CS 530 - Fall 2018 Quiz 1

Instructions: Please answer any 3 of the following 4 questions. Each question counts equally. Put the number of the question which you do NOT want graded here: . (1 point)

1. **(LU decomposition for 2×2 singular matrices.)**

   (i) Show that the all zero 2×2 matrix has an LU decomposition.

   (ii) Show that the 2×2 matrix of all 1's has an LU decomposition.

   (iii) T or F: Every singular 2×2 Boolean matrix has an LU decomposition. Prove or give a counterexample. Note: Entries of a Boolean matrix are 0's or 1's.

2. **(Reducing matrix multiplication to matrix inversion.)** Let I be an algorithm which takes as input a matrix M and outputs its inverse M^{-1}. In class we showed how you can multiply two $n \times n$ matrices A and B by inverting a $3n \times 3n$ matrix C.

   (i) Now show how to do the same thing for non-square matrices A and B where A is $n \times r$ and B is $r \times s$ for any positive natura l numbers n, r, s. (Hint: Use padding of matrices with 0's, similar to what you may have done for Strassen's algorithm.)

3. **(A Strassen question for matrices of size 3^k)** Assume someone discovers an algorithm S which does 3×3 matrix multiplications in 25 multiplications (mults.) instead of 27, and 24 additions (adds.) instead of 9 of 3×3 matrices.

   (i) What is the exact number of mults. and adds. of numbers that S computes when $n = 9$ and you use this algorithm along with divide and conquer to multiply two 9×9 matrices A and B by dividing A and B into 9 3×3 matrices each and doing all of the 3×3 multiplications, both of numbers and of matrices, using algorithm S.
   Pick one answer:
   (a) 625 mults. and 657 adds. (b) 125 mults. and 144 adds. (c) 625 mults. and 576 adds. (d) 625 mults. and 441 adds. (e) 425 mults. and 372 adds. (f) None of the above

   625, 816

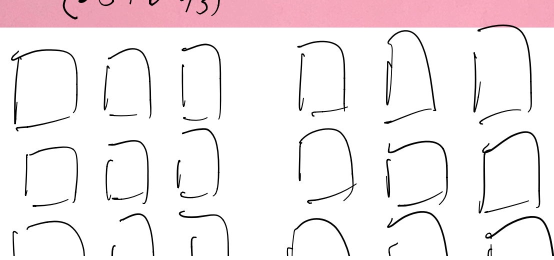
   (ii) What is its complexity (order) of the number of multiplications for using S to multiply two $n \times n$ matrices when $n = 3^k$ for some natu ral number k ? Just write out the value of the order of the number of multiplications used.
   (For this part you can ignore the additions.)

$$T_n = \begin{cases} \Theta(1) \\ 25T(n/3) \end{cases} \qquad n^{\log_3 25}$$

$\square \times \square$ 25, 24

* +

25 × 25



$24 \times 25 + 24 \times 9 = 816$

4. (**Multiple choice - no partial credit.**) Each problem is worth 2 points. It is possible that in some questions there are several correct answers, or none. In that case give all of the answers that are correct.

(i). You are given

$$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 3 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 2 & 3 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & -3 \end{pmatrix}$$

For which $Ai = A1, A2, A3$, or $A4$ is it true that $P(Ai) = LU$?

$$A1 = \begin{pmatrix} 2 & 3 & 0 \\ 4 & 5 & 4 \\ 10 & 12 & 9 \end{pmatrix} \qquad A2 = \begin{pmatrix} 2 & 4 & 10 \\ 3 & 5 & 12 \\ 0 & 4 & 9 \end{pmatrix} \qquad A3 = \begin{pmatrix} 10 & 12 & 9 \\ 2 & 3 & 0 \\ 4 & 5 & 4 \end{pmatrix} \qquad A4 = \begin{pmatrix} 10 & 12 & 9 \\ 4 & 5 & 4 \\ 2 & 3 & 0 \end{pmatrix}$$

(ii). Which of the following statements are true?

A. An upper bound on the complexity of a problem is proven by giving an algorithm to solve the problem.

B. The reduction of matrix inversion to LUP decomposition is done by finding the inverse matrix one row at a time.

C. There is an upper triangular matrix that has no LU decomposition. $L = 1, 1, 1$ 啊

D. Strassen's algorithm only works when the two matrices we are multiplying are non-singular.

(iii). When you list the orders of magnitude $O(n^2 \log n), O((1.2)^{9n}), O(2^{\log^2 n}), O(0.4^{n^2})$ from smallest to largest you get:

A. $O(n^2 \log n) < O((1.2)^{9n}) < O(2^{\log^2 n}) < O(4^{n^2})$

B. $O(2^{\log^2 n}) < O(n^2 \log n) < O(4^{n^2}) < \cancel{O((1.2)^{9n})}$

C. $O(2^{\log^2 n}) < O((1.2)^{9n}) < O(2^{\log^2 n}) < O(4^{n^2})$

D. $O(n^2 \log n) < O(2^{\log^2 n}) < O((1.2)^{9n}) < O(4^{n^2})$

E. None of the above

(iv). Which of the following matrices are singular

$$A = \begin{pmatrix} 2 & 3 & 0 \\ 4 & 5 & 4 \\ 10 & 12 & 9 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 4 & 10 \\ 2 & 8 & 19 \\ 0 & 4 & 9 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 2 & 6 \\ 2 & 0 & 0 \\ 4 & 5 & 4 \end{pmatrix} \qquad D = \begin{pmatrix} 3 & 12 & 9 \\ -1 & 5 & 4 \\ -3 & 3 & 0 \end{pmatrix}$$