CS 530 - Fall 2019
Quiz 2

Instructions: Please answer any 3 of the following 4 questions. Each question counts equally. Write your answers on the Answersheet. If need be, you can also use 1 piece of paper for longer answers. Put an X over the number of the problem on the answer sheet which you do not want graded.

1. The TSP
Recall the first approximation to the traveling salesman problem (TSP) which uses the MWST on a complete Euclidean graph to find a 2-approximation for the TSP problem.

(i). State exactly what a 2-approximation means for the TSP problem.

(ii). Give an example of a non-Euclidean complete graph H where, when you use this approximation method, H it fails to give a 2-approximation to the TSP. Say what the optimal TSP solution is for your graph and explain the answer you obtain when the 2 approximation algorithm is run on H.

(iii). (T or F) There is no approximation algorithm which is better than a 2-approximation for any complete Euclidean graph. 1.5 Appro

2. Freivalds Algorithm for Checking Non-square Matrix Multiplication
You are given the 3 matrices A, B and C below where A is 3×2, B which is 2×3 and C is 3×3.

$$A=\begin{pmatrix} 5 & 1 \\ -1 & 0 \\ 2 & -2 \end{pmatrix} \quad B=\begin{pmatrix} 0 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \quad C=\begin{pmatrix} 3 & 7 & 1 \\ 0 & -1 & 0 \\ -6 & -2 & -2 \end{pmatrix}$$

$AB=C$

Now you run Freivalds algorithm as in the square matrix case but with a random binary column vector V of 3 bits which is generated to test whether $A\times B = C$.
Answer the following 4 short answer questions about this algorithms.

$\begin{matrix} 0 & 1 & 0 \\ 3 & 2 & 1 \end{matrix} \times \begin{matrix} 1 \\ 0 \\ 1 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$

$x=Br \quad y=Ax \quad z=Cr$

(i) What is the output of Freivalds algorithm when the column vector V is $V=\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$?

Yes.

(ii). With V as in (i), is the output of the algorithm correct or incorrect ? Explain why this is so.

(iii). What is the exact error probability of Freivalds algorithm for this particular A, B and C when V is picked at random ? ~~Explain.~~ 0

(iv). (T or F) ~~The error~~ probability for this algorithm on some A, B and C (of the given dimensions) could be bigger than 1/2 as the arrays matrices are not square.

(v). Is this algorithm a Monte Carlo algorithm or a Las Vegas algorithm, or neither. Explain.

3. NP Problems and Graph Coloring

For an integer k, a graph is k-colorable if you can color the vertices of the graph with k different colors in such a way that no two adjacent vertices have the same color. Let GC (Graph Coloring) be the problem of finding, given a graph G, the smallest k for which G is k-colorable. This problem is NP-hard.

(i). The decision version of GC, called DGC is: Given a graph G, does G have a coloring using k or fewer colors.

Describe an efficient verifier V which shows that DCG is in NP.
Specifically, you need not include every detail here but you should say what the input and outputs of V are, then give a brief version the algorithm defining V, and then say what makes V a correct verifier for DGC.

(ii). For a graph G=(V,E) define deg(G) = max {deg(v) | v is a vertex in V }.
Give a proof or a counterexample for the following statement:

Any graph G has a coloring of no more than 1+deg(G) colors.

4. Polynomial Identity Testing (PIT)

Recall the randomized PIT algorithm which can be used to test if two single variable polynomials f(x) and g(x) both of degree 9 are identical. Which of the following is true about our randomized test when the random inputs in the 5 questions below are chosen from a set S of 100 different numbers.

Answer the following 5 True/False questions about this PIT problem. No partial credit is given for this one.

A. If we pick 19 random points from S and find that f(x)×g(x) = 0 for all 19 chosen points then we know (with probability 1) that f(x)×g(x) ≡ 0.

B. If in fact f(x) ≠ g(x), and we choose 1 random r from S to use for the identity test for f(x) and g(x) then the polynomial identity test has error probability ≤ .1 $\frac{9}{100}$

C. If we pick 6 random inputs $r_1, r_2, \ldots, r_6$ from S and we test and find that $f(r_i) = g(r_i)$ for all 6 inputs, then we know that prob(f(x) ≡ g(x)) ≥ 1/2.

已知 超过6个 9个 不同根 $f(x) \equiv g(x) \Leftrightarrow$ 超过9个不同根

D. If we pick 6 random inputs from S and we test if f(x) = g(x) on these 6 inputs and find them not equal on exactly three of them then we only know with probability 9/100 that f(x)≢g(x).

E. If we pick 12 random inputs from S and we test if f(x) = g(x) on these 12 inputs, then we know with certainty (prob = 1) that f(x) = g(x) for all inputs x.

$\cancel{\frac{P(A)}{P(AB)}}$

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Problem 3 that I do not want graded.

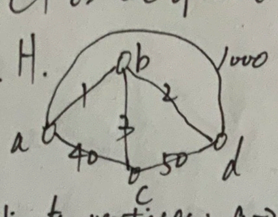
$\frac{19}{20}$

QUIZ 2, ANSWER SHEET

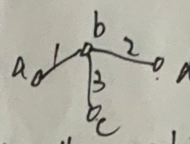
1\. (i) 2 approx means that we can always find a TSP tour with cost $C \le 2|C^*|$ where $|C^*|$ is the optimal cost.

$\frac{10}{10}$

(ii) Example. H.

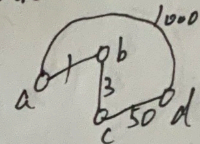


$\Rightarrow$ MST:



A full walk: $a \to b \to c \to b \to d \to b \to a$.

$\Rightarrow$ Delete duplicate vertices: $a \to b \to c \to d \to a$



2 approx tour

Therefore, 2 Approx cost: $1054 = 1+3+50+1000$

optimal: $a \to b \to d \to c \to a$ $93 = 1+2+50+40$

$1000 > 2 \times 93 = 186$

OK

$\therefore$ This algorithm fails to give a two approximation.

(iii) False. For complete Euclidean graph, we have Christofide algorithm, a 1.5 approximation algorithm to solve TSP problem.

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2\. (i) Output: Yes./True

$\frac{9}{10}$

Explain: $x = BV = \begin{pmatrix} 0 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ $y = Ax = \begin{pmatrix} 5 & 1 \\ -1 & 0 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -8 \end{pmatrix}$

$z = CV = \begin{pmatrix} 3 & 7 & 1 \\ 0 & -1 & 0 \\ -6 & -2 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -8 \end{pmatrix}$ $\therefore y = z \Rightarrow AB = C \Rightarrow$ Yes.

(ii) Correct

Explain: $AB = \begin{pmatrix} 5 & 1 \\ -1 & 0 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 7 & 1 \\ 0 & -1 & 0 \\ -6 & -2 & -2 \end{pmatrix} = C$

(iii) Error probability is 0.

Explain: In fact, $AB = C$. Therefore, for all V, we will have $ABV = CV$.

(iv) False.

The incorrect probability is ~~less than~~ $\le \frac{1}{2}$. OK

(V). Monte Carlo algorithm.

Explain: It tries to test a problem by some inputs repeatedly and has an incorrect probability. ? X NOT REALLY

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$\frac{?}{10}$

$\frac{9}{10}$

4. A. True.

Explain: $deg(f(x) \cdot g(x)) = 9 \times 2 = 18$, which means $f(x)g(x)$ has at most 18 different roots if it is not a zero poly. If there is 19 different roots, it has to be zero.

B. True.

Explain: $f(x) - g(x) = 0$ has at most 9 different roots.
The error probability is $\frac{9}{100} \leq \frac{10}{100}$.

C. True. ×−1

Explain: If $f(x) - g(x) = 0$ has 10 or more differents roots,
we can say $f(x) - g(x) \equiv 0$. $\frac{6}{10} > \frac{5}{10} = \frac{1}{2}$

D. False. If $f(r_i) \neq g(r_i)$, then $f(x) \not\equiv g(x)$ with certainty.

~~$f(x) \neq g(x)$~~

E. True. $f(x) - g(x) = 0$ has at most 9 different roots.