarkable prediction in the first place. The most familiar sharp nonlinearities in electronic components are exponential: that is, the output depends the input voltage V as $\exp[eV/k_BT]$, and this dependence arises from the underlying statistical mechanics of the component materials. Such exponential nonlinearities account for the "smearing" of the SIN junction $I(V)$ curve around the nonlinearity at $eV = \Delta$ and the $I(V)$ curve of the familiar semiconductor diode. They may be considered "sharp" only when the temperature is low compared to the voltage range of interest.

In contrast, the nonlinearity of the SIS junction is due to the transition from overlapping to non-overlapping energy gaps. There are *never* any states in the gap, so this transition is arbitrarily sharp at any temperature! In a real physical system, a small resistance in series with the junction will tilt the nonlinearity slightly. Furthermore, quantum mechanics forbids energy transfers smaller than $\hbar\omega$ at frequency ω, so there can be no tunneling for voltages smaller than $\hbar\omega/e$. Even so, a well-prepared SIS junction is extremely nonlinear. SIS devices are among the best available detectors and mixers for $O(100\,\text{GHz})$ signals; they are ubiquitous in submillimeter astronomy.

3   Overview of Experiment

By this point, you should have a rough idea of how you might go about measuring tunnel junction $I(V)$ curves. First, you need to create a junction, consisting of two metals with different superconducting transition temperatures separated by a thin insulating film. In practice, you will do this using a technique called **thermal evaporation**. You'll place the junction in a **cryostat**, a vessel capable of being cooled to very low temperatures, with current and voltage leads connecting your sample to room-temperature electronics.

You can increase the voltage across the junction in small steps, and measure the current at each step, or better yet, write DAQ code to automate this process. At room temperature, the junction should exhibit NIN behavior. Repeating the measurement with your junction temperature between T_{c_1} and T_{c_2}, you should see SIN behavior, and if you cool it further still, you should see SIS behavior. You can then compare these $I(V)$ curves to theoretical predictions from the integral expressions (7) and (8), and find values for the gaps Δ_1 and Δ_2 which yield the best fit of data to theory.

This is more or less how you will actually do the experiment, with one notable caveat: when dealing with small resistances such as those of your tunnel junctions, it turns out to be much easier to vary current and measure voltage than vice versa. An approximate schedule for the four weeks of this lab is presented below:

- Week 1: Familiarize yourself with the measurement apparatus; write and test your DAQ code in MATLAB.

- Week 2: On Monday, make tunnel junctions. Measure the $I(V)$ curve at room temperature. On Wednesday, either measure your junction $I(V)$ curves at cryogenic temperatures or make junctions again if something went wrong the first time.

- Week 3: Perform your cryogenic measurements if you didn't get to them last week. If you did, but your data doesn't look good, you can repeat the evaporation/measurement process. Otherwise, start your analysis.

- Week 4: Analysis.

In general, there's plenty of built-in flexibility, which you can use to retry measurements and/or explore the properties of your junctions in greater detail.

4 Measurement Chain and DAQ

(a) 4-Wire Measurement

I will first try to motivate the claim made in section 3 that to do a tunneling experiment in practice, we must control current and measure voltage, rather than the other way around. For simplicity, let's assume we want to measure the $I(V)$ curve of an ordinary resistor R_1. The simplest way to do this is to set a voltage and measure the resulting current, as shown in figure 6:
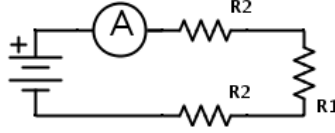

Figure 6: 2-wire resistance measurement.

This is called a **2-wire measurement**, because the voltage source and the ammeter share the same pair of wires to the sample. Note that I have included the (typically small) wire resistance R_2. For $R_2 \ll R_1$, there's nothing wrong with 2-wire measurement. But in the opposite limit, $R_1 \ll R_2$, the current that flows in response to the applied voltage is almost completely determined by the wire resistances: we're no longer measuring the $I(V)$ curve of the component we care about. The junctions we want to measure have more complicated $I(V)$ curves, but the same considerations apply since V/I is small almost everywhere.

*How is the above problem solved by **4-wire measurement**, with a current source and voltmeter in place of the voltage source and ammeter?* To answer this question, it might be useful to think about what resistance an ideal voltmeter should have, and consider which wires will have appreciable current flowing through them and which will not.
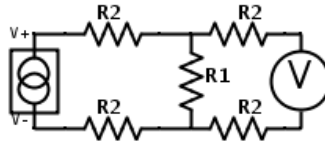

Figure 7: 4-wire resistance measurement.

The goal of the your DAQ system should thus be to produce a current passing through the junction, and measure the resulting voltage. The first question you need to ask to design such a system is "how big do I expect these quantities to be?" For your voltage measurement, this is not too hard to answer. The superconductors you will use have critical temperatures around 7 K and 1 K, so according to equations (1) and (2), the corresponding gap energies will be roughly 10^{-3} and 10^{-4} eV, respectively. Of course, we'd like to measure the $I(V)$ curve above and below the gap as well, and in particular we want to make sure our system can tell the difference between $|\Delta_1 - \Delta_2|/e$ and $(\Delta_1 + \Delta_2)/e$, which may be as small as $100\,\mu$V. To be conservative, we should make sure our system can measure voltages ranging from $\sim 10\,\mu$V to ~ 10 mV.

For the current measurement, it's somewhat less clear what the answer should be. The voltage across a tunnel junction fundamentally determines the current density; the total current also depends on the somewhat arbitrary geometry of the junction. As a crude conservative estimate, the top of the current range of interest might be anywhere between $5\,\mu$A and 5 mA, and you'd like to be able to control this current to a precision of 1% or less.

Lastly, note that controlling current and measuring voltage amounts to flipping the curves in figures 4 and 5 on their sides. This is not a problem for the monotonically increasing NIN and SIN curves, but because of the negative resistance region, the inverse of the SIS curve is not single-valued, as illustrated in figure 8.

As we increase the current from 0, the voltage will increase continuously until the point on the plot indicated by the solid red line; increasing the current beyond this point will cause the voltage to jump up discontinuously in order to remain on the curve, and then increase continuously from there. If we now decrease the current from some large value well above the negative-resistance region, the voltage decreases continuously until it must jump down to the curve at the dashed blue line.
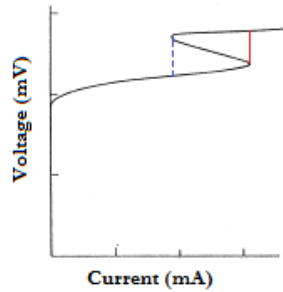
Figure 8: Hysteretic $V(I)$ curve of the SIS junction. Adapted from [1].

The SIS $V(I)$ curve thus exhibits **hysteresis**. Although you will ultimately invert your data and plot $I(V)$ when comparing it to the theory, the fact that you actually measured $V(I)$ implies that you should observe this hysteresis in your data. Note that the Josephson Effect described at the end of section 2 (e) results in a spike in current at 0 voltage, and thus will also manifest in hysteresis. *What kind of V(I) curve would you observe if you added to figure 8 a Josephson supercurrent whose amplitude is the distance to the first tick mark on the current axis?*

(b) Measurement Chain

Your goal in the first part of the lab is to write a MATLAB program to vary the junction current and measure the resulting voltage. This section will describe the hardware you will use to implement this DAQ scheme, shown in the diagram below:
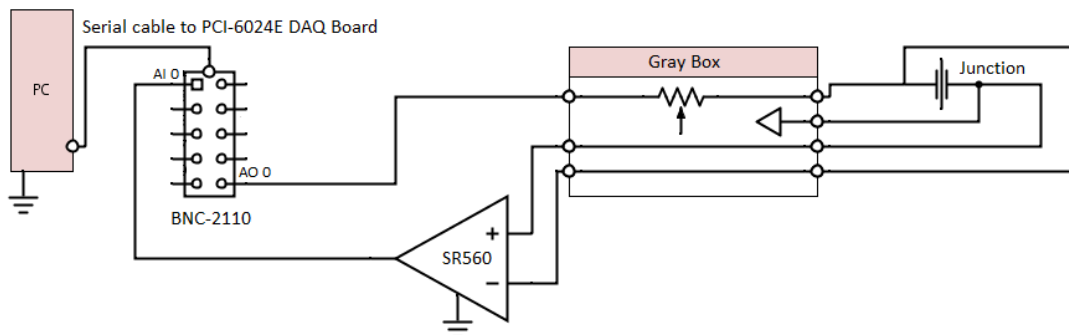

Figure 9: Schematic of measurement chain.

MATLAB interfaces directly with a DAQ board plugged into a PCI slot inside the lab computer. For our purposes, the most important components on this board are an ADC, which converts continuous analog input voltages into digital signals the computer can process, and a DAC, which performs the inverse operation, and thus enables the computer to produce analog output.

There are a few parameters you need to keep in mind to understand the performance of an ADC or DAC. One is the input or output range, which tells you the largest positive and negative voltages the component can faithfully represent. Just as important is the number of ADC or DAC bits, i.e., the size of the binary number that will represent each sample of an analog voltage bounded between $\pm V_{\text{range}}$; the number of bits thus sets the precision of the device. Lastly, it might be important to know the maximum rate at which your device can input or output data.

In general, you want to make sure your code makes efficient use of your device's **dynamic range**, defined as the ratio of maximum to minimum voltage. To take an extreme example, if your DAC has an output range of 1 V and a precision of 1 mV, you don't want to tell it to produce a 2 mV output: this limits the precision of your device to 50%. It's much better practice to produce say a 0.2 V output, and then use an analog circuit to divide this voltage by 100.

The DAQ board you will use for this lab is called the **NI PCI-6024E**, and you can find the values of the parameters defined above

in its datasheet online [3]. *How would you expect a ADC or DAC to represent a voltage that exceeds its range, or falls below the precision threshold determined by the number of bits?* You might run into one or both of these issues in the course of your data collection, so it's useful to develop an intuitive sense of how they would manifest in your data.

The PCI-6024E is connected via a serial cable to another National Instruments device called the **BNC-2110**, which simply routes analog input/output signals to the BNC connectors more commonly used in analog electronics. For this experiment, you will only need to use a single analog output (AO) and a single analog input (AI). Note that below each BNC connector is a switch that may be set to "GS" (grounded source) or "FS" (floating source). Make sure the ports you use are set to "FS."

The cable connected to port AO 0 in the above diagram will carry an analog *voltage* signal controlled by your MATLAB code. We would like to know the junction current as a function of this DAC output voltage, independent of the unknown resistance of the junction; that is, we need a voltage-controlled current source. A simple current source is illustrated in figure 10, with a known internal resistance R_1, and another resistor R_2 representing the junction. *What should the size of R_1 be relative to that of R_2 for this to be a good current source? What happens when this condition is violated?* Note also that the arrow below R_1 in the diagram indicates that this resistor is adjustable. By changing R_1, we can change the overall scale of the current without sacrificing DAC dynamic range.
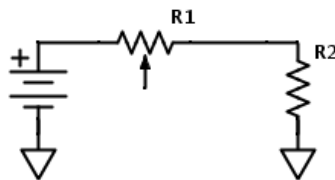

Figure 10: Current source with adjustable range.

The current source you will be using is the gray box with seven BNC ports on the table in JWG 608; the knob on top is used to select one of four resistance settings between 1 kΩ and 1 MΩ. The gray box also functions as a breakout box, enabling lines going from the DAQ board to the measurement cryostat to be connected in a more modular way. The cryostat circuitry will be discussed further in section 5 (e); for now, suffice it to say that the upper two ports on the right side of the gray box connect to the junction current leads, and the lower two are for measuring the voltage across the junction.

This voltage signal is fed to a **SR560 amplifier**, which multiplies the signal by the gain G and passes it through a low-pass filter with time constant τ; both G and τ can be adjusted on the amplifier's front panel. The amplified and filtered signal is then routed to your ADC through the BNC-2110. *Why is this amplifier necessary? What considerations constrain your choice of gain and time constant?* Make sure the SR560 is set to A−B mode and DC Coupling; you can use either output.

Note that the use of two different symbols for ground in figure 9 is intentional. The "triangle" symbol inside the gray box indicates the "circuit ground," consisting of the mutually connected outer conductors of the BNCs used in this experiment, which constitute a ground in the sense that they provide a return path for the junction current and a common reference for voltage measurements. When the BNC-2110 ports are set to "FS," this circuit ground is electrically isolated from the "earth ground" of the wall outlet: the only other component in your measurement chain which receives power from the wall is the SR560, and it has built-in isolation between its signal and power supply circuits.

Isolating the circuit ground is important because the earth ground may be very noisy, and introducing these fluctuating voltages and currents into the circuit ground could swamp the small signals we're looking for. But this kind of isolation only works for noise at very low frequencies: higher-frequency noise can pass through capacitive couplings between connectors that are not physically touching. High-frequency noise is somewhat mitigated by low-pass filters and ferrite beads at several points in the measurement chain.

AC noise can also be reduced by simply moving potential sources of noise further away from your signal lines to reduce the stray capacitance between them. Both the power supply circuit of the SR560 and the thermometry circuit described below are likely sources of capacitively coupled noise, which you can mitigate by unplugging the SR560 from the wall (it can operate on battery power for several hours at a time) and disconnecting your thermometry circuit while your DAQ code is running. The noise-reduction steps described here are probably not necessary for the SIN case, but may be essential to avoid smearing out the fine structure of the SIS $I(V)$ curve.

In addition to the equipment used for DAQ described above, the JWG 608 lab contains a **SR510 lock-in amplifier**, which you can use with a **resistance thermometer** to measure the temperature of your junction. A resistance thermometer uses a four-wire circuit very similar to the one described above to measure the (temperature-dependent) resistance of a Ruthenium Oxide resistor [4]; because our knowledge of the resistor's $R(T)$ curve is only approximate, temperature measurements performed this way are precise to only ~ 0.2 K.

Lastly, on the table in JWG 608 there should also be a Tektronix 2213 oscilloscope, a few Wavetek model 20 function generators, and a few Fluke 8010A multimeters, along with a number of discrete resistors and BNC/banana connectors. *How might you use some of these components to conduct quick tests of your junctions without relying on your much slower DAQ code? How might you build a setup to test your DAQ code without a real junction?*

(c)   Writing Your DAQ Code

Your DAQ code should:

- Initialize the ADC and DAC channels you will use on the DAQ board.

- Loop over a fixed number of measurement steps. At each step:
  - Increment the DAC output voltage by a small fixed amount, thus increasing the current through the junction. The total range that you step over should be symmetric around 0 V, and should make good use of the DAC dynamic range.
  - Make an ADC measurement of the voltage across the junction. Again, be sure to efficiently use the dynamic range.
  - Save your data to an array in Matlab.

- Repeat the loop, this time stepping down from your maximum output voltage rather than up from your minimum. If there is hysteresis, as we expect in the SIS case, your up data and down data will differ.

- Plot your data as $I(V)$.

- Write your data to a file. Make sure you also record information that's critical to the proper interpretation of your data, but which MATLAB has no access to, such as the resistance setting on the current source, the gain on the SR560, and the regime (NIN, SIN, or SIS) in which you are measuring your junction.

- Quantify the error in the measured junction voltage. There are ways to do this within the loop and ways to do it outside the loop; you can do either.

When designing your code, keep in mind the total amount of data you want to collect and the total run time of your code (longer than 5 minutes is excessive). These considerations should enable you to settle on an optimal output step size, and will also impact how you make your ADC measurement.

MATLAB is a very well-documented language, and the extensive help files should be able to answer most questions you have about syntax, even if you haven't programmed in MATLAB before. Typing "help daq" at the prompt will take you to a table of contents for DAQ-related functions. There's also a document in the "data_acquisition" folder on classes*v2 called "daq.txt," which contains a few examples of how to talk to DAQ devices in MATLAB.

5   Cryogenics and Thin Film Deposition

This section is an introduction to two incredibly powerful techniques in experimental physics which you will employ to fabricate tunnel junctions and cool them below their transition temperatures. Both involve complex processes that are best learned through practice; the aim of this document is simply to present enough of the big picture that you understand how each process contributes to your ultimate goal. I recommend you read them in advance of your second week of lab, but don't get too hung up on details in parts (b) and (d); they will probably be clearer on a second pass after you've worked with the apparatus itself.

(a)   Introduction to Cryogenic Liquids

The experimental study of superconductivity has always required cryogenic technology. For most of the history of the field, this invariably meant liquid helium, whose atmospheric boiling point, at 4.2 K, is the lowest of any element. More recently, research

has focused on the properties of high-temperature superconductors with transition temperatures above the boiling point of liquid nitrogen (77 K). Liquid nitrogen is much easier to work with than liquid helium, but high-temperature superconductors have very complicated electronic structure not described by BCS theory: to get a simple quantitative prediction for the tunneling current, we must use lower-T_c superconductors, and thus liquid helium.

Simply submerging a sample in liquid helium at atmospheric pressure is a very reliable way to maintain its temperature at 4.2 K. This is because any boiling liquid stays at constant temperature: all the incoming heat is absorbed in the process of breaking the bonds that allow molecules to escape into the gas phase. For cryogenic liquids whose boiling points are below room temperature, we don't intentionally supply heat, but there is always a residual heat leak from the much warmer environment. The guiding principle of cryogenic experimental design is to minimize this heat leak.

The key to obtaining temperatures other than 4.2 K is that a liquid's boiling point actually depends on the pressure of the surrounding gas. The easiest way to understand this conceptually is to think of boiling as a result of pockets of gas forming in the interior of the liquid; this is only possible if the ambient pressure is less than the pressure inside these pockets. We thus find that boiling requires less energy at lower ambient pressure; equivalently, the boiling point is reduced. In practice, by pumping the gas above a liquid helium cryostat down to a rough vacuum, it's possible to reduce the boiling point to a lower limit of about 1 K.

(b) Helium Cryostat Design

It takes a very small amount of energy to evaporate liquid helium – this fact is intimately related to its low boiling point. Great care must thus be taken to shield the helium from its environment. All cryostat design begins with vacuum-insulated glass or stainless steel walls surrounding the helium to reduce conducted heat, the same principle used in a thermos to keep hot drinks hot. The other major heat load is infrared thermal radiation from room-temperature surfaces. To reduce radiated heat, the helium vessel may be placed inside a second vacuum-insulated vessel containing a liquid nitrogen "radiation shield."

The helium cannot be surrounded by nitrogen on all four sides; the cryostat in JWG 608 is long and skinny to reduce the area of the upper surface, where the heat load is largest. The sample we wish to cool will invariably have physical connections to room temperature, which are themselves unacceptable heat leaks unless they are made out of materials with low thermal conductivity. The sample is typically supported by a thin-walled stainless steel tube, and wiring, which invariably requires a compromise between good electrical conductivity and poor thermal conductivity, is made as thin and delicate as possible.

Helium must be transfered into a cryostat using a special vacuum-jacketed transfer tube – otherwise, most of the the helium would evaporate before even getting to your experiment. By contrast, liquid nitrogen can basically be poured from its dewar into an open flask, and then poured into the outer vessel.

Finally, there must be a vent port on the cryostat through which helium gas can escape, or else the pressure will build up and the cryostat can explode catastrophically – all cryostats have some kind of relief valve to prevent this from happening. The vent must be closed in order to pump the helium gas space to below atmospheric pressure, or your pump will just end up sucking air into the cryostat. It's very important to avoid getting air in the cryostat in general: after you've finished performing measurements at 1 K, fill the cryostat slowly with helium gas to bring it up to atmospheric pressure instead of simply opening the vent port. Water vapor in the air is the main danger here: it freezes very easily on contact with the cold cryostat surfaces, and can form "ice plugs" which block the vent and safety valves, leading to overpressure and explosion.

Cryogenic design is a vast subject and this section has only touched on a few of the more fundamental techniques, but the material presented here is sufficient to maintain a sample at 4.2 K for a few hours with ~10s of liters of liquid helium.

(c) Thin Film Junctions

In practice, superconducting tunnel junctions are made of thin-film sandwiches: ~100 nm films of two different metal electrodes, separated by an insulating oxide layer just a few nm thick. Your junctions in particular will consist of 100 nm of aluminum, 2-3 nm of aluminum oxide, and 200 nm of a lead alloy with a small ($\sim 10\%$) concentration of indium, deposited in that order on a clean glass microscope slide. While it is obvious that the insulating layer must be sufficiently thin for quantum mechanical tunneling to occur at an observable level, it's not intuitively obvious that the electrodes need to be thin as well. The reason is simply that thin film deposition grants the experimenter much better control over junction properties than the use of bulk materials.

There are several reasons to use Al and Pb-In as the junction electrodes. First, their transition temperatures are on oppo-

site sides of the boiling point of liquid helium: aluminum superconducts at 1.2 K and lead at 7.2 K; this makes it possible to test the junctions in both the SIN and SIS regimes with a pumped helium cryostat. Second, pure aluminum exposed to air automatically forms a uniform surface oxide layer at a known rate, and this oxide may be used as the junction insulating barrier. This is considerably simpler and more robust than depositing a separate insulating layer; most importantly, defects in the oxide layer are self-healing, which prevents electrical shorting of the two electrodes. Note that because of the small concentration of indium, we will consistently approximate the properties of the Pb-In alloy by those of lead. The function of the small concentration of indium is simply to improve the chemical stability of the film, which otherwise degrades rapidly upon exposure to air.

To deposit films you will use a technique called thermal evaporation, in which the material to be deposited is heated to a very high temperature until it melts and then evaporates; the resulting metal vapor streams out in all directions and adsorbs on any surface in its path. Evaporation must be conducted in a vacuum chamber for two reasons. First, low pressure ensures that the metal vapor streams freely out from the sample upon evaporation, rather than diffusing slowly through the surrounding gas as it would at atmospheric pressure. Second, and more importantly, the ratio of the residual gas pressure in the chamber to the metal vapor pressure determines the proportion of impurities in the resulting film. In practice this last condition translates to the requirement that the chamber be evacuated to 10^{-7} or 10^{-8} Torr, a factor of 10^{11} or 10^{12} below atmospheric pressure.

(d) Evaporation in Practice

You will conduct the evaporation portion of this lab down the hall in JWG 602. The evaporator is set up to deposit three junctions each on five glass substrates simultaneously. It's important to clean your substrates very thoroughly with the acetone and IPA provided in the lab: your films will be thin enough that any visible dirt or smudging can compromise good electrical contact.

We can restrict deposition to a specific region on the substrate using "masks" which are basically stencils. While evaporating aluminum, you will place your substrates on a set of masks that expose three horizontal strips on each substrate. Then you will let the vacuum chamber up to atmospheric pressure for four minutes, creating a thin oxide layer on each strip of aluminum. During this time, you will move the substrates to another set of masks, and, when the five-minute timer is up, deposit a cross-strip of lead on each substrate.

To heat the source during each round of evaporation, we run a very high current through a tungsten "boat" containing 0.3-0.5 g of either solid aluminum or lead and indium pellets. Tungsten is used for its extremely high melting and boiling points, which guarantee that it doesn't itself evaporate, and indium has a substantially higher boiling point than lead; thus equal weight in solid lead and indium actually yields a vapor consisting mostly of lead. A mechanical shutter may be used to block the path from source to substrate, and should be kept closed until the metals to be evaporated begin to melt: this ensures that impurities on your sources which evaporate at lower temperatures don't get incorporated into your film.

Film thickness may be measured in real time during evaporation using a quartz crystal monitor such as the Maxtek TM-100. This device continuosuly monitors the ~ 5 MHz resonance frequency of a quartz crystal oscillator, which changes slightly as the crystal is coated. Since the internal calibration of the TM-100 has become unreliable with age, we just use it to drive the oscillator, and read out the resonant frequency on a Mastech MS6100 multi-function counter. The resonant frequency f as a function of film thickness d is approximately given by

$$f(d) \approx f_0 \left(1 - \frac{\rho_f f_0 \eta d}{v \rho_0}\right) \tag{9}$$

where f_0 is the resonant frequency of the uncoated crystal, ρ_0 is the density of quartz, ρ_f is the density of the film being deposited, $v = 1.668 \times 10^3$ m/s is a constant with units of velocity on the order of the sound speed of quartz, and η is an emprical correction factor to account for the fact that the deposition rate is higher at the substrate, which is closer to the source than the crystal, by the squared ratio of distances. This first-order approximation is accurate enough for the small film thicknesses we will be depositing.

Finally, I will briefly discuss the vacuum requirements for evaporation. There is no single vacuum pump capable of evacuating a chamber from atmospheric pressure to 10^{-7} Torr. Rather, we will use a **diffusion pump**, which operates between base pressure and 10^{-2} Torr, in series with a purely mechanical **roughing pump**, which has its low-pressure end at 10^{-2} Torr and exhausts to atmosphere; in this configuration, the roughing pump is said to be "backing" the diffusion pump. Crudely speaking, the roughing pump simply takes a certain volume of gas in at one end and expels it at the other; this process is simple and robust but becomes very inefficient at low pressures.

The diffusion pump is much less intuitive, and much more complicated. It uses concentrated jets of oil vapor to preferentially kick gas molecules entering the pump towards the high-pressure end. Cooling water and a small volume of liquid nitrogen

are used to ensure that the oil vapor continues to work as a "pump," rather than drift into the chamber being evaporated. If this all sounds very outlandish, it's because the whole process is contingent on the fact that gas flow is ballistic at low pressures, rather than viscous as at atmospheric pressure. In fact, exposing either end of a diffusion pump to atmospheric pressure can seriously damage it; much of the complexity of the evaporation setup in JWG 602 lies in repeatedly pumping out and letting up the sample chamber while avoiding damage to the diffusion pump.

(e)   Sample Preparation

After deposition you should have five glass slides, each with three horizontal aluminum oxide strips crossed by a vertical strip of lead-indium alloy. Choose the slide that looks best, apply silver paint from a pen on both ends of each strip, as indicated on the right side of figure 11, and let it dry under a heat lamp. Appropriately scaled printouts of this diagram are also available in JWG 608, so you can simply lay your slide on top of the diagram to ensure you apply sislver paint in the right places.

The function of the silver paint is to break down the oxide layers on the ends of the aluminum and lead strips, and provide low-resistance connections to your 4-wire measurement circuit. The junction itself is the small square where the two strips overlap; we can force current to flow through the junction or measure voltage across the junction by connecting the relevant leads at right angles. For example, one possible configuration has the current leads connected to 1A and 7B and the voltage leads to 6A and 8B.
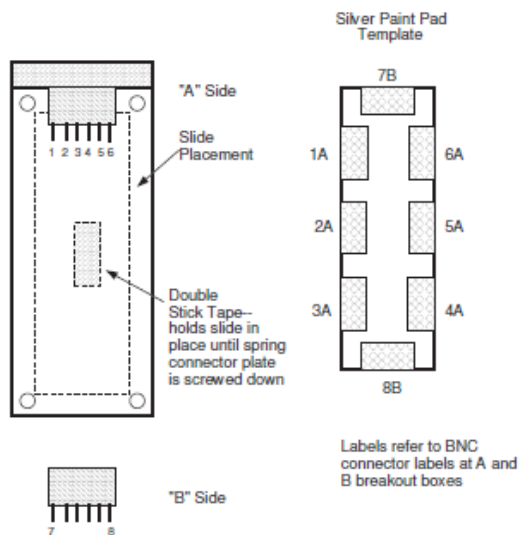

Figure 11: Superconducting tunnel junction sample preparation and connections.

The left side of figure 11 indicates how your chosen slide interfaces electrically and mechanically with the cryostat sample holder. The slide is clamped between two plates held together by screws. As the plates are clamped together, brass spring connectors on the underside of the upper plate press down onto the silver paint pads, providing good electrical contact. Each spring connector is soldered to a wire which runs up the length of the sample holder to room temperature. The bundle of wires culminates in two 10-pin circular connectors on top of the cryostat labeled A and B; attached to each of these is a bundle of 10 BNCs to which you will actually connect your room-temperature measurement equipment.

There are thus a total of 20 signal wires running down into the cryostat, only 12 of which are connected at the low-temperature end, and only eight of which you can use at any given time: four for the junction measurement, and four for the resistance thermometer. The BNCs which are actually connected to your sample are labeled on green tape according to the numbering scheme in figure 11. BNCs with white tape labels are for thermometry, and thermometry leads with the same number are electrically connected; e.g., one of the cables marked "1" should be used for current, and the other for voltage, and likewise with the cables marked "2."

You should make these connections and measure the NIN resistance R_0 of each of your three junctions on the same day that you evaporate your junctions. R_0 will differ from junction to junction based on geometry and purity, and will also increase over time as the films degrade. Junctions are worth testing at cryogenic temperatures if they exhibit $R_0 < 100\ \Omega$ on the day that you actually plan to make your cryogenic measurements.

6 Analysis

Your DAQ code should produce three columns of digital data for each measurement, representing I, V, and the error δV in your voltage measurement. Although you have measured $V(I)$, you want to compare your data to a functional form for $I(V)$, and the χ^2 **analysis** you will use to make this comparison assumes error in the dependent variable only. Thus, you will need to find a way to convert your measured δV into an equivalent δI. *How can you do this using the error propagation you learned in previous lab courses?* Hint: to motivate the form of the correct formula, try sketching an arbitrary function and drawing horizontal error bars, then think about how the same uncertainty could be represented with vertical error bars.

In addition to statistical errors at each point, there are also a few possible systematic errors to be aware of. First, your circuits may contain contact potentials, small DC voltage differences between dissimilar metals in electrical contact. Contact potentials in your voltage (current) circuit will result in small horizontal (vertical) offsets in your $I(V)$ data. To compensate for contact potentials, you should determine the point about which your data is symmetric in both I and V, and subtract whatever offset is necessary to put this at $(0, 0)$.

More subtle effects arise from a large but finite resistance in parallel with your junction, or a small but nonzero resistance in series. Crudely speaking, both will "smear out" your $I(V)$ curve, but in slightly different ways. We will consider only parallel resistances, because their effects on $I(V)$ are much easier to model: just add a term of the form $V/R_\parallel$ to to equations (7) and (8). *Can you visualize how the $I(V)$ curve will change as $R_\parallel$ decreases? In particular, how does the behavior in the negative resistance region change?*

There are a pair of templates for MATLAB scripts on classes*v2 called "SINcurr.m" and "SIScurr.m." These scripts use trapezoidal approximations to the integral expressions (7) and (8) to compute theoretical $I(V)$ curves, given your measured voltage data and values for each of the parameters that enter into the integral. The first setp of your analysis is to come up with initial estimates for each of these parameters.

The parameters $E_{\max}$ and E_{step} determine the fidelity of your integral approximations, and some care must be taken to choose them appropriately: your approximations can depart quite dramatically from the true $I(V)$ curves if you truncate the integral below energies at which there is still appreciable appreciable tunneling, or if your energy resolution is not substantially finer than the smallest energy scale in your problem. Small variations about appropriately chosen values should not systematically alter the shape of your curve, but if $E_{\max}$ is too large or E_{step} is too small, your code may take a very long time to run.

You should already have a measurement of R_0 from your NIN data. For your SIN data, T is fixed at 4.2 K, and for your SIS data, you can obtain an initial estimate of T from the resistance thermometer. This leaves Δ_{Pb} (in the SIN case; Δ_{Pb} and Δ_{Al} in the SIS case) and $R_\parallel$ to be estimated. It's up to you to determine how to make these initial estimates– Giaever's papers suggest one method for getting a good initial estimate of Δ in the SIN case. For each of your data sets, you should quantify the uncertainty in parameters R_0, T, Δ_{Pb}, Δ_{Al}, and $R_\parallel$: for the estimated parameters, this uncertainty will itself be an estimate. Note that Δ_{Pb} and $R_\parallel$ may differ between your SIN and SIS data sets.

Finally, you will use χ^2 minimization to determine the parameter values that yield the best fit of your theoretical $I(V)$ curve to your measured $I(V)$ data. The χ^2 statistic is a normalized measure of the mean squared deviation between two data sets. There are several related definitions; the one you should use is the "reduced χ^2"

$$\chi_\nu^2 = \frac{1}{\nu} \sum_{i=1}^{N} \frac{(y_i - f(x_i, \alpha))^2}{\sigma_i^2} \tag{10}$$

Here, (x_i, y_i) are a set of N measured data points, and σ_i is the error in y_i. f is a theoretical functional form which you expect your $y(x)$ data to follow, which also depends on a set of parameters α. $\nu = N - p$ is called the number of degrees of freedom, where p is the number of adjustable parameters collectively labeled α. The symbol χ^2 with no subscript is typically reserved for the quantity before it is divided by ν.

There is a brief introduction to χ^2 analysis in the "statistical analysis notes" folder on classes*v2, and I very much recommend a set of slides by astronomer Chris Blake available for free online [5]. Here I will just try to motivate an intuitive understanding of why χ_ν^2 is useful. If the function $f(x)$ accurately describes your $y(x)$ data, then the typical size of the difference between $f(x_i)$ and y_i will be your measurement error σ_i. Thus, each of the N terms in the sum will be ~ 1. Typically N will be much larger than p, so $\nu \approx N$, and $\chi_\nu^2 \approx 1$ for a good fit. A worse fit, with larger discrepancies between experiment and theory, will

result in a larger χ_ν^2, and $\chi_\nu^2 < 1$ indicates that you're overfitting or overestimating your errors.

Assuming your data is accurately described by the model, with Gaussian-distributed fluctuations σ_i superimposed, it is straightforward to calculate the resulting statistical distribution of χ_ν^2 about 1. In χ^2 analysis we have the inverse problem: having obtained a given value of χ_ν^2, we want to ask how likely it is that we would obtain this result by chance if the model is accurate; this information is contained in standard "χ^2 tables." In general, the χ_ν^2 distribution is more sharply peaked for larger ν.

You will need to write a script to perform χ^2 minimization. If you do this in MATLAB you can incorporate the templates SINcurr.m and SIScurr.m, but you're welcome to work in another programming language if you prefer. Your script should

- Initialize each of the parameters you want to optimize well below your best guess (that is, further than your estimated uncertainty).

- Compute $I(V)$ using SINcurr.m or SIScurr.m for that set of parameters.

- Compute and save χ_ν^2 for your theoretical curve and your real $I(V)$ data.

- Increment one parameter by an amount substantially smaller than your estimated uncertainty, and repeat the last two steps.

- Repeat this process until the parameter is sufficiently far above your best guess.

The smallest of your χ_ν^2 values corresponds to the optimal choice of the parameter you have been varying, with all other parameters held constant; it may not be the optimal choice for different values of the other parameters. The most thorough way to optimize all n parameters in your model is to compute χ^2 at every point on an n-dimensional grid of parameter values, whose size depends both on how wide a swath of parameter space you want to cover and how finely you step through it. You can of course also perform χ^2 minimization on a finer grid "zoomed in" on your original minimum to optimize your parameters further.

The brute force "grid minimization" described in the previous paragraph can quickly grow to be very computationally intensive, so it is often helpful to reduce the dimensionality of your parameter space by fixing the values of some parameters (provided you can justify your decision to do so!). The only parameter you must obtain via χ^2 minimzation is Δ_{Pb}.

It may have already occurred to you that you will run into a problem when attempting χ^2 analysis on your SIS junction data. Specifically, the theoretical $I(V)$ curve given in equation (8) has a region of negative differential resistance, but your data will exhibit hysteresis rather than negative resistance in this region. In this voltage range, the two curves will simply not overlap very well for any choice of parameters, and the contribution to χ_ν^2 will be very large. Likewise, equation (8) does not describe the Josephson Effect, leading to another large contribution to χ_ν^2 which has nothing to do with how well you're fitting the data.

When you know that the difference between theory and data in a particular region is due to an effect not described by your model rather than to noise or to an improper choice of parameters, the best course of action is to simply exclude the data in this region from the sum in equation (10). You can get a better sense of what points to exclude by plotting the **residuals**

$$\epsilon_i(x_i) = \frac{y_i - f(x_i, \alpha^*)}{\sigma_i} \tag{11}$$

where α^* are the best-fit parameters you've settled on after your χ^2 minimization procedure; the residuals are simply the individual discrepancies that add in quadrature to give you χ^2. If your χ^2 analysis was sound, a plot of the residuals will just look like random noise with most points within 1 or 2 on either side of 0. Any non-random structure in the residuals indicates that your fit does not accurately describing the data in that region.

When presenting your results, you should report the best-fit χ_ν^2, the number of degrees of freedom ν, the best-fit parameter values, and an error for each parameter, defined as the distance you have to move from the best-fit value along the relevant axis in parameter space in order for χ^2 to double. It is also important to include a plot of the residuals for your final fit parameters: this is the clearest way to demonstrate that a good χ^2 is not the result of fortuitous cancellation between overestimated errors in one region and systematic deviations in another.

7 Writeup

For an **informal writeup**, you should include a brief (1-2 paragraph) description of the goals of the experiment. The only equations you need to include are the those you will compare directly to your data (that is, I(V) curves for the NIN, NIS, and SIS cases); no

derivations are required. You should include plots of the theoretical $I(V)$ curves and explain briefly what the different features in the curve correspond to. No background about tunneling or superconductivity is necessary.

This should be followed by a brief (1-2 paragraph) overview of the experimental procedure. The descriptions of the deposition and cooling can be very brief: e.g., indicate the thickness of your evaporated layers and the oxidation time; no need to describe the order in which the chambers were pumped out, the current setting on the evaporator, etc. When describing the measurement itself, you should indicate what was being controlled and what was being measured, and the increments in which you stepped the independent variable. You should then plot your data and explain what it is you're plotting.

The bulk of the effort you put into the writeup should be devoted to analysis. You should qualitatively discuss how errors might arise in your measurement, and explain the steps used to quantify error in your dependent variable. You should also qualitatively discuss how you estimate your initial values for fit parameters from the data, and explain your χ^2 minimization scheme. In a brief (1 paragraph) conclusion you should discuss qualitatively how well your data matches the theory, and possible reasons for any discrepancy.

If this is going to be your **formal writeup**, you should do the above, but expand the theory and experimental procedure sections. You goal is to make this paper legible to somebody with approximately your level of physics education, who is not acquainted with the lab, but willing to do background reading. Approximately six pages is a good target length; this document is not a good model for your formal writeup!

Your hypothetical reader should come away with a sense of what motivated the experiment, and how he or she might go about setting up an analogous measurement. Your introduction should explain what the superconducting energy gap is and why it's interesting, and motivate the models you use for $I(V)$ in different regimes. In the experimental procedure section, diagrams are more useful than big blocks of text, but you should succinctly identify and explain the purpose of each element of your measurement apparatus, and describe your evaporation procedure. You should cite all relevant sources, and you should have an abstract. The analysis should be at the same level for both types of writeup.

References

[1] "Discovery of Electron Tunneling into Superconductors," in *Adventures in Experimental Physics*, edited by B. Maglich (World Science Communications, 1974), Vol. 4.

[2] M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (Dover, Mineola, N.Y., 2004).

[3] National Instruments, PCI-6024E Datasheet, `http://www.ni.com/pdf/products/us/4daqsc202-204_ETC_212-213.pdf`

[4] Lake Shore, RX-102B-CB Datasheet, `http://www.lakeshore.com/products/cryogenic-temperature-sensors/ruthenium-oxide-rox-rtds/models/pages/overview.aspx`

[5] C. Blake, "Hypothesis testing and model-fitting," 2012. Online at `http://astronomy.swin.edu.au/~cblake/StatsLecture3.pdf`