set of edges with no vertice repeated.

maximum matching is Poly.

Bipartite Graph is Poly.

△ Triangle does not have matching.

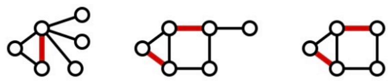
Perfect matching P is a matching in G
catching every vertex exactly once.

Definition [edit]

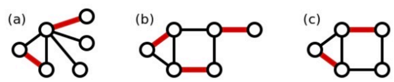
Given a graph *G* = (*V*,*E*), a **matching** *M* in *G* is a set of pairwise non-adjacent edges, none of which are loops; that is, no two edges share a common vertex.

A vertex is **matched** (or **saturated**) if it is an endpoint of one of the edges in the matching. Otherwise the vertex is **unmatched**.

A **maximal matching** is a matching *M* of a graph *G* with the property that if any edge not in *M* is added to *M*, it is no longer a matching, that is, *M* is maximal if it is not a subset of any other matching in graph *G*. In other words, a matching *M* of a graph *G* is maximal if every edge in *G* has a non-empty intersection with at least one edge in *M*. The following figure shows examples of maximal matchings (red) in three graphs.



A **maximum matching** (also known as maximum-cardinality matching[1]) is a matching that contains the largest possible number of edges. There may be many maximum matchings. The **matching number** $\nu(G)$ of a graph G is the size of a maximum matching. Every maximum matching is maximal, but not every maximal matching is a maximum matching. The following figure shows examples of maximum matchings in the same three graphs.



A **perfect matching** (a.k.a. 1-factor) is a matching which matches all vertices of the graph. That is, every vertex of the graph is incident to exactly one edge of the matching. Every perfect matching is maximum and hence maximal. In some literature, the term **complete matching** is used. In the above figure, only part (b) shows a perfect matching. A perfect matching is also a minimum-size edge cover. Thus, $\nu(G) \leq \rho(G)$, that is, the size of a maximum matching is no larger than the size of a minimum edge cover. A perfect matching can only occur when the graph has an even number of vertices.

A **near-perfect matching** is one in which exactly one vertex is unmatched. This can only occur when the graph has an odd number of vertices, and such a matching must be maximum. In the above figure, part (c) shows a near-perfect matching. If, for every vertex in a graph, there is a near-perfect matching that omits only that vertex, the graph is also called factor-critical.

Given a matching *M*,

- an **alternating path** is a path that begins with an unmatched vertex and[2] whose edges belong alternately to the matching and not to the matching.
- an **augmenting path** is an alternating path that starts from and ends on free (unmatched) vertices.

One can prove that a matching is maximum if and only if it does not have any augmenting path. (This result is sometimes called Berge's lemma.)

An induced matching is a matching that is an induced subgraph.[3]

Properties [edit]

In any graph without isolated vertices, the sum of the matching number and the edge covering number equals the number of vertices.[4] If there is a perfect matching, then both the matching number and the edge cover number are $|V| / 2$.

If A and B are two maximal matchings[redacted]this, observe that each edge in $B \setminus A$