

025.10.13: HW2: 习题二: 1-1) 已知: 离散型随机变量: X 的分布律为:

A1 (1) $P(X=1) = \frac{1}{2}$

$P(X=2) = \frac{1}{3}$

$P(X=3) = \frac{1}{6}$

(2) $P(X=0) = \frac{8}{27}$

$P(X=1) = \frac{4}{9}$

$P(X=2) = \frac{2}{9}$

$P(X=3) = \frac{1}{27}$

(3) $P(X=0) = \frac{3}{10}$

$P(X=1) = \frac{3}{5}$

$P(X=2) = \frac{1}{10}$

(4) $P(X=1) = \frac{2}{5}$

$P(X=2) = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

$P(X=3) = 1 - \frac{2}{5} - \frac{6}{25} = \frac{9}{25}$

A8.

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.4, & 0 \leq x < 2 \\ 0.9, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$$

$$P(1 \leq x < 3) = F(3-0) - F(1-0) = 0.9 - 0.4 = 0.5$$

$$A9. P(X=-1) = F(-1) - F(-1-0) = 0.3$$

$$\text{同理 } P(X=1) = 0.4,$$

$$P(X=3) = 0.3$$

$$B2. (1) P(X=0) = 0.8 = \frac{4}{5}$$

$$P(X=1) = 0.2 \times 0.8 = 0.16 = \frac{4}{25}$$

$$P(X=2) = 0.2 \times 0.2 \times 0.8 = \frac{4}{125}$$

$$P(X=4) = 0.2 \times 0.2 \times 0.2 = \frac{1}{125}$$

$$(2) P(X > 2) = \frac{1}{125}$$

(3) 记事件 A: 总得分不低于 2,

B: 玩家得 3 4 分.

$$\text{则 } P(B|A) = \frac{P(BA)}{P(A)} = \frac{\frac{1}{125}}{\frac{1}{125} + \frac{4}{125}} = \frac{1}{5}$$

B6. 记 X 为正常元件个数

$$(1) P(X=3) = C_5^3 \times 0.8^3 \times 0.2^2 = 0.2048$$

$$(2) P(X \geq 4) = C_5^4 \times 0.8^4 \times 0.2 + 0.8^5 = 0.7373$$

$$(3) P(X \leq 2) = 0.2^5 + C_5^1 \times 0.8^1 \times 0.2^4 + C_5^2 \times 0.8^2 \times 0.2^3 \\ = 0.0579$$

$$B7. P(X=0) = (1-P_1)(1-P_2)(1-P_3).$$

$$P(X=1) = P_1(1-P_2)(1-P_3) + P_2(1-P_1)(1-P_3) + P_3(1-P_1)(1-P_2).$$

$$P(X=2) = (1-P_1)P_2P_3 + (1-P_2)P_1P_3 + (1-P_3)P_1P_2$$

$$P(X=3) = P_1P_2P_3.$$

$$P(Y=0) = P_1$$

$$P(Y=1) = (1-P_1)P_2$$

$$P(Y=2) = (1-P_1)(1-P_2)P_3$$

$$P(Y=3) = (1-P_1)(1-P_2)(1-P_3).$$

B12. (1) 设 X 为 10:00 到 12:00 期间收到的短信数

则 $X \sim P(6)$.

$$\therefore P(X=6) = e^{-6} \cdot \frac{6^6}{6!} = \frac{324}{5} e^{-6}.$$

(2) 设事件 A : 收到至少 5 条短信.

B : 恰好收到 6 条短信.

$$\text{则 } P(A) = P(X \geq 5) = 1 - e^{-6} \left(1 + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!} + \frac{6^4}{4!} \right)$$

$$P(B|A) = \frac{324}{5} e^{-6}$$

$$\text{故 } P(B|A) = \frac{P(B)}{P(A)} = \frac{324}{5e^6 - 575}.$$

B16. (1) 当 $x < 0$, $F(x) = 0$;

$$\text{当 } 0 \leq x < 1, F(x) = \frac{x}{2}$$

$$\text{当 } 1 \leq x < 2, F(x) = \frac{1}{2}.$$

$$\text{当 } 2 \leq x < 3, F(x) = \frac{1}{2} + \frac{x-2}{2} = \frac{x-1}{2}.$$

$$\text{当 } x \geq 3, F(x) = 1.$$

$$(2) P(X \leq 2.5) = F(2.5) = \frac{3}{4}.$$