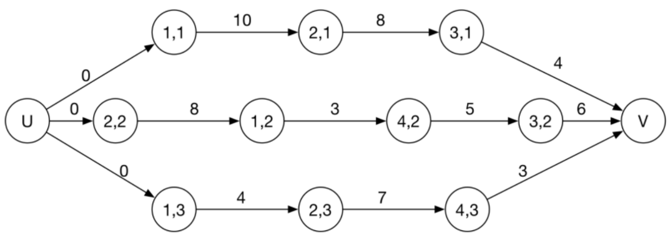
CHAP 5: SHIFTING BOTTLENECK (25pts)

(Machine, Job)

Given the following job shop problem

Job	Sequence	Processing Time
1	1,2,3	$p_{11} = 10,\ p_{21} = 8,\ p_{31} = 4$
2	2,1,4,3	$p_{22} = 8,\ p_{12} = 3,\ p_{42} = 5,\ p_{32} = 6$
3	1,2,4	$p_{13} = 4,\ p_{23} = 7,\ p_{43} = 3$

Step 1: Draw the initial graph of this problem



Total processing time of job 1: 10 + 8 + 4 = **22**

Total processing time of job 2: 8 + 3 + 5 + 6 = **22**

Total processing time of job 3: 4 + 7 + 3 = 14

$$C_{max} = \mathbf{22}$$

Step 2: Apply the 1st iteration to find the sequence

Job	1,1	2,1	3,1	2,2	1,2	4,2	3,2	1,3	2,3	4,3
P	10	8	4	8	3	5	6	4	7	3
ES	0	10	18	0	8	11	16	0	4	11
EC	10	18	22	8	11	16	22	4	11	14
LS	0	10	18	0	8	11	16	8	12	19
LC	10	18	22	8	11	16	22	12	19	22
Sla	0	0	0	0	0	0	0	8	8	8

Machine 1	1,1	1,2	1,3
Processing T	10	3	4
Release (ES)	0	8	0
Due date (LC)	10	11	12
Begin	0	14	10
Finish	10	17	14
Lateness	0	6	2

1,3
4
0
4
8
12
8

Earliest Release Day First

J1 – J3 – J2

Begin J_i = max (Finish J_{i-1}, Release J_i)

Lateness = max (0, Finish – Due date)

M2	2,1	2,2	2,3
P	8	8	7
R	10	0	9
D	18	8	19
B	15	0	8
F	23	8	15
L	5	0	0

M3	3,1	3,2
P	4	6
R	18	16
D	22	22
B	22	16
F	26	22
L	4	0

M4	4,2	4,3
P	5	3
R	11	11
D	16	22
B	11	16
F	16	19
L	0	0

For machine 1, the sequence is (1,1) – (1,3) – (1,2) , $L_{max}(1) = 6$

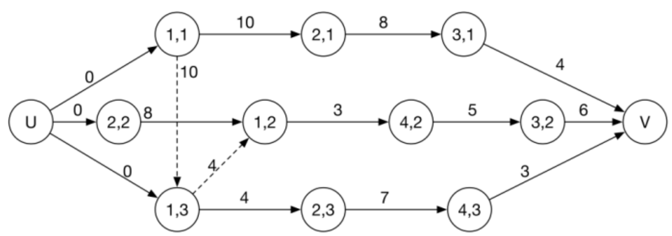
For machine 2, the sequence is (2,2) – (2,3) – (2,1) , $L_{max}(2) = 5$

For machine 3, the sequence is (3,2) – (3,1) , $L_{max}(3) = 4$

For machine 4, the sequence is (4,2) – (4,3) , $L_{max}(4) = 0$

Focus on machine with **largest** bottle neck. Machine **1** is selected to be included in M_0

Step 3: Draw the graph after the first iteration



Cmax = U – (1,1) – (1,3) – (1,2) – (4,2) – (3,2) – V = 28

CHAP 6: PROFILE FITTING HEURISTIC (20pts)

Job	1	2	3
Machine 1	0	1	0
Machine 2	0	0	0
Machine 3	1	0	1
Machine 4	1	1	0

Step 1: Find the first job in the sequence

Select the job with the longest total processing time as the initial job.

Job	1	2	3
Machine 1	0	1	0
Machine 2	0	0	0
Machine 3	1	0	1
Machine 4	1	1	0
Total	2	2	1

Job 1 is selected as the initial job.

Step 2: Find the second job in the sequence

2.1: First column = cumulative processing time of Initial job

2.2: Second column = Processing Time (**J2**)

Job	Leave time	Next job is 2	Leave of next job	Loss
Machine 1	=0+0=0	1		
Machine 2	=0+0=0	0		
Machine 3	=0+1=1	0		
Machine 4	=1+1=2	1		
Total				

2.3: Leave of next job

First machine:

= MAX (**leave time** + **next job PT**, **Leave time (next)**)

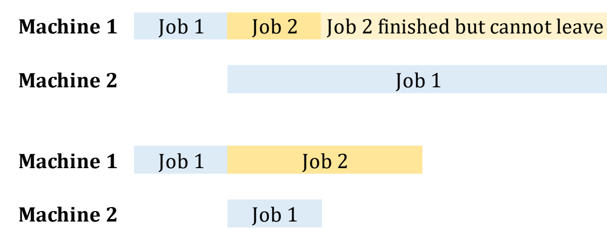
Middle machines:

= MAX (**leave of next job (previous)** + **next job PT**, **Leave time (next)**)

Last machine:

= leave time + Next job PT

Visualization



Job	Leave time	Next job is 2	Leave of next job	Loss
Machine 1	0	1	= max (0+1, 0) = 1	
Machine 2	0	0	= max (1+0, 1) = 1	
Machine 3	1	0	= max (1+0, 2) = 2	
Machine 4	2	1	= 2 + 1 = 3	
Total				

2.4. Last column = Leave of next job – Next job PT – Leave time

Job	Leave time	Next job is 2	Leave of next job	Loss
Machine 1	0	1	1	=1-1-0=0
Machine 2	0	0	1	=1-0-0=1
Machine 3	1	0	2	=2-0-1=1
Machine 4	2	1	3	=3-1-2=0
Total				2

Doing similarly for other jobs that has not yet been scheduled

Job	Leave time	Next job is 3	Leave of next job	Loss
Machine 1	0	0	=max(0+0, 0) = 0	0
Machine 2	0	0	=max(0+0, 1) = 1	1
Machine 3	1	1	=max(1+1, 2) = 2	0
Machine 4	2	0	=2+0 = 2	0
Total				1

2.5 Conclusion

Since the loss regarding to job 3 is the smallest, so job 3 is the next job.

CHAP 9: TIME INTERVAL

1. Multiple machines (25pts)

Job	1	2	3	4
w_j	3	2	2	1
p_j	2	3	1	1
r_j	0	2	1	2
d_j	5	7	6	6
Machine 1	2	0	1	1
Machine 2	1	2	0	1

The number of machine type 1 and 2 are **2**.

Step 1: Calculate the priority of each job

$$I_j = \frac{\left(\sum_{k=1}^{K} \frac{m_{jk}}{N_k}\right) \times p_j}{w_j}$$

m_k: the number of resource type k which is required by job j

N_k: the maximum numer of resource type k

Job	1	2	3	4
I_j	1	1.5	0.25	1

$$I_1 = \frac{\left(\frac{2}{2} + \frac{1}{2}\right) \times 2}{3} = 1 \qquad I_2 = \frac{\left(\frac{0}{2} + \frac{2}{2}\right) \times 3}{2} = 1.5$$

Step 2: Write down the summary table of possible resource usage when no job is assigned

For machine 1

Time	0	1	2	3	4	5	6
J1	2	2	2	2	2		
J2							
J3		1	1	1	1	1	
J4			1	1	1	1	
Use1	2	3	4	4	4	2	0

For machine 2

Time	0	1	2	3	4	5	6
J1	1	1	1	1	1		
J2			2	2	2	2	2
J3							
J4			1	1	1	1	
Use2	1	1	4	4	4	3	2

Summary

Time	0	1	2	3	4	5	6
Use1	2	3	4	4	4	2	0
Use2	1	1	4	4	4	3	2
R1	2	2	2	2	2	2	2
R2	2	2	2	2	2	2	2

Step 3: Assign the first job with the highest priority then write down the table of the update resources and possible resource usage

Highest priority = **Smallest Index value**

3.1: Use1, Use2, R1, R2 are the same as previous

3.2: J_jR_k: loss of machine k if job j start at time t (from ES to LS)

$$\sum_{t}^{t+p-1} \frac{m_{jk}}{R_k} * Use_k$$

3.3: Total

3.4: Assign: choose the time slot with **the smallest value**

Time	0	1	2	3	4	5	6
Use1	2	3	4	4	4	2	0
Use2	1	1	4	4	4	3	2
R1	2	2	2	2	2	2	2
R2	2	2	2	2	2	2	2
J3R1		1.5	2	2	2	1	
Assign						X	

$\boldsymbol{m_{31} = 1 \quad p_3 = 1}$

$$\frac{m_{31}}{R_1(1)} \times Use_1(1) \text{ hoặc } m_{31}\left[\frac{Use_1(1)}{R_1(1)}\right]$$

Assign job 3 to time slot 5.

3.5: Update

From when to when the job occupies in the possible resource usage?

How many resource of each type that the job uses, at which time slot?

Time	0	1	2	3	4	5	6
Use1	2	2	3	3	3	1	0
Use2	1	1	4	4	4	3	2
R1	2	2	2	2	2	1	2
R2	2	2	2	2	2	2	2

Step 4: Assign the 2nd job with the second high priority then write down the table of the update resources and possible resource usage

Time	0	1	2	3	4	5	6
Use1	2	2	3	3	3	1	0
Use2	1	1	4	4	4	3	2
R1	2	2	2	2	2	1	2
R2	2	2	2	2	2	2	2
J1R1	4	5	6	6			
J1R2	1	2.5	4	4			
Total	5	7.5	10	10			
Assign	X						

$\boldsymbol{m_{12} = 2 \quad p_1 = 2}$

$$\frac{m_{11}}{R_1(0)} \times Use_1(0) + \frac{m_{11}}{R_1(1)} \times Use_1(1) \textit{ hoặc } m_{11}\left[\frac{Use_1(0)}{R_1(0)} + \frac{Use_1(1)}{R_1(1)}\right]$$

Update

Time	0	1	2	3	4	5	6
Use1	0	0	1	1	1	1	0
Use2	0	0	3	3	3	3	2
R1	0	0	2	2	2	1	2
R2	1	1	2	2	2	2	2

2. Graph coloring (20pts)

Activities	1	2	3	4	5	6	7
Gary	1	0	0	1	1	0	1
Hamilton	1	1	1	0	0	0	0
Izak	0	0	1	0	1	1	0
Reha	1	0	1	1	1	0	0

Step 1: Find the conflict matrix and degree of each job

Job	1	2	3	4	5	6	7
1	-	1	1	1	1	0	1
2	1	-	1	0	0	0	0
3	1	1	-	1	1	1	0
4	1	0	1	-	1	0	1
5	1	0	1	1	-	1	1
6	0	0	1	0	1	-	0
7	1	0	0	1	1	0	-
Degree	5	2	5	4	5	2	3

Step 2: Apply the graph coloring algorithm to find the schedule

Job	1	2	3	4	5	6	7
Degree	5	2	5	4	5	2	3

Select job with the highest degree

Update the new degree

Job	1	2	3	4	5	6	7
Degree	C1	1	4	3	4	2	2

Job	1	2	3	4	5	6	7
Degree	C1	0	C2	2	3	1	2

Job	1	2	3	4	5	6	7
Degree	C1	0	C2	1	C3	0	1

Job	1	2	3	4	5	6	7
Degree	C1	0	C2	C4	C3	0	0

Possible solutions

	1	3	5	4
2			x	x
6	x			x
7		x		

One of the solutions

1	3	5	4
6	7	2	

Time slot 1: Job 1 & 6

Time slot 2: Job 3 & 7

Time slot 3: Job 5 & 2

Time slot 4: Job 4

CHAP 13: WORKFORCE SCHEDULING (10pts)

Days-off Scheduling

- There are 7 days and each day has two shifts, ie. Night Shift and Day Shift.
- Demands of Day Shift and Night Shift on day d are denoted as Dd and Nd.
- Assumed that there are L labors.
- Among them there are S labors who can work either as supervisor or as labor.

Objective:

Minimize the total usage number of labors

Constraints:

- Each labor works only one shift per day
- Each labor cannot work 2 consecutive shifts
- Each labor works 5 shifts in week
- Apart from the required demand, each shift must have one supervisor.

!When labor works as supervisors, he is not counted when computing Dd or Nd.

Set:

L: set of labors

S: set of labors who can work as supervisors

Parameters

D_d: demand of day shift d

N_d: demand of night shift d

Variables

Y_i: binary variable, $Y_i = 1$ if labor i is used

XD_{id}: binary variable, $XD_{id} = 1$ if labor i works on day shift d as labor

XN_{id}: binary variable, $XN_{id} = 1$ if labor i works on night shift d as labor

XR_{id}: binary variable, $XR_{id} = 1$ if labor i is off on day d

SD_{id}: binary variable, $i \in S, SD_{id} = 1$ if labor i works on day shift d as supervisor

SN_{id}: binary variable, $i \in S, SN_{id} = 1$ if labor i works on night shift d as supervisor

Objective function

$$\text{Minimize} \sum_{i}^{L} Y_i$$

Subject to

C1: Demand has to be served, including 1 supervisor

$$\sum_{i}^{L} XD_{id} \geq D_d + 1 \quad \forall d$$

$$\sum_{i}^{L} XN_{id} \geq N_d + 1 \quad \forall d$$

C2: Each labor works only on shift per day

$$XD_{id} + XN_{id} + XR_{id} = 1 \qquad \forall d, i$$

C3: Each labor cannot work 2 consecutive shifts

$$XN_{id} + XD_{i,d+1} \leq 1 \qquad \forall d, i$$

C4: Each labor works 5 shifts in week

$$\sum_{d}^{D} XR_{id} = 2 \qquad \forall i$$

C5: Each shift must have one supervisor

$$\sum_{i}^{S} SD_{id} = 1 \quad \forall d$$

$$\sum_{i}^{S} SN_{id} = 1 \quad \forall d$$

C6: If labor works as the supervisor, it is still considered that they work in that shift

$$SD_{id} - XD_{id} \leq 0 \qquad \forall d, i$$

$$SN_{id} - XN_{id} \leq 0 \qquad \forall d, i$$

C7: Connect between Y_i and other variables

$$Y_i \geq XD_{id} \quad \forall d, i$$

$$Y_i \geq XN_{id} \quad \forall d, i$$

$$Y_i \leq \sum_{i}^{L} XD_{id} + \sum_{i}^{L} XN_{id} \qquad \forall i$$

Shifting bottle neck

Job	Sequence	Processing Time
1	1,2,3	$p_{1,1} = 2,\ p_{2,1} = 3,\ p_{3,1} = 4$
2	1,3,2	$p_{1,2} = 5,\ p_{3,2} = 4,\ p_{2,2} = 3$
3	2,1,3	$p_{2,3} = 6,\ p_{1,3} = 3,\ p_{3,3} = 5$

Profile fitting heuristic

Job	**1**	**2**	**3**	**4**
Machine 1	3	4	4	3
Machine 2	7	5	5	3
Machine 3	5	3	4	5
Machine 4	5	6	3	4

Multiple machines

Job	**1**	**2**	**3**
w_j	4	4	2
p_j	1	2	2
r_j	0	2	3
d_j	7	7	7
Machine 1	1	2	0
Machine 2	1	1	2

The number of machine type 1 and 2 are **4**.

Graph coloring

Job	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Res 1	1	0	0	0	0	1	0	0
Res 2	0	1	1	0	0	0	0	1
Res 3	1	0	0	1	1	0	1	0
Res 4	0	1	0	1	0	1	0	1
Res 5	0	1	1	0	1	0	0	1