USING LP TO SOLVE MAX ~~MATCHING~~ MATCHING

We reduce MAX MATCHING to solving a LINEAR PROGRAM.

Rules: Given a graph $G = (V, E)$

- Each edge (u, v) of G corresponds to a variable $X_{u,v}$.

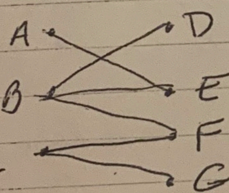
Each vertex of G correspond to an inequality constraint,

An vertex v of G yields $\sum_{u \text{ adjacent to } v} X_{vu} \leq 1$

To this we add non-negativity constraints $X_{uv} \geq 0$, for all vertices

The objective function is:
$$\text{maximize} \sum_{(u,v) \text{ in } E} X_{u,v}$$

EXAMPLE:

If G is


then we have variables X_{AB}, X_{BD}, X_{BE}, X_{BF}, X_{CF}, X_{CG}, X_{DB}, X_{EA}, X_{EB}, X_{FC}, X_{FB}, X_{GC}

The objective is to maximize $\sum_{\substack{\text{all edges} \\ (u,v)}} X_{uv}$

subject to $X_{AE} \leq 1$, $X_{BD} + X_{BE} + X_{BF} \leq 1$
$X_{CF} + X_{CG} \leq 1$, $X_{DB} \leq 1$, $X_{EA} + X_{EB} \leq 1$
$X_{FB} + X_{FC} \leq 1$, $X_{GC} \leq 1$

and all $X_{uv} \geq 0$

For a variable X_{uv} in the LP,

$X_{uv} = 0$ means (u,v) is not in the matching

$X_{uv} = 1$ means (u,v) is in the matching.

Clearly any matching of G gives variable values making all the inequalities true.

The converse also holds: Any assignments of 0's and 1's to the variables of the LP making all the constraints true, and where $X_{u,v} \neq X_{vu}$ for all variables u, v, gives a legal matching of G.

Now the main fact for bipartite graphs G is that, the extreme points for the solution of the LP corresponding to G has all integer values. (These values must be 0 or 1)

Using this, we can prove (omitted here) that for bipartite G, the only extreme pts of the feasible region have integer i.e. 0, 1 values. So solving the LP for bipartite G gives a maximum matching as a solution.