MATHEMATICS

1. MATHEMATICS

Toss Up: Multiple Choice

If the test scores of Mr. Coco's Calculus test are assumed to have a normal distribution, and if the mean test score was 91, what can be said about the median test score of that test?

W) The median was lower than 91

X) The median was higher than 91

Y) The median was 91

Z) The median cannot be determined

Toss Up Answer: Y

Bonus: Multiple Choice

The z-score for Shantanu's physics test grade was found to be 0.375. If he scored a 94 on the test, and the standard deviation was 8, what was the mean test score on the physics test?

W) 91

X) 95

Y) 92

Z) 98

Bonus Answer: W

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2. MATHEMATICS

Toss Up: Short Answer

In simplest terms, find the value of the limit as x approaches 0 of (4^x - 2^x) / x

Bonus Answer: ln(2) (do not accept ln(4) - ln(2))

Bonus: Short Answer

How many real solutions for x does the equation x = ln(x) + 2 have?

Bonus Answer: 2

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3. MATHEMATICS

Toss Up: Multiple Choice

A right triangle has a hypotenuse of length 12. The altitude to the hypotenuse has a length of 8. What is the area of this triangle?

W) 36

X) 48

Y) 60

Z) The triangle is impossible to construct

Toss Up Answer: Z

Bonus: Short Answer

Two REAL numbers x and y satisfy the system of equations: x + y = 12; x^2 + y^2 = 64. Find the value of the product xy.

Bonus Answer: The system has no solutions.

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4. MATHEMATICS

Toss Up: Short Answer

If n is a positive integer, what is the smallest value of n such that n! + 1 is a perfect square?

Bonus Answer: 4

Bonus: Short Answer

If z1 = 3 - 4i and z2 = 7 + i, find the absolute value of z1z2 in simplest terms.

Bonus Answer: 25*sqrt(2)

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5. MATHEMATICS

Toss Up: Short Answer

What is the inverse of the 2x2 matrix (row 1: 6 10), (row 2: 3 5)?

Bonus Answer: The matrix has no inverse (b.c. determinant = 0).

Bonus: Short Answer

The legs of an isosceles triangle have a length of 10, and the altitudes to the legs have a length of 6. In simplified radical form, what is the length of the altitude to the base of the triangle?

Bonus Answer: 3 * sqrt(10) (Do not accept sqrt(90))

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6. MATHEMATICS

Toss Up: Short Answer

What is the 5th non-triangular number?

Bonus Answer: 8

Bonus: Short Answer

Name all the following that are true:

1. An icosahedron has 18 faces.
2. A regular hexahedron has 16 edges.
3. There are only nine regular polyhedra.
4. A regular octahedron has 4 times the volume of a regular tetrahedron with the same side length.

Bonus Answer: 2,4 (an icosahedron has 20 faces; a cube has 12 edges)

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7. MATHEMATICS

Toss Up: Multiple Choice

If θ is an angle such that sin(θ) < 0 and cos (θ) = 0, where in the coordinate plane is it located?

W) Between the 2nd and 3rd quadrants

X) Between the 3rd and 4th quadrants

Y) Between the 1st and 4th quadrants

Z) Between the 1st and 2nd quadrants

Toss Up Answer: Y

Bonus: Short Answer

What is the remainder of x^10+x+1 divided by (x-1)^2 ?

Bonus Answer: 11x-8

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8. MATHEMATICS

Toss Up: Multiple Choice

Given the equation of a conic section, y^2/16 - x^2/25 = 1, what is the length of the conjugate axis?

W) 16

X) 8

Y) 10

Z) 25

Toss Up Answer: Y

Bonus: Multiple Choice

Compute log(base 2) of 2048^2

W) 20

X) 22

Y) 24

Z) 26

Bonus Answer: X

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9. MATHEMATICS

Toss Up: Multiple Choice

A point with coordinates (3,5) is rotated 90 degrees clockwise about the point (2,1) and is then reflected across the y-axis. What are the coordinates of the resulting image?

W) (-5,-3)

X) (-4,0)

Y) (-6,0)

Z) (-5,-1)

Toss Up Answer: Y

Bonus: Multiple Choice

A quadrilateral ABCD has two congruent opposite side segments, AB and CD. Which of the following would have to be true in order for the quadrilateral to be a square?

W) The circumcircle of ABC is centered at the midpoint of AC

X) The medians of ABCD are perpendicular

Y) The perpendicular bisectors of AB and CD are the same line

Z) None of the above.

Bonus Answer: Z

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10. MATHEMATICS

Toss Up: Multiple Choice

Which of the following cannot be the root of a polynomial with rational coefficients?

W) 5*i + 6

X) phi - 1/2, where phi is the golden ratio

Y) sqrt(3+i)

Z) 6 - pi*i

Toss Up Answer: Z

Bonus: Short Answer

What is the name given to a number that is not a root of any non-zero polynomial equation with rational coefficients?

Bonus Answer: Transcendental number (Accept transcendental)

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11. MATHEMATICS

Toss Up: Multiple Choice

Fermat's Last Theorem conjectures that no three positive integers a,b, and c can satisfy the equation a^n + b^n = c^n for any integer value of n greater than 2. It went unproved for more than 300 years until it was finally proved in 1994 by a mathematician who received a substantial monetary prize only this year. What is the name of this mathematician?

W) Grigori Perelman
X) John Nash
Y) Andrew Wiles
Z) Terence Tao
Toss Up Answer: Y

Bonus: Short Answer
Prime numbers of the form 2 ^ (2^n) + 1, where n is a non-negative integer, are known as this.
Bonus Answer: Fermat Primes

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12. MATHEMATICS

Toss Up: Short Answer
If Bruce can do a job in 3 hours, and Clark can do the same job in 4 hours, how long, in hours, will it take them to do the job if they work together? You may leave your answer as a fraction.
Bonus Answer: 12/7

Bonus: Short Answer
What is the eccentricity of an ellipse with a = 5 and b = 4?
Bonus Answer: 3/5

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13. MATHEMATICS

Toss Up: Short Answer
Find the perimeter of a right triangle with legs 11 and 60.
Bonus Answer: 132

Bonus: Short Answer
In a 15-75-90 degrees right triangle with hypotenuse 4, what are the lengths of the shorter and longer leg, respectively? Exact answers please.
Bonus Answer: radical 6 - radical 2, radical 6 + radical 2

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14. MATHEMATICS

Toss Up: Short Answer
If the first term in an arithmetic sequence is 2 and the third term is 6, find the 10th term.
Bonus Answer: 20

Bonus: Short Answer
Compute the sum of the first 100 positive integers.
Bonus Answer: 5050

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15. MATHEMATICS

Toss Up: Short Answer
If the log of n is 10, what is the log of 100n?
Bonus Answer: 12

Bonus: Short Answer
How many ways are there to seat 5 people around a circular table, if rotations are ignored?
Bonus Answer: 24

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16. MATHEMATICS

Toss Up: Multiple Choice

What is the geometric mean of the roots of the polynomial 2x^3 - 2x^2 - 228x - 432
W) 4
X) 6
Y) 8
Z) 10
Toss Up Answer: X

Bonus: Short Answer
The first term of an arithmetic sequence is x, and the xth term is x^2. If the common difference is equal to 7, and none of the terms are equal, what is the 12th term of the sequence?
Bonus Answer: 84

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17. MATHEMATICS

Toss Up: Short Answer
Completely expand (x-2y)^3
Bonus Answer: x^3 - 6x^2*y + 12x*y^2 - 8y^3

Bonus: Short Answer
What is the sums of all the terms in the seventh row of Pascal's triangle?
Bonus Answer: 128

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18. MATHEMATICS

Toss Up: Multiple Choice
Which of the following distances is longest?
W) Half a kilometer
X) Half a decimeter
Y) 600 meters
Z) 40 decameters
Toss Up Answer: Y

Bonus: Short Answer
What is the name of a line that intersects two or more coplanar lines in different points?
Bonus Answer: Transversal

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19. MATHEMATICS

Toss Up: Multiple Choice
The number (sqrt(2))^(sqrt(2)) is:
W) Rational
X) Algebraic irrational
Y) Transcendental
Z) Impossible to tell
Toss Up Answer: Y

Bonus: Short Answer
Let f(x) = ax^7 + bx^3 + cx - 5 , where a,b, and c are real numbers. What is the value of f(7) if f(-7) = 8?
Bonus Answer: -18

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20. MATHEMATICS

Toss Up: Short Answer
What is the limit as x approaches 0 of x^x?

Bonus Answer: Does not exist. (NOT 1)

Bonus: Short Answer

Let f(x) be a function with no real roots. If f(3) = 4, then in what quadrants can f(x) be in?

Bonus Answer: All four quadrants (b.c. there's no specification f has to be continuous)

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21. MATHEMATICS

Toss Up: Multiple Choice

1. What statistical average is most appropriate to use when the quantities being averaged when one or more of the quantities are not necessarily bounded?

W) Geometric mean

X) median

Y) Harmonic mean

Z) Arithmetic mean

Toss Up Answer: Y

Bonus: Short Answer

You pick two cards at random without replacement from a standard, 52 card deck. Compute the probability

exactly one is a heart.

Bonus Answer: 13/34

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22. MATHEMATICS

Toss Up: Multiple Choice

Which of the following expressions in x grows fastest?

W) 3^x [3 to the x]

X) 10(3/2)^x [10 times three halves to the x]

Y) x^2 [x squared]

Z) 2^(2x) [2 to the 2x]

Toss Up Answer: Z

Bonus: Short Answer

Given 504=2^3 x 3^2 x 7 [504 equals 2 cubed times 3 squared times 7], compute the sum of the positive divisors of 504

Bonus Answer: 1560

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23. MATHEMATICS

Toss Up: Short Answer

Which of the following statements concerning prime numbers are true?

1. If a and n are relatively prime than there are infinitely many primes that leave a remainder of a when divided by n.
2. All integers less than p are quadratic residues modulo p
3. There are infinitely many prime numbers

Bonus Answer: 1 and 3

Bonus: Short Answer

If a1+a2+a3+a4+a5=17 for positive integers a1,a2,a3,a4,a5, then maximize the product a1a2a3a4a5?

Bonus Answer: 432

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24. MATHEMATICS

Toss Up: Multiple Choice

Simplify (sin15cos75-cos15sin75)/(cos15cos75+sin15sin75)
W) negative square root of 3
X) 1/2
Y) -1/2
Z) square root 6 minus square root 2 all over 2
Toss Up Answer: W

Bonus: Short Answer
Compute (2+2sqrt(3)i)^6. [2+2 times the square root of 3, i to the 6th power]
Bonus Answer: 4096

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25. MATHEMATICS

Toss Up: Multiple Choice
Compute 3C1+4C2+5C3+6C4+7C5. [3 choose 1 + 4 choose 2 + 5 choose 3 + 6 choose 4 + 7 choose 5]
W) 55
X) 336
Y) 240
Z) 56
Toss Up Answer: W

Bonus: Short Answer
Compute the square root of 5476
Bonus Answer: 74

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26. MATHEMATICS

Toss Up: Short Answer
In Triangle ABC the angle bisector of A intersects BC at D. Given AB=9, AC=21, and DB=15, compute DC.

Bonus Answer: 35

Bonus: Short Answer
In triangle ABC, AB=14, AC=13, and BC=15. Let the incenter be I. Compute CI in simplest radical form.
Bonus Answer: square root of 65

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27. MATHEMATICS

Toss Up: Short Answer
which of the following are true about the number and nature of the roots of the polynomial
x^3 - 29x^2 + 229x

I. 0 is a root
II. There must be a real, positive root
III. There must be three real roots

Bonus Answer: I

Bonus: Short Answer
Solve the following linear system of equations in 3 variables:

2x + y + 4z=9
-x + 2y + 5z=-11

5x + 3y + 3z=-4

Bonus Answer: x=4,y=-11,z=3

Also accept (4,-11,3)

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28. MATHEMATICS

Toss Up: Multiple Choice

MULTIPLE CHOICE: tossup

For an infinite sequence of numbers a_0,a_1, ..., which of the following best describes the polynomial P(x)= sum(i=0 to infinity) a_i*x^i.

W) Reimann-Zeta function

X) Generating function

Y) Newtonian series

Z) Tchebychev polynomial

Toss Up Answer: X

Bonus: Short Answer

In the Pell equation x^2-3y^2=1 the smallest solution is x=2 and y=1. The next smallest solution is x=7 and y=4. Compute the third smallest solution.

Bonus Answer: x=26, y=15

Also accept (26,15)

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29. MATHEMATICS

Toss Up: Multiple Choice

Which of the following is not a group?

W) the integers under multiplication

X) the integers under addition

Y) the symmetries of a regular n-gon

Z) the permutations of integers between 1 and n

Toss Up Answer: W

Bonus: Short Answer

find the smallest positive solution to the following congruences.

x congruent to 3 mod 13

x congruent to 8 mod 11

Bonus Answer: x=107

Also accept: 107

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30. MATHEMATICS

Toss Up: Multiple Choice

What mathematician proved the existence of a straight edge and compass construction of a regular 17-gon?

W) Euler

X) Euclid

Y) Gauss

Z) Galois [Gal-wah]

Toss Up Answer: Y

Bonus: Short Answer

Given sinX=1/3 [sine of x equals one third], compute cos^2(3X) [co-sine squared of three x].

Bonus Answer: -23/27

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31. MATHEMATICS

Toss Up: Short Answer

Tim is 5 times older than his younger brother. In 3 years Tim will be 3 times older than his younger brother. Compute the difference in the ages of Tim and his younger brother.

Bonus Answer: 12

Bonus: Short Answer

How many positive integers n have no solutions to the following equation where a and b are nonnegative integers?
3a+7b=n

Bonus Answer: 6

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32. MATHEMATICS

Toss Up: Multiple Choice

To test products, there is a test that identifies 95% of defective products as defective, but also labels 10% of all working products as defective. If 85% of the products made are not defective, what probability of products the test determines to be defective are actually defective?

W) 39%
X) 84%
Y) 67%
Z) 42%

Toss Up Answer: Y

Bonus: Short Answer

Compute the surface area of a cylinder with radius 8 and height 2

Bonus Answer: 160pi

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33. MATHEMATICS

Toss Up: Multiple Choice

If x+(1/x)=2, find x^128+(1/x)^128
W) 256
X) 128
Y) 64
Z) 2

Toss Up Answer: Z

Bonus: Short Answer

Given the quadratic x^2-20x+9 and its roots p and q, find (1/p)^2+(1/q)^2

Bonus Answer: 382/81

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34. MATHEMATICS

Toss Up: Multiple Choice

Let f be an odd function over the real numbers. What is the value of f at 0?
W) -1
X) 0
Y) 1
Z) There is not enough information to determine an answer

Toss Up Answer: Z

Bonus: Multiple Choice

Which of the following is true?

W) An irrational number raised to an irrational power must have an irrational value.

X) For all positive number x and y, x^ln(y) is equal to y^ln(x)

Y) For all real numbers a,b,c,d with b and d being non-zero, if a/b > c/d, then a*d must be greater than b*c

Z) There are more complex numbers than there are real numbers.

Bonus Answer: X

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35. MATHEMATICS

Toss Up: Short Answer

How many vertices does a dodecahedron have?

Bonus Answer: 20

Bonus: Short Answer

In a+bi form, what is the square root of i?

Bonus Answer: (1/(sqrt2)) + (1/(sqrt2))i

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36. MATHEMATICS

Toss Up: Short Answer

What is 11 base 5 in base 2?

Bonus Answer: 110

Bonus: Short Answer

If log base 10 of x is 100, what is log base 100 of x?

Bonus Answer: 50

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37. MATHEMATICS

Toss Up: Short Answer

What is the sum of the infinite geometric series whose first term is 1 and fourth term is 1/64?

Bonus Answer: 4/3

Bonus: Multiple Choice

What is the value of e to the (pi times i/2)?

W) e^-1

X) 1

Y) -1

Z) i

Bonus Answer: Z

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38. MATHEMATICS

Toss Up: Short Answer

What is the value of the quantity ((sin^4)x) + 2(cos^2)x(sin^2)x + (cos^4)x)^2?

Bonus Answer: 1

Bonus: Multiple Choice

What is the probability of selecting two natural numbers and having them be relatively prime?
W) 1/2
X) 2/3
Y) 6/(pi^2)
Z) 2/pi

Bonus Answer: Y

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39. MATHEMATICS

Toss Up: Short Answer

What is the integral of sec x dx?

Bonus Answer: Ln (abs(sec x + tan x)) + C

Bonus: Multiple Choice

Which of these functions cannot be integrated and represented with elementary functions?
W) (sin^5)x times (cos^6)x dx
X) e^x times x^3 dx
Y) e^(x^2) dx
Z) tan x sin x dx

Bonus Answer: Y

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40. MATHEMATICS

Toss Up: Short Answer

What is (1000! + 999!)/(998!)?

Bonus Answer: 999,999

Bonus: Short Answer

What is the first number with 5 distinct prime factors?

Bonus Answer: 2,310

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41. MATHEMATICS

Toss Up: Short Answer

What is the relationship between the surface area and the volume of a sphere?

Bonus Answer: The surface area is the derivative of the volume/ the volume is the integral of the surface area when the constant is 0

Bonus: Multiple Choice

When the inner diagonal of a cube is 7 times root 3 inches, what is the surface area of the cube?
W) 216 square inches
X) 294 square inches
Y) 343 square inches
Z) 512 square inches

Bonus Answer: X

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42. MATHEMATICS

Toss Up: Multiple Choice

Given two attempts, what is the probability of getting a multiple choice question correct, if there are 5 choices?
W) 1:3

X) 1:4
Y) 2:3
Z) 2:4

Toss Up Answer: Y

Bonus: Multiple Choice

What percentage of data falls within 2 standard deviations of the mean, assuming a normal distribution?
W) 34.10%
X) 47.70%
Y) 68.20%
Z) 95.40%

Bonus Answer: Z

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43. MATHEMATICS

Toss Up: Short Answer

What is the 3rd mercenne prime?

Bonus Answer: 31

Bonus: Multiple Choice

How many terms does the 15th integral of sin x dx have?
W) 1
X) 2
Y) 15
Z) 16

Bonus Answer: Z

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44. MATHEMATICS

Toss Up: Multiple Choice

What is the slope intercept in the equation y = 4x + 3 -2
W) 3
X) 1
Y) -2
Z) 2

Toss Up Answer: X

Bonus: Multiple Choice

If 3x - y = 12, what is (8^x) / (2^y)
W) 2^12
X) 4^4
Y) 8^3
Z) Cannot be determined

Bonus Answer: W

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45. MATHEMATICS

Toss Up: Short Answer

What shape is X^2+1

Bonus Answer: parabola

Bonus: Short Answer

what type of function is x+2x

Bonus Answer: a line

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46. MATHEMATICS

Toss Up: Short Answer

What is the volume of a sphere of radius "R"?

Bonus Answer: (4/3)piR^3

Bonus: Short Answer

Using an x-y coordinate axis, the figure represented by the equation [x^2/36] + [y^2/16] =1 is centered about what x-y coordinate point?

Bonus Answer: (0,0) ; the origin

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47. MATHEMATICS

Toss Up: Short Answer

In a normal distribution, approximately what percentage of the cases, to the nearest whole number, falls within 4 standard deviations of the mean:

Bonus Answer: 100%

Bonus: Multiple Choice

Which of the following properties would you use to compute the chances of rolling either a 7 or an 11 with a pair of dice:

W) multiplicative
X) conditional
Y) independent
Z) additive

Bonus Answer: Z

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48. MATHEMATICS

Toss Up: Multiple Choice

Give the range for the following six values 2, 7, 11, 19, 25, 33:

W) 2
X) 31
Y) 33
Z) 15

Toss Up Answer: X

Bonus: Short Answer

What percent of a circle is 6/5pi radians?

Bonus Answer: 216

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49. MATHEMATICS

Toss Up: Multiple Choice

Which of the following salts is responsible for the browning of pretzels?

W) Sodium Chloride

X) Sodium Hydroxide
Y) Potassium Chloride
Z) Potassium Carbonate
Toss Up Answer: X

Bonus: Short Answer
Consider the reaction A + 2B yields 4C. Assume all species in the reaction are gaseous. If the reaction is at equilibrium, and the concentration of every species is 2 molar, calculate the equilibrium constant of the reaction.
Bonus Answer: 2

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50. MATHEMATICS

Toss Up: Short Answer
You are trying to give 5 apples to 3 friends. You can give any number of apples to each friend, including 0. How many ways are there to share the apples?
Bonus Answer: 56 (its 8C3)

Bonus: Multiple Choice
If a cubic function equals 0 at exactly two points, which of the following must be true?
W) the function passes through the origin
X) there is a double root
Y) the function is even
Z) one of the roots is imaginary

Bonus Answer: X

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51. MATHEMATICS

Toss Up: Short Answer
What is the length of the longest diagonal of a unit cube?
Bonus Answer: root3

Bonus: Short Answer
What is the largest integer that cant be written as the sum of 3's and 4's?
Bonus Answer: 5

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