1. General Concepts (1/2)

True or False

For the true/False answers, give a one sentence explanation of each answer; answers without explanation will not be given any points.

a) Suppose we use polynomial features for linear regression, then the hypothesis is linear in the original features [T/F]

b) Maximum likelihood can be used to derive a closed-form solution to logistic regression [T/F]

c) The gradient descent update for logistic regression is identical to linear regression [T/F]

d) Changing the prior in Linear Discriminant Analysis changes the direction of the decision hyperplane [T/F]

e) One example of a discriminative classification model is logistic regression.

2. General Concepts (2/2)

Short answer questions

Answer the following questions in brief one to two sentence answers.

a) For a training dataset $\mathrm{D} = \{x_i, y_i\}$ where x_i are the inputs and y_i are the output, explain the difference between discriminative and generative classification models.

b) Give one example of a generative model.

c) What is cross-validation?

d) How can we use cross-validation to prevent overfitting? Explain the procedure using the setup of (d).

3. Error Metrics

a) Give one example each of error metrics that can be used to evaluate: classification, regression, clustering.

b) Which are the correct definitions of precision and recall? Here 'actual positives' are examples labeled positive (by humans), and 'predicted positives' are examples for which the algorithm predicts a positive label.

1. $precision = \frac{true\ positives}{predicted\ positives}$
2. $precision = \frac{true\ positives}{actual\ positives}$
3. $recall = \frac{predicted\ positives}{actual\ positives}$
4. $recall = \frac{true\ positives}{actual\ positives}$

4. Bayesian Methods

Alice has a dataset of m points with n-dimensional inputs and scalar outputs. She has trained several regularized linear regression models using regularization parameters $\lambda = e^0, e^{-1}, e^{-2}, e^{-3}$.

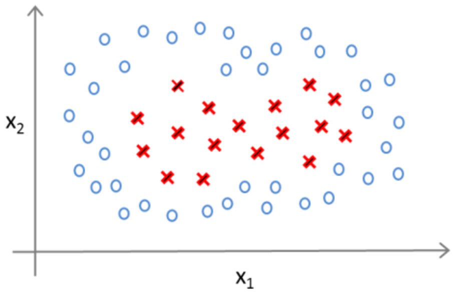
a) Which parameter will lead to highest bias? To highest variance?

b) Alice then decides to use a Bayesian approach to control the complexity of the model. What is the Bayesian equivalent to changing λ?

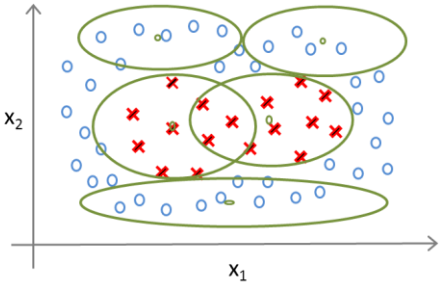
c) Which Bayesian model should she use? Explain what makes the model Bayesian.

5. SVMs and Kernels

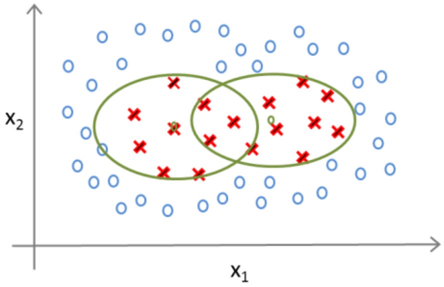
Consider the following dataset of 2-dimensional datapoints:



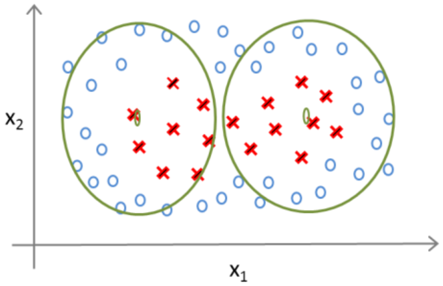
a) Which placement of Gaussian basis functions corresponds to a kernel feature representation for this dataset? Explain your answer in one sentence below.



(A)



(B)



(C)

(D) none of the above

b) How does increasing the variance of the Gaussian σ^2 affect the bias and variance of the resulting Gaussian Kernel SVM classifier? Explain.

c) For $k(x_1, x_2)$ to be a valid kernel, there must be a feature basis function $\varphi(.)$ such that we can write $k(x_1, x_2) = \varphi(x_1)^T \varphi(x_2)$. Suppose $k_1(x_1, x_2)$ and $k_2(x_1, x_2)$ are valid kernels. Prove that the following is also a valid kernel:

$$k(x_1, x_2) = k_1(x_1, x_2) + k_2(x_1, x_2)$$

d) Both SVMs with Gaussian kernels and Neural Networks with at least one hidden layer can be used to learn non-linear decision boundaries, such as the boundary between positive and negative examples in the dataset above. Describe the main similarity and the main difference in how these two approaches achieve this.

e) Explain what slack variables are used for when training SVMs.

6. Overfitting and Regularization

Q6.1 Bias-Variance and λ

Alice has a binary classification dataset of m points with n-dimensional inputs.

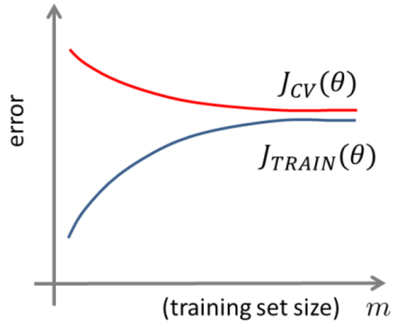
a) She has trained several regularized logistic regression models using regularization parameters $\lambda = 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}$. She computed the cross-validation (CV) and training errors for each value of λ, shown in the table below, but the rows are out of order. Fill in the correct values of λ for each row.

Train error	*CV error*	λ
80%	85%	
40%	45%	
70%	76%	
35%	50%	

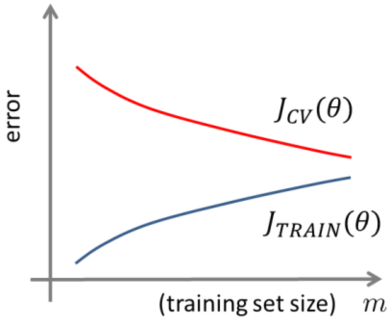
b) Based on these results, which λ should she choose, and why?

c) Which of the four models will have the highest error due to variance? Why?

d) Alice also plotted learning curves for the models with $\lambda = 10^{-1}, 10^{-4}$. Match each plot with the correct value, and explain why it matches.



$\lambda =$



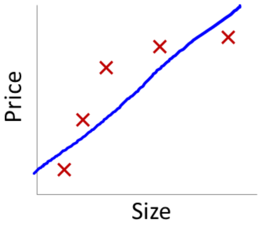
$\lambda =$

Q6.2 Regularization for Linear Regression

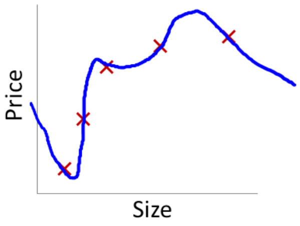
Alice is trying to fit a linear regression model to predict house price based on size using polynomial features. Since her training dataset is very small, she is applying regularization. She fit several models by minimizing the cost function

$$J(\theta) = \frac{1}{2m}\left[\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda\sum_{j=1}^{n}\theta_j^2\right]$$

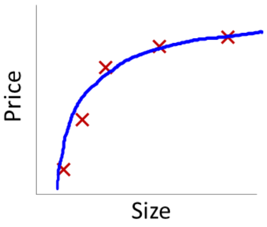
for $\lambda = 10^0, 10^1, 10^2, 10^3$. The following are sketches of the resulting models.



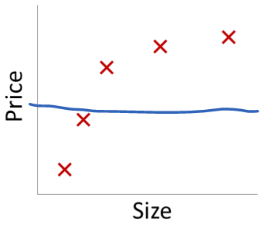
a



b



c



d

a) [3 points] Which value of λ goes with each of the plots? (Write it next to the plot)

b) [3 points] Alice tries her model on a test set. Which model will have the highest error due to bias?

c) [3 points] Which model will have the highest error due to variance?

d) [3 points] Which model, if any, will always have zero test error?

7. Maximum Likelihood Principle

Recall that probabilistic linear regression defines the likelihood of observing outputs $t^{(i)} \in \mathbb{R}$ given inputs $x^{(i)} \in \mathbb{R}^p$, where $i = 1, \dots, m$ and m is the number of samples in the dataset, as

$$p(t_1, \dots, t_m | x_1 \dots, x_m, \theta, \beta) = \prod_{i=1}^{m} N(t^{(i)} | h(x^{(i)}),\ \beta^{-1})$$

where $h(x)$ is the linear regression hypothesis, θ, β are parameters and $N(x|\mu, \sigma^2)$ is the normal (Gaussian) probability density with mean μ and variance σ^2. Here $\beta = \sigma^{-2}$ is the inverse variance of the Gaussian noise that we assume is added to the data.

(a) Find β_{ML}, the maximum likelihood solution for β. *Hint: maximize log likelihood with respect to only* β.

(b) What is the interpretation of the solution β_{ML}? Explain in one sentence.

8. Unsupervised Learning

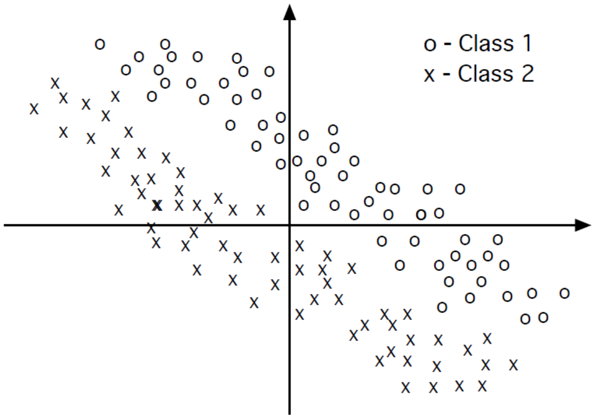
Q8.1 Principle Component Analysis

a) PCA assumes a specific relationship between the unobserved latent coordinates z and the observed data points x. Express this relationship as an equation. Clearly identify and name the parameters which are learned.

b) Name one objective function which could be minimized to learn the parameters of PCA.

c) For a dataset of arbitrary points $x^{(1)}, \dots, x^{(m)}$, specify the steps of the PCA algorithm.

d) Suppose you are given 2D feature vectors for a classification task which are distributed according to the figure below. You apply PCA to the entire dataset. On the figure, draw all the PCA components.

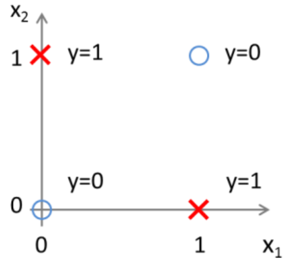


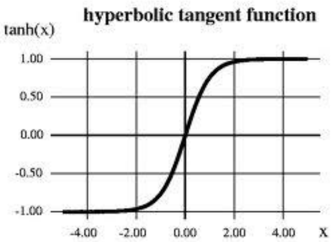
e) In (d) above, could you use PCA components directly to classify the data (without training a classifier)? Explain.

9. Neural Networks

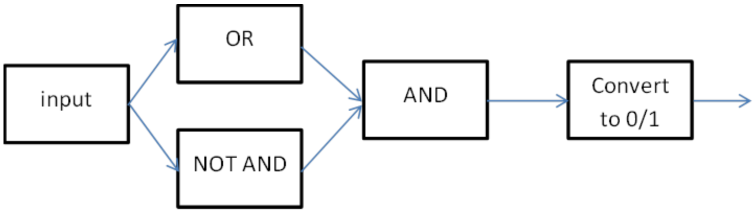
Q9.1 Neural Network for XOR

Design a neural network to solve the XOR problem, i.e. the network should output 1 if only one of the two binary input variables is 1, and 0 otherwise (see left figure). Use the hyperbolic tangent, or *tanh,* activation function in all nodes (right figure), which ranges in [-1, +1].

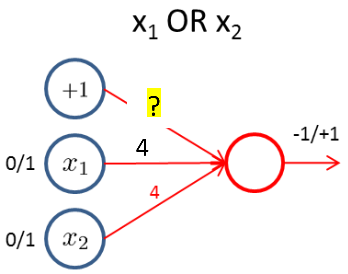


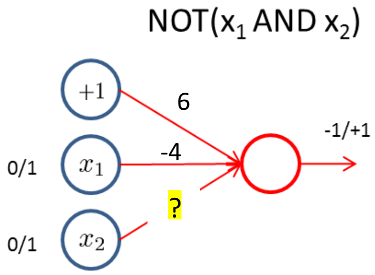


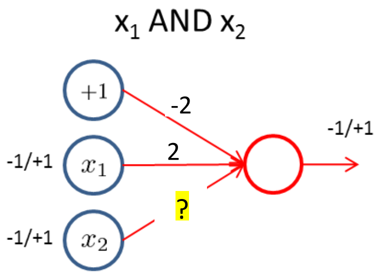
Note that (A XOR B) can be expressed as (A OR B) AND NOT(A AND B)), as illustrated below:

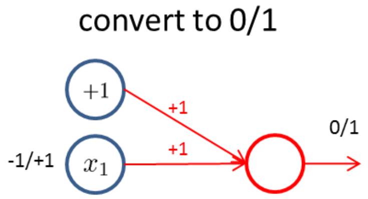


In the diagrams below, we filled in most of the tanh units' parameters. Fill in the remaining parameters, keeping in mind that tanh outputs +1/-1, not 0/1. Note that we need to appropriately change the second layer (the AND node) to take +1/-1 as inputs. Also, we must add an extra last layer to convert the final output from +1/-1 to 0/1. *Hint: assume tanh outputs* -1 *for any input* $x \leq -2$, $+1$ *for any input* $x \geq +2$, 0 *for* $x = 0$.









Q9.2 Computation Graph and Backpropagation

In class, we learned how to take a complex function that consists of multiple nested functions and represent it with a computation graph, which allows us to write down the forward and backward pass used to compute the function gradient.

a) Practice converting different functions $f_\theta\,(x) = f_k(f_{k-1}(\ldots f_1(x))$ of input vector x parametrized by θ to their computation graphs.

b) For the computation graphs obtained in (a), write down the forward pass and the backward pass equations.

Q9.3 Neural Network Architectures

a) Draw a convolutional network with input $x \in R^4$, one hidden layer with 2x1 filters and 2 channels with stride 2, and a fully-connected output layer with one neuron. How many parameters does this network have?

b) What algorithm is used for learning the parameters of a recurrent network? Name the algorithm and sketch out its main steps.