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1 Idempotent

$A \vdash (A \wedge A)$

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$(A \wedge A) \vdash A$

Line	Assumptions	Formula	Justification	References
1	1	$(A \wedge A)$	Assumption Introduction	
2	1	A	Conjunction Elimination	1

$A \vdash (A \vee A)$

Line	Assumptions	Formula	Justification	References
1	1	A	Assumption Introduction	
2	1	$(A \vee A)$	Disjunction Introduction	1

$(A \vee A) \vdash A$

Line	Assumptions	Formula	Justification	References
1	1	$(A \vee A)$	Assumption Introduction	
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2 Commutative

$(A \wedge B) \vdash (B \wedge A)$

Line	Assumptions	Formula	Justification	References
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$(A \vee B) \vdash (B \vee A)$

Line	Assumptions	Formula	Justification	References
1	1	$(A \vee B)$	Assumption Introduction	
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$(A \wedge (B \wedge C)) \vdash ((A \wedge B) \wedge C)$

Line	Assumptions	Formula	Justification	References
1	1	$(A \wedge (B \wedge C))$	Assumption Introduction	
2	1	A	Conjunction Elimination	1
3	1	$(B \wedge C)$	Conjunction Elimination	1
4	1	B	Conjunction Elimination	3
5	1	C	Conjunction Elimination	3
6	1	$(A \wedge B)$	Conjunction Introduction	2, 4
7	1	$((A \wedge B) \wedge C)$	Conjunction Introduction	5, 6

$((A \wedge B) \wedge C) \vdash (A \wedge (B \wedge C))$

Line	Assumptions	Formula	Justification	References
1	1	$((A \wedge B) \wedge C)$	Assumption Introduction	
2	1	$(A \wedge B)$	Conjunction Elimination	1
3	1	A	Conjunction Elimination	2
4	1	B	Conjunction Elimination	2
5	1	C	Conjunction Elimination	1
6	1	$(B \wedge C)$	Conjunction Introduction	4, 5
7	1	$(A \wedge (B \wedge C))$	Conjunction Introduction	3, 6

$(A \vee (B \vee C)) \vdash ((A \vee B) \vee C)$

Line	Assumptions	Formula	Justification	References
1	1	$(A \vee (B \vee C))$	Assumption Introduction	
2	2	A	Assumption Introduction	
3	2	$(A \vee B)$	Disjunction Introduction	2
4	2	$((A \vee B) \vee C)$	Disjunction Introduction	3
5	5	$(B \vee C)$	Assumption Introduction	
6	6	B	Assumption Introduction	
7	7	C	Assumption Introduction	
8	6	$(A \vee B)$	Disjunction Introduction	6
9	6	$((A \vee B) \vee C)$	Disjunction Introduction	8
10	7	$((A \vee B) \vee C)$	Disjunction Introduction	7
11	5	$((A \vee B) \vee C)$	Disjunction Elimination	5, 6, 9, 7, 10
12	1	$((A \vee B) \vee C)$	Disjunction Elimination	1, 2, 4, 5, 11

$((A \vee B) \vee C) \vdash (A \vee (B \vee C))$

Line	Assumptions	Formula	Justification	References
1	1	$((A \lor B) \lor C)$	Assumption Introduction	
2	2	$(A \lor B)$	Assumption Introduction	
3	3	A	Assumption Introduction	
4	3	$(A \lor (B \lor C))$	Disjunction Introduction	3
5	5	B	Assumption Introduction	
6	5	$(B \lor C)$	Disjunction Introduction	5
7	5	$(A \lor (B \lor C))$	Disjunction Introduction	6
8	2	$(A \lor (B \lor C))$	Disjunction Elimination	2, 3, 4, 5, 7
9	9	C	Assumption Introduction	
10	9	$(B \lor C)$	Disjunction Introduction	9
11	9	$(A \lor (B \lor C))$	Disjunction Introduction	10
12	1	$(A \lor (B \lor C))$	Disjunction Elimination	1, 2, 8, 9, 11

4 Distributive

$(A \land (B \lor C)) \vdash ((A \land B) \lor (A \land C))$

Line	Assumptions	Formula	Justification	References
1	1	$(A \land (B \lor C))$	Assumption Introduction	
2	1	A	Conjunction Elimination	1
3	1	$(B \lor C)$	Conjunction Elimination	1
4	4	B	Assumption Introduction	
5	1, 4	$(A \land B)$	Conjunction Introduction	2, 4
6	1, 4	$((A \land B) \lor (A \land C))$	Disjunction Introduction	5
7	7	C	Assumption Introduction	
8	1, 7	$(A \land C)$	Conjunction Introduction	2, 7
9	1, 7	$((A \land B) \lor (A \land C))$	Disjunction Introduction	8
10	1	$((A \land B) \lor (A \land C))$	Disjunction Elimination	3, 4, 6, 7, 9

$((A \land B) \lor (A \land C)) \vdash (A \land (B \lor C))$

Line	Assumptions	Formula	Justification	References
1	1	$((A \land B) \lor (A \land C))$	Assumption Introduction	
2	2	$(A \land B)$	Assumption Introduction	
3	2	A	Conjunction Elimination	2
4	2	B	Conjunction Elimination	2
5	2	$(B \lor C)$	Disjunction Introduction	4
6	2	$(A \land (B \lor C))$	Conjunction Introduction	3, 5
7	7	$(A \land C)$	Assumption Introduction	
8	7	A	Conjunction Elimination	7
9	7	C	Conjunction Elimination	7
10	7	$(B \lor C)$	Disjunction Introduction	9
11	7	$(A \land (B \lor C))$	Conjunction Introduction	8, 10
12	1	$(A \land (B \lor C))$	Disjunction Elimination	1, 2, 6, 7, 11

$(A \lor (B \land C)) \vdash ((A \lor B) \land (A \lor C))$

Line	Assumptions	Formula	Justification	References
1	1	$(A \vee (B \wedge C))$	Assumption Introduction	
2	2	A	Assumption Introduction	
3	2	$(A \vee B)$	Disjunction Introduction	2
4	2	$(A \vee C)$	Disjunction Introduction	2
5	2	$((A \vee B) \wedge (A \vee C))$	Conjunction Introduction	3, 4
6	6	$(B \wedge C)$	Assumption Introduction	
7	6	B	Conjunction Elimination	6
8	6	$(A \vee B)$	Disjunction Introduction	7
9	6	C	Conjunction Elimination	6
10	6	$(A \vee C)$	Disjunction Introduction	9
11	6	$((A \vee B) \wedge (A \vee C))$	Conjunction Introduction	8, 10
12	1	$((A \vee B) \wedge (A \vee C))$	Disjunction Elimination	1, 2, 5, 6, 11

$((A \vee B) \wedge (A \vee C)) \vdash (A \vee (B \wedge C))$

Line	Assumptions	Formula	Justification	References
1	1	$((A \vee B) \wedge (A \vee C))$	Assumption Introduction	
2	1	$(A \vee B)$	Conjunction Elimination	1
3	1	$(A \vee C)$	Conjunction Elimination	1
4	4	A	Assumption Introduction	
5	5	B	Assumption Introduction	
6	6	C	Assumption Introduction	
7	4	$(A \vee (B \wedge C))$	Disjunction Introduction	4
8	5, 6	$(B \wedge C)$	Conjunction Introduction	5, 6
9	5, 6	$(A \vee (B \wedge C))$	Disjunction Introduction	8
10	1, 6	$(A \vee (B \wedge C))$	Disjunction Elimination	2, 4, 7, 5, 9
11	1	$(A \vee (B \wedge C))$	Disjunction Elimination	3, 4, 7, 6, 10

5 de Morgan's Laws

$(\neg A \vee \neg B) \vdash \neg(A \wedge B)$

Line	Assumptions	Formula	Justification	References
1	1	$(\neg A \vee \neg B)$	Assumption Introduction	
2	2	$\neg A$	Assumption Introduction	
3	3	$\neg B$	Assumption Introduction	
4	4	$(A \wedge B)$	Assumption Introduction	
5	4	A	Conjunction Elimination	4
6	2, 4	$\bot$	Negation Elimination	5, 2
7	2	$\neg(A \wedge B)$	Negation Introduction	4, 6
8	4	B	Conjunction Elimination	4
9	3, 4	$\bot$	Negation Elimination	8, 3
10	3	$\neg(A \wedge B)$	Negation Introduction	4, 9
11	1	$\neg(A \wedge B)$	Disjunction Elimination	1, 2, 7, 3, 10

$\neg(A \wedge B) \vdash (\neg A \vee \neg B)$

Line	Assumptions	Formula	Justification	References
1	1	$\neg(A \wedge B)$	Assumption Introduction	
2	2	$\neg(\neg A \vee \neg B)$	Assumption Introduction	
3	3	$\neg A$	Assumption Introduction	
4	3	$(\neg A \vee \neg B)$	Disjunction Introduction	3
5	2, 3	$\bot$	Negation Elimination	4, 2
6	2	$\neg\neg A$	Negation Introduction	3, 5
7	7	$\neg B$	Assumption Introduction	
8	7	$(\neg A \vee \neg B)$	Disjunction Introduction	7
9	2, 7	$\bot$	Negation Elimination	8, 2
10	2	$\neg\neg B$	Negation Introduction	7, 9
11	2	$(\neg\neg A \wedge \neg\neg B)$	Conjunction Introduction	6, 10
12	2	$\neg\neg A$	Conjunction Elimination	11
13	2, 3	$\bot$	Negation Elimination	3, 12
14	2	A	Reducto Absurdium	3, 13
15	2	$\neg\neg B$	Conjunction Elimination	11
16	2, 7	$\bot$	Negation Elimination	7, 15
17	2	B	Reducto Absurdium	7, 16
18	2	$(A \wedge B)$	Conjunction Introduction	14, 17
19	1, 2	$\bot$	Negation Elimination	18, 1
20	1	$(\neg A \vee \neg B)$	Reducto Absurdium	2, 19

$(\neg A \wedge \neg B) \vdash \neg(A \vee B)$

Line	Assumptions	Formula	Justification	References
1	1	$(\neg A \wedge \neg B)$	Assumption Introduction	
2	1	$\neg A$	Conjunction Elimination	1
3	1	$\neg B$	Conjunction Elimination	1
4	4	$(A \vee B)$	Assumption Introduction	
5	5	$\neg B$	Assumption Introduction	
6	6	B	Assumption Introduction	
7	5, 6	$\bot$	Negation Elimination	6, 5
8	5, 6	A	Falsum	7
9	9	A	Assumption Introduction	
10	4, 5	A	Disjunction Elimination	4, 9, 9, 6, 8
11	1, 4, 5	$\bot$	Negation Elimination	10, 2
12	1, 4	B	Reducto Absurdium	5, 11
13	1, 4	$\bot$	Negation Elimination	12, 3
14	1	$\neg(A \vee B)$	Negation Introduction	4, 13

$\neg(A \vee B) \vdash (\neg A \wedge \neg B)$

Line	Assumptions	Formula	Justification	References
1	1	$\neg(A \vee B)$	Assumption Introduction	
2	2	A	Assumption Introduction	
3	2	$(A \vee B)$	Disjunction Introduction	2
4	1, 2	$\bot$	Negation Elimination	3, 1
5	1	$\neg A$	Negation Introduction	2, 4
6	6	B	Assumption Introduction	
7	6	$(A \vee B)$	Disjunction Introduction	6
8	1, 6	$\bot$	Negation Elimination	7, 1
9	1	$\neg B$	Negation Introduction	6, 8
10	1	$(\neg A \wedge \neg B)$	Conjunction Introduction	5, 9

6 Double Negation

$\neg\neg A \vdash A$

Line	Assumptions	Formula	Justification	References
1	1	$\neg\neg A$	Assumption Introduction	
2	2	$\neg A$	Assumption Introduction	
3	1, 2	$\bot$	Negation Elimination	2, 1
4	1	A	Reducto Absurdium	2, 3

$A \vdash \neg\neg A$

Line	Assumptions	Formula	Justification	References
1	1	A	Assumption Introduction	
2	2	$\neg A$	Assumption Introduction	
3	1, 2	$\bot$	Negation Elimination	1, 2
4	1	$\neg\neg A$	Negation Introduction	2, 3

7 Implication Definition

$(\neg A \vee B) \vdash (A \to B)$

Line	Assumptions	Formula	Justification	References
1	1	$(\neg A \vee B)$	Assumption Introduction	
2	2	$\neg A$	Assumption Introduction	
3	3	A	Assumption Introduction	
4	2, 3	$\bot$	Negation Elimination	3, 2
5	2, 3	B	Falsum	4
6	2	$(A \to B)$	Implication Introduction	3, 5
7	7	B	Assumption Introduction	
8	3, 7	$(A \wedge B)$	Conjunction Introduction	7, 3
9	3, 7	B	Conjunction Elimination	8
10	7	$(A \to B)$	Implication Introduction	3, 9
11	1	$(A \to B)$	Disjunction Elimination	1, 2, 6, 7, 10

$(A \to B) \vdash (\neg A \vee B)$

Line	Assumptions	Formula	Justification	References
1	1	$(A \to B)$	Assumption Introduction	
2	2	$\neg(\neg A \vee B)$	Assumption Introduction	
3	3	A	Assumption Introduction	
4	1, 3	B	Implication Elimination	3, 1
5	1, 3	$(\neg A \vee B)$	Disjunction Introduction	4
6	1, 2, 3	$\bot$	Negation Elimination	5, 2
7	1, 2	$\neg A$	Negation Introduction	3, 6
8	1, 2	$(\neg A \vee B)$	Disjunction Introduction	7
9	1, 2	$\bot$	Negation Elimination	8, 2
10	1	$(\neg A \vee B)$	Reducto Absurdium	2, 9

8 Transposition

$(A \to B) \vdash (\neg B \to \neg A)$

Line	Assumptions	Formula	Justification	References
1	1	$(A \to B)$	Assumption Introduction	
2	2	$\neg B$	Assumption Introduction	
3	3	A	Assumption Introduction	
4	1, 3	B	Implication Elimination	3, 1
5	1, 2, 3	$\bot$	Negation Elimination	4, 2
6	1, 2	$\neg A$	Negation Introduction	3, 5
7	1	$(\neg B \to \neg A)$	Implication Introduction	2, 6

$(\neg B \to \neg A) \vdash (A \to B)$

Line	Assumptions	Formula	Justification	References
1	1	$(\neg B \to \neg A)$	Assumption Introduction	
2	2	A	Assumption Introduction	
3	3	$\neg A$	Assumption Introduction	
4	2, 3	$\bot$	Negation Elimination	2, 3
5	2	$\neg\neg A$	Negation Introduction	3, 4
6	6	$\neg B$	Assumption Introduction	
7	1, 6	$\neg A$	Implication Elimination	6, 1
8	1, 2, 6	$\bot$	Negation Elimination	7, 5
9	1, 2	B	Reducto Absurdium	6, 8
10	1	$(A \to B)$	Implication Introduction	2, 9

9 Exportation

$((A \wedge B) \to C) \vdash (A \to (B \to C))$

Line	Assumptions	Formula	Justification	References
1	1	$((A \land B) \to C)$	Assumption Introduction	
2	2	A	Assumption Introduction	
3	3	B	Assumption Introduction	
4	2, 3	$(A \land B)$	Conjunction Introduction	2, 3
5	1, 2, 3	C	Implication Elimination	4, 1
6	1, 2	$(B \to C)$	Implication Introduction	3, 5
7	1	$(A \to (B \to C))$	Implication Introduction	2, 6

$(A \to (B \to C)) \vdash ((A \land B) \to C)$

Line	Assumptions	Formula	Justification	References
1	1	$(A \to (B \to C))$	Assumption Introduction	
2	2	$(A \land B)$	Assumption Introduction	
3	2	A	Conjunction Elimination	2
4	2	B	Conjunction Elimination	2
5	1, 2	$(B \to C)$	Implication Elimination	3, 1
6	1, 2	C	Implication Elimination	4, 5
7	1	$((A \land B) \to C)$	Implication Introduction	2, 6

10 Trivial Proof

$B \vdash (A \to B)$

Line	Assumptions	Formula	Justification	References
1	1	B	Assumption Introduction	
2	2	A	Assumption Introduction	
3	1, 2	$(A \land B)$	Conjunction Introduction	1, 2
4	1, 2	B	Conjunction Elimination	3
5	1	$(A \to B)$	Implication Introduction	2, 4

$\neg A \vdash (A \to B)$

Line	Assumptions	Formula	Justification	References
1	1	$\neg A$	Assumption Introduction	
2	1	$(\neg A \lor B)$	Disjunction Introduction	1
3	3	A	Assumption Introduction	
4	1, 3	$\bot$	Negation Elimination	3, 1
5	1, 3	B	Falsum	4
6	1	$(A \to B)$	Implication Introduction	3, 5

$A \vdash (\neg A \to B)$

Line	Assumptions	Formula	Justification	References
1	1	A	Assumption Introduction	
2	1	$(A \lor B)$	Disjunction Introduction	1
3	3	$\neg A$	Assumption Introduction	
4	1, 3	$\bot$	Negation Elimination	1, 3
5	1, 3	B	Falsum	4
6	1	$(\neg A \to B)$	Implication Introduction	3, 5

$(A \to (B \to C)) \vdash (B \to (A \to C))$

Line	Assumptions	Formula	Justification	References
1	1	$(A \to (B \to C))$	Assumption Introduction	
2	2	B	Assumption Introduction	
3	3	A	Assumption Introduction	
4	1, 3	$(B \to C)$	Implication Elimination	3, 1
5	1, 2, 3	C	Implication Elimination	2, 4
6	1, 2	$(A \to C)$	Implication Introduction	3, 5
7	1	$(B \to (A \to C))$	Implication Introduction	2, 6

$\neg(A \vee B) \vdash \neg A$

Line	Assumptions	Formula	Justification	References
1	1	$\neg(A \vee B)$	Assumption Introduction	
2	2	A	Assumption Introduction	
3	2	$(A \vee B)$	Disjunction Introduction	2
4	1, 2	$\bot$	Negation Elimination	3, 1
5	1	$\neg A$	Negation Introduction	2, 4

$\vdash (A \vee \neg A)$

Line	Assumptions	Formula	Justification	References
1	1	$\neg(A \vee \neg A)$	Assumption Introduction	
2	2	A	Assumption Introduction	
3	2	$(A \vee \neg A)$	Disjunction Introduction	2
4	1, 2	$\bot$	Negation Elimination	3, 1
5	1	$\neg A$	Negation Introduction	2, 4
6	1	$(A \vee \neg A)$	Disjunction Introduction	5
7	1	$\bot$	Negation Elimination	6, 1
8		$(A \vee \neg A)$	Reducto Absurdium	1, 7

11 Modus Tollens

$(A \to B), \neg B \vdash \neg A$

Line	Assumptions	Formula	Justification	References
1	1	$(A \to B)$	Assumption Introduction	
2	2	$\neg B$	Assumption Introduction	
3	3	A	Assumption Introduction	
4	1, 3	B	Implication Elimination	3, 1
5	1, 2, 3	$\bot$	Negation Elimination	4, 2
6	1, 2	$\neg A$	Negation Introduction	3, 5

12 Hypothetical Syllogism

$(A \to B), (B \to C) \vdash (A \to C)$

Line	Assumptions	Formula	Justification	References
1	1	$(A \to B)$	Assumption Introduction	
2	2	$(B \to C)$	Assumption Introduction	
3	3	A	Assumption Introduction	
4	1, 3	B	Implication Elimination	3, 1
5	1, 2, 3	C	Implication Elimination	4, 2
6	1, 2	$(A \to C)$	Implication Introduction	3, 5

13 Disjunctive Syllogism

$(A \lor B), \neg B \vdash A$

Line	Assumptions	Formula	Justification	References
1	1	$(A \lor B)$	Assumption Introduction	
2	2	$\neg B$	Assumption Introduction	
3	3	B	Assumption Introduction	
4	2, 3	$\bot$	Negation Elimination	3, 2
5	2, 3	A	Falsum	4
6	6	A	Assumption Introduction	
7	1, 2	A	Disjunction Elimination	1, 6, 6, 3, 5

14 Construction Dilemma

$((A \to B) \land (C \to D)), (A \lor C) \vdash (B \lor D)$

Line	Assumptions	Formula	Justification	References
1	1	$((A \to B) \land (C \to D))$	Assumption Introduction	
2	2	$(A \lor C)$	Assumption Introduction	
3	1	$(A \to B)$	Conjunction Elimination	1
4	1	$(C \to D)$	Conjunction Elimination	1
5	5	A	Assumption Introduction	
6	1, 5	B	Implication Elimination	5, 3
7	1, 5	$(B \lor D)$	Disjunction Introduction	6
8	8	C	Assumption Introduction	
9	1, 8	D	Implication Elimination	8, 4
10	1, 8	$(B \lor D)$	Disjunction Introduction	9
11	1, 2	$(B \lor D)$	Disjunction Elimination	2, 5, 7, 8, 10

15 Destructive Dilemma

$((A \to B) \land (C \to D)), (\neg B \lor \neg D) \vdash (\neg A \lor \neg C)$

Line	Assumptions	Formula	Justification	References
1	1	$((A \to B) \land (C \to D))$	Assumption Introduction	
2	2	$(\neg B \lor \neg D)$	Assumption Introduction	
3	1	$(A \to B)$	Conjunction Elimination	1
4	1	$(C \to D)$	Conjunction Elimination	1
5	5	$\neg(\neg A \lor \neg C)$	Assumption Introduction	
6	6	$\neg A$	Assumption Introduction	
7	6	$(\neg A \lor \neg C)$	Disjunction Introduction	6
8	5, 6	$\bot$	Negation Elimination	7, 5
9	5	A	Reducto Absurdium	6, 8
10	1, 5	B	Implication Elimination	9, 3
11	11	$\neg C$	Assumption Introduction	
12	11	$(\neg A \lor \neg C)$	Disjunction Introduction	11
13	5, 11	$\bot$	Negation Elimination	12, 5
14	5	C	Reducto Absurdium	11, 13
15	1, 5	D	Implication Elimination	14, 4
16	16	$\neg B$	Assumption Introduction	
17	1, 5, 16	$\bot$	Negation Elimination	10, 16
18	1, 5, 16	$\neg D$	Falsum	17
19	19	$\neg D$	Assumption Introduction	
20	1, 2, 5	$\neg D$	Disjunction Elimination	2, 16, 18, 19, 19
21	1, 2, 5	$\bot$	Negation Elimination	15, 20
22	1, 2	$(\neg A \lor \neg C)$	Reducto Absurdium	5, 21

16 Qualifier Negation

$\neg\forall x F(x) \vdash \exists x \neg F(x)$

Line	Assumptions	Formula	Justification	References
1	1	$\neg\forall x F(x)$	Assumption Introduction	
2	2	$\neg\exists x \neg F(x)$	Assumption Introduction	
3	3	$\neg F(c)$	Assumption Introduction	
4	3	$\exists x \neg F(x)$	Existential Introduction	3
5	2, 3	$\bot$	Negation Elimination	4, 2
6	2	$F(c)$	Reducto Absurdium	3, 5
7	2	$\forall x F(x)$	Universal Introduction	6
8	1, 2	$\bot$	Negation Elimination	7, 1
9	1	$\exists x \neg F(x)$	Reducto Absurdium	2, 8

$\exists x \neg F(x) \vdash \neg\forall x F(x)$

Line	Assumptions	Formula	Justification	References
1	1	$\exists x \neg F(x)$	Assumption Introduction	
2	2	$\forall x F(x)$	Assumption Introduction	
3	2	$F(c)$	Universal Elimination	2
4	4	$\neg F(c)$	Assumption Introduction	
5	2, 4	$\bot$	Negation Elimination	3, 4
6	1, 2	$\bot$	Existential Elimination	1, 4, 5
7	1	$\neg \forall x F(x)$	Negation Introduction	2, 6

$\neg \exists x F(x) \vdash \forall x \neg F(x)$

Line	Assumptions	Formula	Justification	References
1	1	$\neg \exists x F(x)$	Assumption Introduction	
2	2	$F(c)$	Assumption Introduction	
3	2	$\exists x F(x)$	Existential Introduction	2
4	1, 2	$\bot$	Negation Elimination	3, 1
5	1	$\neg F(c)$	Negation Introduction	2, 4
6	1	$\forall x \neg F(x)$	Universal Introduction	5

$\forall x \neg F(x) \vdash \neg \exists x F(x)$

Line	Assumptions	Formula	Justification	References
1	1	$\forall x \neg F(x)$	Assumption Introduction	
2	2	$\exists x F(x)$	Assumption Introduction	
3	3	$F(c)$	Assumption Introduction	
4	1	$\neg F(c)$	Universal Elimination	1
5	1, 3	$\bot$	Negation Elimination	3, 4
6	1, 2	$\bot$	Existential Elimination	2, 3, 5
7	1	$\neg \exists x F(x)$	Negation Introduction	2, 6

17 Qualifier Unification

$(\forall x F(x) \land \forall x G(x)) \vdash \forall x (F(x) \land G(x))$

Line	Assumptions	Formula	Justification	References
1	1	$(\forall x F(x) \land \forall x G(x))$	Assumption Introduction	
2	1	$\forall x F(x)$	Conjunction Elimination	1
3	1	$F(c)$	Universal Elimination	2
4	1	$\forall x G(x)$	Conjunction Elimination	1
5	1	$G(c)$	Universal Elimination	4
6	1	$(F(c) \land G(c))$	Conjunction Introduction	3, 5
7	1	$\forall x (F(x) \land G(x))$	Universal Introduction	6

$\forall x (F(x) \land G(x)) \vdash (\forall x F(x) \land \forall x G(x))$

Line	Assumptions	Formula	Justification	References
1	1	$\forall x(F(x) \land G(x))$	Assumption Introduction	
2	1	$(F(c) \land G(c))$	Universal Elimination	1
3	1	$F(c)$	Conjunction Elimination	2
4	1	$\forall x F(x)$	Universal Introduction	3
5	1	$G(c)$	Conjunction Elimination	2
6	1	$\forall x G(x)$	Universal Introduction	5
7	1	$(\forall x F(x) \land \forall x G(x))$	Conjunction Introduction	4, 6

$(\exists x F(x) \lor \exists x G(x)) \vdash \exists x(F(x) \lor G(x))$

Line	Assumptions	Formula	Justification	References
1	1	$(\exists x F(x) \lor \exists x G(x))$	Assumption Introduction	
2	2	$\exists x F(x)$	Assumption Introduction	
3	3	$F(c)$	Assumption Introduction	
4	3	$(F(c) \lor G(c))$	Disjunction Introduction	3
5	3	$\exists x(F(x) \lor G(x))$	Existential Introduction	4
6	2	$\exists x(F(x) \lor G(x))$	Existential Elimination	2, 3, 5
7	7	$\exists x G(x)$	Assumption Introduction	
8	8	$G(c)$	Assumption Introduction	
9	8	$(F(c) \lor G(c))$	Disjunction Introduction	8
10	8	$\exists x(F(x) \lor G(x))$	Existential Introduction	9
11	7	$\exists x(F(x) \lor G(x))$	Existential Elimination	7, 8, 10
12	1	$\exists x(F(x) \lor G(x))$	Disjunction Elimination	1, 2, 6, 7, 11

$\exists x(F(x) \lor G(x)) \vdash (\exists x F(x) \lor \exists x G(x))$

Line	Assumptions	Formula	Justification	References
1	1	$\exists x(F(x) \lor G(x))$	Assumption Introduction	
2	2	$(F(c) \lor G(c))$	Assumption Introduction	
3	3	$F(c)$	Assumption Introduction	
4	3	$\exists x F(x)$	Existential Introduction	3
5	3	$(\exists x F(x) \lor \exists x G(x))$	Disjunction Introduction	4
6	6	$G(c)$	Assumption Introduction	
7	6	$\exists x G(x)$	Existential Introduction	6
8	6	$(\exists x F(x) \lor \exists x G(x))$	Disjunction Introduction	7
9	2	$(\exists x F(x) \lor \exists x G(x))$	Disjunction Elimination	2, 3, 5, 6, 8
10	1	$(\exists x F(x) \lor \exists x G(x))$	Existential Elimination	1, 2, 9

18 Qualifier Transposition

$\forall x \forall y F(x, y) \vdash \forall y \forall x F(x, y)$

Line	Assumptions	Formula	Justification	References
1	1	$\forall x \forall y F(x, y)$	Assumption Introduction	
2	1	$\forall y F(c, y)$	Universal Elimination	1
3	1	$F(c, d)$	Universal Elimination	2
4	1	$\forall x F(x, d)$	Universal Introduction	3
5	1	$\forall y \forall x F(x, y)$	Universal Introduction	4

$\exists x \exists y F(x, y) \vdash \exists y \exists x F(x, y)$

Line	Assumptions	Formula	Justification	References
1	1	$\exists x \exists y F(x, y)$	Assumption Introduction	
2	2	$\exists y F(c, y)$	Assumption Introduction	
3	3	$F(c, d)$	Assumption Introduction	
4	3	$\exists x F(x, d)$	Existential Introduction	3
5	3	$\exists y \exists x F(x, y)$	Existential Introduction	4
6	2	$\exists y \exists x F(x, y)$	Existential Elimination	2, 3, 5
7	1	$\exists y \exists x F(x, y)$	Existential Elimination	1, 2, 6

19 Trivial Proof with Qualifier

$\forall x F(x) \vdash \exists x F(x)$

Line	Assumptions	Formula	Justification	References
1	1	$\forall x F(x)$	Assumption Introduction	
2	1	$F(c)$	Universal Elimination	1
3	1	$\exists x F(x)$	Existential Introduction	2

$\exists y \forall x F(x, y) \vdash \forall x \exists y F(x, y)$

Line	Assumptions	Formula	Justification	References
1	1	$\exists y \forall x F(x, y)$	Assumption Introduction	
2	2	$\forall x F(x, c)$	Assumption Introduction	
3	2	$F(d, c)$	Universal Elimination	2
4	2	$\exists y F(d, y)$	Existential Introduction	3
5	2	$\forall x \exists y F(x, y)$	Universal Introduction	4
6	1	$\forall x \exists y F(x, y)$	Existential Elimination	1, 2, 5