CS 530 - Fall 2019
Final Exam

NPM

First section: Short answer (6 points each)

Answer any 3 of the following 4 questions. Each question counts equally.

Write your answers on the Answersheet, and write the number of the problem you do NOT want graded at the top of the page. If need be, you can also use 1 piece of paper for longer answers.

1. T/F or multiple choice questions. Answer all 4. No partial credit.

   (i). Consider any 3 by 3 or larger square matrix ~~M of~~ rationals which is non-singular and anti-diagonal. (Anti-diagonal means the only non-zero elements can be those on diagonal going from the upper right element to the lower left element.)

   T or F: No M as above has an LU decomposition.

   (ii). Which of the following statements about the maximum matching problem is false.

   a. Any maximum matching is also maximal

   b. Any perfect matching is maximum.

   c. The maximum weighted matching problem is NP-complete

   d. Any maximum weighted matching must be a perfect matching.

   e. Some maximum matchings may have infinite weight

   0.3 0.8

   0.3 0.8 ~~0.7 0.4 0~~

   (iii). T/F There is an instance of bin packing instance I where the NF bin packing algorithm results in an answer which is greater than 1 3/4 times the optimal packing for I.

   (iv). T/F: Every singular 2 by 2 matrix has an LU decomposition. Briefly explain your reasoning.

   1 0   1 0

   0 1   1 1

2. Answer both problems

   (i). Give an example of a 2 or 3 standard form variable LP where the feasible region is unbounded and where the maximal value of the objective function comes from a unique feasible point.

   Sketch a picture of the LP's feasible region.

   Label each line segment making up the edges of the feasible region with the constraint it represents. show obj and max feasible point

   (ii). In class (and in the textbook) we discussed a 2-approximation algorithm, call it A, for the vertex cover problem. Connected graph

   Give a description of a collection of graphs G_n, where each graph G_n has at least n vertices and where $A(G_n) = 2 \times OPT(G_n)$

   Note You should describe G_n either by giving its vertices and edges or describing it in English or drawing it's picture.

3. Which of the following problems are in NP? (One point for each, no partial credit)

(i). The problem of finding the smallest number of colors which can correctly color the vertices of a graph.
with each edge having two differently colored endpoints. NPH

(ii). The problem of deciding if a graph contains an Euler cycle. NPH

ver

(iii). The problem pairs deciding, given a bipartite graph G and a matching M for G, if M can be extended to a maximum matching.

HK

(iv). The weighted vertex cover problem. (That is finding a vertex cover which has the smallest total weight.) NPH

(v). Deciding if a given matching in a unweighted bipartite graph G has weight exactly 1/2 of the largest matching in G.

(vi). Deciding if a given permutation of the set {1,2,...,n} is odd.

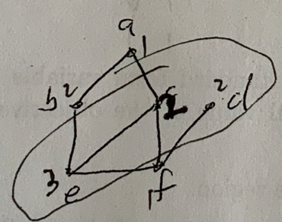
4. Answer the following 3 questions about the graph G below.

(i). What is the minimum number of colors t so that G would be t colorable. (Here any edge of G is colored by different colors at its endpoints.) 2

(ii). Suppose that we randomly choose one of four colors for every vertex of G. What is the expected value of the number of edges which are correctly colored.

(iii). Find a max cut of G. What is its size? How many different max cuts does G have ?

G =



7 edges

ef – abcd 4

Only 1 max cut

$a \to b \to e \to c \to f \to d$

$1 \times \frac{3}{4} \times \frac{3}{4} \times \frac{2}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{27}{256}$

$\frac{9}{16} \times \frac{1}{4} \times \frac{3}{4}$

$7 \times \left(\frac{3}{4}\right)^7$

$$\begin{array}{r} 16 \\ \times 16 \\ \hline 96 \\ 16 \\ \hline 256 \end{array}$$

$c \to a \to b$

af – bcde

Section 2: Longer answer: (10 points each)
Answer 2 of the following 3 questions. Each question counts the same. ≤ 0.1

5. Assume you have a list of N, N > 100,000, numbers from 1 to 1000 and you know that more that 60% of the N numbers are the same (say T).

   (i). Write an Monte Carlo algorithm which determines the number T with high probability. Specifically your algorithm should.

   -run for some constant number of steps not depending on N, and

   -output the value T with probability at least .~~6~~ ≤ 0.1

   You should state the time complexity and error bounds of your algorithm and explain why your algorithm achieves these time and error bounds.

   (ii). Is it possible to find a n efficient (constant time) Las Vegas algorithm to solve this problem ? Why or why not.

6. Consider the following standard form LP:

   maximize 10x + 5y

   subject to the constraints

   (0,10) (30,0)

   x+3y ≤ 30 $\quad x+3y-30\leq 0$

   (0,12) (12,0) x + y ≤ 12

   2x+1/2y ≤ 16 (0,32)

   x,y ≥ 0 (8,0)

   Answer questions i - v below

   i. Write the slack form of the LP above.

   ii. Draw a sketch of the feasible region of this LP.

   Circle the extreme points of this region in your sketch

   iii. How many extreme points are there for this LP ? List them.

   iv. Find an optimal value for this LP. Label and circle it in your sketch.

   v. Is the optimal value unique ? Why or why not ?

   vi. What is the basic solution you find for this LP ? Is this basic solution feasible ? why or why not ?

7. Do both (i) and (ii).

   (i). In class we showed, given any polynomial f with n variables and degree d where f *neq* 0 $\neq$ and where r is chosen randomly from a set of numbers S of size s then prob (f(r) = 0) ≤ d/s. Define a polynomial h(x) and a set S where n=1 d=3 and s=5 and for which pr(h(r) = 0) is exactly d/s.

   (ii). Give an example of two non-zero 3 by 3 matrices A and B where testing whether A = B can be done using Frievalds method and the error probability is exactly = 3/8.

$n=1 \quad d=3 \quad s=5$ $\qquad \leq \frac{4}{8}$

$h(x)=x^3$