Modeling Sequences

RNNs

Outline

- Applications on modeling sequences of data
- Recurrent Neural Networks Models (RNNs)
- Vanishing (and exploding) gradients
- Long Short-Term Memory Models (LSTMs)
- **Announcements:** Pre-lecture Material for Mar 6
  PS3, due Mar 16

Sequences of Data

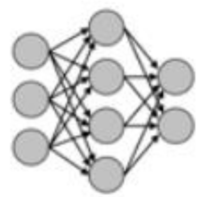
- Sequences in our world:
  - Audio
  - Text
  - Video
  - Weather
  - Stock market

- Sequential data is why we build RNN architectures.

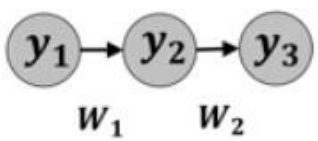
- RNNs are tools for making predictions about sequences.

Recap: Feed-Fwd Networks

- Forward:



$$y_i = g\left(\sum_j W_{ij} x_j + b_i\right)$$



$$\boldsymbol{y}_k = g(W \boldsymbol{y}_{k-1} + \boldsymbol{b})$$

- Backward:

Alternative Notation

E: error

C: cost

L: loss

Error

$$\frac{\partial \boxed{E}}{\partial W} = \frac{\partial E}{\partial g} \frac{\partial g}{\partial a} \frac{\partial a}{\partial W}$$

Limitations of Feed-Fwd Networks

- Limitations of feed-forward networks
  - **Fixed length**

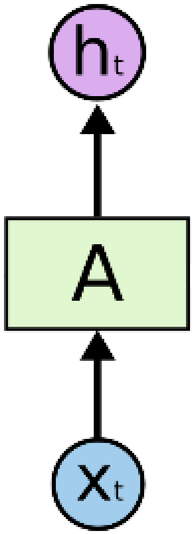
    Inputs and outputs are of fixed lengths
  - **Independence**

    Data (example: images) are independent of one another

Advantages of RNN Models

- What feed-forward networks cannot do
  - **Variable length**
    "We would like to accommodate temporal sequences of various lengths."
  - **Temporal dependence**
    "To predict where a pedestrian is at the next point in time, this depends on where he/she were in the previous time step(s)."

Vanilla Neural Network (NN)

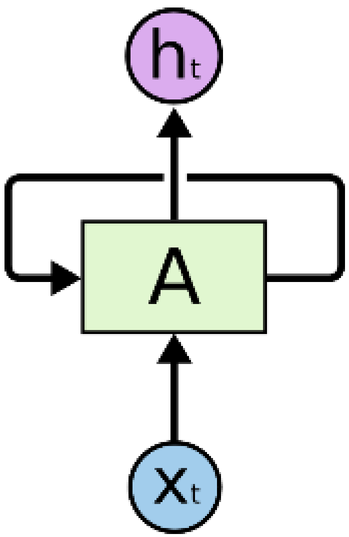


- NN

x_t: input/event
h_t: output/prediction
A : chunk of NN

Every input is treated independently.

Recurrent Neural Network (RNN)



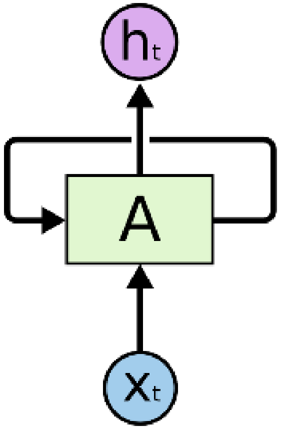
- RNN

The loop allows information to be passed from one time step to the next.

Now we are modeling the dynamics.

Recurrent Neural Network (RNN)

- A recurrent neural network can be thought of as multiple copies of the same network, each passing a message to a successor.



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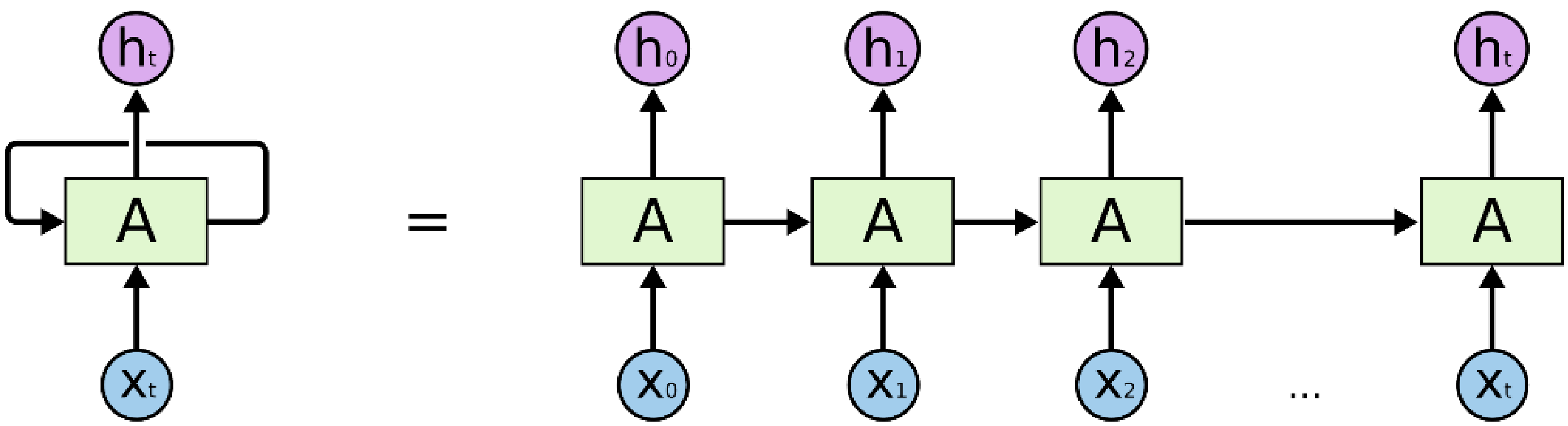
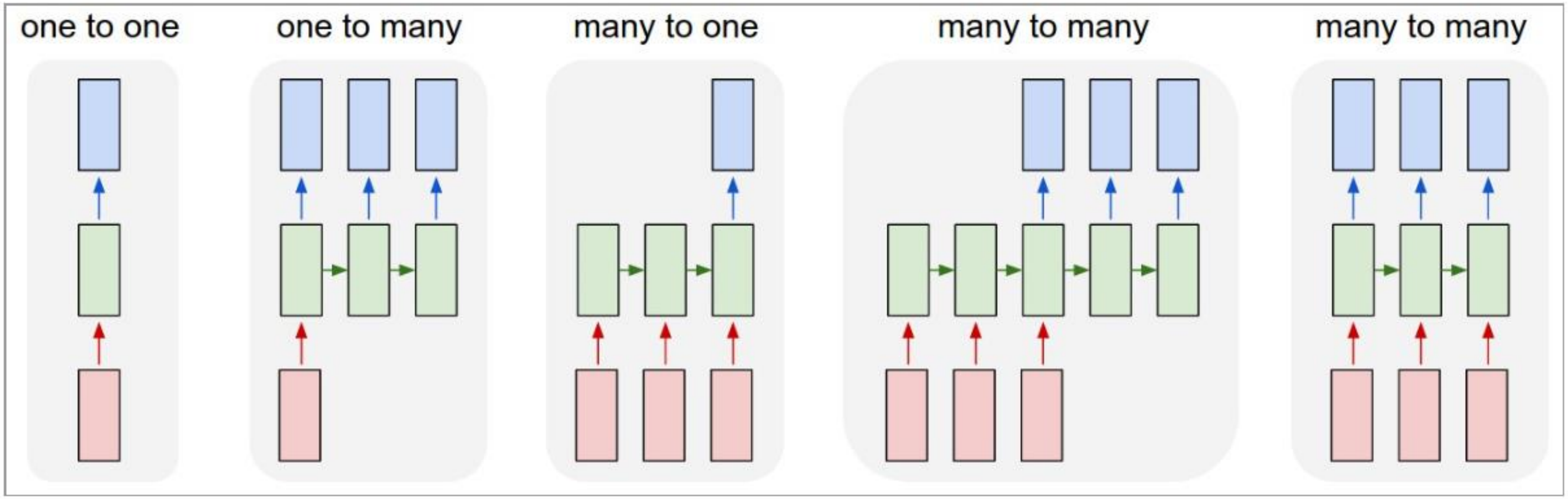


Chris Olah, Blog, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

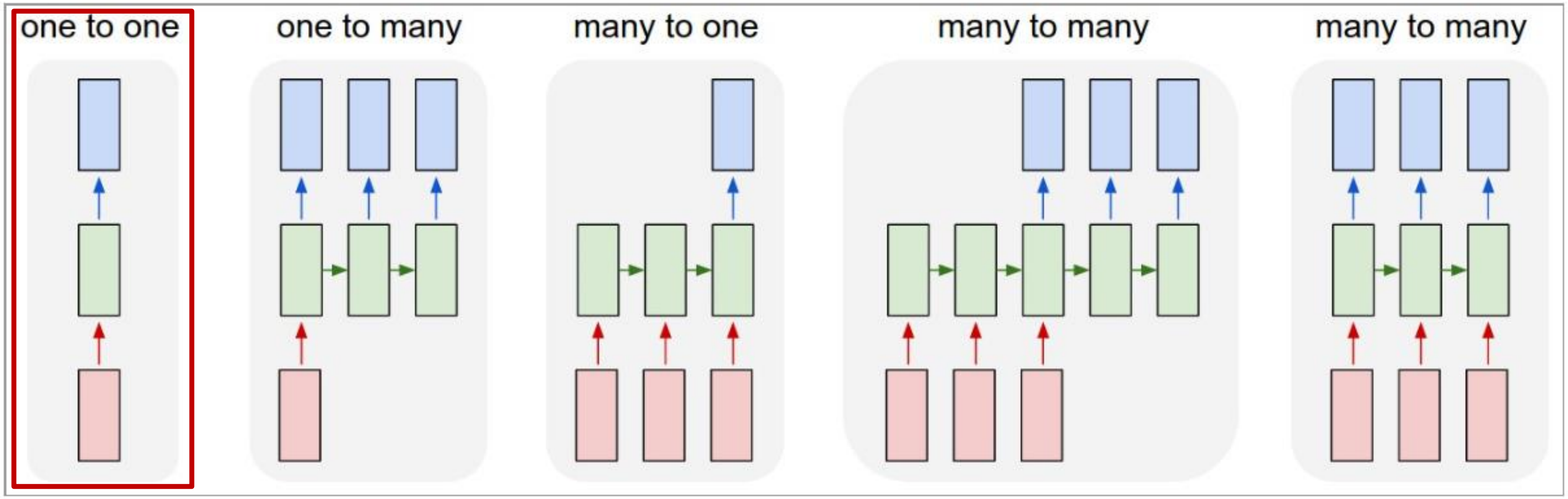
RNN Architectures



Vanilla NN

Recurrent NNs

One-to-one



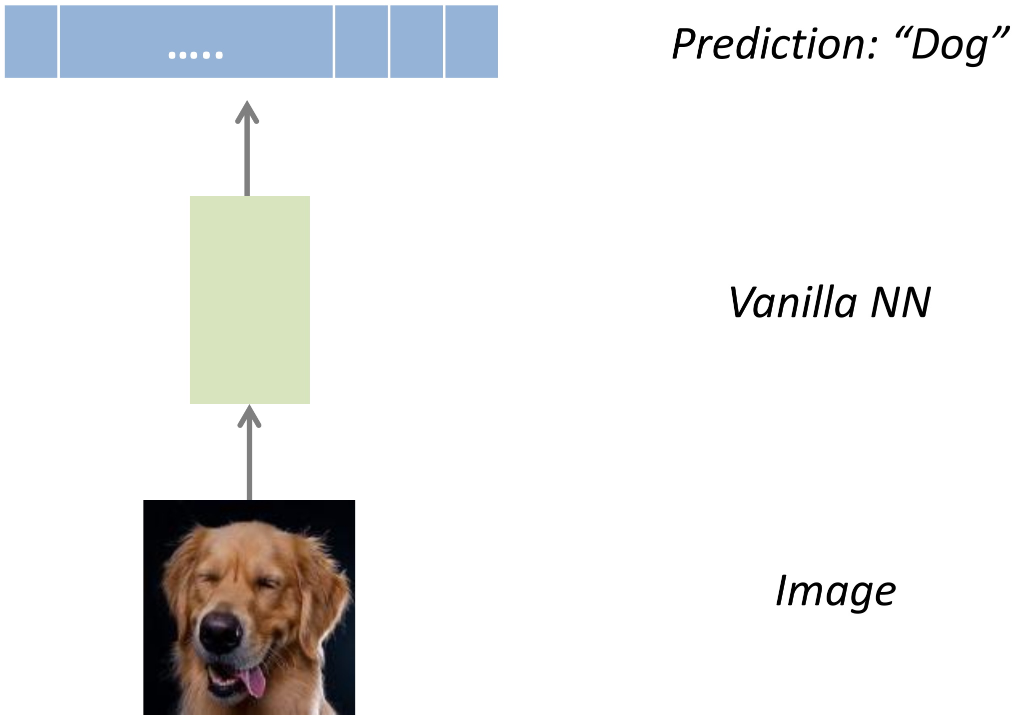
Vanilla mode of processing without RNN

Example: Image classification

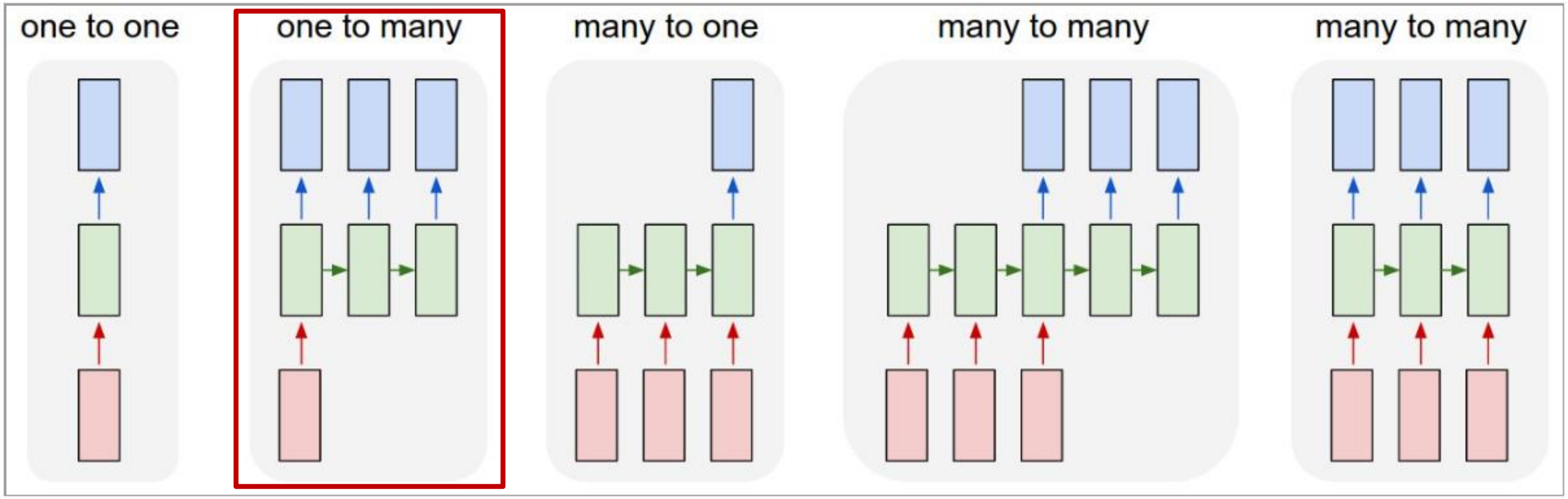
Example: One-to-one

Vanilla mode of processing without RNN

Example: Image classification



One-to-many



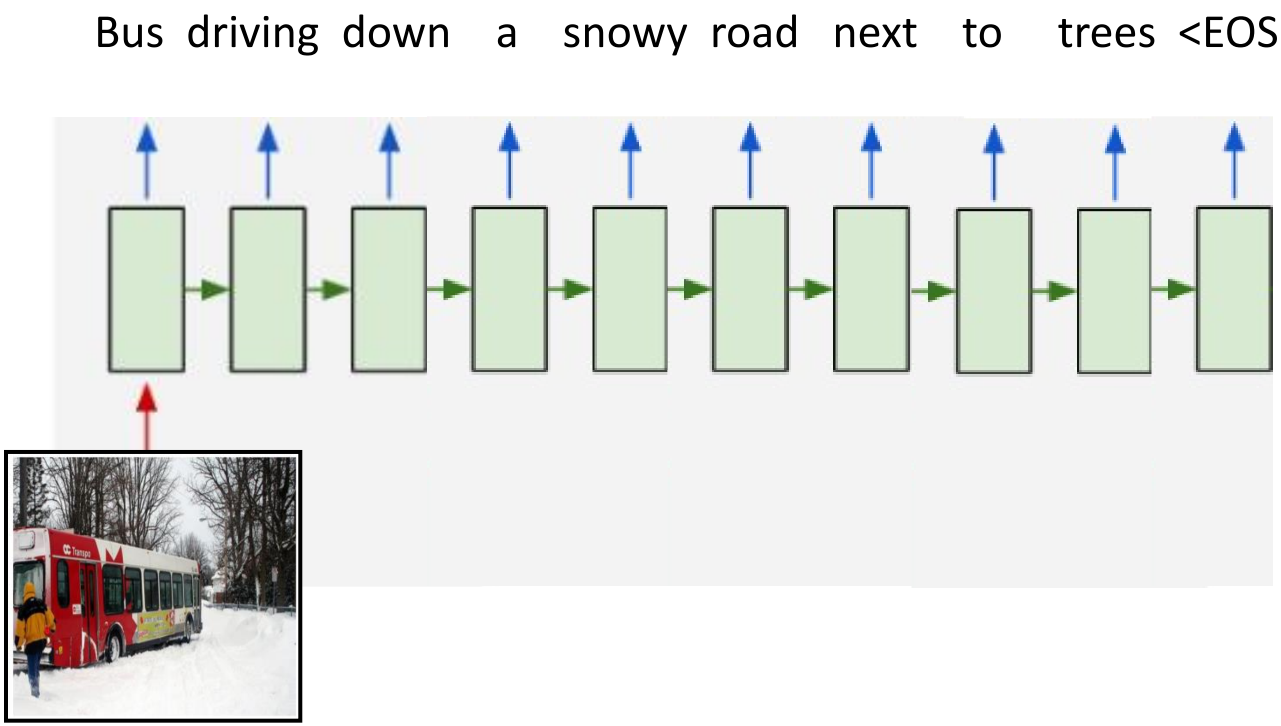
Sequence output

Example: Image captioning

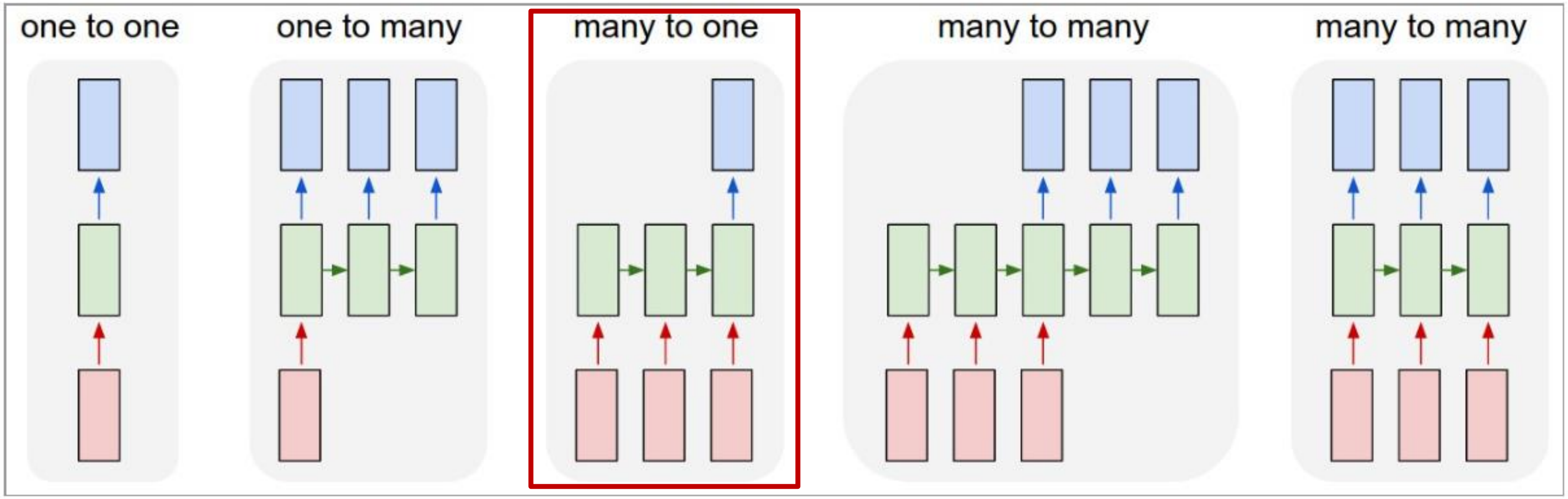
Example: One-to-many

Sequence output

Example: Image Captioning



Many-to-one



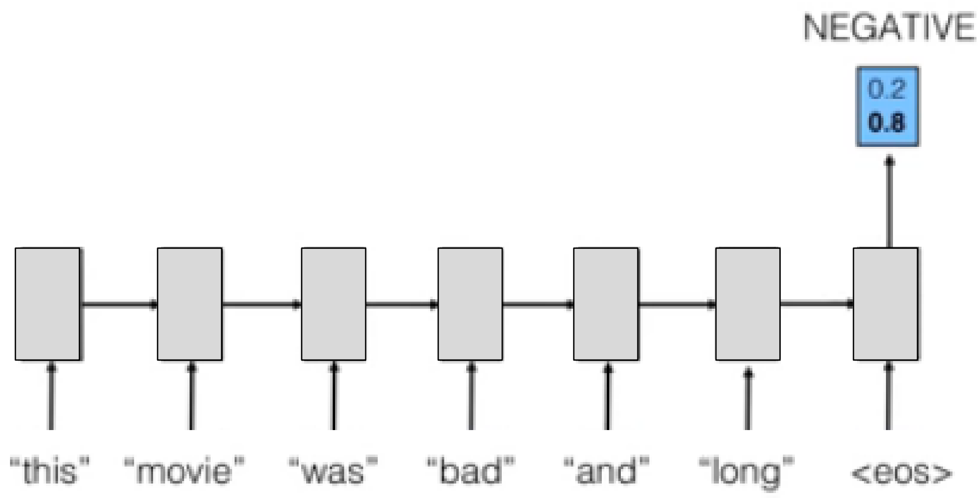
Sequence input

Examples: Sentiment analysis
Action recognition

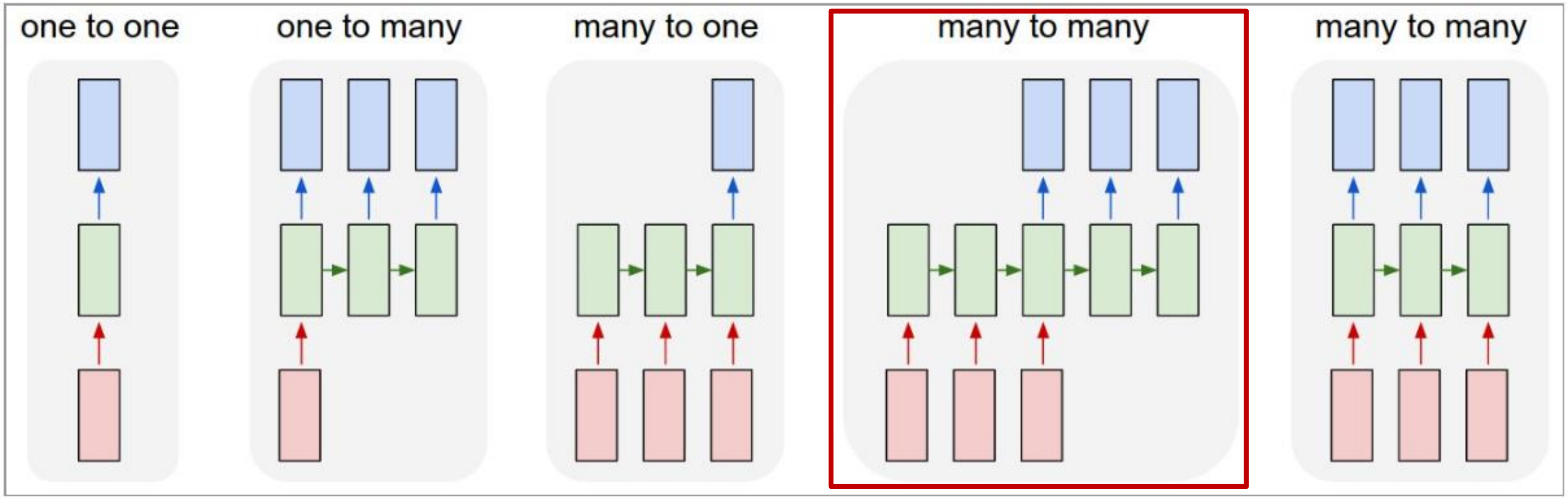
Example: Many-to-one

Sequence input

Example: Sentiment analysis



Many-to-many



Sequence input and sequence output

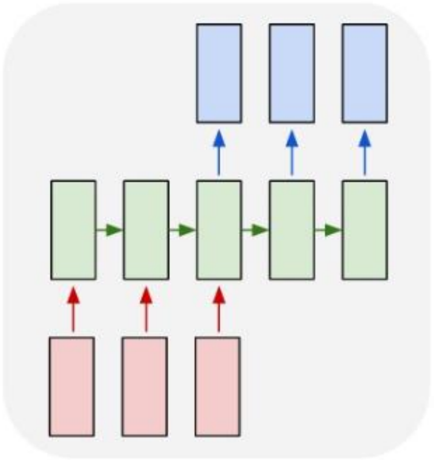
Example: Machine translation

Example: Many-to-many

Sequence input and sequence output

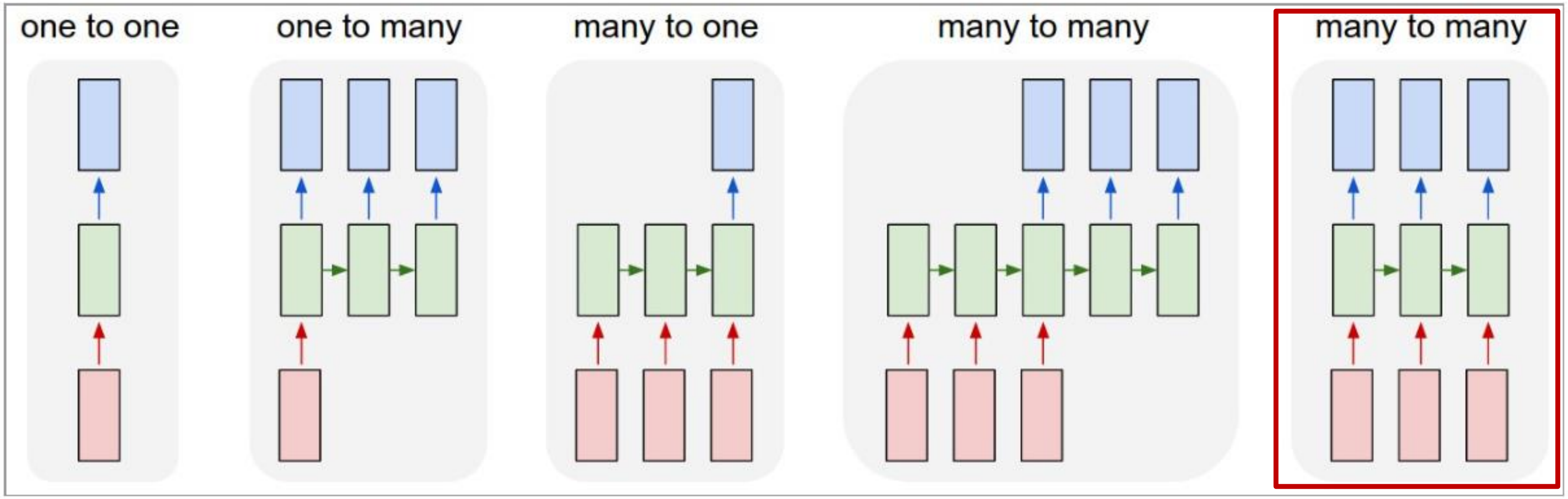
Example: Machine translation

French Translation



English Sentence

Synced Many-to-many

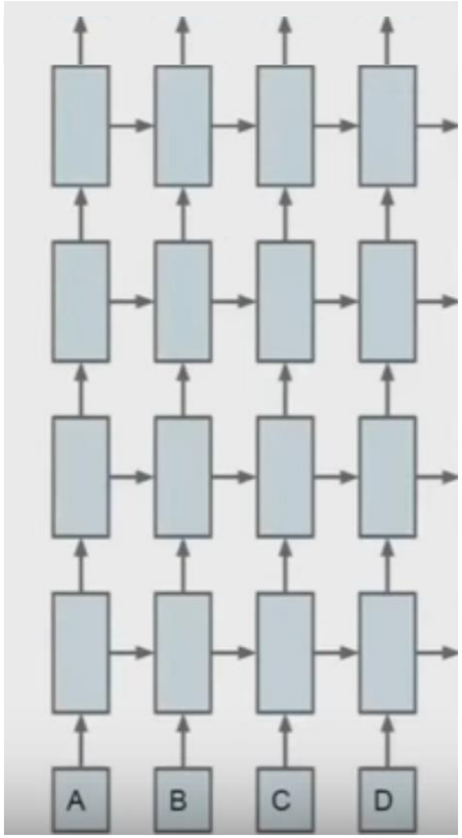


Synced sequence input and output

Examples: Tracking

Early action detection

Deep RNNs



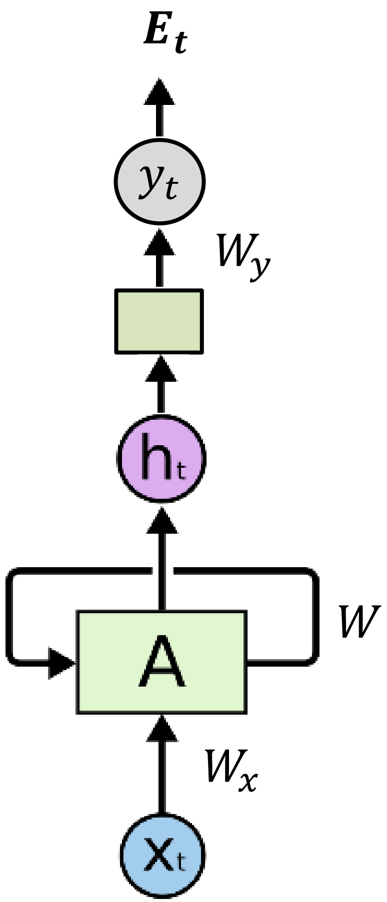
- Stacking RNNs
- More ways of propagating information!
- Requires a lot of data!

Fwd RNN TT

- Forward pass through time

$$h_t = W\phi(h_{t-1}) + W_x x_t$$

$$y_t = W_y \phi(h_t)$$



Recurrent Neural Network (RNN)

Aside: Forward pass

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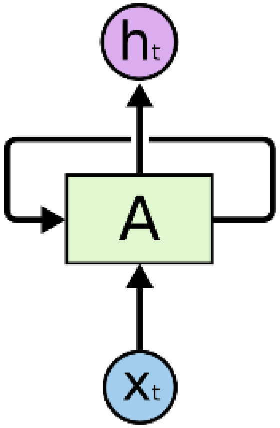
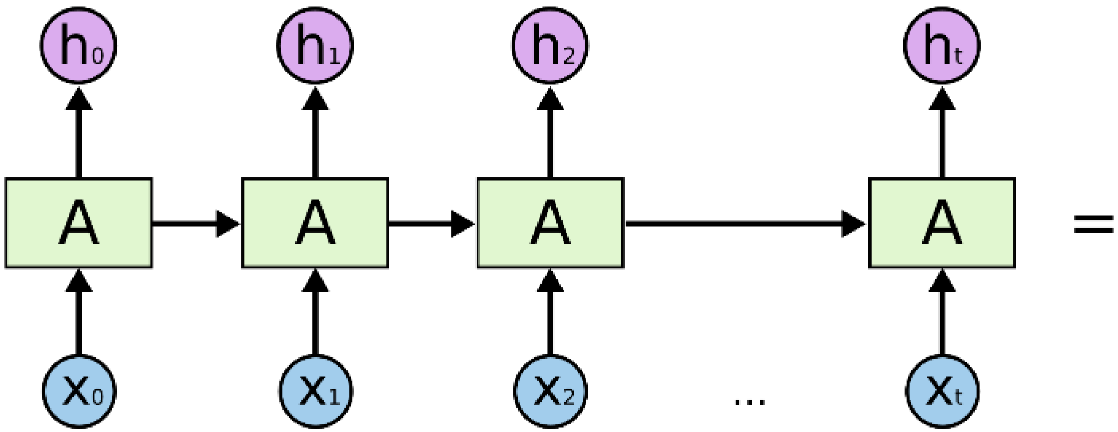


Recurrent Neural Network (RNN)

- Error or cost is computed for each prediction.



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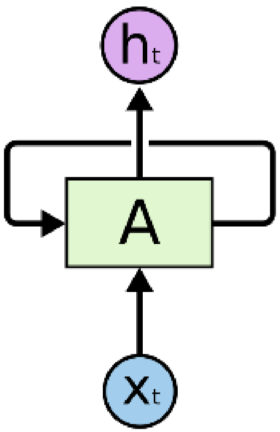
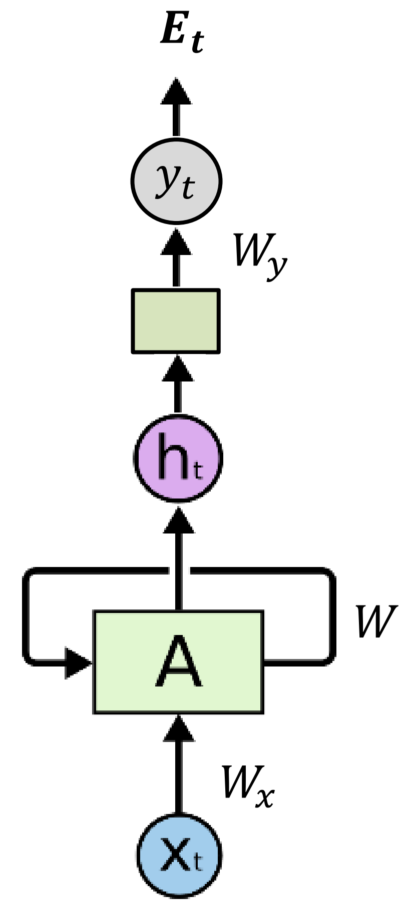


BP TT

- Backpropagation through time

$$\frac{\partial E}{\partial W} = \sum_{t=1}^{T} \frac{\partial E_t}{\partial W}$$

Aside: Forward pass



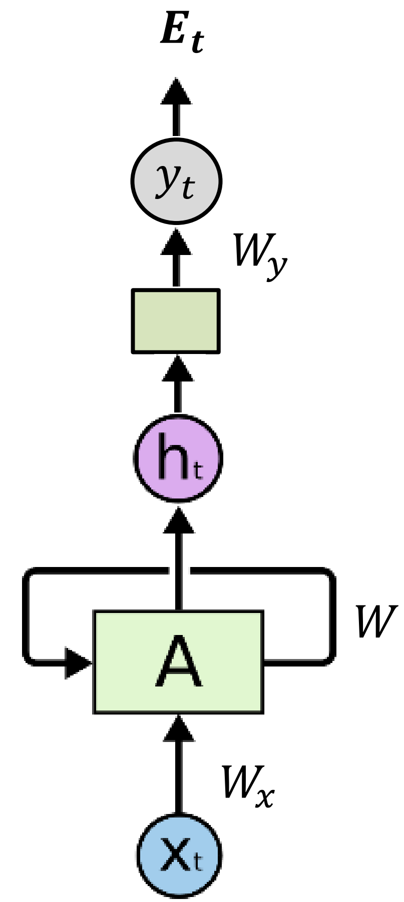
BP TT

- Backpropagation through time

$$\frac{\partial E}{\partial W} = \sum_{t=1}^{T} \boxed{\frac{\partial E_t}{\partial W}}$$

$$\frac{\partial E_t}{\partial W} = \sum_{k=1}^{t} \frac{\partial E_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W}$$

Aside: Forward pass



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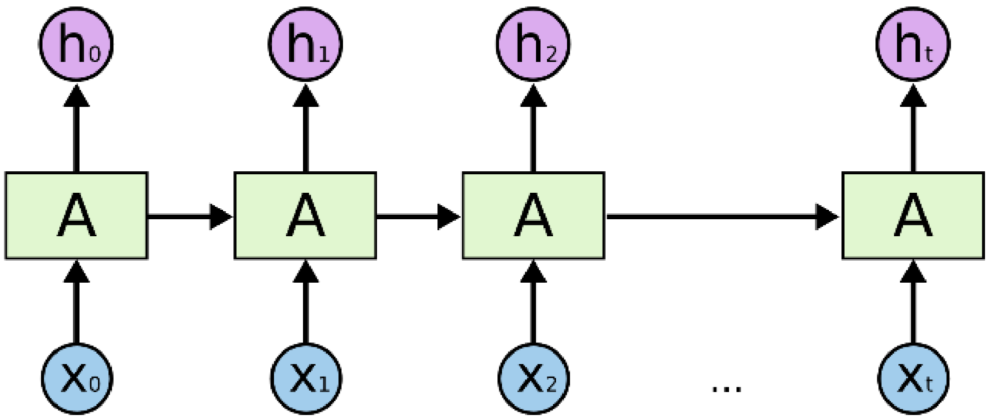
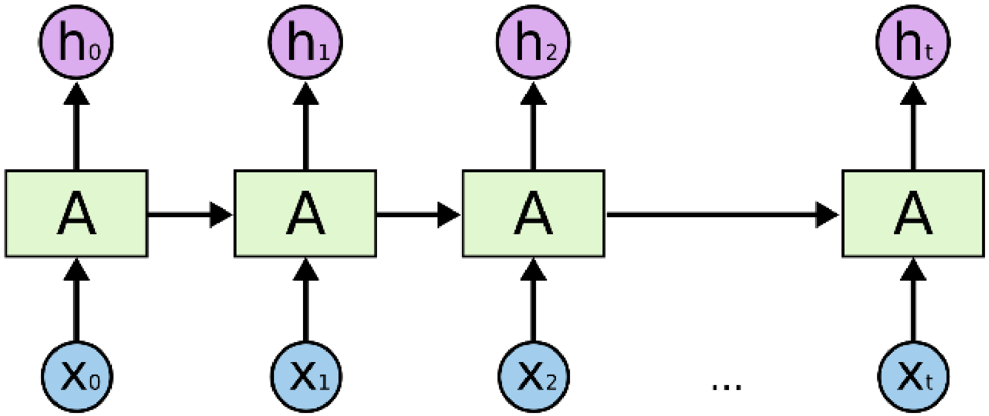


$$\frac{\partial E_t}{\partial W} = \sum_{k=1}^{t} \frac{\partial E_t}{\partial y_t}\frac{\partial y_t}{\partial h_t}\frac{\partial h_t}{\partial h_k}\frac{\partial h_k}{\partial W}$$

BP TT



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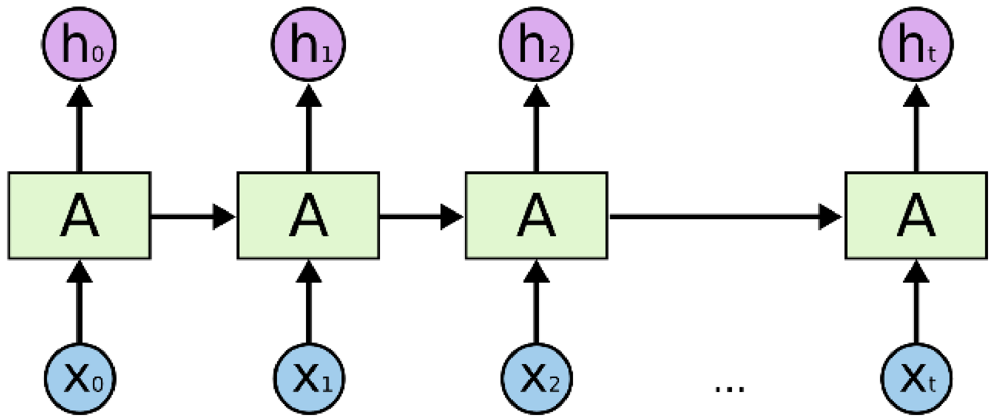

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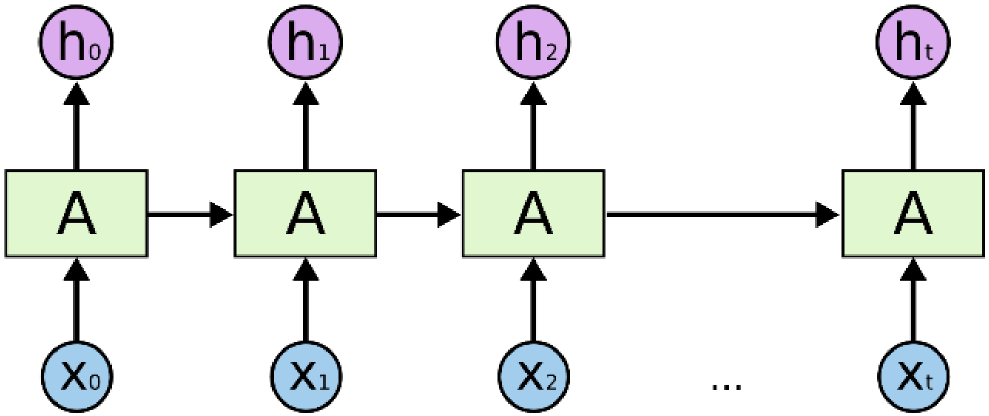
BP TT



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$$\frac{\partial h_t}{\partial h_k} = \prod_{i=k+1}^{t} \frac{\partial h_i}{\partial h_{i-1}}$$

For example @ $t = 2$,

$$\frac{\partial h_2}{\partial h_0} = \prod_{i=1}^{2} \frac{\partial h_i}{\partial h_{i-1}} = \frac{\partial h_1}{\partial h_0} \frac{\partial h_2}{\partial h_1}$$

Vanishing (and Exploding) Gradients

$$\frac{\partial E_t}{\partial W} = \sum_{k=1}^{t} \frac{\partial E_t}{\partial y_t} \frac{\partial y_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W}$$

$$\frac{\partial h_t}{\partial h_k} = \prod_{i=k+1}^{t} \frac{\partial h_i}{\partial h_{i-1}}$$

Aside: Forward pass

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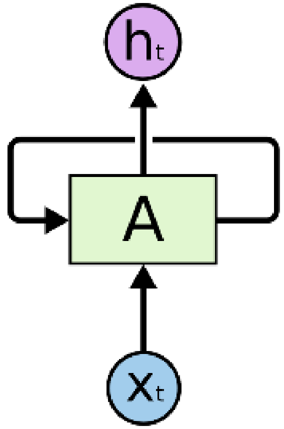

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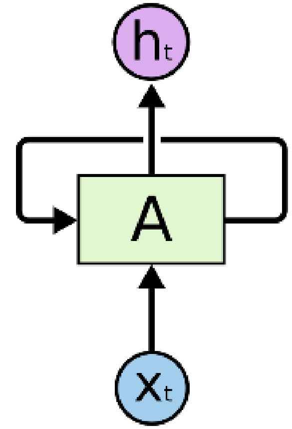

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$$\left\| \frac{\partial h_i}{\partial h_{i-1}} \right\|$$

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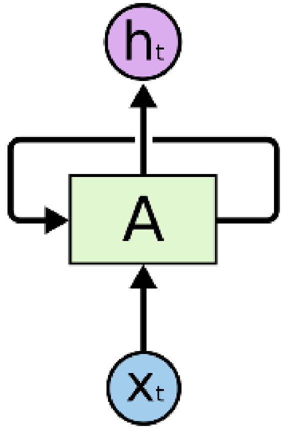

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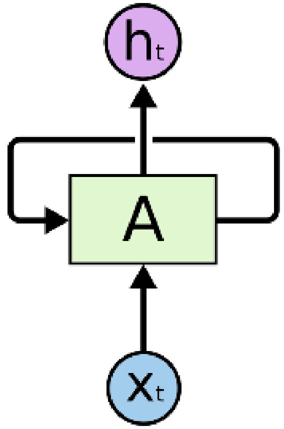

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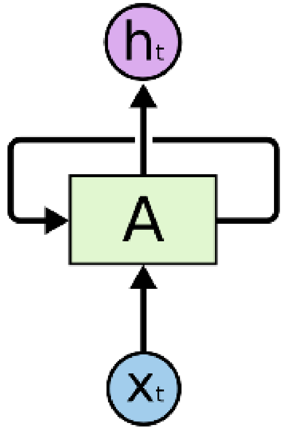

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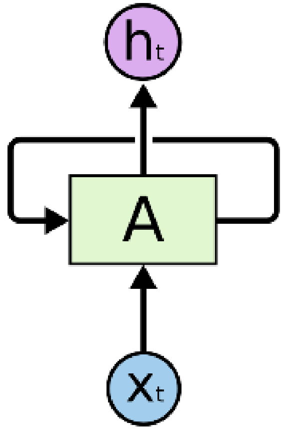

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<1 vanishing

>1 exploding

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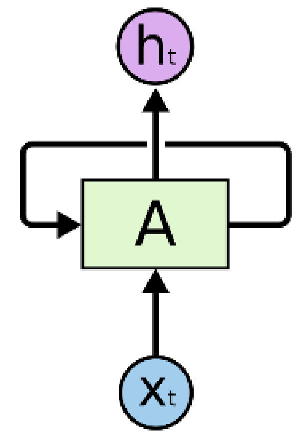


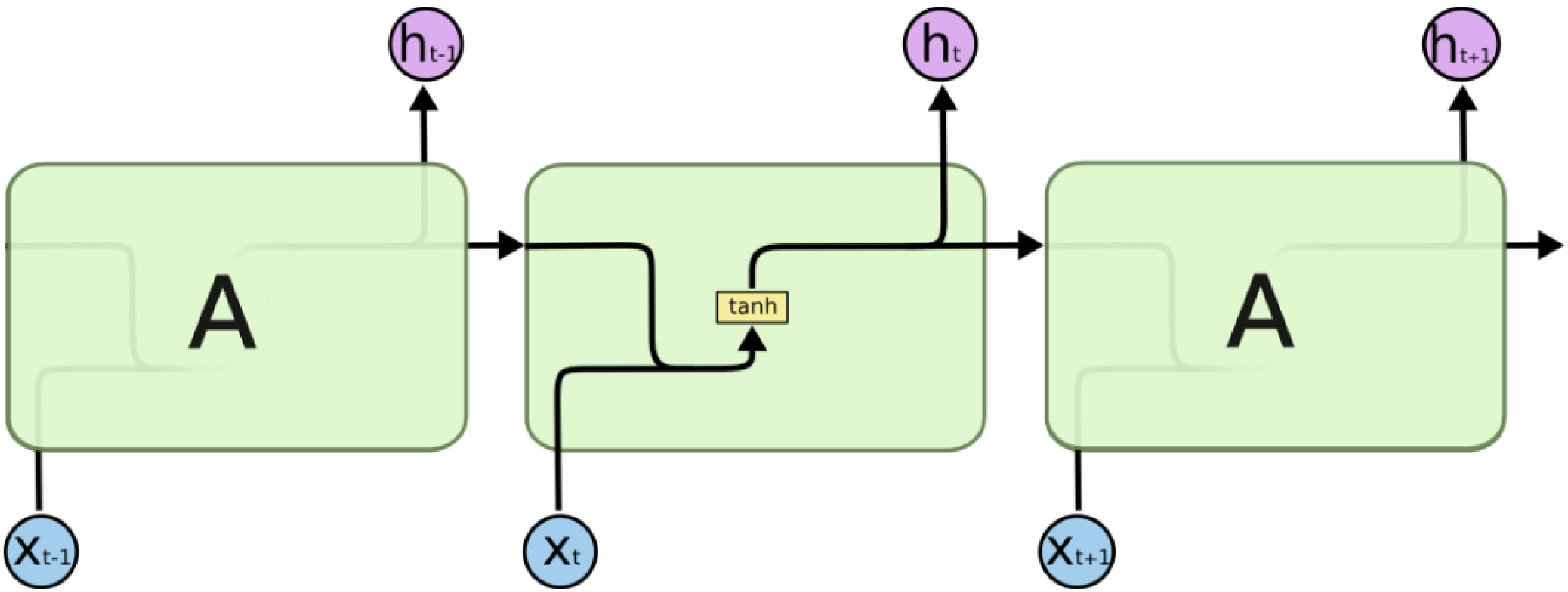
Vanishing (and Exploding) Gradients

- Exploding Gradients
  - Easy to detect
  - Clip the gradient at a threshold

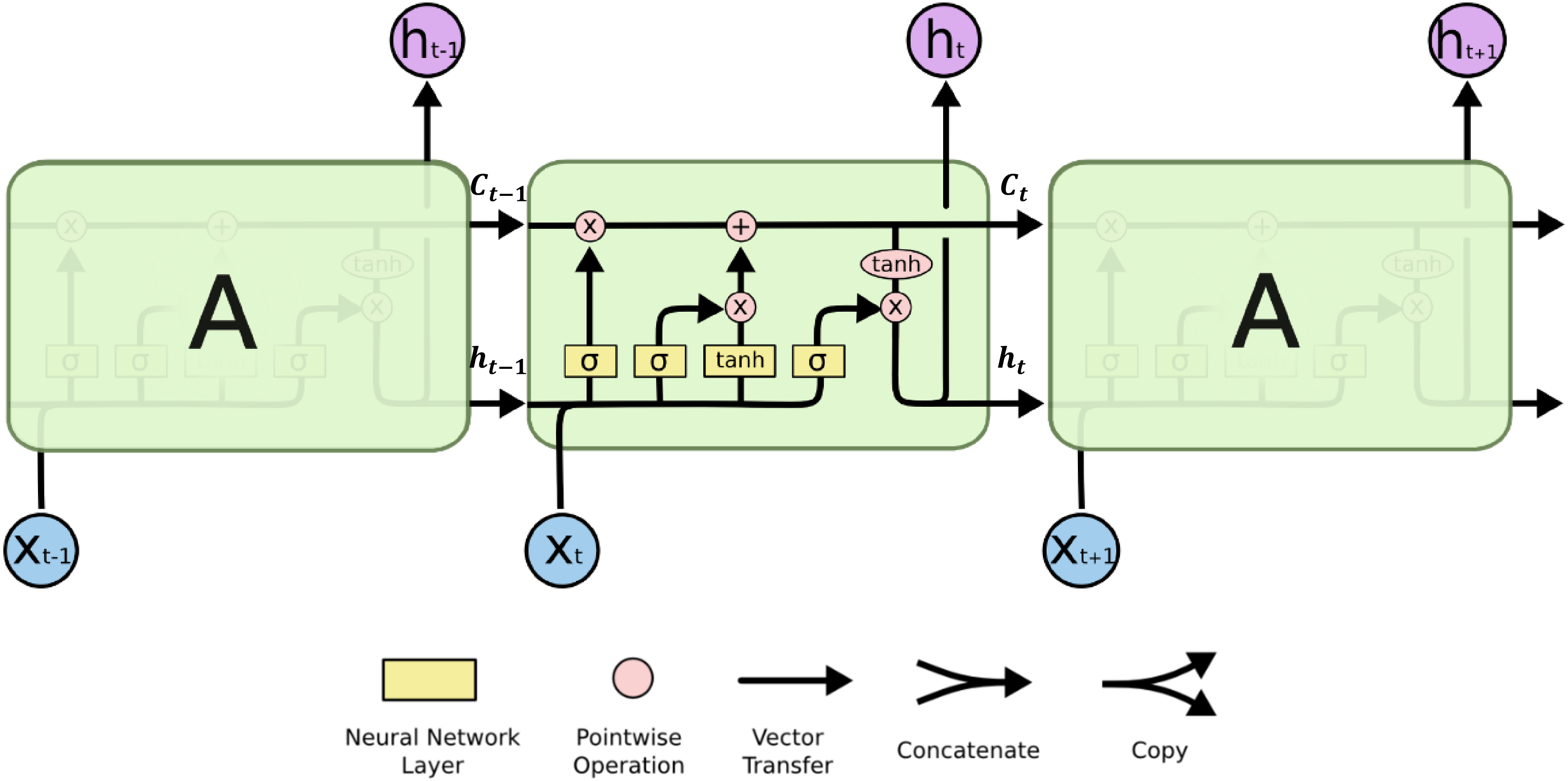
- Vanishing Gradients
  - More difficult to detect
  - Architectures designed to combat the problem of vanishing gradients. Example: LSTMs by *Schmidhuber et al.*

RNNs

- In a standard RNN the repeating module has a simple structure. Example:

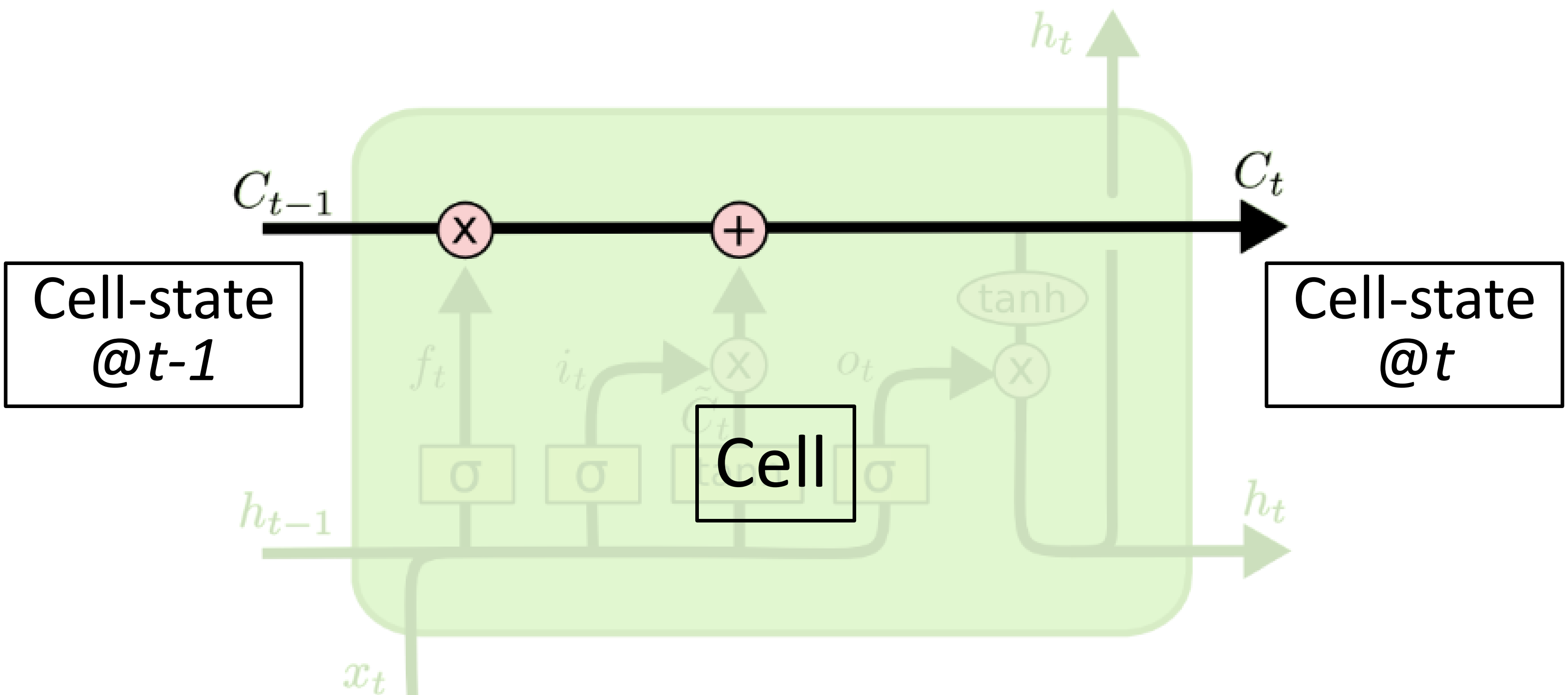


LSTMs



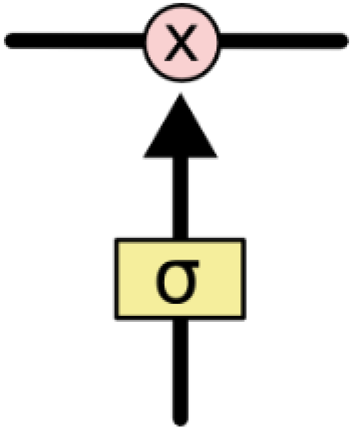
h_{t-1}
h_t
h_{t+1}
C_{t-1}
C_t
h_{t-1}
h_t
A
A
σ
σ
tanh
σ
tanh
x_{t-1}
x_t
x_{t+1}
Neural Network Layer
Pointwise Operation
Vector Transfer
Concatenate
Copy

LSTM Memory / Cell State



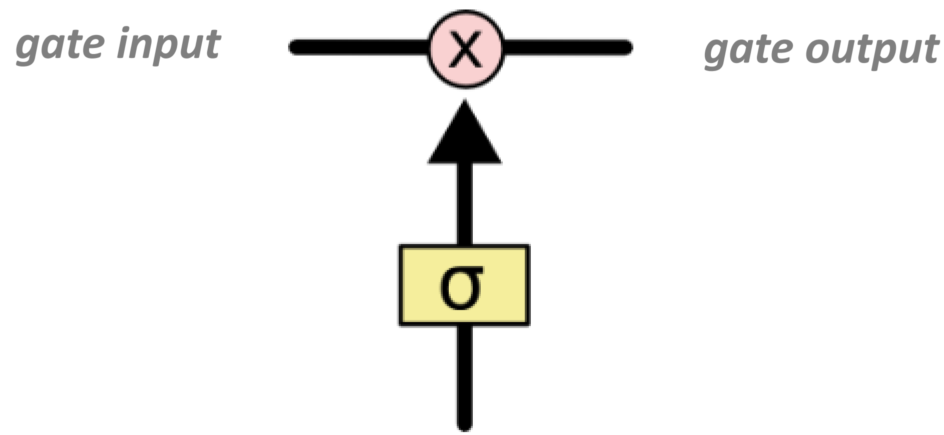
Gate

- Composed of a sigmoid neural net layer and a pointwise multiplication operation.



Gate

- sigmoid: outputs numbers between:
  - zero "let nothing through," and
  - one, "let everything through!"
- Example:



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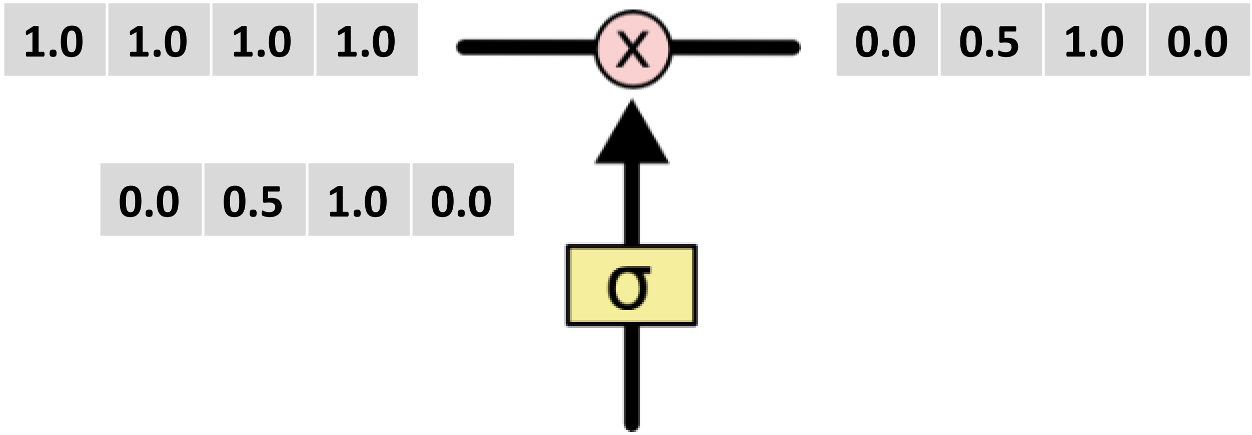

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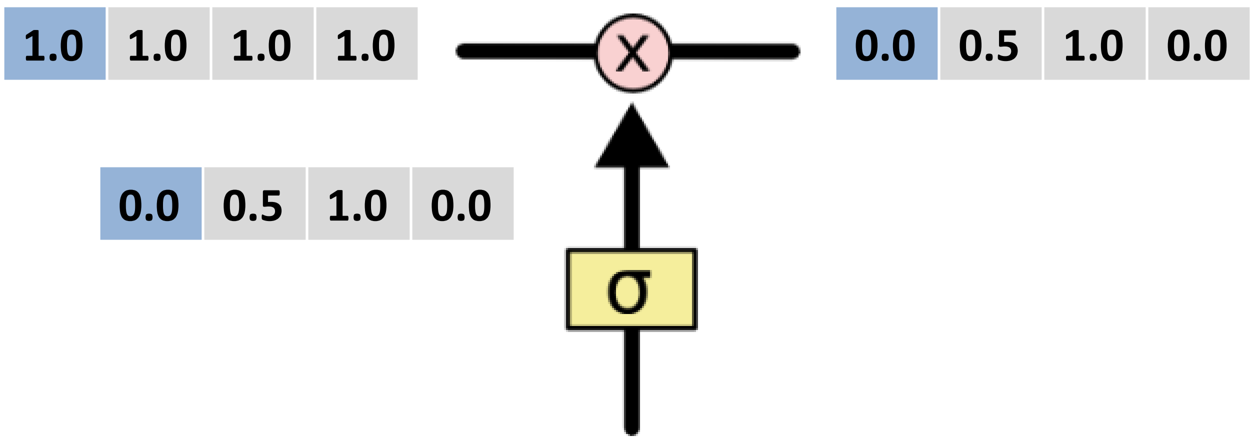

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- sigmoid: outputs numbers between:
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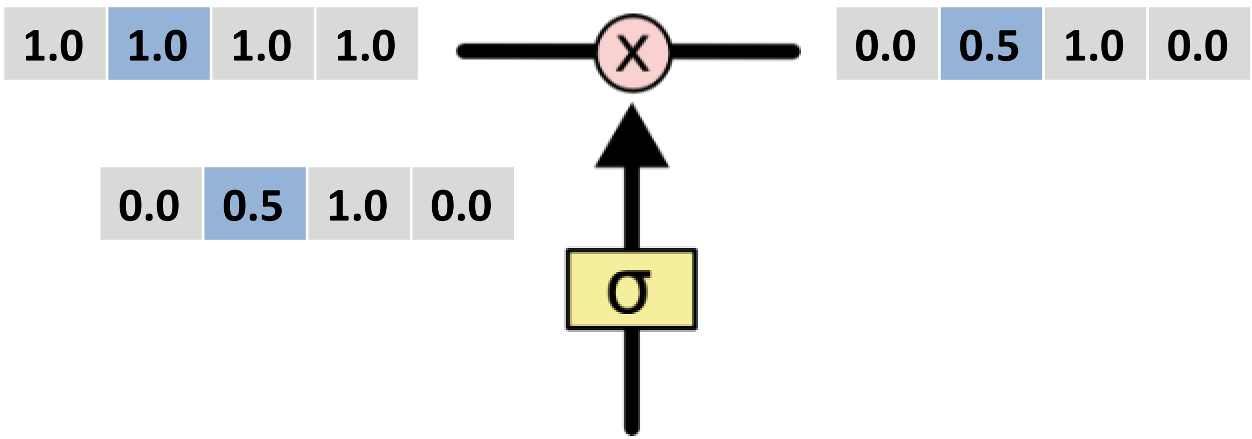
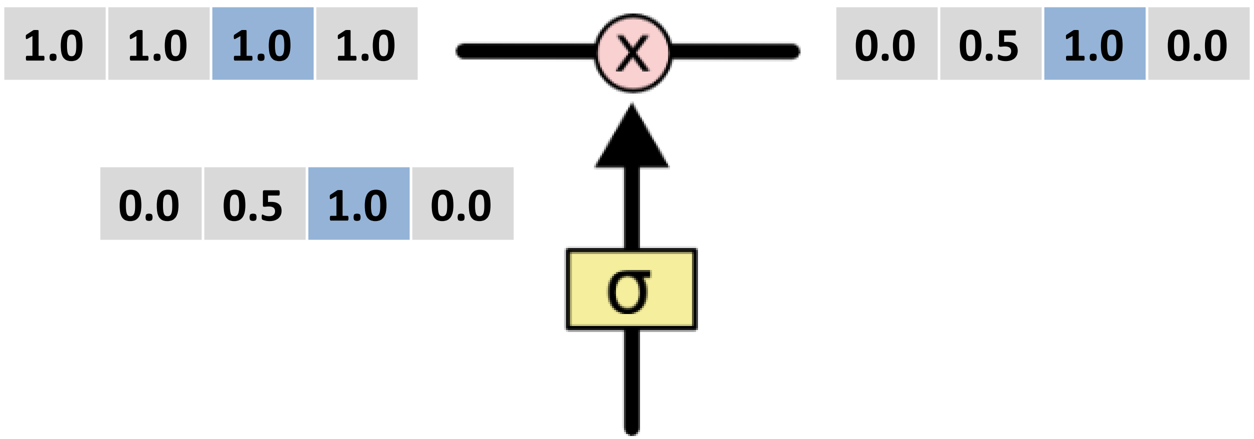
  - one, "let everything through!"
- Example:



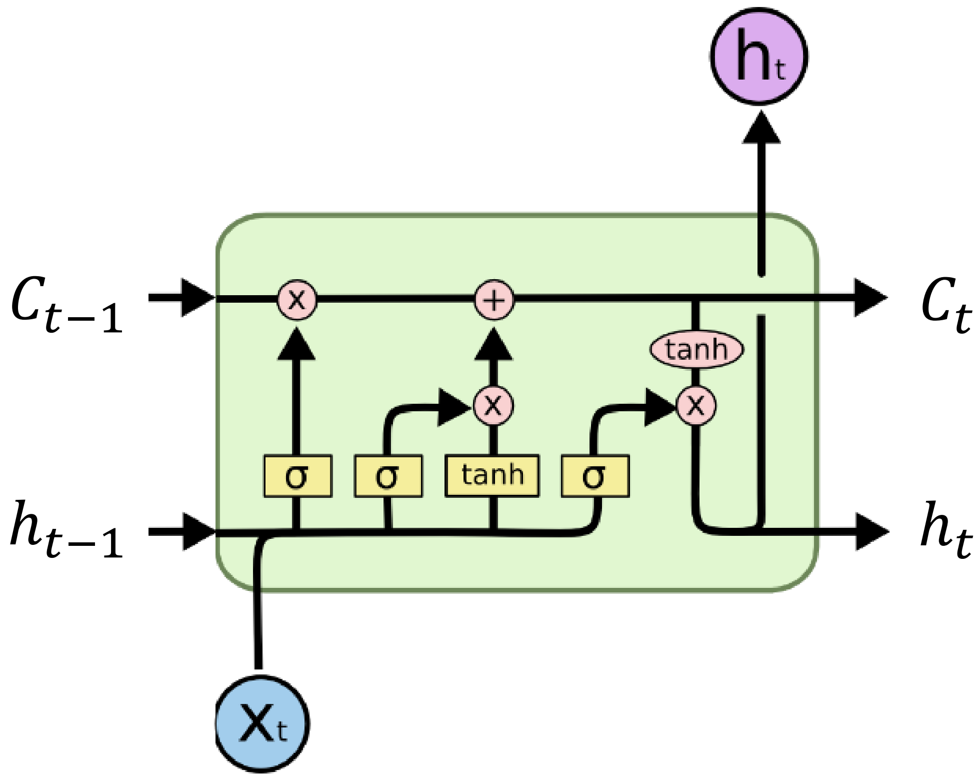
Gate

- sigmoid: outputs numbers between:
  - zero "let nothing through," and
  - one, "let everything through!"
- Example:



LSTM Gates

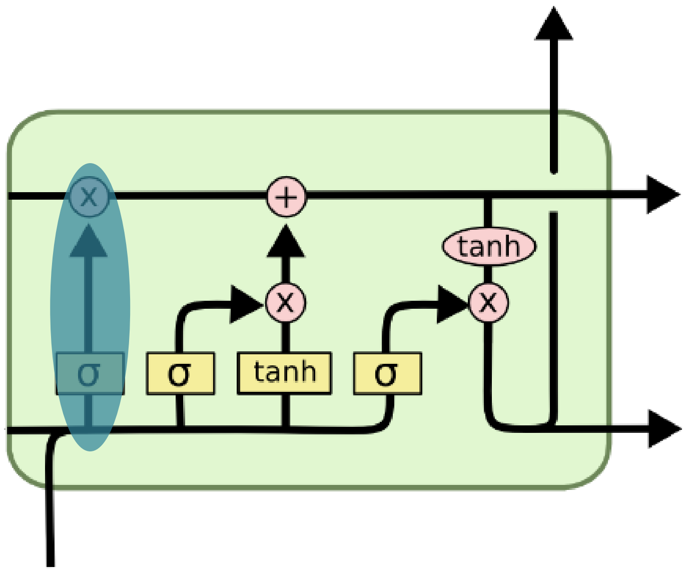
- An LSTM has three of these gates.



LSTM Gates

- An LSTM has three of these gates.

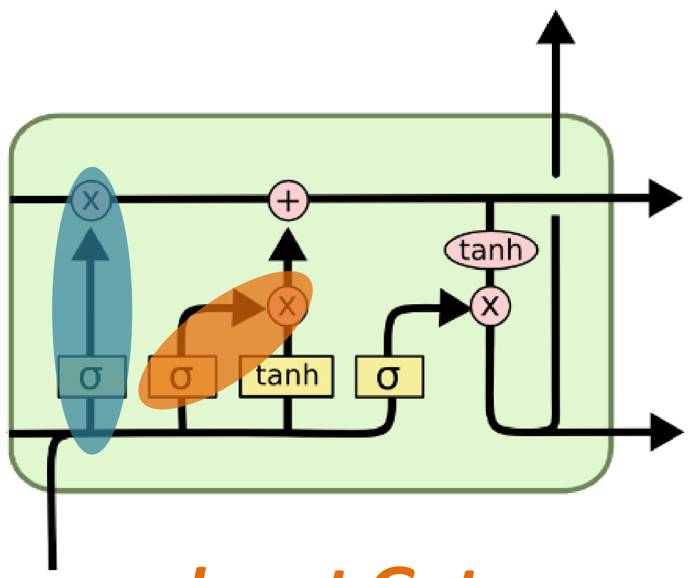
Forget gate



LSTM Gates

- An LSTM has three of these gates.

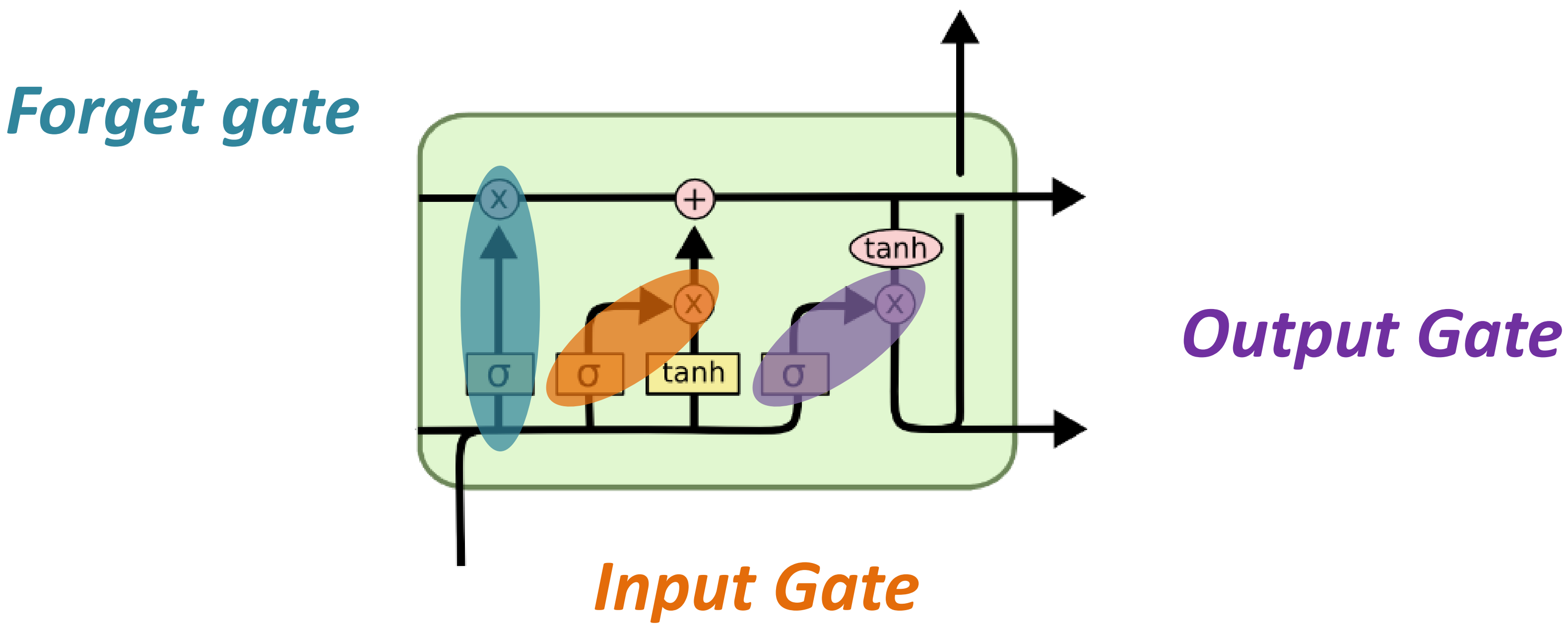
Forget gate



Input Gate

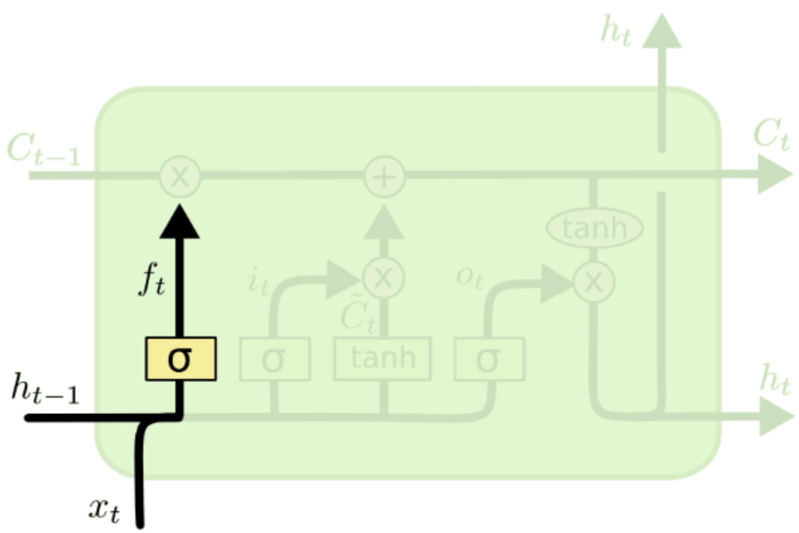
LSTM Gates

- An LSTM has three of these gates.



Step 1: Forget Gate

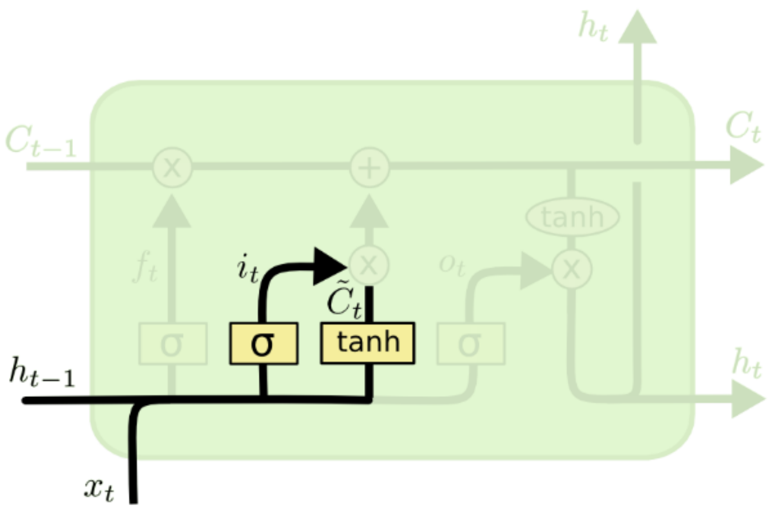
- Decide what to forget/ignore.



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \ + \ b_f\right)$$

Step 2: Input Gate

- Propose new candidate memory $\tilde{C}_t$.
- Modulate the proposal.

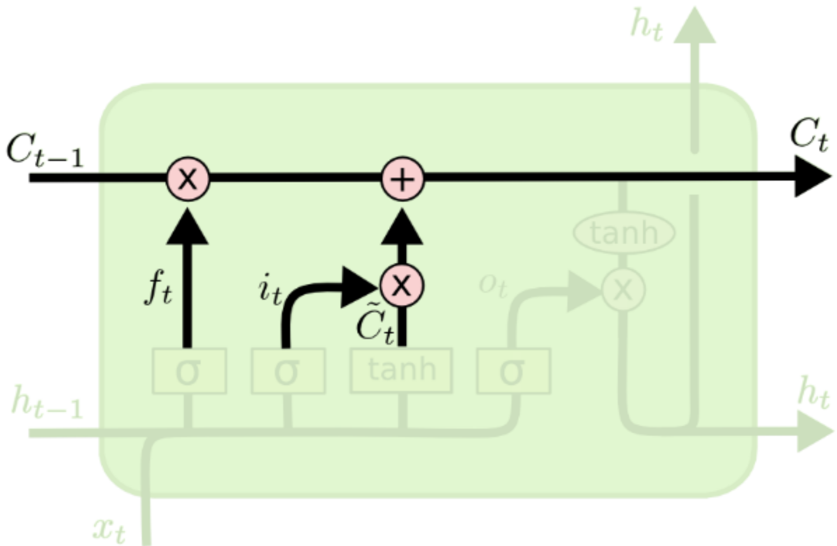


$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \ + \ b_C)$$

$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \ + \ b_i\right)$$

Step 3: Memory Update

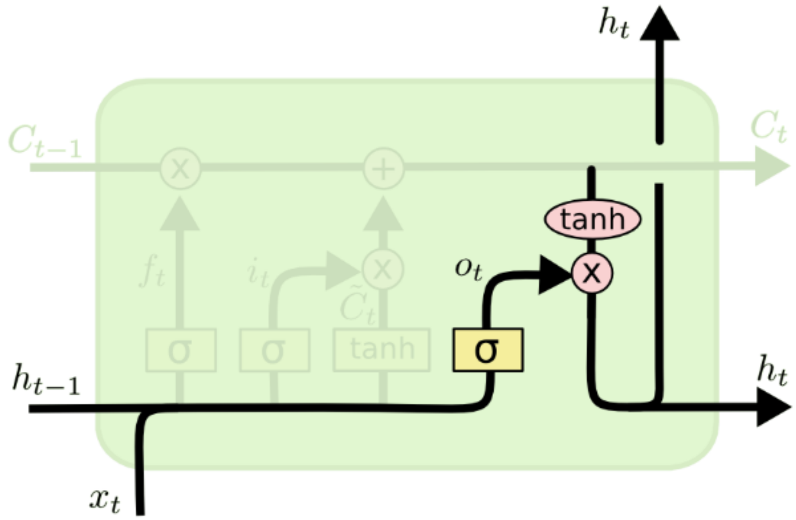
- Perform the memory update.



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

Step 4: Output Gate

- Decide what to output based on the memory.



$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$
$$h_t = o_t * \tanh\left(C_t\right)$$

LSTM BP TT

Forward Pass: $h^t = o^t \odot \tanh(c^t)$

Given $\delta h^t = \dfrac{\partial E}{\partial h^t}$, find $\delta o^t, \delta c^t$

$$\frac{\partial E}{\partial o_i^t} = \frac{\partial E}{\partial h_i^t} \cdot \frac{\partial h_i^t}{\partial o_i^t} = \delta h_i^t \cdot \tanh(c_i^t)$$

$$\therefore \delta o^t = \delta h^t \odot \tanh(c^t)$$

$$\frac{\partial E}{\partial c_i^t} = \frac{\partial E}{\partial h_i^t} \cdot \frac{\partial h_i^t}{\partial c_i^t} = \delta h_i^t \cdot o_i^t \cdot (1 - \tanh^2(c_i^t))$$

$$\therefore \delta c^t += \delta h^t \odot o^t \odot (1 - \tanh^2(c^t))$$

Forward Pass: $c^t = i^t \odot a^t + f^t \odot c^{t-1}$

Given $\delta c^t = \dfrac{\partial E}{\partial c^t}$, find $\delta i^t, \delta a^t, \delta f^t, \delta c^{t-1}$

$$\frac{\partial E}{\partial i_i^t} = \frac{\partial E}{\partial c_i^t} \cdot \frac{\partial c_i^t}{\partial i_i^t} = \delta c_i^t \cdot a_i^t$$

$$\therefore \delta i^t = \delta c^t \odot a^t$$

$$\frac{\partial E}{\partial f_i^t} = \frac{\partial E}{\partial c_i^t} \cdot \frac{\partial c_i^t}{\partial f_i^t} = \delta c_i^t \cdot c_i^{t-1}$$

$$\therefore \delta f^t = \delta c^t \odot c^{t-1}$$

$$\frac{\partial E}{\partial a_i^t} = \frac{\partial E}{\partial c_i^t} \cdot \frac{\partial c_i^t}{\partial a_i^t} = \delta c_i^t \cdot i_i^t$$

$$\therefore \delta a^t = \delta c^t \odot i^t$$

$$\frac{\partial E}{\partial c_i^{t-1}} = \frac{\partial E}{\partial c_i^t} \cdot \frac{\partial c_i^t}{\partial c_i^{t-1}} = \delta c_i^t \cdot f_i^t$$

$$\therefore \delta c^{t-1} = \delta c^t \odot f^t$$

Forward Pass: $z^t = \begin{bmatrix} \hat{a}^t \\ \hat{i}^t \\ \hat{f}^t \\ \hat{o}^t \end{bmatrix} = W \times I^t$

Given $\delta a^t, \delta i^t, \delta f^t, \delta o^t$, find δz^t

$$\begin{aligned} \delta \hat{a}^t &= \delta a^t \odot (1 - \tanh^2(\hat{a}^t)) \\ \delta \hat{i}^t &= \delta i^t \odot i^t \odot (1 - i^t) \\ \delta \hat{f}^t &= \delta f^t \odot f^t \odot (1 - f^t) \\ \delta \hat{o}^t &= \delta o^t \odot o^t \odot (1 - o^t) \\ \delta z^t &= \left[\delta \hat{a}^t, \delta \hat{i}^t, \delta \hat{f}^t, \delta \hat{o}^t\right]^T \end{aligned}$$

Forward Pass: $z^t = W \times I^t$

Given δz^t, find $\delta W^t, \delta h^{t-1}$

$$\delta I^t = W^T \times \delta z^t$$

As $I^t = \begin{bmatrix} x^t \\ h^{t-1} \end{bmatrix}$,

δh^{t-1} can be retrieved from δI^t

$$\delta W^t = \delta z^t \times (I^t)^T$$

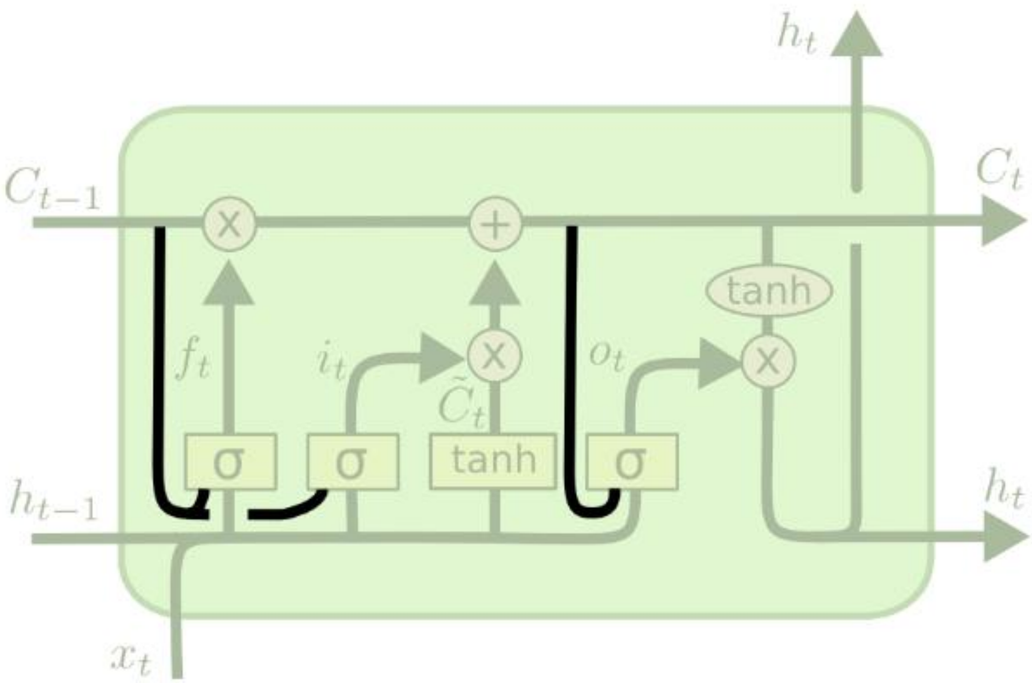
If input x has T time-steps, i.e. $x = [x^1, x^2, \cdots, x^T]$, then

$$\delta W = \sum_{t=1}^{T} \delta W^t$$

W is then updated using an appropriate Stochastic Gradient Descent solver.

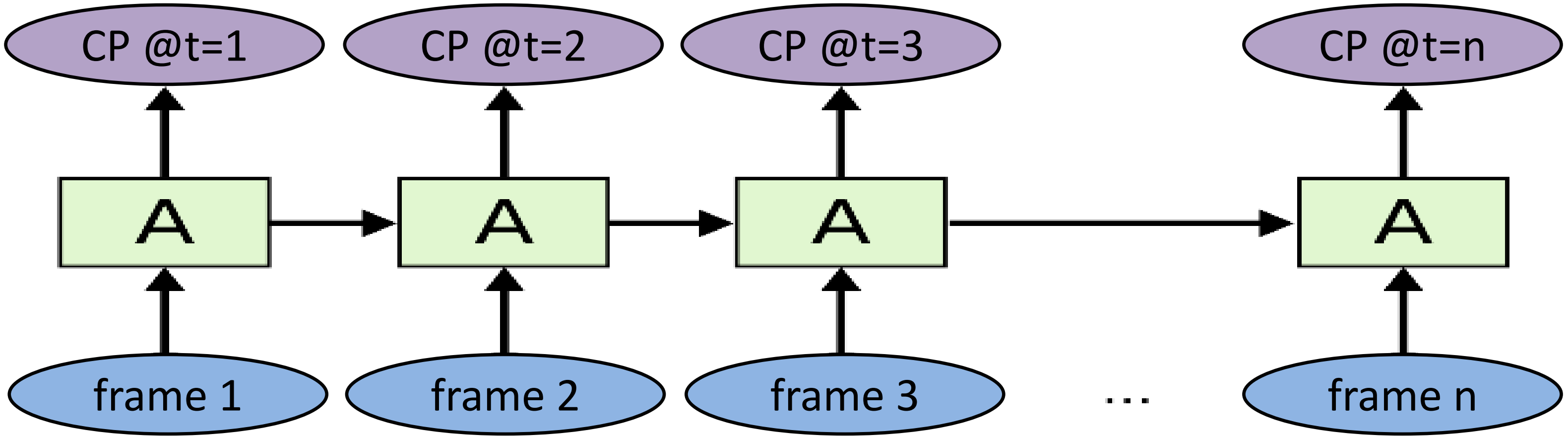
LSTM Variants

- Example: PeepHole Model

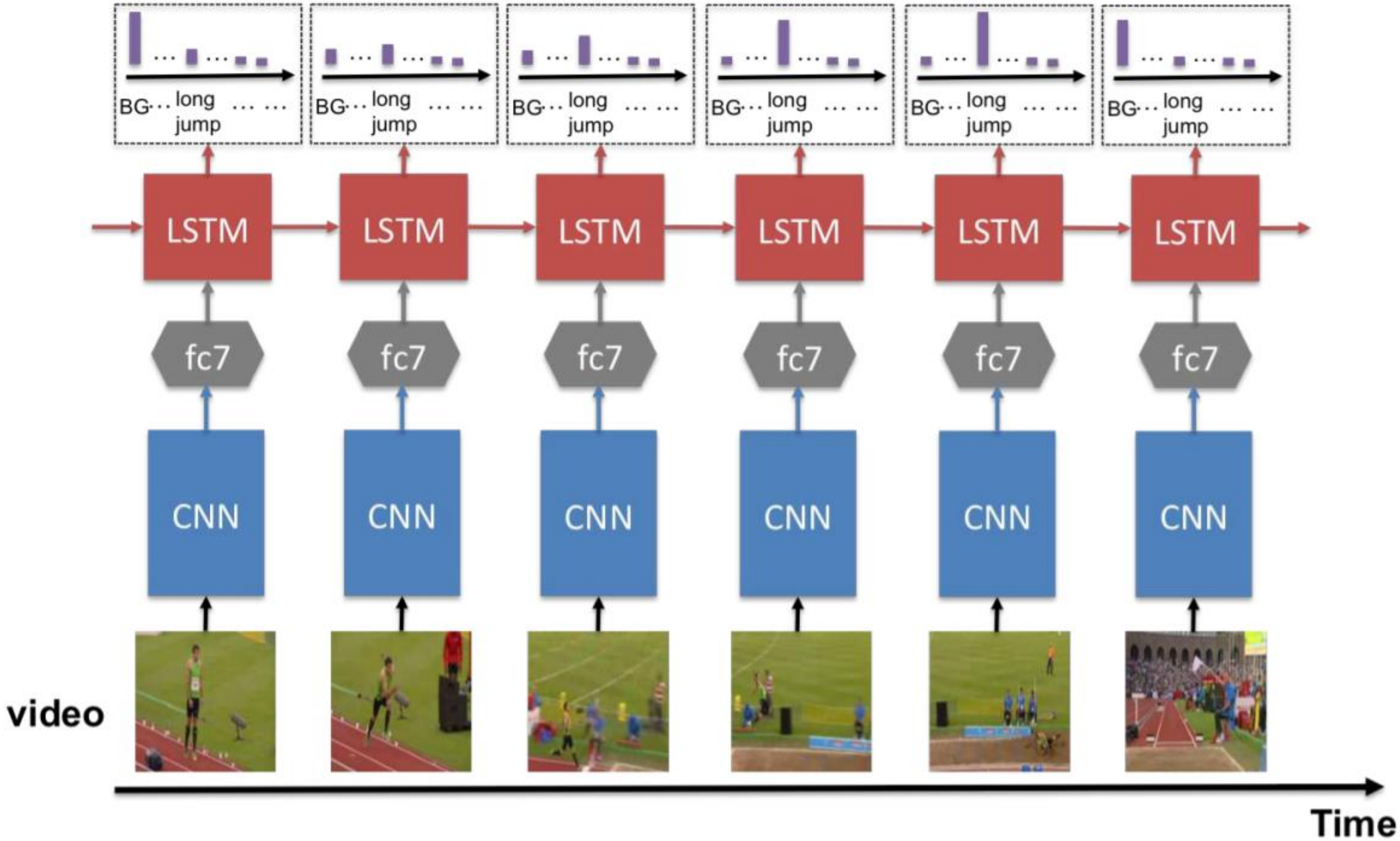


Application 1: Video Classification

- CP: conditional class probability
- frame i could be a feature describing frame i, example: CNN feature



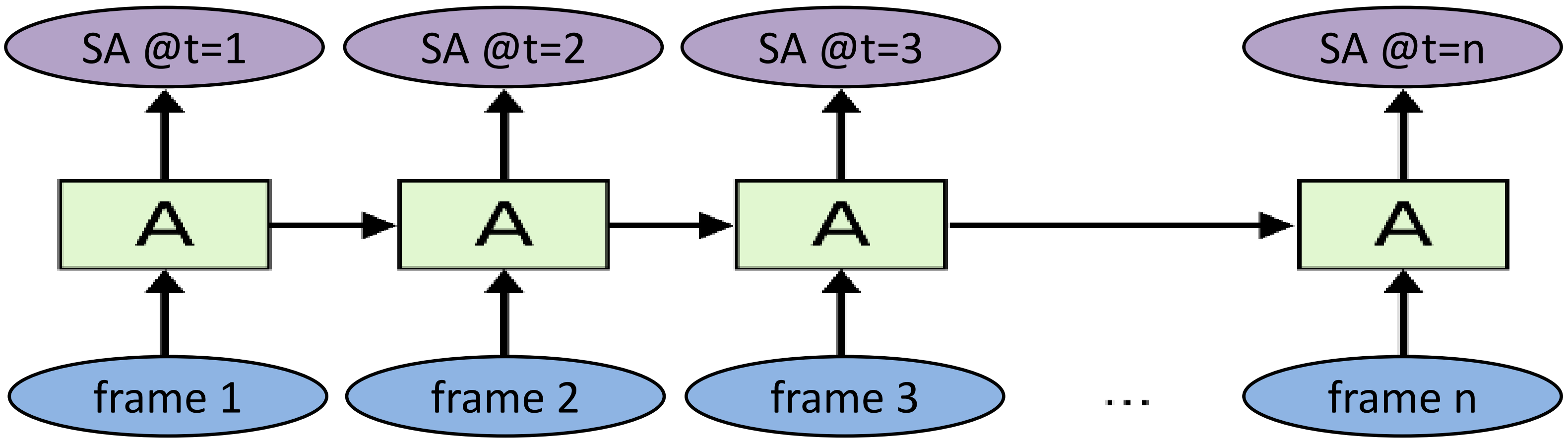
Application 1: Video Classification



Shugao Ma et al., CVPR'16, http://www.cv-foundation.org/openaccess/content_cvpr_2016/papers/Ma_Learning_Activity_Progression_CVPR_2016_paper.pdf

Application 2: Self-Driving Cars

- SA: steering angle
- frame i could be a feature describing frame i, example: 3D-CNN feature



Application 2: Self-Driving Cars

DeepTesla

Application 2: Self-Driving Cars

- Udacity winning team: Team Komanda
  - x_t: 3D convolution of image sequence
  - h_t: steering angle, speed, torque