特征值 eigenvalue 和 特征向量 eigenvector

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特征值eigenvalue和特征向量eigenvector

在数学上，特别是线性代数中，对于一个给定的方阵A，它的**特征向量**（eigenvector，也译**固有向量**或**本征向量**）v 经过这个线性变换[a]之后，得到的新向量仍然与原来的v 保持在同一条直线上，但其长度或方向也许会改变。即

$$Av = \lambda v,$$

λ为标量，即特征向量的长度在该线性变换下缩放的比例，称λ 为其**特征值**（本征值）。如果特征值为正，则表示v 在经过线性变换的作用后方向也不变；如果特征值为负，说明方向会反转；如果特征值为0，则是表示缩回零点。但无论怎样，仍在同一条直线上。图1给出了一个以著名油画《蒙

“特征”一词译自德语的eigen

在一定条件下（如其矩阵形式为实对称矩阵的线性变换），一个变换可以由其特征值和特征向量完全表述，也就是说：所有的特征向量组成了这向量空间的一组基底。一个**特征空间**（eigenspace）是具有相同特征值的特征向量与一个同维数的零向量的集合.

Now consider the linear transformation of n-dimensional vectors defined by an n by n matrix A,

$$Av = w,$$

or

$$\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

where, for each row,

$$w_i = A_{i1}v_1 + A_{i2}v_2 + \cdots + A_{in}v_n = \sum_{j=1}^{n} A_{ij}v_j.$$

If it occurs that v and w are scalar multiples, that is if

$$Av = w = \lambda v, \tag{1}$$

then v is an **eigenvector** of the linear transformation A and the scale factor λ is the **eigenvalue** corresponding to that eigenvector. Equation (1) is the **eigenvalue equation** for the matrix A.

如此看来，一个矩阵，可以有多

Equation (**1**) can be stated equivalently as

$$(A - \lambda I)v = 0, \qquad (2)$$

where I is the n by n identity matrix and 0 is the zero vector.

Eigenvalues and the characteristic polynomial [edit]

Main article: Characteristic polynomial

Equation (**2**) has a nonzero solution *v* if and only if the determinant of the matrix ($A - \lambda I$) is zero. Therefore, the eigenvalues of *A* are values of λ that satisfy the equation

$$|A - \lambda I| = 0 \qquad (3)$$

Using Leibniz' rule for the determinant, the left-hand side of Equation (**3**) is a polynomial function of the variable λ and the degree of this polynomial is *n*, the order of the matrix *A*. Its coefficients depend on the entries of *A*, except that its term of degree *n* is always $(-1)^n\lambda^n$. This polynomial is called the *characteristic polynomial* of *A*. Equation (**3**) is called the *characteristic equation* or the *secular equation* of *A*.

The fundamental theorem of algebra implies that the characteristic polynomial of an *n*-by-*n* matrix *A*, being a polynomial of degree *n*, can be factored into the product of *n* linear terms,

$$|A - \lambda I| = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\cdots(\lambda_n - \lambda), \qquad (4)$$

where each λ_i may be real but in general is a complex number. The numbers λ_1, λ_2, ... λ_n, which may not all have distinct values, are roots of the polynomial and are the eigenvalues of *A*.

举个例子，如何先求特征值，再求各个特征值所对应的特征向量。

As a brief example, which is described in more detail in the examples section later, consider the matrix

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

Taking the determinant of $(A - \lambda I)$, the characteristic polynomial of A is

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 3 - 4\lambda + \lambda^2.$$

Setting the characteristic polynomial equal to zero, it has roots at $\lambda = 1$ and $\lambda = 3$, which are the two eigenvalues of A. The eigenvectors corresponding to each eigenvalue can be found by solving for the components of v in the equation $Av = \lambda v$. In this example, the eigenvectors are any nonzero scalar multiples of

$$v_{\lambda=1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad v_{\lambda=3} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

特征值的神奇性质

1. **原矩阵的对角线元素之和等于其所有特征值之和。**
2. **原矩阵的行列式等于特征之的连乘。**
3. **原矩阵的秩 (rank)等于非零特征值的个数。**
4. **对角矩阵的特征值就是其对角线上的元素。**

- The trace of A, defined as the sum of its diagonal elements, is also the sum of all eigenvalues,

$$\operatorname{tr}(A) = \sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{n} \lambda_i = \lambda_1 + \lambda_2 + \cdots + \lambda_n.$$ [31][32][33]

- The determinant of A is the product of all its eigenvalues,

$$\det(A) = \prod_{i=1}^{n} \lambda_i = \lambda_1 \lambda_2 \cdots \lambda_n.$$ [31][34][35]