

MATHEMATICS

1. MATHEMATICS

Toss Up: Short Answer

How many non empty subsets are there of the set $\{1,2,6,9\}$?

Bonus Answer: 15

Bonus: Short Answer

What are the roots of x^4-5x^2+4 ?

Bonus Answer: 1,-1,4,-4

2. MATHEMATICS

Toss Up: Short Answer

What number is a factor of $y^3 + 10y^2 + 123y + 5$?

Bonus Answer: 1, -1

Bonus: Short Answer

What is the area of a equilateral triangle inscribed in a circle with a radius of 6?

Bonus Answer: $(27(\text{radical } 3))$

3. MATHEMATICS

Toss Up: Multiple Choice

What is the height of an equilateral triangle with side length 4?

W) 8

X) 4

Y) $2\text{rad}3$

Z) $4\text{rad}3$

Toss Up Answer: Y

Bonus: Short Answer

What is the area of an isosceles right triangle with hypotenuse $10\text{rad}2$?

Bonus Answer: 50

4. MATHEMATICS

Toss Up: Multiple Choice

Which of the following has the largest amplitude?

W) $9\sin(x) - 2$

X) $2\cos(6x) + 5$

Y) $\cos(2x)$

Z) $4\sin(10x)$

Toss Up Answer: W

Bonus: Short Answer

What power of 10 is closest to 255?

Bonus Answer: 10 to the 16

5. MATHEMATICS

Toss Up: Short Answer

What is the derivative of x^2 ?

Bonus Answer: $2x$

Bonus: Short Answer

What is the derivative of $42x^2$

Bonus Answer: $84x$

6. MATHEMATICS**Toss Up: Short Answer**

For all positive integers n , what is the degree of the polynomial $(x^2 - 3)^{(2n)}$ in terms of n .

Bonus Answer: $4n$

Bonus: Short Answer

State the solutions, in increasing value, for the equation $(x+1)^3 - x^3 - x^2 = 0$

Bonus Answer: (Only accept the answers read in this order): -1 , $-1/2$

7. MATHEMATICS**Toss Up: Short Answer**

Compute $100^2 - 98 \cdot 102$

Bonus Answer: 4

Bonus: Multiple Choice

The absolute value of $n^2 - (n+x)(n-x)$, where n and x are positive integers, is always:

W) $-(x^2)$

X) n^2

Y) $2 \cdot x \cdot n$

Z) x^2

Bonus Answer: Z

8. MATHEMATICS**Toss Up: Short Answer**

Find the product of all primes less than 11.

Bonus Answer: 210

Bonus: Multiple Choice

Find the number of triangular numbers less than 2016.

W) 58

X) 60

Y) 62

Z) 64

Bonus Answer: Y

9. MATHEMATICS**Toss Up: Multiple Choice**

Which three numbers satisfy the Pythagorean Theorem?

W) 1, 1, and $\sqrt{3}$

X) 3, 4, and 5.01

Y) $\pi/3$, $\pi/4$, $\pi/5$

Z) $1/3$, $1/4$, $5/12$

Toss Up Answer: Z

Bonus: Short Answer

Triangle ABC has side lengths of 10, 10, and $10\sqrt{2}$. State all distinct median lengths of the triangle in increasing value.

Bonus Answer: $5\sqrt{2}$, $5\sqrt{5}$ (only accept the numbers said in that order).

10. MATHEMATICS**Toss Up: Short Answer**

What is the full name of the French mathematician who invented the Cartesian coordinate plane?

Bonus Answer: Rene Descartes (You may also accept Descartes Rene, but do not accept Descartes or Rene)

Bonus: Short Answer

What is the name of the value of x in the ordered pair (x,y)

Bonus Answer: Abscissa

11. MATHEMATICS**Toss Up: Short Answer**

Find the limit of $(x^2 - x - 2)/(x + 1)$ as x approaches -1 .

Bonus Answer: -3

Bonus: Short Answer

Find the limit of $(x^2 + 6x + 5)/x^3$ as x approaches infinite

Bonus Answer: 0

12. MATHEMATICS**Toss Up: Short Answer**

There are 27 people in a party. If 16 people wanted ice cream and 17 people wanted chocolate, at most how many people wanted only ice cream?

Bonus Answer: 10

Bonus: Short Answer

How many non congruent rectangles are there with an area of 324 and positive integer side lengths?

Bonus Answer: 8

13. MATHEMATICS**Toss Up: Short Answer**

If $x^5 - 4x^4 + 3x^2 - 2x + 1 = 0$, find the sum of all five of the roots.

Bonus Answer: 4

Bonus: Short Answer

If polynomial $P(x)$ leaves a remainder of 5 when divided by $x-1$ and a remainder of 7 when divided by $x+1$, find the remainder when $P(x)$ is divided by x^2-1 .

Bonus Answer: $-x+6$

14. MATHEMATICS**Toss Up: Multiple Choice**

Which of the following shapes is always cyclic?

W) A parallelogram

X) A rhombus

Y) An obtuse triangle

Z) A pentagon

Toss Up Answer: Y

Bonus: Short Answer

In a triangle with side lengths 10, $10\sqrt{3}$, and 20, find the length of the angle bisector which intersects the side of length $10\sqrt{3}$.

Bonus Answer: $20\sqrt{3}/3$

15. MATHEMATICS

Toss Up: Short Answer

In what type of triangle is the angle bisector, the altitude and the median the same line?

Bonus Answer: Isosceles or equilateral triangle

Bonus: Short Answer

Find the other roots of $x^3 + 6x^2 - 13x - 42$ if one of them is -2

Bonus Answer: -7, 3

16. MATHEMATICS

Toss Up: Multiple Choice

A set is countable:

W) If and only if it is finite.

X) If and only if a surjection can be made between the set and the natural numbers.

Y) If it has the same cardinality as some subset of the natural numbers.

Z) If it has the same cardinality as some subset of the real numbers.

Toss Up Answer: Y

Bonus: Short Answer

Who was the famous German mathematician that invented Set Theory and effectively proved the uncountableness of the reals?

Bonus Answer: Georg Cantor (accept Cantor)

17. MATHEMATICS

Toss Up: Short Answer

What is $64x^3 + 27y^3$ factored in simplest form?

Bonus Answer: $(4x+3y)(16x^2-12xy+9y^2)$

Bonus: Short Answer

If i is the square root of -1, what is the value of i^{97} ?

Bonus Answer: i

18. MATHEMATICS

Toss Up: Short Answer

If Robert can paint a fence in 12 hours and George can paint a fence in 6 hours how long will they take to paint the fence together?

Bonus Answer: 4 hours

Bonus: Multiple Choice

What is the coefficient of the 3rd term in the expansion of $(x+y)^7$

W) 1

X) 7

Y) 21

Z) 35

Bonus Answer: Y

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19. MATHEMATICS

Toss Up: Multiple Choice

Simplify the following expression: $(x^3+27)/(x+3)$ if $x=12$

W) 125

X) 117

Y) 136

Z) 109

Toss Up Answer: X

Bonus: Short Answer

Find all the real solutions of $x^4-18=-2$

Bonus Answer: $x=2, x=-2$

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20. MATHEMATICS

Toss Up: Short Answer

What is the sum of the interior angles of a nonagon?

Bonus Answer: 1260 degrees

Bonus: Short Answer

How many edges does an octahedral prism have?

Bonus Answer: 24 edges

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21. MATHEMATICS

Toss Up: Short Answer

Convert CD from hexadecimal to decimal

Bonus Answer: 205

Bonus: Short Answer

Convert CD from hexadecimal to binary

Bonus Answer: 11001101

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22. MATHEMATICS

Toss Up: Short Answer

What is the slope of the tangent line to $e^{(2x)}$ at $x=3$

Bonus Answer: $2e^6$

Bonus: Short Answer

What is the slope of the tangent line to $\ln(2x)$ at $x=3$

Bonus Answer: $1/3$

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23. MATHEMATICS

Toss Up: Short Answer

What is the area of the circle $x^2+y^2=6/\pi^2$ in terms of π

Bonus Answer: $6/\pi$

Bonus: Short Answer

What is the area of the circle $x^2 - 4x + y^2 - 6y - 2 = 0$ in terms of π

Bonus Answer: 15π

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24. MATHEMATICS

Toss Up: Short Answer

How many distinct roots does $x^3 - 6x^2 + 32$ have

Bonus Answer: 2

Bonus: Short Answer

What is the remainder when $x^3 - 6x^2 + 32$ is divided by $x - 1$

Bonus Answer: 27

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25. MATHEMATICS

Toss Up: Short Answer

How many seconds are in a day

Bonus Answer: 86400

Bonus: Multiple Choice

How many days are in a second

W) $1.16 \cdot 10^{-3}$

X) $1.16 \cdot 10^{-4}$

Y) $1.16 \cdot 10^{-5}$

Z) $1.16 \cdot 10^{-6}$

Bonus Answer: Y

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26. MATHEMATICS

Toss Up: Short Answer

What is $\tan(\arcsin(9/41))$

Bonus Answer: $9/40$

Bonus: Short Answer

What is $\sin(\operatorname{arccot}(\tan(\arccos(3/5))))$

Bonus Answer: $3/5$

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27. MATHEMATICS

Toss Up: Short Answer

What is the amplitude of $y = 4\sin(5x + 3)$

Bonus Answer: 4

Bonus: Short Answer

What is the period of $y = 4\sin(5x + 3)$ rounded to the nearest 100th

Bonus Answer: 1.26

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28. MATHEMATICS

Toss Up: Short Answer

Prime factorize 273.

Bonus Answer: $3 \cdot 7 \cdot 13$

Bonus: Multiple Choice

Which of the following numbers is a root of unity?

W) $1 + i$

X) $\frac{1}{2} + \frac{1}{2}(\sqrt{3})(i)$

Y) $3 + 4i$

Z) $\frac{1}{2}(\sqrt{2}) + \frac{3}{2}(\sqrt{2})(i)$

Bonus Answer: X

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29. MATHEMATICS

Toss Up: Short Answer

How many ways are there to distribute 4 identical blue balls and 3 identical red balls among 3 people?

Bonus Answer: 315 (accept $21 \cdot 15$)

Bonus: Short Answer

What are all possible shapes that are yielded by inverting a circle across another circle?

Bonus Answer: circle, line

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30. MATHEMATICS

Toss Up: Multiple Choice

How many zeroes does the equation $x^3 + 6x^2 + 11x + 6 = 0$ have?

W) 1

X) 2

Y) 3

Z) 4

Toss Up Answer: Y

Bonus: Short Answer

Find the solution(s) of the following system of linear equations: $2x + 4y = 6$, $x + 2y = 5$

Bonus Answer: No solutions

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31. MATHEMATICS

Toss Up: Multiple Choice

If x is positive, what is the minimum value of $2x + (2/x)$?

W) 1

X) 2

Y) 3

Z) 4

Toss Up Answer: Z

Bonus: Multiple Choice

What is $f(15)$ if $f(x) = 2\sin(x)\cos(x)$, where x is in degrees?

W) $(\sqrt{3})/2$

X) $1/(\sqrt{3})$

Y) $1/2$

Z) 1

Bonus Answer: Y

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32. MATHEMATICS

Toss Up: Short Answer

$$(mn)^2 - p^2$$

Bonus Answer: $(mn - p)(mn + p)$

Difference of Squares

Bonus: Short Answer

Factor $a^4 + 4b^4$

Bonus Answer: $(a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$

Solution:

$$a^4 + 4b^4 = a^4 + 4a^2b^2 + 4b^4 - 4a^2b^2$$

$$(a^2 + 2b^2)^2 - (2ab)^2 = (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$$

Difference of Squares

(This is also the Sophie Germain Identity)

33. MATHEMATICS

Toss Up: Multiple Choice

$f(x) = x^4 + 5x^8 + 7x^6 + 5$ Which of the following is true about $f(x)$?

W) $f(x)$ is odd, only

X) $f(x)$ is even, only

Y) $f(x)$ is neither odd nor even

Z) $f(x)$ is both even and odd

Toss Up Answer: X

Bonus: Short Answer

Simplify: $\cos(\sin^{-1}(3/5))$

Bonus Answer: $4/5$

Solution:

Draw a 3-4-5 right triangle and find the angle that $\sin^{-1}(3/5)$ is equal to. Then, take its cosine.

34. MATHEMATICS

Toss Up: Multiple Choice

Which of the following types of symmetry does $x^2 + (y-1)^2 = 36$ have?

W) Y-axis symmetry, only

X) X-axis symmetry, only

Y) Both x and y axis symmetry

Z) Neither x nor y axis symmetry

Toss Up Answer: Y

Bonus: Multiple Choice

Which of the following types of symmetry does $x^2 - y^2 = 36$ have?

W) Y-axis symmetry, only

X) X-axis symmetry, only

Y) Both x and y axis symmetry

Z) Neither x nor y axis symmetry

Bonus Answer: Y

35. MATHEMATICS

Toss Up: Multiple Choice

What is the area of an equilateral triangle with side lengths 2?

- W) 1
- X) $\sqrt{3}$
- Y) $\sqrt{6}$
- Z) 2

Toss Up Answer: X

Bonus: Multiple Choice

If the roots of $P(x) = 64x^2 - 24x + 2$ are A and B, what is the harmonic mean of A and B in simplest fractional form?

- W) $\frac{1}{3}$
- X) $\frac{1}{6}$
- Y) $\frac{1}{12}$
- Z) $\frac{1}{24}$

Bonus Answer: X

36. MATHEMATICS**Toss Up: Multiple Choice**

Given $P(x) = x^2 + ax + b$ and $P(1) = 9$, what is $a + b$?

- W) 8
- X) 10
- Y) 9
- Z) 1

Toss Up Answer: W

Bonus: Multiple Choice

Factor the following completely: $(4x^4) - (20x^2) + 16$

- W) $(x+1)(x-1)(x+2)(x-2)$
- X) $(x+1)(x+1)(x-2)(x-2)$
- Y) $4(x+1)(x-1)(x+2)(x-2)$
- Z) $4(x+1)(x+1)(x-2)(x-2)$

Bonus Answer: Y

37. MATHEMATICS**Toss Up: Multiple Choice**

Simplify: $(24 \cdot 63) / (72 \cdot 21)$

- W) 1
- X) 3
- Y) 7
- Z) $\frac{1}{3}$

Toss Up Answer: W

Bonus: Short Answer

Simplify $(333(x^4 - x^2)) / (9(x^2 + 2x + 1)(x^2))$

Bonus Answer: $(x-1) / ((37)(x+1))$

38. MATHEMATICS

Toss Up: Short Answer

Compute: $\sqrt{(3 \cdot 4 \cdot 5 - 60)^2}$

Bonus Answer: 0

Bonus: Short Answer

Compute $(34/17 - 2 + 5) \cdot (96 + -144) \cdot (1/(46 - 130/5))$

Bonus Answer: -12

39. MATHEMATICS

Toss Up: Short Answer

What is the slope of a line parallel to $45x = 9y + 5$?

Bonus Answer: 5

Bonus: Multiple Choice

How many vertical asymptotes does $f(x) = (x^2 + 2x + 1)/(x^3 - x)$ have?

W) 0

X) 1

Y) 2

Z) 3

Bonus Answer: Y

40. MATHEMATICS

Toss Up: Multiple Choice

Which of the following is true?

W) -1 is congruent to 1 modulo 5

X) -1 is congruent to 2 modulo 5

Y) -1 is congruent to 3 modulo 5

Z) -1 is congruent to 4 modulo 5

Toss Up Answer: Z

Bonus: Short Answer

Which is larger, 8^{10} or 3^{30}

Bonus Answer: 3^{30}

41. MATHEMATICS

Toss Up: Short Answer

What is the solution for square root of the quantity $x+5$ equals $x+3$

Bonus Answer: -1 *(do not except: [-1 and -4] or [-3])

Bonus: Short Answer

In a certain language of a certain people of a certain world, to frack a number is to find the average of the number and its square. For example, the frack of 2 is 3 since it is the average of 2 and 2 squared. what is the frack of 100?

Bonus Answer: 2020

42. MATHEMATICS

Toss Up: Multiple Choice

The sum $(1+3+3^2+\dots+3^n)$ is equal to which of the following?

W) $2 \cdot 3^n$

X) $4 \cdot 3^n$

Y) $(3^{(n+1)}+1)/2$

Z) $(3^{(n+1)}-1)/2$

Toss Up Answer: Z

Bonus: Short Answer

In space, what is the graph of the equation $x^2 = y$?

Bonus Answer: A parabolic cylinder

43. MATHEMATICS

Toss Up: Short Answer

What is the name for the rule that states that when you have a limit of indeterminate form as a fraction, it is equal to the limit of the derivative of the numerator over the derivative of the denominator?

Bonus Answer: L'hospital's rule

Bonus: Short Answer

What famous function, named after a mathematician, takes the infinite sum of the reciprocals of the natural numbers raised to an argument?

Bonus Answer: Riemann zeta function

44. MATHEMATICS

Toss Up: Multiple Choice

A statement about a relationship between natural quantities. The statement has been tested many times, and has not been contradicted. What is it?

W) Fact

X) Hypothesis

Y) Law

Z) Theory

Toss Up Answer: Y

Bonus: Multiple Choice

A framework that includes many well-tested and verified hypotheses about one or more aspects of the natural world.

W) Fact

X) Hypothesis

Y) Law

Z) Theory

Bonus Answer: Z

45. MATHEMATICS

Toss Up: Short Answer

The surface area of a sphere is how many times the area of a circle?

Bonus Answer: 4

Bonus: Short Answer

What is the area of a cone?

Bonus Answer: $(\pi)r^2h$

46. MATHEMATICS

Toss Up: Short Answer

What type of number is defined as a number that is not a root of a polynomial with rational coefficients?

Bonus Answer: Transcendental numbers

Bonus: Short Answer

Of the following, which of these numbers of transcendental? π , e , i , ϕ , $e \times i$, $2^{\sqrt{2}}$

Bonus Answer: π , e , $e \times i$, $2^{\sqrt{2}}$

47. MATHEMATICS

Toss Up: Short Answer

Given $y = |x - 6|$, what is the derivative at $x = 6$?

Bonus Answer: The derivative does not exist

Bonus: Short Answer

Find the derivative of x^3 / x^2

Bonus Answer: 1

48. MATHEMATICS

Toss Up: Short Answer

Find the derivative of: $4x^3 + 18x + 2$

Bonus Answer: $8x^2 + 18$

Bonus: Short Answer

At what point is the slope of the tangent line to the parabola $y = 3x^2 + 5x + 23$ equal to 0?

Bonus Answer: $(-5/6, 441/36)$

49. MATHEMATICS

Toss Up: Short Answer

Write an equation for a sine function with an amplitude of $5/3$, a period of $\pi/2$, and a vertical shift of 4 units up.

Bonus Answer: $y = (5/3)\sin(4x) + 4$

Bonus: Short Answer

Two large and 1 small pumps can fill a swimming pool in 4 hours. One large and 3 small pumps can also fill the same swimming pool in 4 hours. How many hours will it take 4 large and 4 small pumps to fill the swimming pool.

Bonus Answer: 1 hour and 40 minutes

50. MATHEMATICS

Toss Up: Multiple Choice

If a regular hexagon has a side length of 2, what is its area?

W) $3\sqrt{6}$

X) $6\sqrt{3}$

Y) $\sqrt{18}$

Z) $6\sqrt{2}$

Toss Up Answer: X

Bonus: Multiple Choice

If $f(x)$ is $(\ln(x^2))/(e^x)$, what is $f'(x)$ in simplest form?

W) $((x^2)/2x + \ln(x^2))/(e^{2x})$

X) $((2/x) - \ln(x^2))/(e^2)$
Y) $(\ln(4/x))/(e^{2x})$
Z) $((2x/(x^2)) - \ln(x^2))/(e^{2x})$

Bonus Answer: X

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51. MATHEMATICS

Toss Up: Multiple Choice

$3^{12} = 27^x$. What is the value of x?

W) $x=12$

X) $x=4$

Y) $x=6$

Z) $x=3$

Toss Up Answer: X

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Bonus: Short Answer

$3^{12} + 3^{11} + 3^{10} = b \cdot 3^a$ What is the positive difference between the product and the sum of a and b?

Bonus Answer: 107

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52. MATHEMATICS

Toss Up: Multiple Choice

Donald Trump has $10x+5$ apples where $x=3$. How many apples does he have if he eats two of them?

W) 35

X) 39

Y) 30

Z) 33

Toss Up Answer: Z

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Bonus: Short Answer

Donald Trump has a tower consisting of $0.00405 \cdot 10^{45}$ grams of gold. How many kilograms of gold is in his tower?
Express in scientific notation.

Bonus Answer: $4.05 \cdot 10^{39}$ (kilograms of gold)

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53. MATHEMATICS

Toss Up: Short Answer

What is the volume of a sphere of radius "R"?

Bonus Answer: $(\frac{4}{3}) \pi R^3$

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Bonus: Short Answer

Using an x-y coordinate axis, a parabola is given by the equation $y = x^2$. Give the x-y coordinates of the focal point for this parabola.

Bonus Answer: (0, 1/4)

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54. MATHEMATICS

Toss Up: Short Answer

Using an x-y coordinate axis, a parabola is represented by the equation $x^2 = 6y$. The vertex of this parabola is at what coordinate point?

Bonus Answer: (0,0)

Bonus: Short Answer

For a right triangle, the $\sin(A)$ is $3/5$. To what value is the $\tan(A)$ equal?

Bonus Answer: $3/4$
