2025.10.30 HW5 习题三

A18. (1) $f_X(x)=\begin{cases} x+\frac{1}{2}, & 0<x<1 \\ 0, & \text{其他} \end{cases}$

$f_Y(y)=\begin{cases} \frac{y}{4}+\frac{1}{4}, & 0<y<2 \\ 0, & \text{其他} \end{cases}$

(2) $f_{X|Y}(x|y)=\dfrac{f(x,y)}{f_Y(y)}=\begin{cases} \frac{2x+y}{y+1}, & 0<x<1,0<y<2 \\ 0, & \text{其他} \end{cases}$

当$0<y<2$时,

$f_{Y|X}(y|x)=\dfrac{f(x,y)}{f_X(x)}=\begin{cases} \frac{2x+y}{4x+2}, & 0<x<1,0<y<2 \\ 0, & \text{其他} \end{cases}$

当$0<x<1$时,

(3) $P(X\le 0.5\mid Y=0.5)=\int_{-\infty}^{0.5} f_{X|Y}(x|0.5)\,dx$

$=\int_0^{0.5}\frac{2x+0.5}{1.5}dx=\frac{1}{3}$

$P(Y\le 0.5\mid X=0.5)=\int_{-\infty}^{0.5} f_{Y|X}(y|0.5)\,dy$

$=\int_0^{0.5}\frac{1+y}{4}dy=\frac{5}{32}$

A19. (1) $f(x,y)=f_X(x)f_{Y|X}(y|x)=\begin{cases} \lambda^2 e^{-\lambda x-y/x}, & x>0,y>0 \\ 0, & \text{其他} \end{cases}$

(2) $F_{Y|X}(y|x)=\int_{-\infty}^{y} f_{Y|X}(u|x)\,du=\begin{cases}1-e^{-y/x}, & y>0\\ 0, & y\le 0\end{cases}$

(3) $P(Y>1\mid X=1)=1-F_{Y|X}(1|1)=e^{-1}$

A25. 由 $P(X=0,Y=0)=P(X=0)P(Y=0)$

$0.1=(0.1+0.2+0.1)(0.1+0.05+a)$

$\Rightarrow a=0.1$

同理得 $b=0.2$, $c=0.1$

A26. (1) 不独立. 由A18知 $f_X(x)=\begin{cases}x+\frac{1}{2}, & 0<x<1\\ 0, & \text{其他}\end{cases}$

$f_Y(y)=\begin{cases}\frac{y+1}{4}, & 0\le y<2\\ 0, & \text{其他}\end{cases}$

当 $0<x<1$, $0<y<2$ 时,

而 $f(x,y)=\frac{2x+y}{4}\neq f_X(x)\cdot f_Y(y)=\frac{(2x+1)(y+1)}{8}$.

(2) 独立. $f_X(x)=\begin{cases}2x, & 0<x<1\\ 0, & \text{其他}\end{cases}$

$f_Y(y)=\begin{cases}\frac{1}{2}x, & 0<y<2\\ 0, & \text{其他}\end{cases}$

$f(x,y)=xy=f_X(x)f_Y(y)=xy$.

(3) 不独立. $f_X(x)=\begin{cases}\frac{3}{4}x(2-x), & 0<x<2\\ 0, & \text{其他}\end{cases}$

$$f_Y(y)=\begin{cases}\frac{3y^2}{8}, & 0<y<2\\ 0, & \text{其他}.\end{cases}$$

而当 $0<x<y<2$ 时，$f(x,y)=\frac{3x}{4}\neq f_X(x)f_Y(y)=\frac{9}{32}x(2-x)y^2$.

B3. 比值为 $\lambda=\frac{1-0.1-0.1}{1}=0.8$

$$F(x,y)=\begin{cases}0, & x<0\text{或}y<0.\\ 0.1+0.8xy, & 0\le x<1,\ 0\le y<1\\ 0.1+0.8x, & 0\le x<1,\ y\ge 1.\\ 0.1+0.8y, & 0\le y<1,\ x\ge 1\\ 1, & x\ge 1,\ y\ge 1.\end{cases}$$

B5. (1) $f(x,y)=f_X(x)\cdot f_{Y|X}(y|x)=\begin{cases}\frac{2(4-y)}{(3-x)^2}, & 1<x<2,\ x+1<y<4\\ 0, & \text{其他}.\end{cases}$

$$P(Y<3)=\int_{-\infty}^{+\infty}\int_{-\infty}^{3}f(x,y)\,dy\,dx.$$

$$=\int_1^2\int_{x+1}^{3}\frac{2(4-y)}{(3-x)^2}\,dy\,dx.$$

$$=\int_1^2\frac{x^2-6x+8}{(3-x)^2}\,dx=\frac{1}{2}$$

(2) $f_{Y|y}(y) = \begin{cases} y-2, & 2<y<3 \\ 4-y, & 3<y<4 \\ 0, & \text{其他} \end{cases}$

(3) $P(X \le 1.5 \mid Y=3) = \int_{-\infty}^{1.5} f(x,3)\,dx$

$= \int_{1}^{1.5} \frac{2}{(3-x)^2}\,dx = \frac{1}{3}$

B8. (1) $P(Z=k) = C_n^k p^k (1-p)^{n-k}$, $k = 0, 1, \cdots, n$.

(2) $P(W=k) = C_{m+n}^k p^k (1-p)^{m+n-k}$, $k = 0, 1, \cdots, m+n$