2025.10.23 HW4 习题三.

A9. (1)

X \ Y	1	2	3	4	5	6	$P(X=i)$
1	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
2	0	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
3	0	0	$\frac{3}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
4	0	0	0	$\frac{4}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
5	0	0	0	0	$\frac{5}{36}$	$\frac{1}{36}$	$\frac{1}{6}$
6	0	0	0	0	0	$\frac{6}{36}$	$\frac{1}{6}$
$P(Y=j)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$	1

(2) $P(X=1 \mid Y=6)=\frac{1}{11}$

$P(X=2 \mid Y=6)=\frac{1}{11}$

$P(X=3 \mid Y=6)=\frac{1}{11}$

$P(X=4 \mid Y=6)=\frac{1}{11}$

$P(X=5 \mid Y=6)=\frac{1}{11}$

$P(X=6 \mid Y=6)=\frac{6}{11}$.

A11. (1) $F(0,1)=0.1+0.1=0.2$

$F(1,1.5)=0.1+0.1+0+0.2=0.4$

$F(2.1, 1.1) = 0.1 + 0.1 + 0 + 0.2 + 0.2 + 0 = 0.6$

(2)
$$F_X(x) = \begin{cases} 0, & x < 0 \\ 0.2, & 0 \le x < 1 \\ 0.6, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$

A12. (1)

X\Y	0	1
1	0.1	0.2
2	0.3	0.4

(2) 边际分布律为 $P(X=1) = 0.3$ $P(Y=0) = 0.4$

$P(X=2) = 0.7$ $P(Y=1) = 0.6$

$\therefore P(X=1 \mid Y=0) = \frac{0.1}{0.4} = \frac{1}{4}$

$P(X=2 \mid Y=0) = \frac{0.3}{0.4} = \frac{3}{4}$

$$\therefore F_{X|Y}(x|0) = \begin{cases} 0, & x < 1 \\ 0.25, & 1 \le x < 2 \\ 1, & x \ge 2. \end{cases}$$

A13. (1) $P(B\bar{A}) = 0.5 \times 0.7 = 0.35$, $P(BA) = 0.4 - 0.35 = 0.05$

$P(\bar{B}A) = 0.3 - 0.05 = 0.25$

X\Y	0	1
0	0.35	0.35
1	0.25	0.05

(2) $F_X(x)=\begin{cases}0, & x<0\\ 0.7, & 0\le x<1\\ 1, & x\ge 1.\end{cases}$

(3) $F_{Y|X}(y|1)=\begin{cases}0, & y<0\\ \frac{5}{6}, & 0\le y<1\\ 1, & y\ge 1\end{cases}$

B2. 即 $P(X=k)=e^{-\lambda}\frac{\lambda^k}{k!}$

(1) $P(X=i,Y=j)=\frac{e^{-\lambda}\lambda^i}{i!}C_i^j 0.1^j 0.9^{i-j}$

$i=0,1,\dots;\ j=0,1,\dots,i$

(2) $P(Y=j)=\sum_{i=j}^{\infty}P(X=i,Y=j)$

$=\sum_{i=j}^{\infty}\frac{e^{-\lambda}\lambda^i 0.1^j 0.9^{i-j}}{(i-j)!\,j!}$

$=e^{-\lambda}\cdot e^{0.9\lambda}\cdot\frac{(0.1)^j\lambda^j}{j!}$

$=\frac{e^{-0.1\lambda}(0.1\lambda)^j}{j!}$, $j=0,1,2\dots$