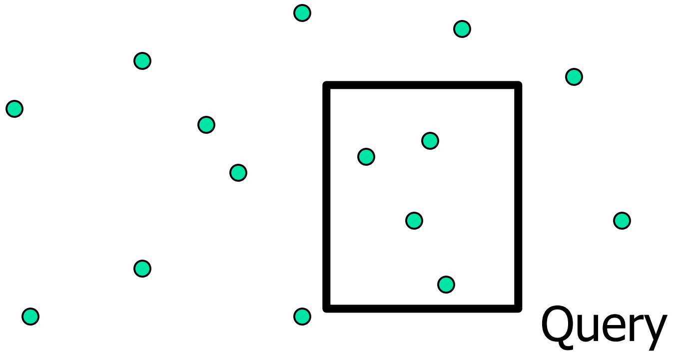
Spatial Indexing I

Point Access Methods

Many slides are based on slides provided by Prof. Christos Faloutsos (CMU) and Prof. Evimaria Terzi (BU)

The problem: Range query (or range reporting)

- Given a point set and a rectangular query, find the points enclosed in the query
- We allow insertions/deletions on line



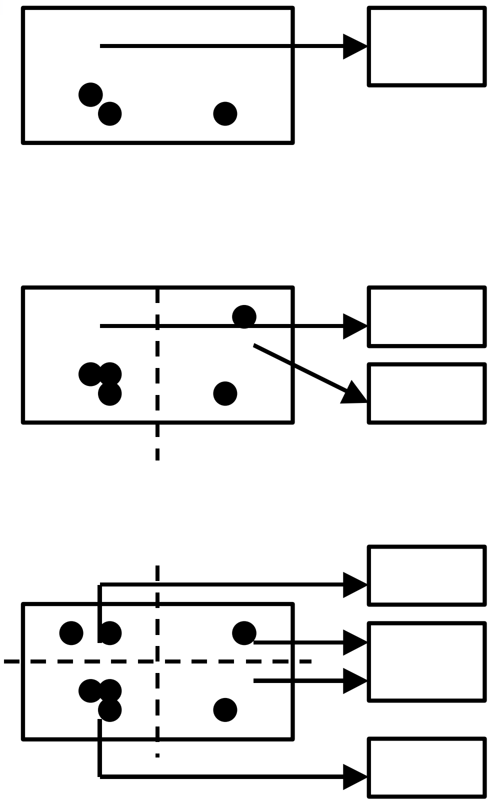
Outline

- Grid file
- kd-tree based methods
- Space filing curves
  - Quadtrees

Grid File

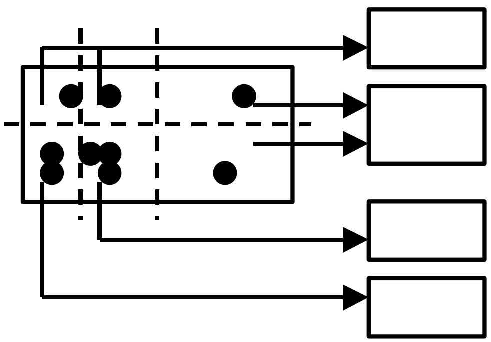
- Hashing methods for multidimensional points (extension of Extensible hashing)
- Idea: Use a grid to partition the space→ each cell is associated with one page
- Two disk access principle (exact match)

Grid File



- Start with one bucket for the whole space.
- Select dividers along each dimension. Partition space into cells
- Dividers cut all the way.

Grid File

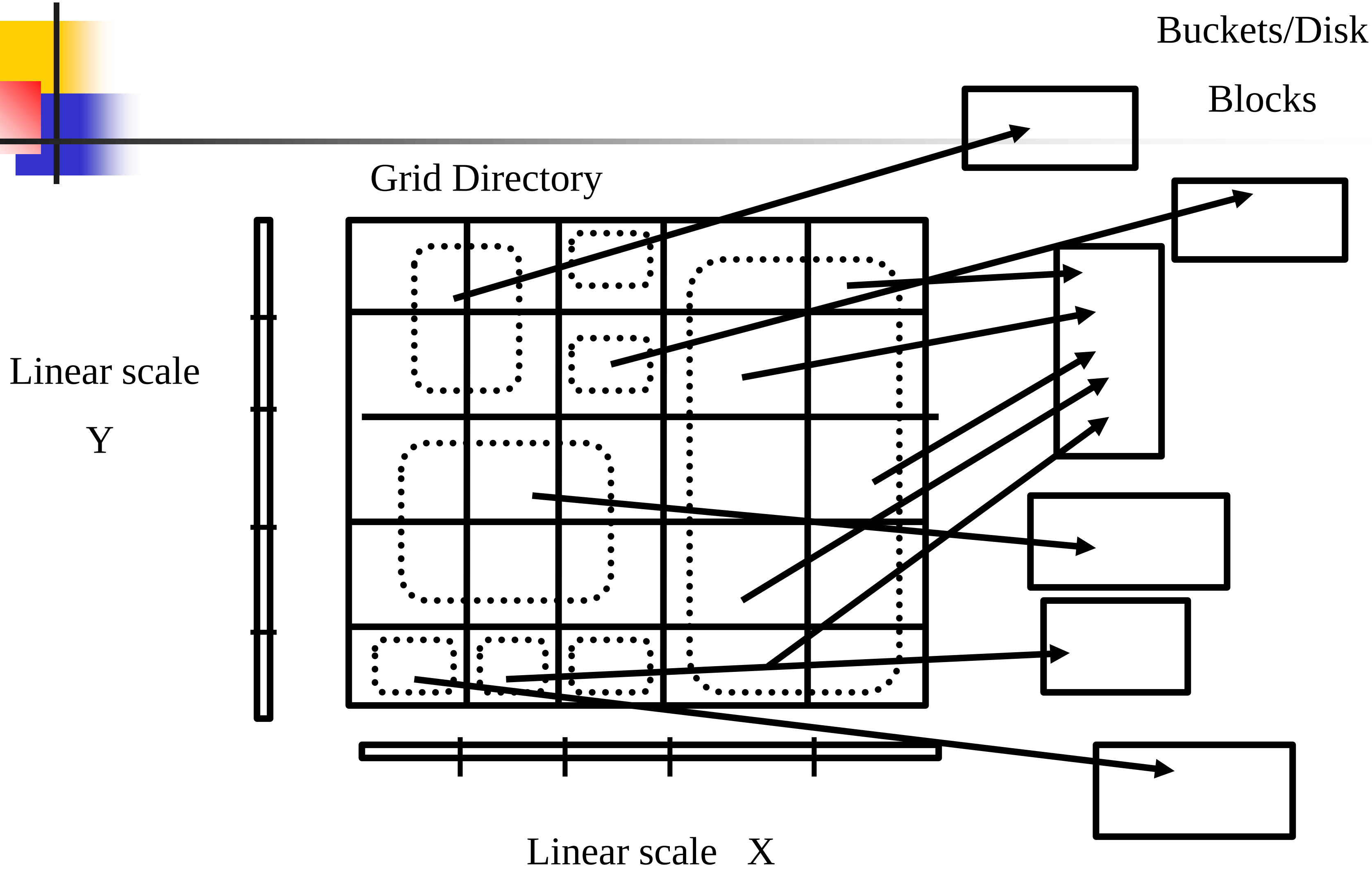


- Each cell corresponds to 1 disk page.
- Many cells can point to the same page.
- Cell directory potentially exponential in the number of dimensions

Grid File Implementation

- Dynamic structure using a grid directory
  - Grid array: a 2 dimensional array with pointers to buckets (this array can be large, disk resident) G(0,…, nx-1, 0, …, ny-1)
  - Linear scales: Two 1 dimensional arrays that used to access the grid array (main memory) X(0, …, nx-1), Y(0, …, ny-1)

Example


Buckets/Disk
Blocks
Grid Directory
Linear scale
Y
Linear scale X

Grid File Search

- Exact Match Search: at most 2 I/Os assuming linear scales fit in memory.
  - First use liner scales to determine the index into the cell directory
  - access the cell directory to retrieve the bucket address (may cause 1 I/O if cell directory does not fit in memory)
  - access the appropriate bucket (1 I/O)
- Range Queries:
  - use linear scales to determine the index(es) into the cell directory.
  - Access the cell directory to retrieve the bucket addresses of buckets to visit.
  - Access the buckets.

Grid File Insertions

- Determine the bucket into which insertion must occur.
- If space in bucket, insert.
- Else, split bucket
  - how to choose a good dimension to split?
- If bucket split causes a cell directory to split do so and adjust linear scales.
- insertion of these new entries potentially requires a complete reorganization of the cell directory--- expensive!!!

Grid File Deletions

- Deletions may decrease the space utilization. Merge buckets
- We need to decide which cells to merge and a merging threshold
- Buddy system and neighbor system
  - A bucket can merge with only one buddy in each dimension
  - Merge adjacent regions if the result is a rectangle

Tree-based PAMs

- Most of tb-PAMs are based on kd-trees
- kd-tree (J. Bentley, 1975) is a main memory binary tree for indexing k-dimensional points
  - Needs to be adapted for the disk model
- Levels rotate among the dimensions, partitioning the space based on a value for that dimension
- kd-tree is not necessarily balanced for dynamic datasets

2-dimensional kd-trees

- A data structure to support range queries in $\mathbf{R}^2$
  - Not the most efficient solution in theory
  - Used a lot it in practice

- Preprocessing time: **O(nlogn)**
- Space complexity: **O(n)**
- Query time: $\mathbf{O(n^{1/2}+P)}$ **(worst case)**
  - **P** number of points in the answer

2-dimensional kd-trees

- Idea: Split the point set alternating by x-coordinate and by y-coordinates
  - split by x-coordinate: split by a vertical line that has half the points left or on, and half right
  - split by y-coordinate: split by a horizontal line that has half the points below or on, and half above
- We get a binary tree:
  - Size **O(n)**
  - Depth **O(logn)**
  - Construction time **O(nlogn)**

BuildKdtree

Algorithm BuildKdTree(P,*depth*)

if P contains only one point

then return a leaf storing this point

else if *depth* is even

then Split P with a vertical line l through the median x-coordinate

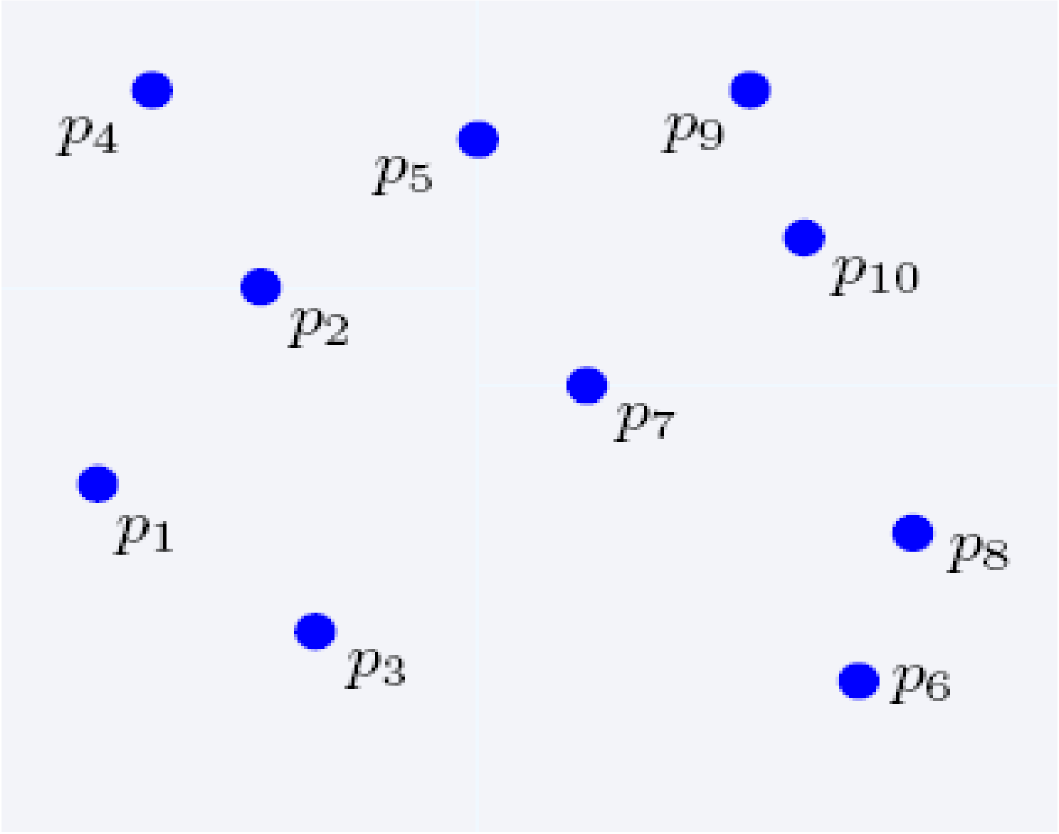
into P_1 (left of or on l) and P_2 (right of l)

else Split P with a horizontal line l through the median y-coordinate

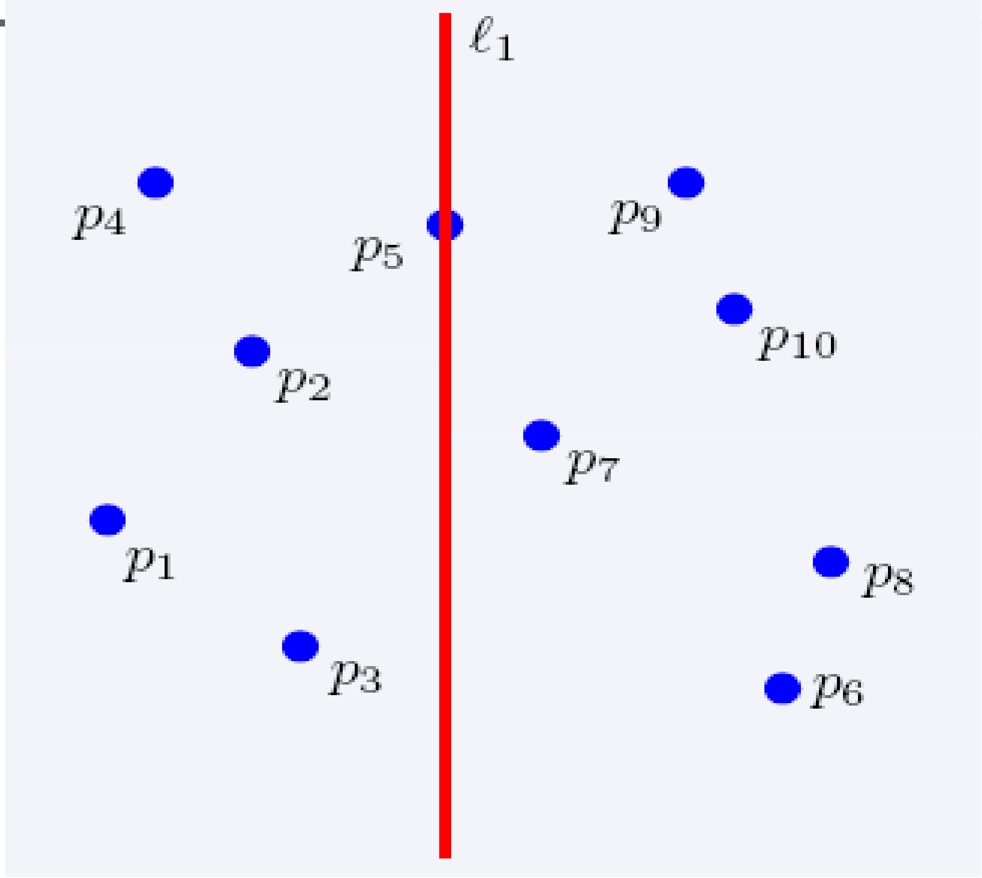
into P_1 (below or on l) and P_2 (above l)

BuildKdTree(P, *depth* + 1)

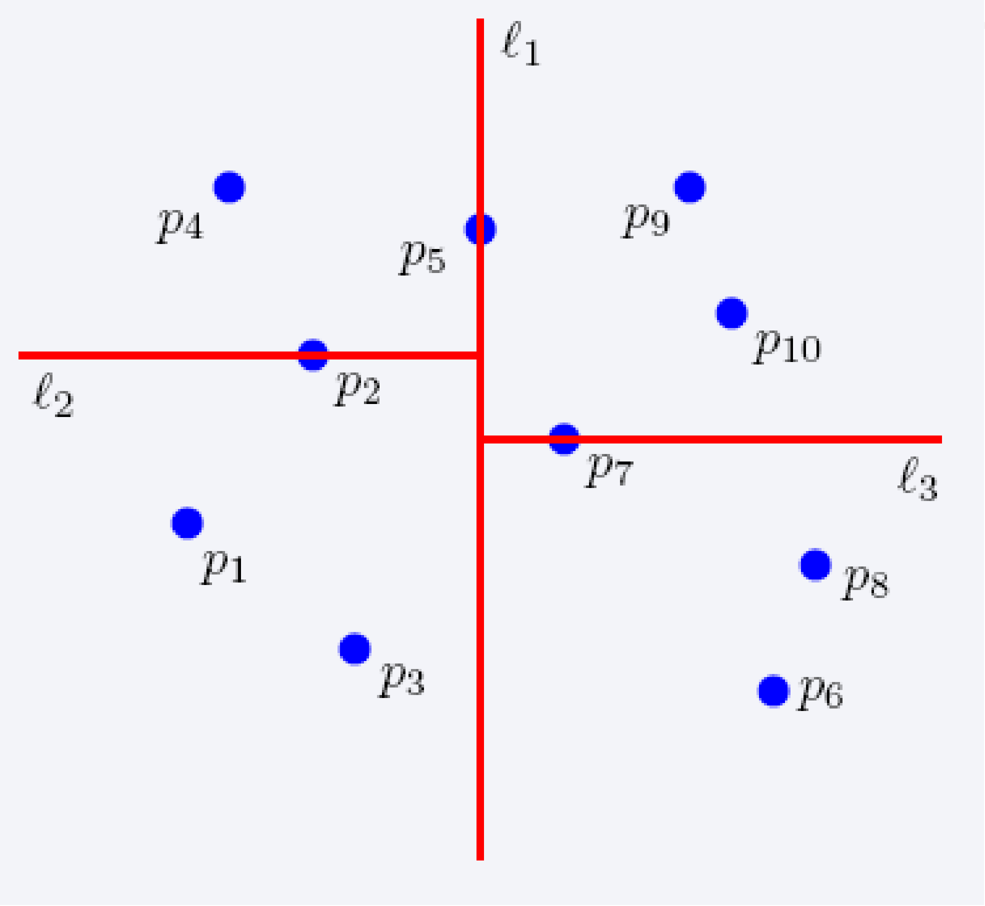
Construction of kd-trees



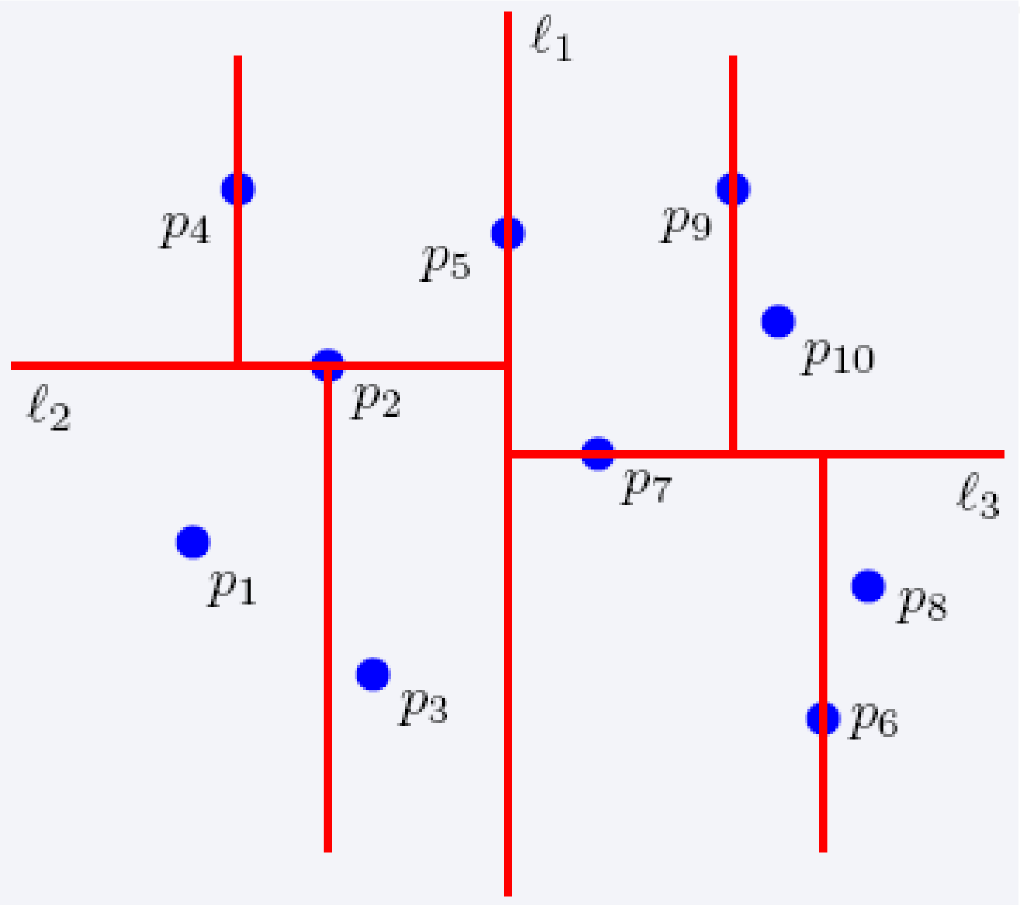
Construction of kd-trees



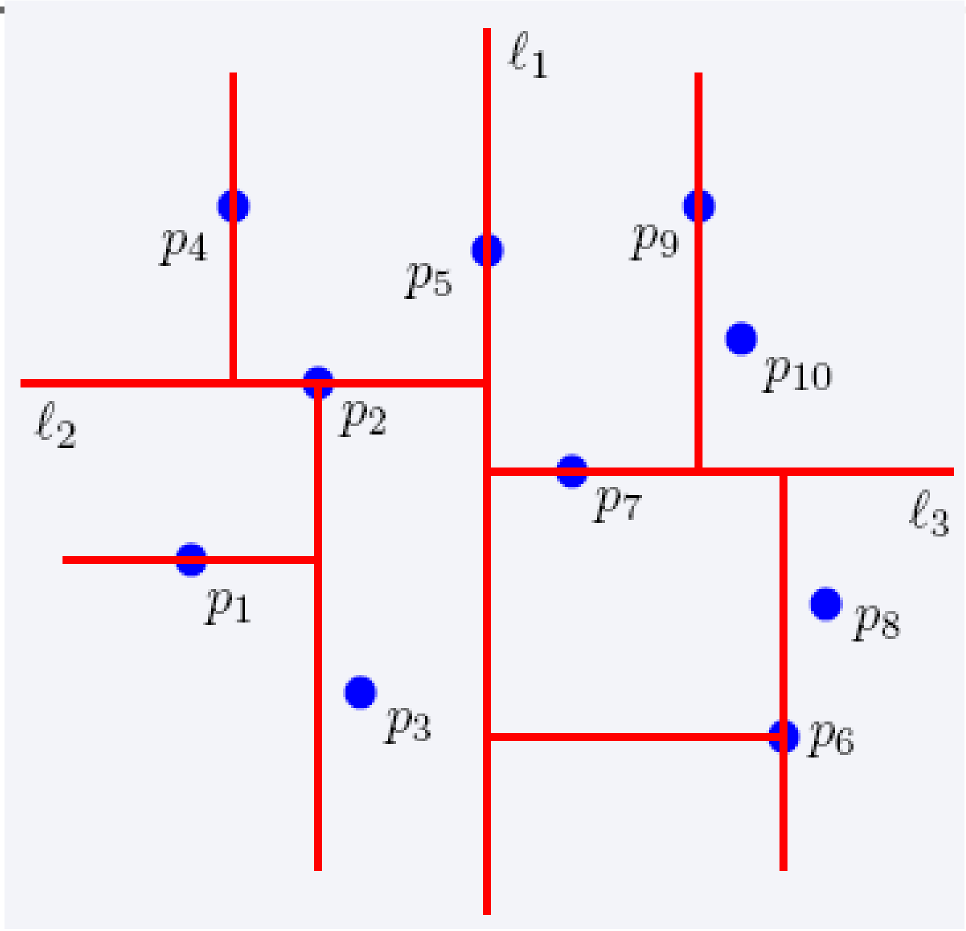
Construction of kd-trees



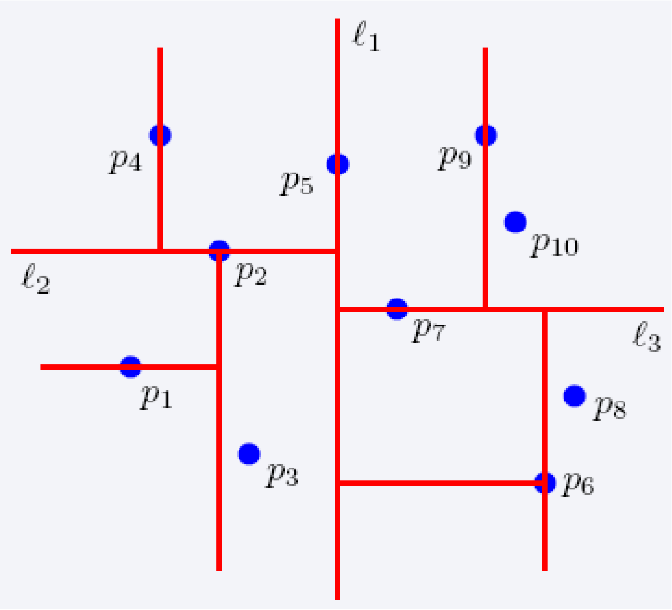
Construction of kd-trees

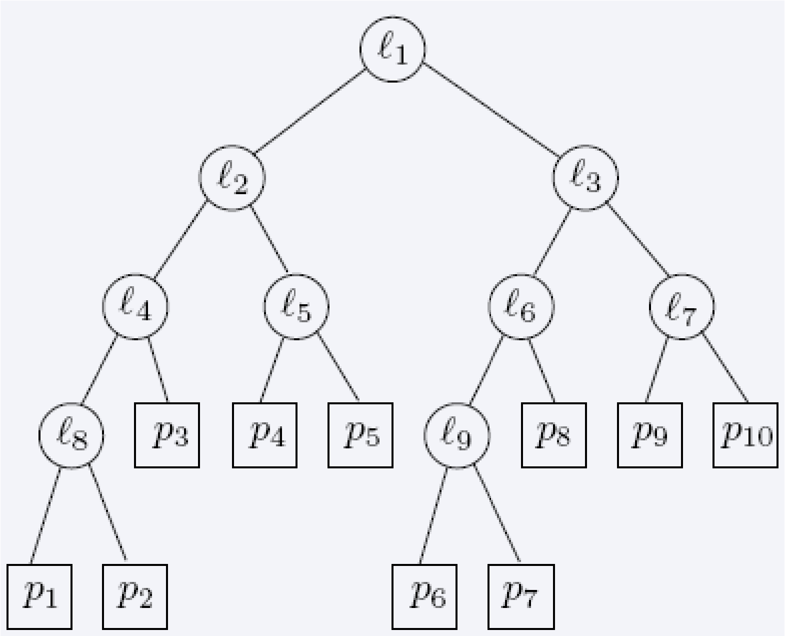


Construction of kd-trees



The complete kd-tree





Construction cost

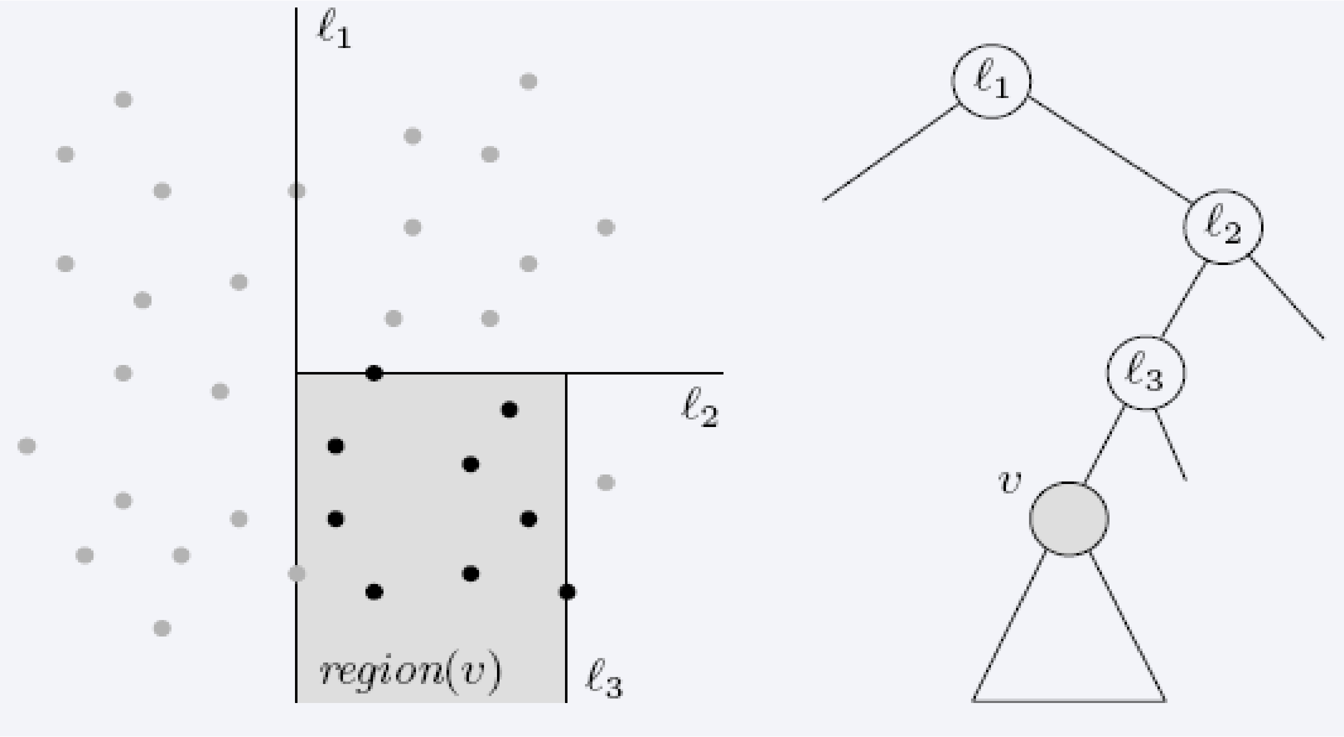
- Finding median is O(n)
- Also: $T(1) = O(1)$

$$T(n) = 2\,T(n/2) + O(n)$$

(Master Theorem, MT)

=> $T(n) = O(n\log n)$

Region of node **v**



Region(v) : the subtree rooted at **v** stores the points in black dots

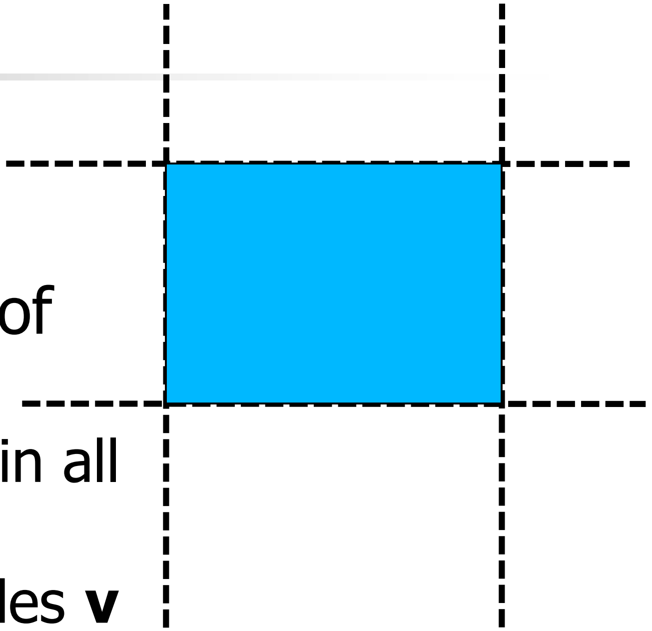
Searching in kd-trees

- Range-searching in **2-d**
  - Given a set of **n** points, build a data structure that for any query rectangle **R** reports all point in **R**

kd-tree: range queries

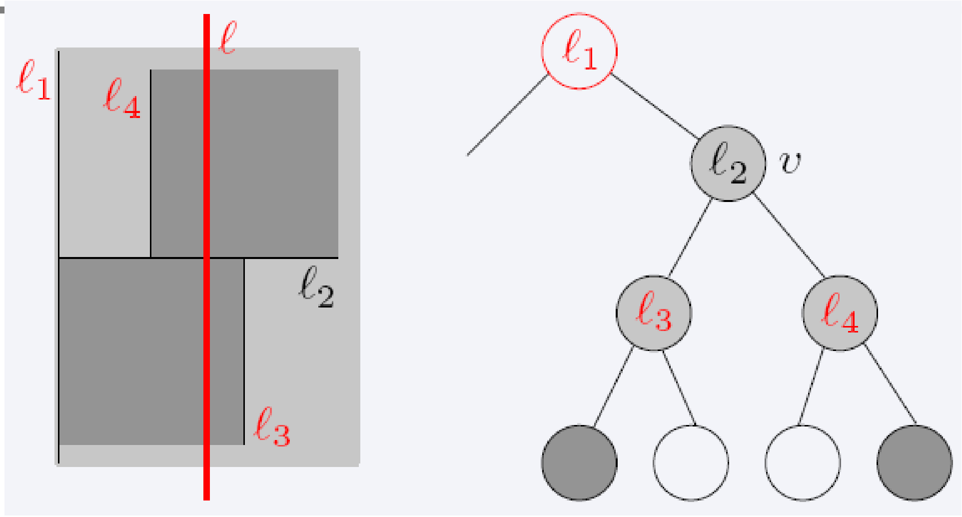
- Recursive procedure starting from **v = root**
- Search **(v,R)**
  - If **v** is a leaf, then report the point stored in **v** if it lies in **R**
  - Otherwise, if **Reg(v)** is contained in **R**, report all points in the **subtree(v)**
  - Otherwise:
    - If **Reg(left(v))** intersects **R**, then **Search(left(v),R)**
    - If **Reg(right(v))** intersects **R**, then **Search(right(v),R)**

Query time analysis



- We will show that **Search** takes at most **$O(n^{1/2}+P)$** time, where **P** is the number of reported points
  - The total time needed to report all points in all sub-trees is **O(P)**
  - We just need to bound the number of nodes **v** such that **region(v)** intersects **R** but is not contained in **R** (i.e., boundary of R intersects the boundary of **region(v)**)
  - **gross overestimation**: bound the number of **region(v)** which are crossed by any of the **4** horizontal/vertical lines

- **Q(n):** max number of regions in an n-point kd-tree intersecting a (say, vertical) line?



- If ℓ intersects **region(v)** (due to vertical line splitting), then after two levels it intersects **2** regions (due to 2 vertical splitting lines)
- The number of regions intersecting ℓ is **Q(n)=2+2Q(n/4), MT** → **Q(n)=(n$^{1/2}$)**

Query time (Cont'd)

- Range-searching in **2d kd-tree**
- A range has four sides. In the worst case, every side intersects $O(n^{1/2})$ regions (nodes to follow)
- So, total query time is:

  $Q(n) = (n^{1/2} + P)$

d-dimensional kd-trees

- A data structure to support range queries in $\mathbf{R}^d$

- Preprocessing time: $\mathbf{O(n\log n)}$
- Space complexity: $\mathbf{O(n)}$
- Query time: $\mathbf{O(n^{1-1/d}+P)}$

Construction of the **d**-dimensional kd-trees

- The construction algorithm is similar as in **2-d**
- At the root we split the set of points into two subsets of same size by a hyper-plane vertical to $\mathbf{x_1}$-axis
- At the children of the root, the partition is based on the second coordinate: $\mathbf{x_2}$-coordinate
- At depth **d**, we start all over again by partitioning on the first coordinate
- The recursion stops until there is only one point left, which is stored as a leaf

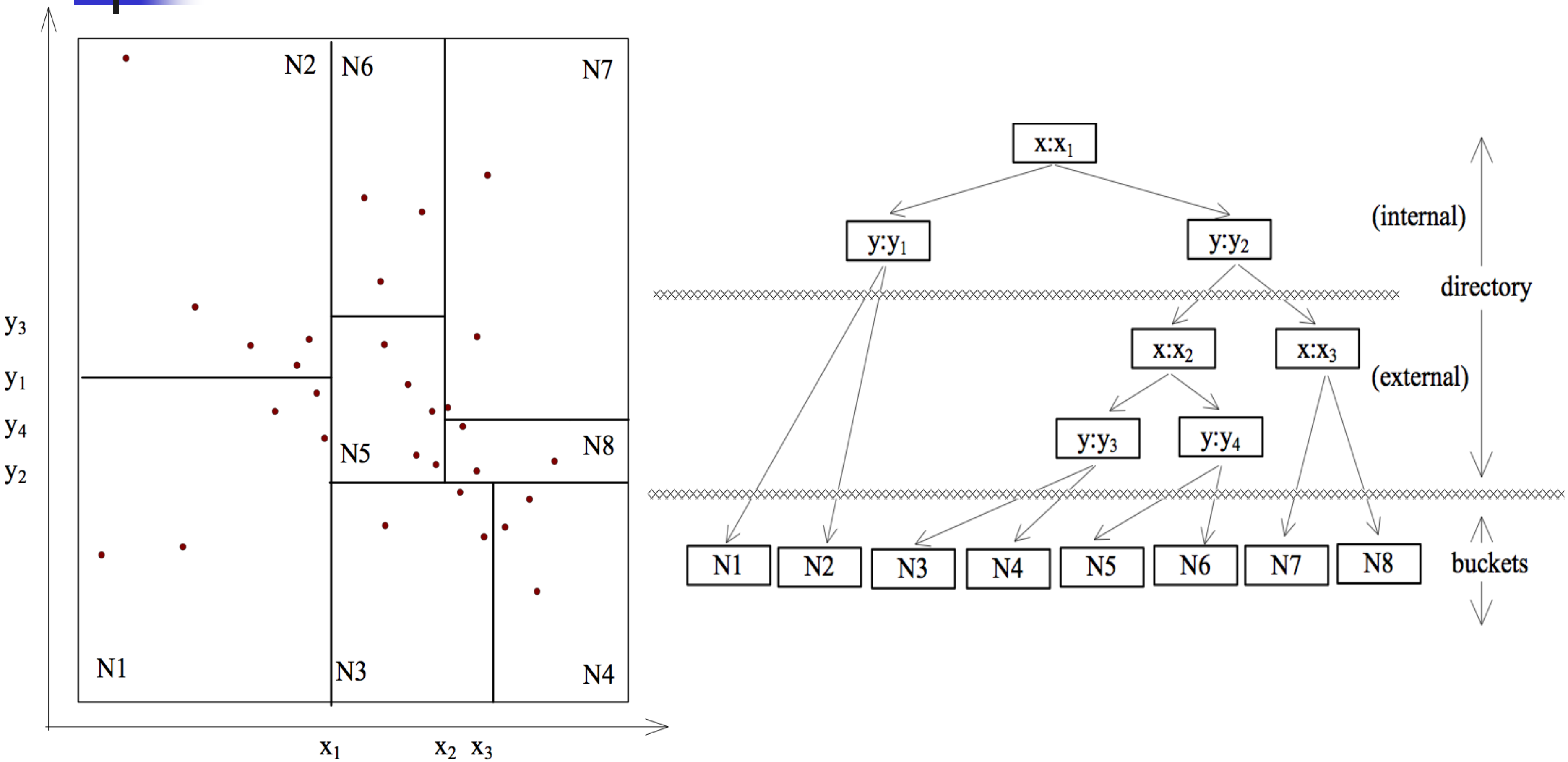
External memory kd-trees (kdB-tree)

- Pack many interior nodes (forming a subtree) into a block using BFS-traversal.
  - it may not be feasible to group nodes at lower level into a block productively.
  - Many interesting papers on how to optimally pack nodes into blocks recently published.
- Similar to B-tree, tree nodes split many ways instead of two ways
  - insertion becomes quite complex and expensive.
  - No storage utilization guarantee since when a higher level node splits, the split has to be propagated all the way to leaf level resulting in many empty blocks.

LSD-tree

- Local Split Decision – tree
- Use kd-tree to partition the space. Each partition contains up to B points. The kd-tree is stored in main-memory.
- If the kd-tree (directory) is large, we store a sub-tree on disk
- Goal: the structure must remain balanced: external balancing property

Example: LSD-tree



LSD-tree: main points

- Split strategies:
  - Data dependent
  - Distribution dependent
- Paging algorithm
- Two types of splits: bucket splits and internal node splits

PAMs

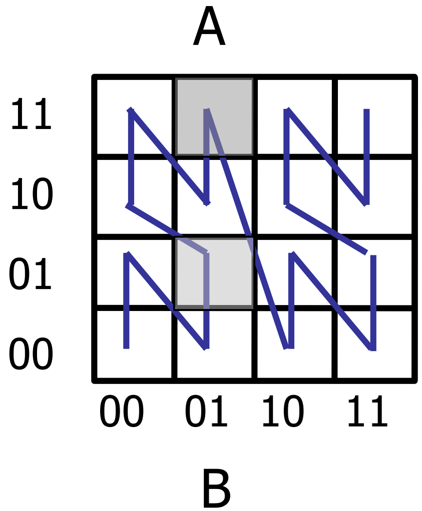
- Point Access Methods
  - Multidimensional Hashing: Grid File
    - Exponential growth of the directory
  - Hierarchical methods: kd-tree based
    - Storing in external memory is tricky but possible
  - Space Filling Curves: Z-ordering
    - Map points from 2-dimensions to 1-dimension. Use a B+-tree to index the 1-dimensional points

Z-ordering

- Basic assumption: Finite precision in the representation of each co-ordinate, K bits (2^K values)
- The address space is a square (<u>image</u>) and represented as a $2^K \times 2^K$ array
- Each element is called a <u>pixel</u>

Z-ordering

- Impose a linear ordering on the pixels of the image → 1 dimensional problem



Z_A = shuffle(x_A, y_A) = shuffle("01", "11")

= 0111 = $(7)_{10}$

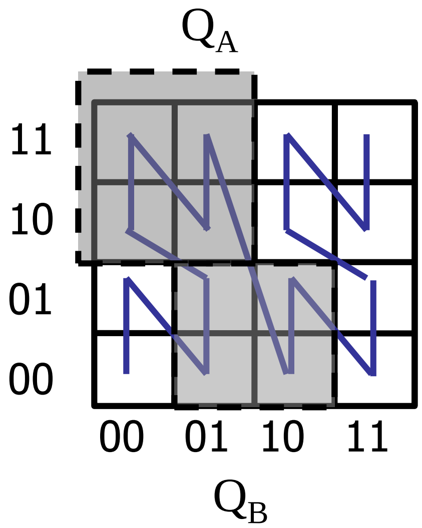
Z_B = shuffle("01", "01") = 0011

Z-ordering

- Given a point (x, y) and the precision K find the pixel for the point and then compute the z-value
- Given a set of points, use a B+-tree to index the z-values
- A range (rectangular) query in 2-d is mapped to a set of ranges in 1-d

Queries

- Find the z-values that contained in the query and then the ranges



$Q_A \rightarrow$ range [4, 7]

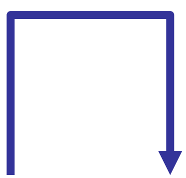
$Q_B \rightarrow$ ranges [2,3] and [8,9]

Hilbert Curve

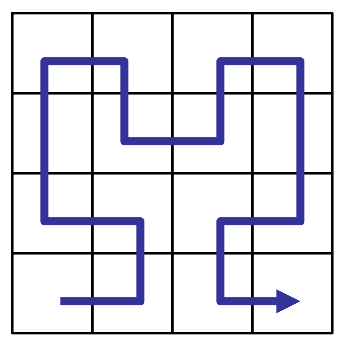
- We want points that are close in 2d to be close in the 1d
- Note that in 2d there are 4 neighbors for each point where in 1d only 2.
- Z-curve has some “jumps” that we would like to avoid
- Hilbert curve avoids the jumps : recursive definition

Hilbert Curve- example

- It has been shown that in general Hilbert is better than the other space filling curves for retrieval [Jag90]
- Hi (order-i) Hilbert curve for $2^i x 2^i$ array

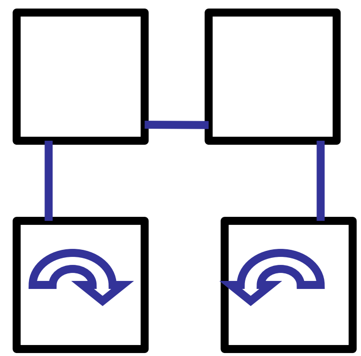


H1



H2

...



H(n+1)

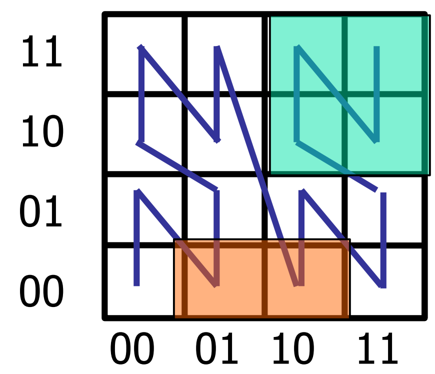
Handling Regions

- A region breaks into one or more pieces, each one with different z-value
- Works for raster representations (pixels)
- We try to minimize the number of pieces in the representation: precision/space overhead trade-off

$Z_{R1} = 0010 = (2)$
$Z_{R2} = 1000 = (8)$

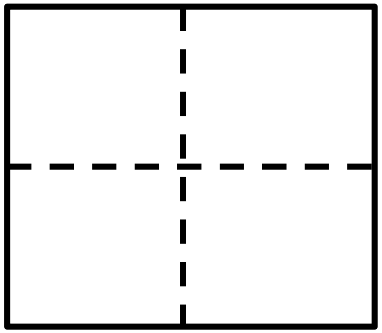
$Z_G = 11$

("11" is the common prefix)



Z-ordering for Regions

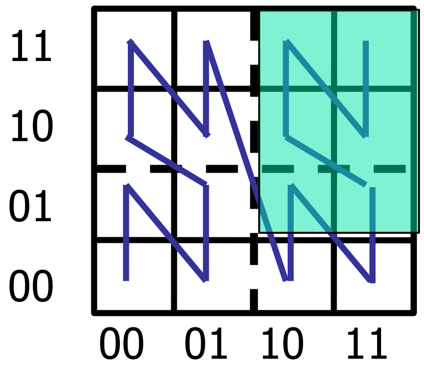
- Break the space into 4 equal quadrants: level-1 blocks
- Level-i block: one of the four equal quadrants of a level-(i-1) block
- Pixel: level-K blocks, image level-0 block
- For a level-i block: all its pixels have the same prefix up to 2i bits; the z-value of the block



Quadtree

- Object is recursively divided into blocks until:
  - Blocks are homogeneous
  - Pixel level
- Quadtree: '0' stands for S and W

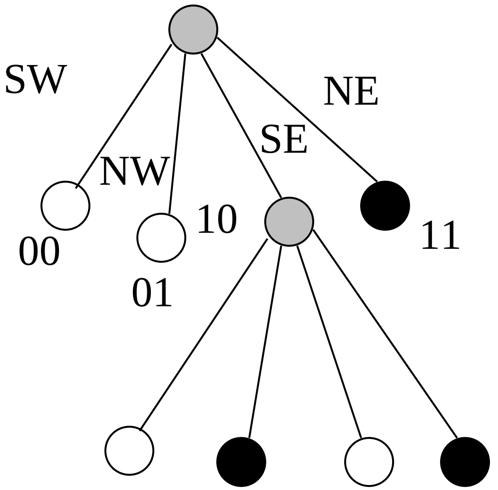
  '1' stands for N and E



11

1001

1011



Region Quadtrees

- Implementations
  - FL (Fixed Length)
  - FD (Fixed length-Depth)
  - VL (Variable length)
- Use a B+-tree to index the z-values and answer range queries

Linear Quadtree (LQ)

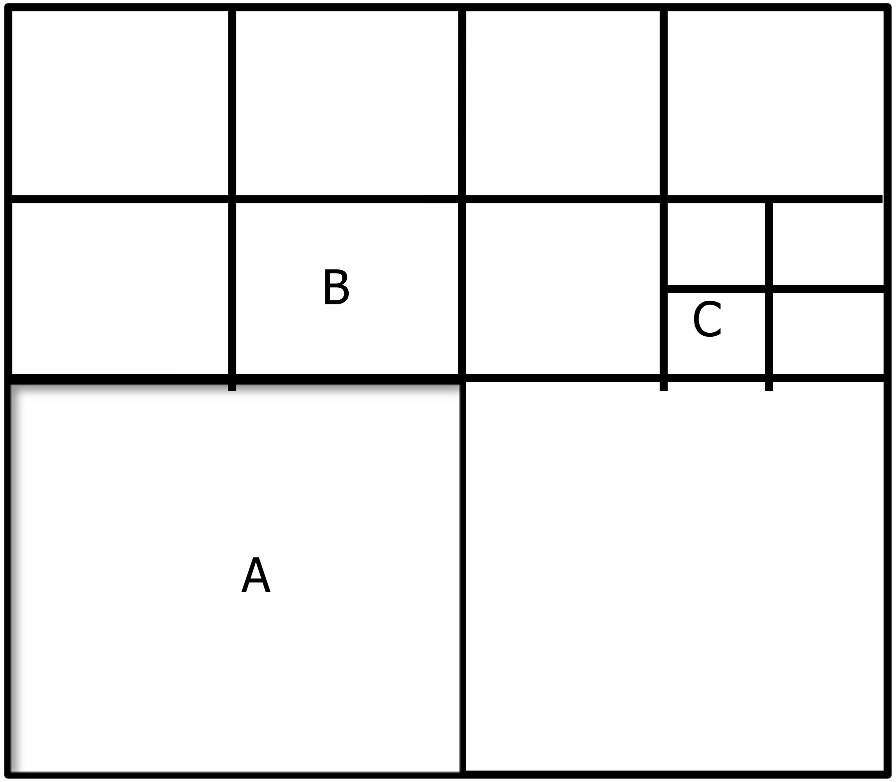
- Assume we use n-bits in each dimension (x,y) (so we have $2^n x 2^n$ pixels)
- For each object O, compute the z-values of this object: z_1, z_2, z_3, …, z_k (each value can have between 0 and 2n bits)
- For each value z_i we append at the end the level l of this value (level l =log($|z_i|$))
- We create a value with 2n+l bits for each z-value and we insert it into a B+-tree (l= $\log_2$(h))

Z-value, l | Morton block

A: 00, 01 = 00000001
B: 0110, 10 = 01100010
C: 111000,11 = 11100011
n=3

A:1, B:98, C: 227

Insert into B+-tree using Mb



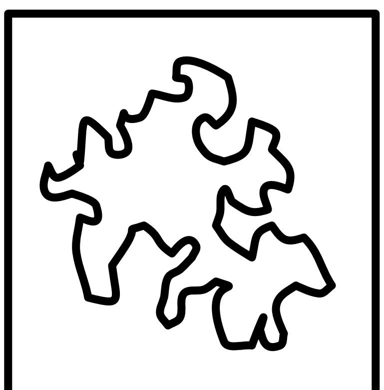
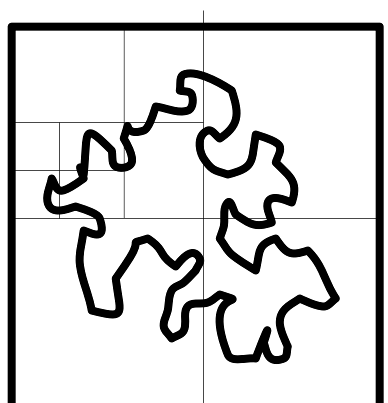
Query Alg

```
WindowQ(query w, quadtree block b)
{     Mb = Morton block of b;
      If b is totally enclosed in w {
         Compute Mbmax
         Use B+-tree to find all objects with M values between Mb<=M<= Mbmax
           add to result
      } else {
         Find all objects with Mb in the B+-tree
              add to result
         Decompose b into four quadrants sw, nw, se, ne
         For child in {sw, nw, se, ne}
              if child overlaps with w
                      WindowQ(w, child)
      }
}
```

z-ordering - analysis

Q: How many pieces (‘quad-tree blocks’) per region?

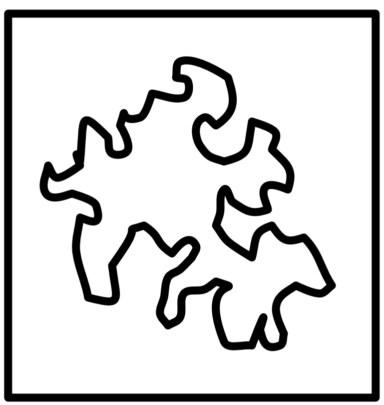
A: proportional to perimeter (surface etc)

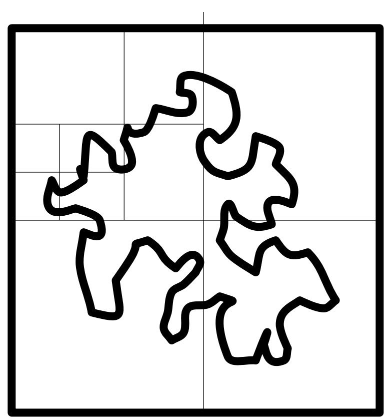



z-ordering - analysis

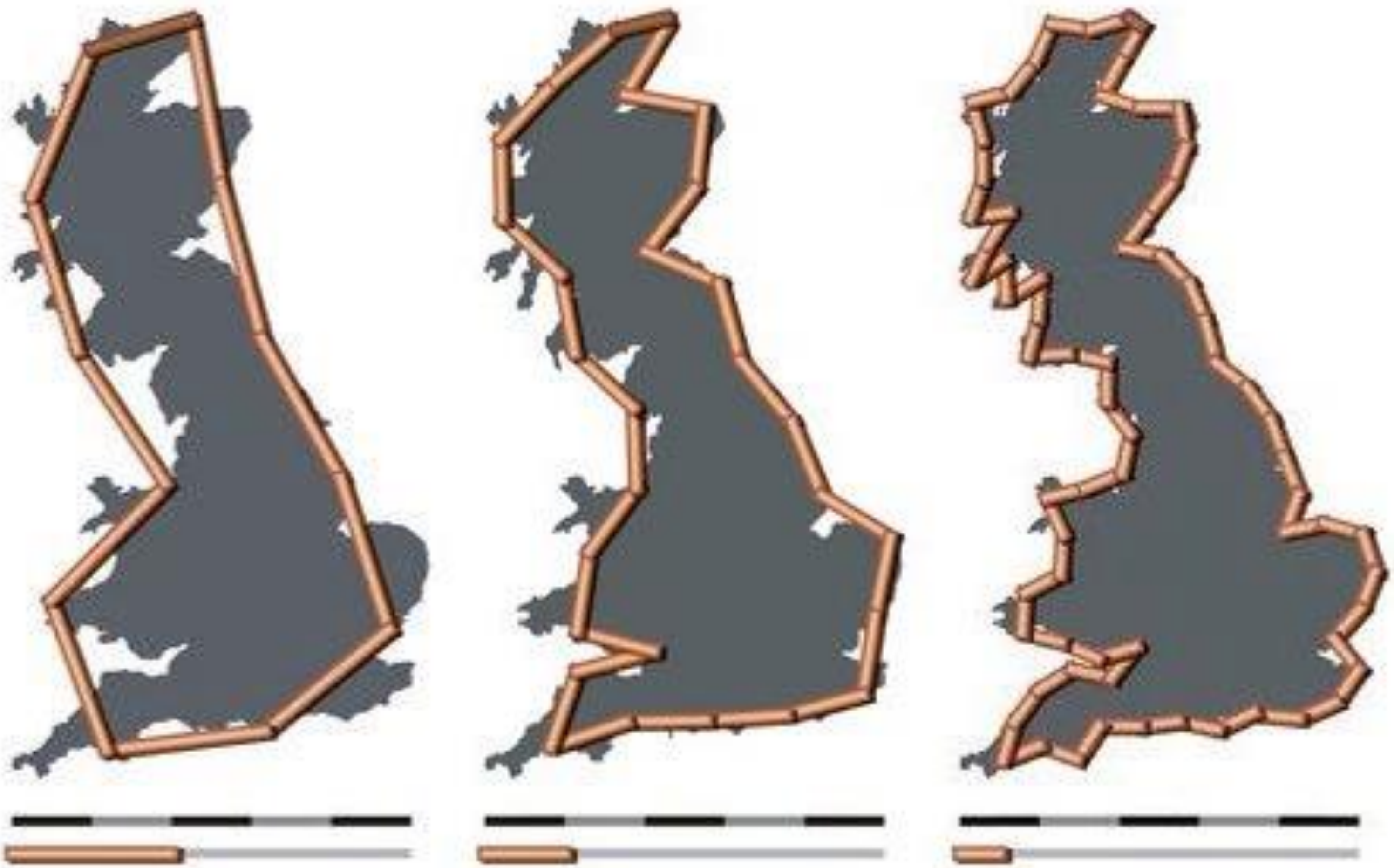
(How long is the coastline, say, of Britain?

Paradox: The answer changes with the yard-stick -> fractals ...)





http://en.wikipedia.org/wiki/How_Long_Is_the_Coast_of_Britain%3F_Statistical_Self-Similarity_and_Fractional_Dimension

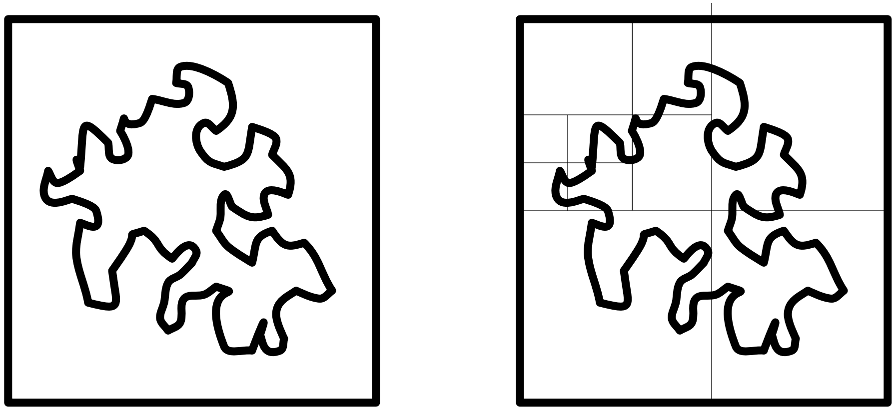


Unit: 200 km, 100 km and 50 km in length.
The resulting coastline is about 2350 km, 2775 km and 3425 km

https://commons.wikimedia.org/wiki/File:Britain-fractal-coastline-50km.png#/media/File:Britain-fractal-coastline-combined.jpg

z-ordering - analysis

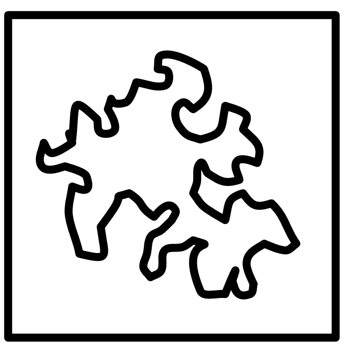
Q: Should we decompose a region to full detail (and store in B-tree)?

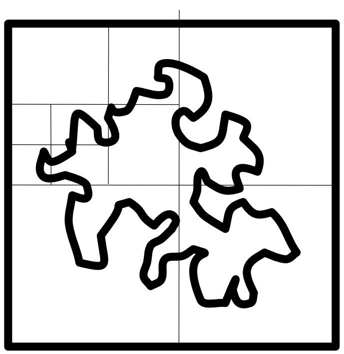


z-ordering - analysis

Q: Should we decompose a region to full detail (and store in B-tree)?

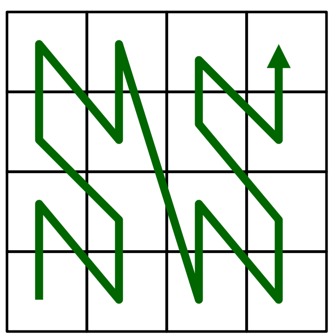
A: NO! approximation with 1-5 pieces/z-values is best [Orenstein90]

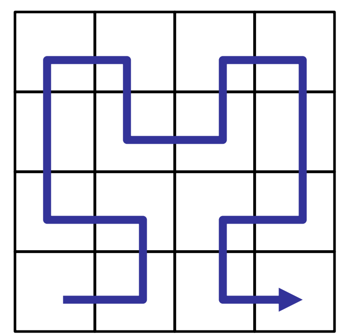




z-ordering - analysis

Q: how to measure the 'goodness' of a curve?

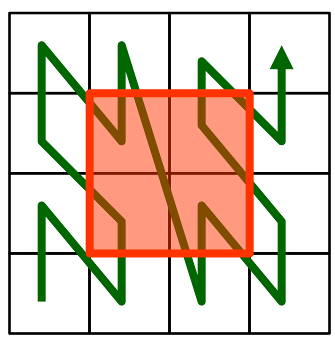


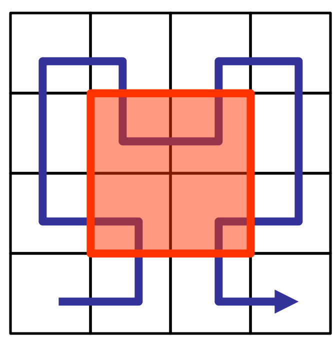


z-ordering - analysis

Q: how to measure the ‘goodness’ of a curve?

A: e.g., avg. # of runs, for range queries





4 runs

3 runs

(#runs ~ #disk accesses on B-tree)

z-ordering - analysis

Q: So, is Hilbert really better?

A: 27% fewer runs, for 2-d (similar for 3-d)

Q: are there formulas for #runs, #of quadtree blocks etc?

A: Yes, see a paper by [Jagadish '90]

H.V. Jagadish. Linear clustering of objects with multiple attributes. SIGMOD 1990. http://www.cs.ucr.edu/~tsotras/cs236/W15/hilbert-curve.pdf