1 Setup

In the `lorenz_63` notebook, we apply a greedy D-optimal experimental design algorithm to the Lorenz-63 sensitivity matrix `L63data.mat`.

The goal is to determine the most informative observable–time pairs for estimating the system parameters $\beta = [\sigma, \rho, \beta]$.
Let

$$y = X\beta + \varepsilon \tag{1}$$

where:

- $X \in \mathbb{R}^{1200\times 3}$ is the sensitivity matrix derived from partial derivatives of the observables $x(t), y(t), z(t)$ with respect to parameters σ, ρ, β
- Each row of X represents one observable at one time point

2 Implementation Details

We implement the **Fedorov exchange algorithm** as described in St. John and Draper (1975), following equations (14)–(16). For our Lorenz-63 data:

- The design matrix X has dimensions 1200×3, composed of sensitivity data for three observables (x, y, z) at 400 time points each
- We initialize with random weights that sum to 1
- At each iteration, we:
  1. Compute the dispersion matrix $M = X^T \text{diag}(w) X$ and its inverse M^{-1}
  2. Calculate d values for each row j using $d_j = X_j^T M^{-1} X_j$ per equation (14)
  3. Find the index with maximum d value
  4. Compute $\alpha_i = \frac{d_{max}-p}{p(d_{max}-1)}$ using equation (15), where $p = 3$
  5. Update weights using equation (16): $w_{new} = (1-\alpha_i)w + \alpha_i e_{max}$
- The algorithm converges when $\alpha_i < 0.01$.

3 Interpretation

The algorithm identifies the most informative observable–time pairs for parameter estimation in the Lorenz-63 system. The results show that the y observable provides the highest information gain (66.3% of total weight), followed by the z observable (33.3%), while the x observable contributes minimally (0.4%).