B16. (1) 当 $x<0$, $F(x)=0$;

当 $0\le x<1$, $F(x)=\frac{x}{2}$

当 $1\le x<2$, $F(x)=\frac{1}{2}$.

当 $2\le x<3$, $F(x)=\frac{1}{2}+\frac{x-2}{2}=\frac{x-1}{2}$.

当 $x\ge 3$, $F(x)=1$.

(2) $P(X\le 2.5)=F(2.5)=\frac{3}{4}$.

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A12. $(x-3)(x+1)<0 \Rightarrow -1<x<3$

$\therefore P\{X^2-2X-3<0\}=\frac{3}{4}$

A15. (1) $P\{X\le 0\}=\Phi\left(\frac{0-1}{2}\right)=\Phi(-0.5)\approx 0.3085$

$P\{|X-1|\le 2\}=\Phi\left(\frac{3-1}{2}\right)-\Phi\left(\frac{-1-1}{2}\right)=2\Phi(1)-1\approx 0.6826$

(2) $a=1$

(3) $P\{|X|\le 2\}=\Phi\left(\frac{2-1}{2}\right)-\Phi\left(\frac{-2-1}{2}\right)=\Phi(0.5)-\Phi(-1.5)$

≈ 0.6247.

A17. (1) Y的分布律: $P(Y=-3)=0.3$

$P(Y=-1)=0.1$

$P(Y=1)=0.2$

$P(Y=3)=0.4$

Z的分布律：$P(Z=0)=0.1$

$P(Z=1)=0.5$

$P(Z=4)=0.4$

(2) $F_Z(z)=\begin{cases} 0, & z<0 \\ 0.1, & 0\le z<1 \\ 0.6, & 1\le z<4 \\ 1, & z\ge 4. \end{cases}$

B17. (1) $\int_{-\infty}^{+\infty} f(x)dx = 8c-\frac{8}{3}c=\frac{16}{3}c=1$

$\Rightarrow c=\frac{3}{16}$

(2) $F(x)=\begin{cases} 0, & x<0 \\ -\frac{x^3}{16}+\frac{3x}{4}, & 0\le x<2 \\ 1, & x\ge 2. \end{cases}$

(3) $P\{-1<X<1\}=F(1-0)-F(-1)$

$=\frac{11}{16}-0=\frac{11}{16}$

(4) 记事件$\{-1<X<1\}$发生了X次则 $X\sim B(5,\frac{11}{16})$

$P(X=2)=C_5^2\left(\frac{11}{16}\right)^2\left(\frac{5}{16}\right)^3=\frac{75625}{524288}$

B24. (1) $\alpha = 0.1\times P(X<200) + 0.001\times P(200\leq X\leq 240) + 0.2\times P(X>240)$

$= 0.1\Phi(-\frac{4}{5}) + 0.001(\Phi(\frac{4}{5}) - \Phi(-\frac{4}{5})) + 0.2(1-\Phi(\frac{4}{5}))$

$= 0.0641$

(2) $\beta = \dfrac{0.2\times P(X>240)}{\alpha} = 0.6607$

(3) $\theta = [1-0.1^3 - C_3^1\times 0.1^2\times 0.9]P(X<200) + [1-0.001^3 - C_3^1\times 0.001^2\times 0.999]P(200\leq X\leq 240) + [1-0.2^3 - C_3^1\times 0.2^2\times 0.8]P(X>240) = 0.9720$.

B29. 记甲产品寿命为X，则$X\sim Exp(\frac{1}{3})$.

记乙产品寿命为Y，则$Y\sim Exp(\frac{1}{6})$.

(1) 记事件A：该产品寿命大于6年.

则$P(A) = 0.4P(X>6) + 0.6P(Y>6)$

$= 0.4e^{-2} + 0.6e^{-1}$

(2)

$$P(X>1\mid X>\tfrac{1}{3}) = \frac{P(X>1)}{P(X>\frac{1}{3})} = \frac{e^{-\frac{1}{3}\times 1}}{e^{-\frac{1}{3}\times\frac{1}{3}}} = e^{-\frac{2}{9}}.$$

(3) 记该混合产品寿命为Z.

则
$$P(Z>1\mid Z>\tfrac{1}{3}) = \frac{P(Z>1)}{P(Z>\frac{1}{3})} = \frac{0.4e^{-\frac{1}{3}\times 1}+0.6e^{-\frac{1}{6}\times 1}}{0.4e^{-\frac{1}{3}\times\frac{1}{3}}+0.6e^{-\frac{1}{6}\times\frac{1}{3}}} = \frac{2e^{-\frac{1}{3}}+3e^{-\frac{1}{6}}}{2e^{-\frac{1}{9}}+3e^{-\frac{1}{18}}}$$

B33. (1) $\int_{-\infty}^{+\infty} f(x)dx = 12c - 3c = 9c = 1 \Rightarrow c = \frac{1}{9}$.

(2) $F_Y(y) = P(3X \le y) = P(X \le \frac{y}{3})$.

由 $F_X(x) = \begin{cases} 0, & x < -1 \\ -\frac{1}{27}x^3 + \frac{4}{9}x + \frac{11}{27}, & -1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$

$\therefore F_Y(y) = \begin{cases} 0, & y < -3 \\ -\frac{1}{729}y^3 + \frac{4}{27}y + \frac{11}{27}, & -3 \le y < 6 \\ 1, & y \ge 6. \end{cases}$

$f_Y(y) = \begin{cases} 0, & y < -3 \\ -\frac{1}{243}y^2 + \frac{4}{27}, & -3 \le y < 6. \\ 0, & y \ge 6. \end{cases}$

(3) $F_Z(z) = \begin{cases} 0, & z < 0. \\ -\frac{2}{27}z^3 + \frac{8}{9}z, & 0 \le z < 1. \\ -\frac{1}{27}z^3 + \frac{4}{9}z + \frac{11}{27}, & 1 \le z < 2. \\ 1, & z \ge 2. \end{cases}$

$\therefore f_Z(z) = \begin{cases} 0, & z < 0 \\ -\frac{2}{9}z^2 + \frac{8}{9}, & 0 \le z < 1. \\ -\frac{1}{9}z^2 + \frac{4}{9}, & 1 \le z < 2 \\ 0, & z \ge 2. \end{cases}$

B36. $F_X(x)=\begin{cases}0, & x<0\\ \dfrac{2x}{3\pi}, & 0\le x<\dfrac{3\pi}{2}\\ 1, & x\ge\dfrac{3\pi}{2}.\end{cases}$

$F_Y(y)=P(\cos X\le y)$

$$=\begin{cases}0, & y<-1\\ \dfrac{4}{3}-\dfrac{4\arccos y}{3\pi}, & -1\le y<0.\\ 1-\dfrac{2\arccos y}{3\pi}, & 0\le y<1\\ 1, & y\ge 1.\end{cases}$$

B39. (1) $F_Y(y)=P(e^X\le y)=P(X\le \ln y)$.

$$\therefore f_Y(y)=\begin{cases}\dfrac{1}{\sqrt{2\pi}\,y}e^{-\frac{1}{2}(\ln y)^2}, & y>0\\ 0, & 其他.\end{cases}$$

(2) $F_Z(z)=P(\ln|X|<z)=P(-e^z<X<e^z)$.

$$\therefore f_Z(z)=\sqrt{\frac{2}{\pi}}e^{z-\frac{1}{2}e^{2z}},\ -\infty<z<+\infty.$$