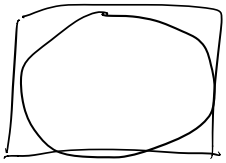
1. Numerie Algorithm : Use random num to estimate some values.
Estimate value of $\tilde{\pi}$.



2. Monte carlo Algorithm error prob $\leq \frac{1}{2}$
eg: Freivald algorithm, PIT,
generate large prime flip coin max cut

3. Las Vegas Algorithm : Answer is always correct,
but may run a long time.
There are some random numbers,
which may cause the algorithm to be not poly,
but prob(algorith run for a long time) is small.

Example : quick sort

PIT + perfect matching + Computing the determinant of matrix
or the permanent of matrix

M $n \times n$ matrix

$$\det(M) = \sum_{\tilde{\pi} \in \text{permutation of } \{1,2,\cdots,n\}} (-1)^{\text{sign}(\tilde{\pi})} \prod_{i=1}^{n} M_{i,\tilde{\pi}(i)}$$

$\tilde{\pi} = (2,1,4,3,6,5 \cdots, n, n-1)$

$\tilde{\pi}(1) = 2 \quad \tilde{\pi}(2) = 1$

$$\text{sign}\,\tilde{\pi} = \begin{cases} 0 & \text{if } \tilde{\pi} \text{ is even permutation} \\ 1 & \text{odd} \end{cases}$$ 交換回数

$$\begin{pmatrix} a & b \\ 4 & 5 \end{pmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$(1,2) \quad (2,1)$

$M_i, \tilde{\pi}_i$