

# MATHEMATICS

## 1. MATHEMATICS

Writer: American Heritage

Toss Up: Short Answer

If  $f(x) = \sqrt{(e^x \tan(x) + 1)}$ , find  $f(x) \cdot f'(x)$  [read as If  $f(x)$  equals the square root of quantity  $e$  to the power of  $x$  times tangent of  $x$  plus one, closed quantity, find  $f$  of  $x$  times  $f$  prime of  $x$ ].

Bonus Answer:  $[e^x(\tan x + \sec^2 x)]/2$ . Accept:  $(e^x \tan x + e^x \sec^2 x)/2$ , or  $e^x(\tan^2 + \tan x + 1)/2$

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Bonus: Short Answer

If  $f(x)$  is an even function such that the integral from 0 to 4 of  $f(x)dx$  equals 31, and the integral from 0 to 8 of  $f(x)dx$  equals 42, evaluate the integral from -4 to -8 of  $f(x)dx$

Bonus Answer: 9

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## 2. MATHEMATICS

Writer: American Heritage

Toss Up: Short Answer

A car travels at 20mph for the first half of its trip, and 30 mph for the second half. What is the average speed for the trip in miles per hour?

Bonus Answer: 24 mph

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Bonus: Short Answer

Christopher takes 4 hours to paint a wall by himself. Tyger being lazy takes 6 hours to paint the same wall by himself. If Christopher and Tyger work together, but take a 1 hour break to play Super Smash Bros, how long does it take them to finish the wall?

Bonus Answer: 3 hours and 24 minutes (Accept 17/5 hours)

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## 3. MATHEMATICS

Writer: American Heritage

Toss Up: Short Answer

Evaluate the intergral from negative pi to pi of  $(x^2) \cdot \sin x \, dx$

Bonus Answer: 0

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Bonus: Multiple Choice

Evaluate the limit as  $x$  approaches zero of the fraction with numerator  $e^x - \cos(x)$  and denominator  $\sin(x)$ .

W) 1

X) 0

Y) -1

Z) DNE

Bonus Answer: X

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## 4. MATHEMATICS

Writer: Stuyvesant

Toss Up: Multiple Choice

What statistical average is most appropriate to use when the quantities being averaged when one or more of the quantities are not necessarily bounded?

W) Geometric Mean

X) Median

Y) Harmonic Mean

Z) Arithmetic Mean

**Toss Up Answer: Z**

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**Bonus: Short Answer**

You pick two cards at random without replacement from a standard, 52 card deck. Compute the probability that exactly one is a heart.

**Bonus Answer: 13/34**

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## 5. MATHEMATICS

**Writer: Stuyvesant**

**Toss Up: Multiple Choice**

Which of the following expressions in  $x$  grows fastest?

W)  $(3)^x$

X)  $2^{(2x)}$

Y)  $(x)^2$

Z)  $10^{(3/2)^x}$

**Toss Up Answer: X**

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**Bonus: Short Answer**

Given  $504 = 2^3 \times 3^2 \times 7$  [504 equals 2 cubed times 3 squared times 7], compute the sum of the positive divisors of 504.

**Bonus Answer: 1560**

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## 6. MATHEMATICS

**Writer: Stuyvesant**

**Toss Up: Multiple Choice**

If the test scores of Mr. Coco's Calculus test are assumed to have a normal distribution, and if the mean test score was 91, what can be said about the median test score of that test?

W) The median was lower than 91

X) The median was higher than 91

Y) The median was 91

Z) The median cannot be determined

**Toss Up Answer: Y**

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**Bonus: Multiple Choice**

The z-score for Steven's physics test grade was found to be 0.375. If he scored a 94 on the test, and the standard deviation was 8, what was the mean test score on the physics test?

W) 91

X) 95

Y) 92

Z) 98

**Bonus Answer: W**

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## 7. MATHEMATICS

**Writer: Stuyvesant**

**Toss Up: Multiple Choice**

Simplify  $(\sin 15 \cos 75 - \cos 15 \sin 75) / (\cos 15 \cos 75 + \sin 15 \sin 75)$

W) negative square root of 3

X)  $1/2$

Y)  $-1/2$

Z) square root of 6 minus square root of 2 all over 2

**Toss Up Answer: W**

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**Bonus: Short Answer**

Compute  $(2+2\sqrt{3}i)^6$ . [the quantity 2+2 times the square root of 3 time i to the 6th power]

**Bonus Answer: 4096**

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## 8. MATHEMATICS

**Writer: Centennial**

**Toss Up: Short Answer**

If X and Y are independent random variables with standard deviations of a and b respectively, what is the standard deviation of the quantity  $X+2Y$ ?

**Bonus Answer:  $\sqrt{a^2+4b^2}$**

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**Bonus: Short Answer**

If X and Y are independent random variables with expected values c and d respectively, what is the expected value of the quantity  $X+2Y$ ?

**Bonus Answer:  $c+2d$**

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## 9. MATHEMATICS

**Writer: Centennial**

**Toss Up: Short Answer**

What is the name for the group of four datasets with nearly identical summary statistics, but drastically different scatterplots?

**Bonus Answer: Anscombe's quartet**

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**Bonus: Short Answer**

X is a random variable with a uniform distribution with width w of values. In terms of w, what is the standard deviation of X?

**Bonus Answer:  $\sqrt{w^2/12}$**

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## 10. MATHEMATICS

**Writer: Centennial**

**Toss Up: Short Answer**

What is the exact average value of  $x^2+3x-2$  from  $x=3$  to  $x=6$ ?

**Bonus Answer:  $195/6$**

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**Bonus: Multiple Choice**

With which of the following nth terms will an infinite series converge?

W)  $1/n$

X)  $\ln n$

Y)  $\sin n$

Z)  $1/n^3$

**Bonus Answer: Z**

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## 11. MATHEMATICS

**Writer: Granada Hills**

**Toss Up: Short Answer**

What is the sum of coefficients of the expression  $(x+1)^3$  (cubed)

**Bonus Answer: 8**

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**Bonus: Short Answer**

A student is riding a roller coaster. The track resembles the equation  $y = \sin(t)$ , where  $y$  is the height and  $t$  is the time elapsed in seconds. At what time, rounded to the nearest second, does the student reach the first peak of the roller coaster? Assume degrees.

**Bonus Answer: 90**

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## 12. MATHEMATICS

**Writer: Granada Hills**

**Toss Up: Short Answer**

Assuming random chance decides admission, and the acceptance rate for University A is 10% and the acceptance rate for University B is 20%, what is the probability that a student will be rejected from both to the nearest hundredth.

**Bonus Answer: 0.72 (Accept 72%)**

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**Bonus: Multiple Choice**

A teacher wants to buy pizza for the very hardworking kids of Science Bowl. Which of the following options will provide the least square inches of pizza per dollar?

- W) 2 10 inch diameter pizzas for \$15
- X) 2 square pizzas with side length 10 inches for \$20
- Y) 1 14 inch diameter pizza for \$12
- Z) 1 12 inch diameter pizza for \$10

**Bonus Answer: X**

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## 13. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Short Answer**

In a triangle, the sides are 13, 14, and 15. Find the altitude of the triangle.

**Bonus Answer: 12**

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**Bonus: Multiple Choice**

What is 2017 to the second power?

- W) 4068290
- X) 4068269
- Y) 4068279
- Z) 4068289

**Bonus Answer: Z**

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## 14. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Multiple Choice**

What is 10 factorial

- W) 370431
- X) 390216
- Y) 429764
- Z) 3628800

**Toss Up Answer: Z**

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**Bonus: Multiple Choice**

what type of series is this 1,1,2,3....

- W) geometric
- X) arithmetic
- Y) taylor
- Z) none

**Bonus Answer: Z**

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## 15. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Short Answer**

$(1+(1/X))^2 + 6(1+(1/X)) + 8 = 0$  solve for x

**Bonus Answer:  $1/(1 \pm \sqrt{8/5})$**

**Explanation:** substitute  $(1 + (1/X))$  for another variable like y. Then it's just a simple quadratic that you can factor

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**Bonus: Multiple Choice**

$2(X^{5/3}) + 64 = 0$  solve for X

- W) 8
- X)  $4^5$
- Y)  $(-2)^3$
- Z)  $3^3$

**Bonus Answer: Y**

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## 16. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Short Answer**

$\sin(x+20)=\cos(50)$

**Bonus Answer:  $X = 20$**

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**Bonus: Short Answer**

$X^2+4x+4$

**Bonus Answer:  $X=2$**

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## 17. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Multiple Choice**

$Y=11$  find x in  $y=4(x^2)-3$

- W) 3
- X) 4
- Y) 2
- Z) 5

**Toss Up Answer: Y**

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**Bonus: Short Answer**

Find x,  $45 - x = 21^2$

**Bonus Answer: -396**

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## 18. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Short Answer**

What type of function looks like a logarithmic function when reflected over line  $y=x$ ?

**Bonus Answer: Exponential function**

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**Bonus: Short Answer**

Solve for x in  $32 = 2^x$

**Bonus Answer: 5**

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## 19. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Multiple Choice**

If the height of a pole is equal to 24 meters and it's shadow is equal to 3.5 meters. what is the height of a man with a shadow of 10.5 meters?

W) 18 meters

X) 72 yards

Y) 72 meters

Z) 78 yards

**Toss Up Answer: Y**

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**Bonus: Multiple Choice**

What is the value of x when  $3x$  to the power of 4 equals 19683?

W) 7

X) 8

Y) 9

Z) 10

**Bonus Answer: Y**

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## 20. MATHEMATICS

**Writer: Lakeside**

**Toss Up: Short Answer**

Consider a function  $f(x)$  (read: eff of ex). If  $f(x)$  is a positive monotone decreasing function on the interval from 1 to infinity and the integral from one to infinity of  $f(x)$  equals a finite number, what can be said about the convergence of an infinite series defined by the summation of the values of  $f(x)$  from 1 to infinity?

**Bonus Answer: IT CONVERGES**

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**Bonus: Multiple Choice**

Which of the following eccentricities describes a circle?

W)  $e=0$

X)  $0 < e < 1$

Y)  $e=1$

Z)  $e > 1$

**Bonus Answer: W**

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## 21. MATHEMATICS

**Writer: Lakeside**

**Toss Up: Short Answer**

What theorem states that there are no integer solutions for the equation  $x^n + y^n = z^n$  when  $n$  is greater than 2, famously proposed in the margins of a book by its namesake?

**Bonus Answer: FERMAT'S LAST THEOREM**

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**Bonus: Short Answer**

What is the name given an  $n$ th root of a number which often yields an irrational number?

**Bonus Answer: SURD**

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## 22. MATHEMATICS

**Writer: Lakeside**

**Toss Up: Short Answer**

What is the probability of getting at least one tails when flipping 5 fair coins?

**Bonus Answer: 31/32 (Accept: .96875)**

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**Bonus: Multiple Choice**

Multiple Choice At  $x = -1/2$ , the graph of  $y = x^3$  is:

- W) increasing and concave up
- X) increasing and concave down
- Y) decreasing and concave up
- Z) decreasing and concave down

**Bonus Answer: X**

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## 23. MATHEMATICS

**Writer: Oxford**

**Toss Up: Short Answer**

In a 2 by 2 matrix with first row  $a, b$  and second row  $c, d$ , what is the name commonly associated with the value  $ad - bc$ ?

**Bonus Answer: Determinant**

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**Bonus: Multiple Choice**

What is the closest distance from the point  $(6, 8)$  to the line,  $3x + 2y - 1 = 0$ ?

- W) 2.3
- X) 3.3
- Y) 4.3
- Z) 5.3

**Bonus Answer: X**

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## 24. MATHEMATICS

**Writer: Oxford**

**Toss Up: Multiple Choice**

A complex number multiplied by its complex conjugate is equivalent to which of the following?

- W) The argument
- X) The argument squared
- Y) The magnitude
- Z) The magnitude squared

**Toss Up Answer: Z**

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**Bonus: Short Answer**

The Hardy-Ramanujan is given as the smallest positive integer that can be expressed as the sum of two different pairs of perfect positive cubes. What is the Hardy-Ramanujan number?

**Bonus Answer: 1729**

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## 25. MATHEMATICS

**Writer: Oxford**

**Toss Up: Multiple Choice**

Given the polynomial,  $24-50x+35x^2-10x^3+x^4$ , which of the following is not a possible root by the rational root theorem?

- W) 2
- X) 3
- Y) 4
- Z) 5

**Toss Up Answer: Z**

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**Bonus: Short Answer**

How many cubic polynomials,  $f(x)$  with positive integral coefficients have  $f(1)=6$ ?

**Bonus Answer: 84 (equivalent to 9C3 by stars and bars)**

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## 26. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Multiple Choice**

Find  $x$  when  $(x^2)-4x-12 = 0$

- W)  $x=\{6, -2\}$
- X)  $x=\{6, 2\}$
- Y)  $x=\{-6, 2\}$
- Z)  $x=\{-6, -2\}$

**Toss Up Answer: Y**

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**Bonus: Short Answer**

Given: triangle  $abc$  is a right angle,  $d$  is the midpoint of line  $ab$  and  $e$  is the midpoint of line  $Bc$ ,  $de \parallel ac$   $de=7$ ,  $ac=14$ ,  $ad=5$  find  $db$

**Bonus Answer: 5**

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## 27. MATHEMATICS

**Writer: Brooklyn Tech**

**Toss Up: Multiple Choice**

What is the following in degrees?  $7\pi/8$

- W) 157 degrees
- X) 155 degrees
- Y) 157.5 degrees
- Z) 200 degrees

**Toss Up Answer: Y**



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**Bonus: Short Answer**

Find the following in radian: 280 degrees

**Bonus Answer:**  $14\pi/9$

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**28. MATHEMATICS**

**Writer:** Brooklyn Tech

**Toss Up:** Short Answer

What is the exact value of  $\sin(225)$  and  $\cos(225)$ ?

**Bonus Answer:**  $-(\sqrt{2})/2$  for both

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**Bonus: Short Answer**

What is the exact value of the period of the sine wave in radian form:  $y=4\sin(5x)$

**Bonus Answer:**  $2\pi/5$

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**29. MATHEMATICS**

**Writer:** Brooklyn Tech

**Toss Up:** Short Answer

What is the value of  $\theta$  if  $\cos(\theta)=-\sqrt{2}/2$ ?

**Bonus Answer:** 225 degrees or  $5\pi/4$ . Adding 360 degrees any amount of times to 225 degrees or  $2\pi$  any amount of times to  $5\pi/4$  is acceptable.

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**Bonus: Short Answer**

Find the value of  $\sin(\theta)$  in the expression  $\sin(\arctan(x))$

**Bonus Answer:**  $x$

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**30. MATHEMATICS**

**Writer:** Brooklyn Tech

**Toss Up:** Short Answer

Graph theory is the study of graphs (mathematical structures that represent the relationship between objects through vertices or points and edges or connections between points). Describe the relationship of vertices, edges, and faces through a mathematical equation.

**Bonus Answer:**  $V$  (the number of vertices) -  $E$  (number of edges) +  $F$  (number of faces) = 2

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**Bonus: Short Answer**

Name the type of path that doesn't repeat any edges and has the same initial and final vertex.

**Bonus Answer:** Euler Circuit

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**31. MATHEMATICS**

**Writer:** Venice

**Toss Up:** Multiple Choice

Which of the following could be the correlation coefficient for a negative, weak linear relationship?

W) -0.8

X) 0.8

Y) -0.2

Z) 0.2

**Toss Up Answer:** Y

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**Bonus: Short Answer**

Blue has 10 Pokémon in his box. How many different teams of 6 Pokémon can Blue make?

Bonus Answer: 210

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## 32. MATHEMATICS

Writer: Venice

Toss Up: Short Answer

What is the average value of the function  $y=4x + 5$  over the interval  $x=2$  to  $x=6$ ?

Bonus Answer: 21

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Bonus: Multiple Choice

What is the volume of the solid made by rotating the area enclosed by the functions  $y=x^2$ ,  $x=1$ ,  $x=2$ , and the  $x$ -axis around the  $x$ -axis?

W)  $\pi/5$

X)  $\pi/3$

Y)  $7\pi/3$

Z)  $31\pi/5$

Bonus Answer: Z

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## 33. MATHEMATICS

Writer: Venice

Toss Up: Short Answer

If a bag contains 5 red marbles, 3 green marbles, and 2 blue marbles, what is the probability of taking out a red and then blue marble without replacement?

Bonus Answer:  $1/9$

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Bonus: Multiple Choice

McDonald's sells McNuggets in boxes of 6, 9, and 20. What is the greatest number of McNuggets that cannot be bought with combinations of these boxes?

W) 19

X) 25

Y) 31

Z) 43

Bonus Answer: Z

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