MATHEMATICS

1. MATHEMATICS

Toss Up: Multiple Choice

If the test scores of Mr. Coco's Calculus test are assumed to have a normal distribution, and if the mean test score was 91, what can be said about the median test score of that test?

- W) The median was lower than 91
- X) The median was higher than 91
- Y) The median was 91
- Z) The median cannot be determined

Toss Up Answer: Y

Bonus: Multiple Choice

The z-score for Shantanu's physics test grade was found to be 0.375. If he scored a 94 on the test, and the standard deviation was 8, what was the mean test score on the physics test?

W) 91

X) 95

Y) 92

Z) 98

Bonus Answer: W

2. MATHEMATICS

Toss Up: Short Answer

In simplest terms, find the value of the limit as x approaches 0 of $(4^x - 2^x) / x$

Bonus Answer: In(2) (do not accept In(4) - In(2))

Bonus: Short Answer

How many real solutions for x does the equation x = ln(x) + 2 have?

Bonus Answer: 2

3. MATHEMATICS

Toss Up: Multiple Choice

A right triangle has a hypotenuse of length 12. The altitude to the hypotenuse has a length of 8. What is the area of this triangle?

W) 36

X) 48

Y) 60

Z) The triangle is impossible to construct

Toss Up Answer: Z

Bonus: Short Answer

Two REAL numbers x and y satisfy the system of equations: x + y = 12; $x^2 + y^2 = 64$. Find the value of the product xy.

Bonus Answer: The system has no solutions.

4. MATHEMATICS

Toss Up: Short Answer

If n is a positive integer, what is the smallest value of n such that n! + 1 is a perfect square?

Bonus Answer: 4

Bonus: Short Answer

If z1 = 3 - 4i and z2 = 7 + i, find the absolute value of z1z2 in simplest terms.

Bonus Answer: 25*sqrt(2)

5. MATHEMATICS

Toss Up: Short Answer

What is the inverse of the 2x2 matrix (row 1: 6 10), (row 2: 3 5)? Bonus Answer: The matrix has no inverse (b.c. determinant = 0).

Bonus: Short Answer

The legs of an isosceles triangle have a length of 10, and the altitudes to the legs have a length of 6. In simplified radical form, what is the length of the altitude to the base of the triangle?

Bonus Answer: 3 * sqrt(10) (Do not accept sqrt(90))

6. MATHEMATICS

Toss Up: Short Answer

What is the 5th non-triangular number?

Bonus Answer: 8

Bonus: Short Answer

Name all the following that are true:

- 1. An icosahedron has 18 faces.
- 2. A regular hexahedron has 16 edges.
- 3. There are only nine regular polyhedra.
- 4. A regular octahedron has 4 times the volume of a regular tetrahedron with the same side length.

Bonus Answer: 2,4 (an icosahedron has 20 faces; a cube has 12 edges)

7. MATHEMATICS

Toss Up: Multiple Choice

If θ is an angle such that $\sin(\theta) < 0$ and $\cos(\theta) = 0$, where in the coordinate plane is it located?

- W) Between the 2nd and 3rd quadrants
- X) Between the 3rd and 4th quadrants
- Y) Between the 1st and 4th quadrants
- Z) Between the 1st and 2nd quadrants

Toss Up Answer: Y

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Bonus: Short Answer

What is the remainder of x^10+x+1 divided by $(x-1)^2$?

Bonus Answer: 11x-8

8. MATHEMATICS

Toss Up: Multiple Choice

Given the equation of a conic section, $y^2/16 - x^2/25 = 1$, what is the length of the conjugate axis?

W) 16

X) 8

Y) 10

Z) 25

Toss Up Answer: Y

Bonus: Multiple Choice

Compute log(base 2) of 2048^2

W) 20

X) 22

Y) 24

Z) 26

Bonus Answer: X

9. MATHEMATICS

Toss Up: Multiple Choice

A point with coordinates (3,5) is rotated 90 degrees clockwise about the point (2,1) and is then reflected across the y-axis. What are the coordinates of the resulting image?

W) (-5,-3)

X) (-4,0)

Y) (-6,0)

Z) (-5,-1)

Toss Up Answer: Y

Bonus: Multiple Choice

A quadrilateral ABCD has two congruent opposite side segments, AB and CD. Which of the following would have to be true in order for the quadrilateral to be a square?

- W) The circumcircle of ABC is centered at the midpoint of AC
- X) The medians of ABCD are perpendicular
- Y) The perpendicular bisectors of AB and CD are the same line
- Z) None of the above.

Bonus Answer: Z

10. MATHEMATICS

Toss Up: Multiple Choice

Which of the following cannot be the root of a polynomial with rational coefficients?

W) 5*i + 6

X) phi - 1/2, where phi is the golden ratio

Y) sqrt(3+i)

Z) 6 - pi*i

Toss Up Answer: Z

Bonus: Short Answer

What is the name given to a number that is not a root of any non-zero polynomial equation with rational coefficients? Bonus Answer: Transcendental number (Accept transcendental)

11. MATHEMATICS

Toss Up: Multiple Choice

Fermat's Last Theorem conjectures that no three positive integers a,b, and c can satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2. It went unproved for more than 300 years until it was finally proved in 1994 by a mathematician who received a substantial monetary prize only this year. What is the name of this mathematician?

W) Grigori Perelman

X) John Nash

Y) Andrew Wiles

Z) Terence Tao

Toss Up Answer: Y

Bonus: Short Answer

Prime numbers of the form 2 ^ (2^n) + 1, where n is a non-negative integer, are known as this.

Bonus Answer: Fermat Primes

12. MATHEMATICS

Toss Up: Short Answer

If Bruce can do a job in 3 hours, and Clark can do the same job in 4 hours, how long, in hours, will it take them to do the job if they work together? You may leave your answer as a fraction.

Bonus Answer: 12/7

Bonus: Short Answer

What is the eccentricity of an ellipse with a = 5 and b = 4?

Bonus Answer: 3/5

13. MATHEMATICS

Toss Up: Short Answer

Find the perimeter of a right triangle with legs 11 and 60.

Bonus Answer: 132

Bonus: Short Answer

In a 15-75-90 degrees right triangle with hypotenuse 4, what are the lengths of the shorter and longer leg,

respectively? Exact answers please.

Bonus Answer: radical 6 - radical 2, radical 6 + radical 2

14. MATHEMATICS

Toss Up: Short Answer

If the first term in an arithmetic sequence is 2 and the third term is 6, find the 10th term.

Bonus Answer: 20

Bonus: Short Answer

Compute the sum of the first 100 positive integers.

Bonus Answer: 5050

15. MATHEMATICS

Toss Up: Short Answer

If the log of n is 10, what is the log of 100n?

Bonus Answer: 12

Bonus: Short Answer

How many ways are there to seat 5 people around a circular table, if rotations are ignored?

Bonus Answer: 24

16. MATHEMATICS

Toss Up: Multiple Choice

What is the geometric mean of the roots of the polynomial 2x³ - 2x² - 228x - 432

W) 4

X) 6

Y) 8

Z) 10

Toss Up Answer: X

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Bonus: Short Answer

The first term of an arithmetic sequence is x, and the xth term is x^2 . If the common difference is equal to 7, and none of the terms are equal, what is the 12th term of the sequence?

Bonus Answer: 84

17. MATHEMATICS

Toss Up: Short Answer
Completely expand (x-2y)^3

Bonus Answer: x^3 - 6x^2*y + 12x*y^2 - 8y^3

Bonus: Short Answer

What is the sums of all the terms in the seventh row of Pascal's triangle?

Bonus Answer: 128

18. MATHEMATICS

Toss Up: Multiple Choice

Which of the following distances is longest?

W) Half a kilometer

X) Half a decimeter

Y) 600 meters

Z) 40 decameters

Toss Up Answer: Y

Bonus: Short Answer

What is the name of a line that intersects two or more coplanar lines in different points?

Bonus Answer: Transversal

19. MATHEMATICS

Toss Up: Multiple Choice

The number (sqrt(2))^(sqrt(2)) is:

W) Rational

X) Algebraic irrational

Y) Transcendental

Z) Impossible to tell Toss Up Answer: Y

Bonus: Short Answer

Let $f(x) = ax^7 + bx^3 + cx - 5$, where a,b, and c are real numbers. What is the value of f(7) if f(-7) = 8?

Bonus Answer: -18

20. MATHEMATICS

Toss Up: Short Answer

What is the limit as x approaches 0 of x^x?

Bonus Answer: Does not exist. (NOT 1)

Bonus: Short Answer

Let f(x) be a function with no real roots. If f(3) = 4, then in what quadrants can f(x) be in? Bonus Answer: All four quadrants (b.c. there's no specification f has to be continuous)

21. MATHEMATICS

Toss Up: Multiple Choice

- 1. What statistical average is most appropriate to use when the quantities being averaged when one or more of the quantities are not necessarily bounded?
- W) Geometric mean
- X) median
- Y) Harmonic mean
- Z) Arithmetic mean

Toss Up Answer: Y

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Bonus: Short Answer

You pick two cards at random without replacement from a standard, 52 card deck. Compute the probability

exactly one is a heart. Bonus Answer: 13/34

22. MATHEMATICS

Toss Up: Multiple Choice

Which of the following expressions in x grows fastest?

W) 3^x [3 to the x]

X) $10(3/2)^x$ [10 times three halves to the x]

Y) x^2 [x squared]

Z) 2 $^{(2x)}$ [2 to the 2x]

Toss Up Answer: Z

Bonus: Short Answer

Given 504=2³ x 3² x 7 [504 equals 2 cubed times 3 squared times 7], compute the sum of the positive divisors of 504

_ .

Bonus Answer: 1560

23. MATHEMATICS

Toss Up: Short Answer

Which of the following statements concerning prime numbers are true?

- 1. If a and n are relatively prime than there are infinitely many primes that leave a remainder of a when divided by n.
- 2. All integers less than p are quadratic residues modulo p
- 3. There are infinitely many prime numbers

Bonus Answer: 1 and 3

Bonus: Short Answer

If a1+a2+a3+a4+a5=17 for positive integers a1,a2,a3,a4,a5, then maximize the product a1a2a3a4a5?

Bonus Answer: 432

24. MATHEMATICS

Toss Up: Multiple Choice

Simplify (sin15cos75-cos15sin75)/(cos15cos75+sin15sin75)

W) negative square root of 3

X) 1/2

Y) -1/2

Z) square root 6 minus square root 2 all over 2

Toss Up Answer: W

Bonus: Short Answer

Compute (2+2sqrt(3)i)^6. [2+2 times the square root of 3, i to the 6th power]

Bonus Answer: 4096

25. MATHEMATICS

Toss Up: Multiple Choice

Compute 3C1+4C2+5C3+6C4+7C5. [3 choose 1 + 4 choose 2 + 5 choose 3 + 6 choose 4 + 7 choose 5]

W) 55

X) 336

Y) 240

Z) 56

Toss Up Answer: W

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Bonus: Short Answer

Compute the square root of 5476

Bonus Answer: 74

26. MATHEMATICS

Toss Up: Short Answer

In Triangle ABC the angle bisector of A intersects BC at D. Given AB=9, AC=21, and DB=15, compute DC.

Bonus Answer: 35

Bonus: Short Answer

In triangle ABC, AB=14, AC=13, and BC=15. Let the incenter be I. Compute CI in simplest radical form.

Bonus Answer: square root of 65

27. MATHEMATICS

Toss Up: Short Answer

which of the following are true about the number and nature of the roots of the polynomial $x^3 - 29x^2 + 229x$

- I. 0 is a root
- II. There must be a real, positive root
- III. There must be three real roots

Bonus Answer: I

Bonus: Short Answer

Solve the following linear system of equations in 3 variables:

2x + y + 4z = 9

-x + 2y + 5z = -11

5x + 3y + 3z = -4

Bonus Answer: x=4,y=-11,z=3

Also accept (4,-11,3)

28. MATHEMATICS

Toss Up: Multiple Choice

MULTIPLE CHOICE: tossup

For an infinite sequence of numbers $a_0,a_1,...$, which of the following best describes the polynomial P(x) = sum(i=0) to infinity) a $i*x^i$.

W) Reimann-Zeta function

- X) Generating function
- Y) Newtonian series
- Z) Tchebychev polynomial

Toss Up Answer: X

Bonus: Short Answer

In the Pell equation $x^2-3y^2=1$ the smallest solution is x=2 and y=1. The next smallest solution is x=7 and y=4. Compute the third smallest solution.

Bonus Answer: x=26, y=15

Also accept (26,15)

29. MATHEMATICS

Toss Up: Multiple Choice

Which of the following is not a group?

W) the integers under multiplication

X) the integers under addition

Y) the symmetries of a regular n-gon

Z) the permutations of integers between 1 and n

Toss Up Answer: W

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Bonus: Short Answer

find the smallest positive solution to the following congruences.

x congruent to 3 mod 13 x congruent to 8 mod 11 Bonus Answer: x=107

Also accept: 107

30. MATHEMATICS

Toss Up: Multiple Choice

What mathematician proved the existence of a straight edge and compass construction of a regular 17-gon?

- W) Euler
- X) Euclid
- Y) Gauss
- Z) Galois [Gal-wah] **Toss Up Answer: Y**

Bonus: Short Answer

Given $\sin X = 1/3$ [sine of x equals one third], compute $\cos^2(3X)$ [co-sine squared of three x].

Bonus Answer: -23/27

31. MATHEMATICS

Toss Up: Short Answer

Tim is 5 times older than his younger brother. In 3 years Tim will be 3 times older than his younger brother. Compute the difference in the ages of Tim and his younger brother.

Bonus Answer: 12

Bonus: Short Answer

How many positive integers n have no solutions to the following equation where a and b are nonnegative integers? 3a+7b=n

Bonus Answer: 6

32. MATHEMATICS

Toss Up: Multiple Choice

To test products, there is a test that identifies 95% of defective products as defective, but also labels 10% of all working products as defective. If 85% of the products made are not defective, what probability of products the test determines to be defective are actually defective?

W) 39%

X) 84%

Y) 67%

Z) 42%

Toss Up Answer: Y

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Bonus: Short Answer

Compute the surface area of a cylinder with radius 8 and height 2

Bonus Answer: 160pi

33. MATHEMATICS

Toss Up: Multiple Choice

If x+(1/x)=2, find $x^128+(1/x)^128$

W) 256

X) 128

Y) 64

Z) 2

Toss Up Answer: Z

1000 GP / 110WG1. 2

Bonus: Short Answer

Given the quadratic $x^2-20x+9$ and its roots p and q, find $(1/p)^2+(1/q)^2$

Bonus Answer: 382/81

34. MATHEMATICS

Toss Up: Multiple Choice

Let f be an odd function over the real numbers. What is the value of f at 0?

W) -1

X) 0

Y) 1

Z) There is not enough information to determine an answer

Toss Up Answer: Z

Bonus: Multiple Choice

Which of the following is true?

- W) An irrational number raised to an irrational power must have an irrational value.
- X) For all positive number x and y, $x^{n}(y)$ is equal to $y^{n}(x)$
- Y) For all real numbers a,b,c,d with b and d being non-zero, if a/b > c/d, then a*d must be greater than b*c
- Z) There are more complex numbers than there are real numbers.

Bonus Answer: X

35. MATHEMATICS

Toss Up: Short Answer

How many vertices does a dodecahedron have?

Bonus Answer: 20

Bonus: Short Answer

In a+bi form, what is the square root of i? Bonus Answer: (1/(sqrt2)) + (1/(sqrt2))i

36. MATHEMATICS

Toss Up: Short Answer

What is 11 base 5 in base 2?

Bonus Answer: 110

Bonus: Short Answer

If log base 10 of x is 100, what is log base 100 of x?

Bonus Answer: 50

37. MATHEMATICS

Toss Up: Short Answer

What is the sum of the infinite geometric series whose first term is 1 and fourth term is 1/64?

Bonus Answer: 4/3

Bonus: Multiple Choice

What is the value of e to the (pi times i/2)?

W) e^-1

X) 1

Y) -1

Z) i

Bonus Answer: Z

38. MATHEMATICS

Toss Up: Short Answer

What is the value of the quantity $((\sin^4)x) + 2(\cos^2)x(\sin^2)x + (\cos^4)x)^2$?

Bonus Answer: 1

Bonus: Multiple Choice

What is the probability of selecting two natural numbers and having them be relatively prime?

W) 1/2

X) 2/3

Y) 6/(pi^2)

Z) 2/pi

Bonus Answer: Y

39. MATHEMATICS

Toss Up: Short Answer

What is the integral of sec x dx?

Bonus Answer: Ln (abs(sec x + tan x)) + C

Bonus: Multiple Choice

Which of these functions cannot be integrated and represented with elementary functions?

W) (sin^5)x times (cos^6)x dx

X) e^x times x^3 dx

Y) e^(x^2) dx

Z) tan x sin x dx

Bonus Answer: Y

40. MATHEMATICS

Toss Up: Short Answer

What is (1000! + 999!)/(998!)? **Bonus Answer: 999,999**

Bonus: Short Answer

What is the first number with 5 distinct prime factors?

Bonus Answer: 2.310

41. MATHEMATICS

Toss Up: Short Answer

What is the relationship between the surface area and the volume of a sphere?

Bonus Answer: The surface area is the derivative of the volume/ the volume is the integral of the surface area when

the constant is 0

Bonus: Multiple Choice

When the inner diagonal of a cube is 7 times root 3 inches, what is the surface area of the cube?

W) 216 square inches

X) 294 square inches

Y) 343 square inches

Z) 512 square inches

Bonus Answer: X

42. MATHEMATICS

Toss Up: Multiple Choice

Given two attempts, what is the probability of getting a multiple choice question correct, if there are 5 choices?

W) 1:3

X) 1:4 Y) 2:3 Z) 2:4 Toss Up Answer: Y **Bonus: Multiple Choice** What percentage of data falls within 2 standard deviations of the mean, assuming a normal distribution? W) 34.10% X) 47.70% Y) 68.20% Z) 95.40% **Bonus Answer: Z** 43. MATHEMATICS Toss Up: Short Answer What is the 3rd mercenne prime? **Bonus Answer: 31 Bonus: Multiple Choice** How many terms does the 15th integral of sin x dx have? W) 1 X) 2 Y) 15 Z) 16 **Bonus Answer: Z** 44. MATHEMATICS Toss Up: Multiple Choice What is the slope intercept in the equation y = 4x + 3 - 2W) 3 X) 1 Y) -2 Z) 2 Toss Up Answer: X **Bonus: Multiple Choice** If 3x - y = 12, what is $(8^x) / (2^y)$ W) 2¹² X) 4⁴ Y) 8³

Z) Cannot be determined

Bonus Answer: W

45. MATHEMATICS

Toss Up: Short Answer

What shape is X^2+1

Bonus Answer: parabola

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Bonus: Short Answer

what type of function is x+2x

Bonus Answer: a line

46. MATHEMATICS

Toss Up: Short Answer

What is the volume of a sphere of radius "R"?

Bonus Answer: (4/3)piR^3

Bonus: Short Answer

Using an x-y coordinate axis, the figure represented by the equation $[x^2/36] + [y^2/16] = 1$ is centered about what x-y

coordinate point?

Bonus Answer: (0,0); the origin

47. MATHEMATICS

Toss Up: Short Answer

In a normal distribution, approximately what percentage of the cases, to the nearest whole number, falls within 4 standard deviations of the mean:

Bonus Answer: 100%

Bonus: Multiple Choice

Which of the following properties would you use to compute the chances of rolling either a 7 or an 11 with a pair of dice:

W) multiplicative

X) conditional

Y) independent

Z) additive

Bonus Answer: Z

48. MATHEMATICS

Toss Up: Multiple Choice

Give the range for the following six values 2, 7, 11, 19, 25, 33:

W) 2

X) 31

Y) 33

Z) 15

Toss Up Answer: X

Bonus: Short Answer

What percent of a circle is 6/5pi radians?

Bonus Answer: 216

49. MATHEMATICS

Toss Up: Multiple Choice

Which of the following salts is responsible for the browning of pretzels?

W) Sodium Chloride

X) Sodium Hydroxide

Y) Potassium Chloride

Z) Potassium Carbonate

Toss Up Answer: X

Bonus: Short Answer

Consider the reaction A + 2B yields 4C. Assume all species in the reaction are gaseous. If the reaction is at equilibrium, and the concentration of every species is 2 molar, calculate the equilibrium constant of the reaction.

Bonus Answer: 2

50. MATHEMATICS

Toss Up: Short Answer

You are trying to give 5 apples to 3 friends. You can give any number of apples to each friend, including 0. How many ways are there to share the apples?

Bonus Answer: 56 (its 8C3)

Bonus: Multiple Choice

If a cubic function equals 0 at exactly two points, which of the following must be true?

W) the function passes through the origin

X) there is a double root

Y) the function is even

Z) one of the roots is imaginary

Bonus Answer: X

51. MATHEMATICS

Toss Up: Short Answer

What is the length of the longest diagonal of a unit cube?

Bonus Answer: root3

Bonus: Short Answer

What is the largest integer that cant be written as the sum of 3's and 4's?

Bonus Answer: 5

52. MATHEMATICS

Toss Up: Multiple Choice

What is the limit as x approaches 0 of x times In x?

W) -1

X) 1

Y) 0

Z) The limit does not exist.

Toss Up Answer: Y

Bonus: Short Answer

What is the derivative of arcsec x? Bonus Answer: $1/(x((x^2)-1)^{-5})$

53. MATHEMATICS

Toss Up: Multiple Choice

Which of these numbers are transcendental?

W) i

X) root 2

Y) e to the pi i

Z) pi

Toss Up Answer: Z

Bonus: Short Answer

What conic section does this equation create? 25x^2-150x-16y^2-128y+481=0

Bonus Answer: Two intersecting lines/ a degenerate hyperbola
