

MATHEMATICS

1. MATHEMATICS

Toss Up: Multiple Choice

If the test scores of Mr. Coco's Calculus test are assumed to have a normal distribution, and if the mean test score was 91, what can be said about the median test score of that test?

- W) The median was lower than 91
- X) The median was higher than 91
- Y) The median was 91
- Z) The median cannot be determined

Toss Up Answer: Y

Bonus: Multiple Choice

The z-score for Shantanu's physics test grade was found to be 0.375. If he scored a 94 on the test, and the standard deviation was 8, what was the mean test score on the physics test?

- W) 91
- X) 95
- Y) 92
- Z) 98

Bonus Answer: W

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2. MATHEMATICS

Toss Up: Short Answer

In simplest terms, find the value of the limit as x approaches 0 of $(4^x - 2^x) / x$

Bonus Answer: $\ln(2)$ (do not accept $\ln(4) - \ln(2)$)

Bonus: Short Answer

How many real solutions for x does the equation $x = \ln(x) + 2$ have?

Bonus Answer: 2

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3. MATHEMATICS

Toss Up: Multiple Choice

A right triangle has a hypotenuse of length 12. The altitude to the hypotenuse has a length of 8. What is the area of this triangle?

- W) 36
- X) 48
- Y) 60
- Z) The triangle is impossible to construct

Toss Up Answer: Z

Bonus: Short Answer

Two REAL numbers x and y satisfy the system of equations: $x + y = 12$; $x^2 + y^2 = 64$. Find the value of the product xy .

Bonus Answer: The system has no solutions.

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4. MATHEMATICS

Toss Up: Short Answer

If n is a positive integer, what is the smallest value of n such that $n! + 1$ is a perfect square?

Bonus Answer: 4

Bonus: Short Answer

If $z_1 = 3 - 4i$ and $z_2 = 7 + i$, find the absolute value of $z_1 z_2$ in simplest terms.

Bonus Answer: $25\sqrt{2}$

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5. MATHEMATICS

Toss Up: Short Answer

What is the inverse of the 2×2 matrix (row 1: 6 10), (row 2: 3 5)?

Bonus Answer: The matrix has no inverse (b.c. determinant = 0).

Bonus: Short Answer

The legs of an isosceles triangle have a length of 10, and the altitudes to the legs have a length of 6. In simplified radical form, what is the length of the altitude to the base of the triangle?

Bonus Answer: $3 * \sqrt{10}$ (Do not accept $\sqrt{90}$)

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6. MATHEMATICS

Toss Up: Short Answer

What is the 5th non-triangular number?

Bonus Answer: 8

Bonus: Short Answer

Name all the following that are true:

1. An icosahedron has 18 faces.
2. A regular hexahedron has 16 edges.
3. There are only nine regular polyhedra.
4. A regular octahedron has 4 times the volume of a regular tetrahedron with the same side length.

Bonus Answer: 2,4 (an icosahedron has 20 faces; a cube has 12 edges)

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7. MATHEMATICS

Toss Up: Multiple Choice

If θ is an angle such that $\sin(\theta) < 0$ and $\cos(\theta) = 0$, where in the coordinate plane is it located?

- W) Between the 2nd and 3rd quadrants
X) Between the 3rd and 4th quadrants
Y) Between the 1st and 4th quadrants
Z) Between the 1st and 2nd quadrants

Toss Up Answer: Y

Bonus: Short Answer

What is the remainder of $x^{10} + x + 1$ divided by $(x-1)^2$?

Bonus Answer: $11x - 8$

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8. MATHEMATICS

Toss Up: Multiple Choice

Given the equation of a conic section, $y^2/16 - x^2/25 = 1$, what is the length of the

conjugate axis?

- W) 16
- X) 8
- Y) 10
- Z) 25

Toss Up Answer: Y

Bonus: Multiple Choice

Compute $\log(\text{base } 2)$ of 2048^2

- W) 20
- X) 22
- Y) 24
- Z) 26

Bonus Answer: X

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9. MATHEMATICS

Toss Up: Multiple Choice

A point with coordinates (3,5) is rotated 90 degrees clockwise about the point (2,1) and is then reflected across the y-axis. What are the coordinates of the resulting image?

- W) (-5,-3)
- X) (-4,0)
- Y) (-6,0)
- Z) (-5,-1)

Toss Up Answer: Y

Bonus: Multiple Choice

A quadrilateral ABCD has two congruent opposite side segments, AB and CD. Which of the following would have to be true in order for the quadrilateral to be a square?

- W) The circumcircle of ABC is centered at the midpoint of AC
- X) The medians of ABCD are perpendicular
- Y) The perpendicular bisectors of AB and CD are the same line
- Z) None of the above.

Bonus Answer: Z

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10. MATHEMATICS

Toss Up: Multiple Choice

Which of the following cannot be the root of a polynomial with rational coefficients?

- W) $5i + 6$
- X) $\phi - 1/2$, where ϕ is the golden ratio
- Y) $\sqrt{3+i}$
- Z) $6 - \pi i$

Toss Up Answer: Z

Bonus: Short Answer

What is the name given to a number that is not a root of any non-zero polynomial

equation with rational coefficients?

Bonus Answer: Transcendental number (Accept transcendental)

11. MATHEMATICS

Toss Up: Multiple Choice

Fermat's Last Theorem conjectures that no three positive integers a, b , and c can satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2. It went unproved for more than 300 years until it was finally proved in 1994 by a mathematician who received a substantial monetary prize only this year. What is the name of this mathematician?

W) Grigori Perelman

X) John Nash

Y) Andrew Wiles

Z) Terence Tao

Toss Up Answer: Y

Bonus: Short Answer

Prime numbers of the form $2^{2^n} + 1$, where n is a non-negative integer, are known as this.

Bonus Answer: Fermat Primes

12. MATHEMATICS

Toss Up: Short Answer

If Bruce can do a job in 3 hours, and Clark can do the same job in 4 hours, how long, in hours, will it take them to do the job if they work together? You may leave your answer as a fraction.

Bonus Answer: 12/7

Bonus: Short Answer

What is the eccentricity of an ellipse with $a = 5$ and $b = 4$?

Bonus Answer: 3/5

13. MATHEMATICS

Toss Up: Short Answer

Find the perimeter of a right triangle with legs 11 and 60.

Bonus Answer: 132

Bonus: Short Answer

In a 15-75-90 degrees right triangle with hypotenuse 4, what are the lengths of the shorter and longer leg, respectively? Exact answers please.

Bonus Answer: radical 6 - radical 2, radical 6 + radical 2

14. MATHEMATICS

Toss Up: Short Answer

If the first term in an arithmetic sequence is 2 and the third term is 6, find the 10th term.

Bonus Answer: 20

Bonus: Short Answer

Compute the sum of the first 100 positive integers.

Bonus Answer: 5050

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15. MATHEMATICS

Toss Up: Short Answer

If the log of n is 10, what is the log of 100n?

Bonus Answer: 12

Bonus: Short Answer

How many ways are there to seat 5 people around a circular table, if rotations are ignored?

Bonus Answer: 24

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16. MATHEMATICS

Toss Up: Multiple Choice

What is the geometric mean of the roots of the polynomial $2x^3 - 2x^2 - 228x - 432$

W) 4

X) 6

Y) 8

Z) 10

Toss Up Answer: X

Bonus: Short Answer

The first term of an arithmetic sequence is x, and the xth term is x^2 . If the common difference is equal to 7, and none of the terms are equal, what is the 12th term of the sequence?

Bonus Answer: 84

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17. MATHEMATICS

Toss Up: Short Answer

Completely expand $(x-2y)^3$

Bonus Answer: $x^3 - 6x^2y + 12xy^2 - 8y^3$

Bonus: Short Answer

What is the sums of all the terms in the seventh row of Pascal's triangle?

Bonus Answer: 128

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18. MATHEMATICS

Toss Up: Multiple Choice

Which of the following distances is longest?

W) Half a kilometer

X) Half a decimeter

Y) 600 meters

Z) 40 decameters

Toss Up Answer: Y

Bonus: Short Answer

What is the name of a line that intersects two or more coplanar lines in different points?

Bonus Answer: Transversal

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19. MATHEMATICS

Toss Up: Multiple Choice

The number $(\sqrt{2})^{(\sqrt{2})}$ is:

- W) Rational
- X) Algebraic irrational
- Y) Transcendental
- Z) Impossible to tell

Toss Up Answer: Y

Bonus: Short Answer

Let $f(x) = ax^7 + bx^3 + cx - 5$, where a, b , and c are real numbers. What is the value of $f(7)$ if $f(-7) = 8$?

Bonus Answer: -18

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20. MATHEMATICS

Toss Up: Short Answer

What is the limit as x approaches 0 of x^x ?

Bonus Answer: Does not exist. (NOT 1)

Bonus: Short Answer

Let $f(x)$ be a function with no real roots. If $f(3) = 4$, then in what quadrants can $f(x)$ be in?

Bonus Answer: All four quadrants (b.c. there's no specification f has to be continuous)

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21. MATHEMATICS

Toss Up: Multiple Choice

1. What statistical average is most appropriate to use when the quantities being averaged when one or more of the quantities are not necessarily bounded?

- W) Geometric mean
- X) median
- Y) Harmonic mean
- Z) Arithmetic mean

Toss Up Answer: Y

Bonus: Short Answer

You pick two cards at random without replacement from a standard, 52 card deck. Compute the probability

exactly one is a heart.

Bonus Answer: 13/34

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22. MATHEMATICS

Toss Up: Multiple Choice

Which of the following expressions in x grows fastest?

- W) 3^x [3 to the x]
- X) $10(3/2)^x$ [10 times three halves to the x]
- Y) x^2 [x squared]
- Z) $2^{(2x)}$ [2 to the $2x$]

Toss Up Answer: Z

Bonus: Short Answer

Given $504 = 2^3 \times 3^2 \times 7$ [504 equals 2 cubed times 3 squared times 7], compute the sum of the positive divisors of 504

Bonus Answer: 1560

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23. MATHEMATICS

Toss Up: Short Answer

Which of the following statements concerning prime numbers are true?

1. If a and n are relatively prime then there are infinitely many primes that leave a remainder of a when divided by n .
2. All integers less than p are quadratic residues modulo p
3. There are infinitely many prime numbers

Bonus Answer: 1 and 3

Bonus: Short Answer

If $a_1 + a_2 + a_3 + a_4 + a_5 = 17$ for positive integers a_1, a_2, a_3, a_4, a_5 , then maximize the product $a_1 a_2 a_3 a_4 a_5$?

Bonus Answer: 432

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24. MATHEMATICS

Toss Up: Multiple Choice

Simplify $(\sin 15^\circ \cos 75^\circ - \cos 15^\circ \sin 75^\circ) / (\cos 15^\circ \cos 75^\circ + \sin 15^\circ \sin 75^\circ)$

- W) negative square root of 3
- X) $1/2$
- Y) $-1/2$
- Z) square root 6 minus square root 2 all over 2

Toss Up Answer: W

Bonus: Short Answer

Compute $(2 + 2\sqrt{3}i)^6$. [2+2 times the square root of 3, i to the 6th power]

Bonus Answer: 4096

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25. MATHEMATICS

Toss Up: Multiple Choice

Compute $3C1 + 4C2 + 5C3 + 6C4 + 7C5$. [3 choose 1 + 4 choose 2 + 5 choose 3 + 6 choose 4 + 7 choose 5]

- W) 55
- X) 336
- Y) 240
- Z) 56

Toss Up Answer: W

Bonus: Short Answer

Compute the square root of 5476

Bonus Answer: 74

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26. MATHEMATICS

Toss Up: Short Answer

In Triangle ABC the angle bisector of A intersects BC at D. Given $AB=9$, $AC=21$, and $DB=15$, compute DC.

Bonus Answer: 35

Bonus: Short Answer

In triangle ABC, AB=14, AC=13, and BC=15. Let the incenter be I. Compute CI in simplest radical form.

Bonus Answer: square root of 65

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27. MATHEMATICS

Toss Up: Short Answer

which of the following are true about the number and nature of the roots of the polynomial

$$x^3 - 29x^2 + 229x$$

- I. 0 is a root
- II. There must be a real, positive root
- III. There must be three real roots

Bonus Answer: I

Bonus: Short Answer

Solve the following linear system of equations in 3 variables:

$$\begin{aligned} 2x + y + 4z &= 9 \\ -x + 2y + 5z &= -11 \\ 5x + 3y + 3z &= -4 \end{aligned}$$

Bonus Answer: x=4,y=-11,z=3

Also accept (4,-11,3)

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28. MATHEMATICS

Toss Up: Multiple Choice

MULTIPLE CHOICE: tossup

For an infinite sequence of numbers a_0, a_1, \dots , which of the following best describes the polynomial $P(x) = \sum_{i=0}^{\infty} a_i x^i$.

- W) Reimann-Zeta function
- X) Generating function
- Y) Newtonian series
- Z) Tchebychev polynomial

Toss Up Answer: X

Bonus: Short Answer

In the Pell equation $x^2 - 3y^2 = 1$ the smallest solution is $x=2$ and $y=1$. The next smallest solution is $x=7$ and $y=4$. Compute the third smallest solution.

Bonus Answer: x=26, y=15

Also accept (26,15)

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29. MATHEMATICS

Toss Up: Multiple Choice

Which of the following is not a group?

- W) the integers under multiplication
- X) the integers under addition
- Y) the symmetries of a regular n-gon
- Z) the permutations of integers between 1 and n

Toss Up Answer: W

Bonus: Short Answer

find the smallest positive solution to the following congruences.

x congruent to 3 mod 13

x congruent to 8 mod 11

Bonus Answer: x=107

Also accept: 107

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30. MATHEMATICS

Toss Up: Multiple Choice

What mathematician proved the existence of a straight edge and compass construction of a regular 17-gon?

- W) Euler
- X) Euclid
- Y) Gauss
- Z) Galois [Gal-wah]

Toss Up Answer: Y

Bonus: Short Answer

Given $\sin X = 1/3$ [sine of x equals one third], compute $\cos^2(3X)$ [co-sine squared of three x].

Bonus Answer: -23/27

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31. MATHEMATICS

Toss Up: Short Answer

Tim is 5 times older than his younger brother. In 3 years Tim will be 3 times older than his younger brother. Compute the difference in the ages of Tim and his younger brother.

Bonus Answer: 12

Bonus: Short Answer

How many positive integers n have no solutions to the following equation where a and b are nonnegative integers?

$$3a + 7b = n$$

Bonus Answer: 6

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32. MATHEMATICS

Toss Up: Multiple Choice

To test products, there is a test that identifies 95% of defective products as defective, but also labels 10% of all working products as defective. If 85% of the products made are not defective, what probability of products the test determines to be defective are actually defective?

- W) 39%
- X) 84%
- Y) 67%
- Z) 42%

Toss Up Answer: Y

Bonus: Short Answer

Compute the surface area of a cylinder with radius 8 and height 2

Bonus Answer: 160pi

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33. MATHEMATICS

Toss Up: Multiple Choice

If $x + (1/x) = 2$, find $x^{128} + (1/x)^{128}$

- W) 256
- X) 128
- Y) 64
- Z) 2

Toss Up Answer: Z

Bonus: Short Answer

Given the quadratic $x^2 - 20x + 9$ and its roots p and q , find $(1/p)^2 + (1/q)^2$

Bonus Answer: 382/81

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34. MATHEMATICS

Toss Up: Multiple Choice

Let f be an odd function over the real numbers. What is the value of f at 0?

- W) -1
- X) 0
- Y) 1
- Z) There is not enough information to determine an answer

Toss Up Answer: Z

Bonus: Multiple Choice

Which of the following is true?

- W) An irrational number raised to an irrational power must have an irrational value.
- X) For all positive number x and y , $x^{\ln(y)}$ is equal to $y^{\ln(x)}$
- Y) For all real numbers a, b, c, d with b and d being non-zero, if $a/b > c/d$, then $a*d$ must be greater than $b*c$
- Z) There are more complex numbers than there are real numbers.

Bonus Answer: X

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35. MATHEMATICS

Toss Up: Short Answer

How many vertices does a dodecahedron have?

Bonus Answer: 20

Bonus: Short Answer

In $a+bi$ form, what is the square root of i ?

Bonus Answer: $(1/(\sqrt{2})) + (1/(\sqrt{2}))i$

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36. MATHEMATICS

Toss Up: Short Answer

What is 11 base 5 in base 2?

Bonus Answer: 110

Bonus: Short Answer

If $\log_{10} x$ is 100, what is $\log_{100} x$?

Bonus Answer: 50

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37. MATHEMATICS

Toss Up: Short Answer

What is the sum of the infinite geometric series whose first term is 1 and fourth term is $1/64$?

Bonus Answer: $4/3$

Bonus: Multiple Choice

What is the value of e to the $(\pi \text{ times } i/2)$?

W) e^{-1}

X) 1

Y) -1

Z) i

Bonus Answer: Z

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38. MATHEMATICS

Toss Up: Short Answer

What is the value of the quantity $((\sin^4 x) + 2(\cos^2 x)(\sin^2 x) + (\cos^4 x))^{1/2}$?

Bonus Answer: 1

Bonus: Multiple Choice

What is the probability of selecting two natural numbers and having them be relatively prime?

W) $1/2$

X) $2/3$

Y) $6/(\pi^2)$

Z) $2/\pi$

Bonus Answer: Y

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39. MATHEMATICS

Toss Up: Short Answer

What is the integral of $\sec x \, dx$?

Bonus Answer: $\ln |\sec x + \tan x| + C$

Bonus: Multiple Choice

Which of these functions cannot be integrated and represented with elementary functions?

W) $(\sin^5 x) \text{ times } (\cos^6 x) \, dx$

X) e^x times $x^3 dx$

Y) $e^{(x^2)} dx$

Z) $\tan x \sin x dx$

Bonus Answer: Y

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40. MATHEMATICS

Toss Up: Short Answer

What is $(1000! + 999!)/(998!)$?

Bonus Answer: 999,999

Bonus: Short Answer

What is the first number with 5 distinct prime factors?

Bonus Answer: 2,310

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41. MATHEMATICS

Toss Up: Short Answer

What is the relationship between the surface area and the volume of a sphere?

Bonus Answer: The surface area is the derivative of the volume/ the volume is the integral of the surface area when the constant is 0

Bonus: Multiple Choice

When the inner diagonal of a cube is 7 times root 3 inches, what is the surface area of the cube?

W) 216 square inches

X) 294 square inches

Y) 343 square inches

Z) 512 square inches

Bonus Answer: X

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42. MATHEMATICS

Toss Up: Multiple Choice

Given two attempts, what is the probability of getting a multiple choice question correct, if there are 5 choices?

W) 1:3

X) 1:4

Y) 2:3

Z) 2:4

Toss Up Answer: Y

Bonus: Multiple Choice

What percentage of data falls within 2 standard deviations of the mean, assuming a normal distribution?

W) 34.10%

X) 47.70%

Y) 68.20%

Z) 95.40%

Bonus Answer: Z

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43. MATHEMATICS

Toss Up: Short Answer

What is the 3rd mercenne prime?

Bonus Answer: 31

Bonus: Multiple Choice

How many terms does the 15th integral of $\sin x$ dx have?

- W) 1
- X) 2
- Y) 15
- Z) 16

Bonus Answer: Z

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44. MATHEMATICS

Toss Up: Multiple Choice

What is the slope intercept in the equation $y = 4x + 3 - 2$

- W) 3
- X) 1
- Y) -2
- Z) 2

Toss Up Answer: X

Bonus: Multiple Choice

If $3x - y = 12$, what is $(8^x) / (2^y)$

- W) 2^{12}
- X) 4^4
- Y) 8^3
- Z) Cannot be determined

Bonus Answer: W

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45. MATHEMATICS

Toss Up: Short Answer

What shape is $x^2 + 1$

Bonus Answer: parabola

Bonus: Short Answer

what type of function is $x + 2x$

Bonus Answer: a line

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46. MATHEMATICS

Toss Up: Short Answer

What is the volume of a sphere of radius "R"?

Bonus Answer: $(4/3)\pi R^3$

Bonus: Short Answer

Using an x-y coordinate axis, the figure represented by the equation $[x^2/36] + [y^2/16] = 1$ is centered about what x-y coordinate point?

Bonus Answer: (0,0) ; the origin

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47. MATHEMATICS

Toss Up: Short Answer

In a normal distribution, approximately what percentage of the cases, to the nearest whole number, falls within 4 standard deviations of the mean:

Bonus Answer: 100%

Bonus: Multiple Choice

Which of the following properties would you use to compute the chances of rolling either a 7 or an 11 with a pair of dice:

- W) multiplicative
- X) conditional
- Y) independent
- Z) additive

Bonus Answer: Z

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48. MATHEMATICS

Toss Up: Multiple Choice

Give the range for the following six values 2, 7, 11, 19, 25, 33:

- W) 2
- X) 31
- Y) 33
- Z) 15

Toss Up Answer: X

Bonus: Short Answer

What percent of a circle is $6/5\pi$ radians?

Bonus Answer: 216

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49. MATHEMATICS

Toss Up: Multiple Choice

Which of the following salts is responsible for the browning of pretzels?

- W) Sodium Chloride
- X) Sodium Hydroxide
- Y) Potassium Chloride
- Z) Potassium Carbonate

Toss Up Answer: X

Bonus: Short Answer

Consider the reaction $A + 2B \rightleftharpoons 4C$. Assume all species in the reaction are gaseous. If the reaction is at equilibrium, and the concentration of every species is 2 molar, calculate the equilibrium constant of the reaction.

Bonus Answer: 2

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50. MATHEMATICS

Toss Up: Short Answer

You are trying to give 5 apples to 3 friends. You can give any number of apples to each friend, including 0. How many ways are there to share the apples?

Bonus Answer: 56 (its $8C3$)

Bonus: Multiple Choice

If a cubic function equals 0 at exactly two points, which of the following must be true?

- W) the function passes through the origin
- X) there is a double root
- Y) the function is even
- Z) one of the roots is imaginary

Bonus Answer: X

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51. MATHEMATICS

Toss Up: Short Answer

What is the length of the longest diagonal of a unit cube?

Bonus Answer: $\sqrt{3}$

Bonus: Short Answer

What is the largest integer that can't be written as the sum of 3's and 4's?

Bonus Answer: 5

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