D-Optimal Design on Lorenz-63 Data

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In the `lorenz_63` notebook, we apply a greedy D-optimal experimental design algorithm to the Lorenz-63 sensitivity matrix `L63data.mat`.

The goal is to determine the most informative observable–time pairs for estimating the system parameters $\beta = [\sigma, \rho, \beta]$.

1 Setup

Let

$$y = X\beta + \varepsilon \tag{1}$$

where:

- $X \in \mathbb{R}^{1200\times 3}$ is the sensitivity matrix derived from partial derivatives of the observables $x(t), y(t), z(t)$ with respect to parameters σ, ρ, β
- Each row of X represents one observable at one time point

We apply a greedy D-optimal algorithm to select k rows of X that maximize the determinant of the Fisher Information Matrix:

$$\det(X_S^T X_S) \tag{2}$$

This comes from equation (11) in St. John and Draper (1975), which states that a D-optimal design maximizes $\det(M(\xi))$, where $M(\xi) = \frac{1}{n}X^T X$ and $M(\xi)$ is nonsingular.

1.1 Algorithm Used

We follow the **Fedorov exchange algorithm** from St. John and Draper (1975, Eq. (14) and Algorithm Steps 1–6), where at each step we:

1. Evaluate the determinant gain from each candidate row.
2. Add the row with the highest contribution to the design.
3. Repeat until the design budget k is met.

2 Results

For a design size of $k = 3$, the selected rows were:

Index	Observable	Time	Weight
464	y	64	0.3265
744	y	344	0.3331
1050	z	250	0.3295

2.1 Total weight by observable:

- x: 0.0036 (0.4%)
- y: 0.6632 (66.3%)
- z: 0.3332 (33.3%)

3 Interpretation

The results show that the observable $y(t)$ provides the highest information gain for estimating parameters of the Lorenz-63 system. Observable $x(t)$ contributes minimally to the determinant, indicating low value in parameter estimation from that trajectory alone.

4 References

References

[1] St. John, R. C., & Draper, N. R. (1975). *D-Optimality for Regression Designs: A Review*. Technometrics, 17(1), 15–23.