International University - VNUHCM
School of Industrial and Engineering Management (IEM)

Midterm Examination
Date: 14/6/2023; Duration: 120 minutes
Open book; **Offline,** Laptops/Cell-phone/… **are not allowed.**

<table>
<tr><td colspan="3">SUBJECT: Simulation Models in Industrial Engineering (ID: IS028IU)</td></tr>
<tr><td>Approval by the School/Department of IEM
Signature

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Student ID:</td></tr>
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INSTRUCTIONS: the total of point is 100 (equivalent to 20 % of the course)

1. *Purpose:*
    - Identify, formulate and solve complex problems *in manufacturing and service systems* by *performing discrete-event system simulation and applying knowledge of statistics*
    - *Use simulation as a tool* in the process of engineering design to produce solutions that meet specified needs with consideration of *economic factors*.
2. *Requirements:*
    - Read carefully each question and answer it following the requirements.
    - Write the answers and draw models CLEAN and TIDY directly in the exam paper.

QUESTIONS

Question 1 (10pts)

Match the context and the distribution:

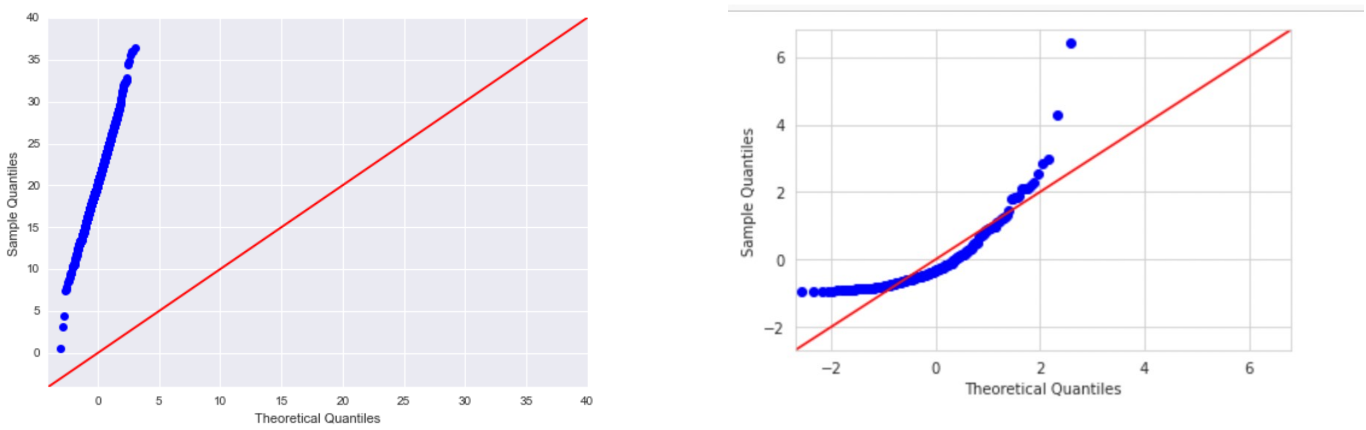
Data Context	Distribution
(a) Arrival process	(1)Weibull: time to failure for components
(b) Number of defective items in a sample	(2)Normal distribution: a process that is the sum of a number of component processes.
(c) Assembly time	(3)Poisson: number of independent events that occur in a fixed amount of time or space
(d) Reliability study	(4)Exponential: Time between independent events, or a process time that is memoryless
	(5)Binomial: number of successes in n trials

Answer

(a)-(3),(4) **2.5pt**
(b)-(5) **2.5pt**
(c)-(2) **2.5pt**
(d) – (1) **2.5pt**

Question 2 (22 pts)

a. Give comments on the Q_Q plots below (Figure 1) **(6 pts)**



(i) (ii)

Figure 1

Answer:

Figure 1(i) The sample distribution is similar to the theoretical one, but different parameters **(4pt)**

Figure 1(ii) The sample distribution is different with the theoretical one **(2pt)**

b. Test Poisson distribution of the below data **(10 pts)**

x	Observed frequency, Oi
0	7
1	6
2	11
3	10
4	8
5	3
6	2
7	3

Answer

n = 50 → Ei = npi >= 5 (Chi square test)

Hypothesis:

H0: Data follows Poisson distribution

H1: Data does not follow Poisson distribution

Test statistic:

Mean of data set:

$$\bar{X} = \alpha = \frac{0*7+1*6+2*11...+7*3}{50} = \frac{138}{50} = 2.76$$

$$E_i = np(x) = n * \frac{e^{-\alpha} * \alpha^x}{x!}$$

xi	Oi		Ei		(Oi-Ei)^2/Ei
0	7	13	3.164588	11.89885	0.1019028
1	6		8.734264		
2	11	11	12.05328	12.05328	0.092042
3	10	10	11.08902	11.08902	0.1069497
4	8	8	7.651425	7.651425	0.01588
5	3	8	4.223587	6.932474	0.1643874
6	2		1.94285		
7	3		0.766038		
sum	50	50	50	50	0.4811618

Test critical:

Dof = k-s-1 = 5-1-1 = 3

$$\chi^2_{0.05,3} = 7.81$$

⇨ Test statistic < test critical → can not reject H0

c. What are the differences in usage of K-S test and Chi-square test? **(6 pts)**

Answer

- K-S test: small data set (<50), only continuous distribution, no parameter is estimated
-Chi square test: large data set (>=50), for both discrete and continuous distribution

Question 3 (30 pts)

Total customers served per day is chosen to validate a simulation model of a coffee shop. The average total customers of the real system is 169. Results of 5 replications from the simulation are as below:

i	1	2	3	4	5
Y	168	154	186	165	147

a. Conduct a statistical test to check the model validity. Use the level of significance alpha =0.05 **(12 pts)**
b. Assume that a difference of 20 customers is significant. What is the probability of accepting an invalid model in the above conclusion? What is the power of the test? Give comments on the number of replications. **(10 pts)**
c. Figure 2 shows the results from Arena. Based on the Confidence Interval of the "# of instore customers", get a conclusion on the model validity assuming that difference of 20 customers is significant **(8 pts)**

Replications: 5 Time Units: Minutes

User Specified

Counter

Count	Average	Half Width	Minimum Average	Maximum Average
# of bills	193.00	0.00	193.00	193.00
# of customer loss	0.00	0.00	0.00	0.00
# of instore customers	164.00	18.52	147.00	186.00
# of takeaways	191.00	15.26	174.00	208.00

Figure 2

Answer

a.

Hypothesis:

H0: E(Y) = 169

H1: E(Y) =/ 169

Test statistics: $\mu_0 = 169, n = 5$

$$\bar{Y} = \frac{sum(Y)}{5} = 164$$

$$S = \sqrt{\frac{\sum_{i=1}^{5}(Y_i - \bar{Y})^2}{n-1}} = 14.9164$$

$$H_0 = \left|\frac{\bar{Y} - \mu_0}{S/\sqrt{n}}\right| = \left|\frac{164 - 169}{14.9164/\sqrt{5}}\right| = 0.7495$$

Critical value:

$$t_{\frac{\alpha}{2}, n-1} = t_{0.025,4} = 2.78$$

⇨ Test statistic < critical value, can not reject H0

b.

$$\varepsilon = 20$$

True difference between E(Y) and $\mu(\varepsilon)$:

$$\delta = \frac{\varepsilon}{\sigma} = \frac{20}{14.9164} = 1.34\,, n = 5$$

From OC curve:

$$\beta = 0.375 = 37.5\%$$

⇨ The power of test: $1 - \beta = 62.5\%$

So, should increase the number of replication at least 7 for the power at least 0.8
More replication needed

c.

CI: $\bar{Y} \pm HW$
LB: $\bar{Y} - HW = 164 - 18.52 = 145.48$
UB: $\bar{Y} + HW = 164 + 18.52 = 182.52$

So, the error:

$$|LB - \mu_0| = |145.48 - 169| = 23.52$$

⇨ Worst-case error

$$|UB - \mu_0| = |182.52 - 169| = 13.52$$

⇨ Best-case error

Question 4 (18 pts)

a. Suggest a simulation type (terminating or steady state) for the following case: **(6 pts)**
   - Simulation of a call center → terminating
   - Simulation of inventory in warehouse → steady state
   - Simulation of an assembly line → terminating
   - Simulation of cross docking activities → terminating

b. Classify the following statistics (count, tally, time persistent): **(6 pts)**
   - Total customers of a day → count
   - Number of customers in the system → time persistent
   - Waiting time of customers → tally
   - Cycle time (Time between 2 products out of the system) → tally

c. How many replications we should run in order to reduce the Half Width of "# of instore customers" in Figure 2 by half? **(6 pts)**

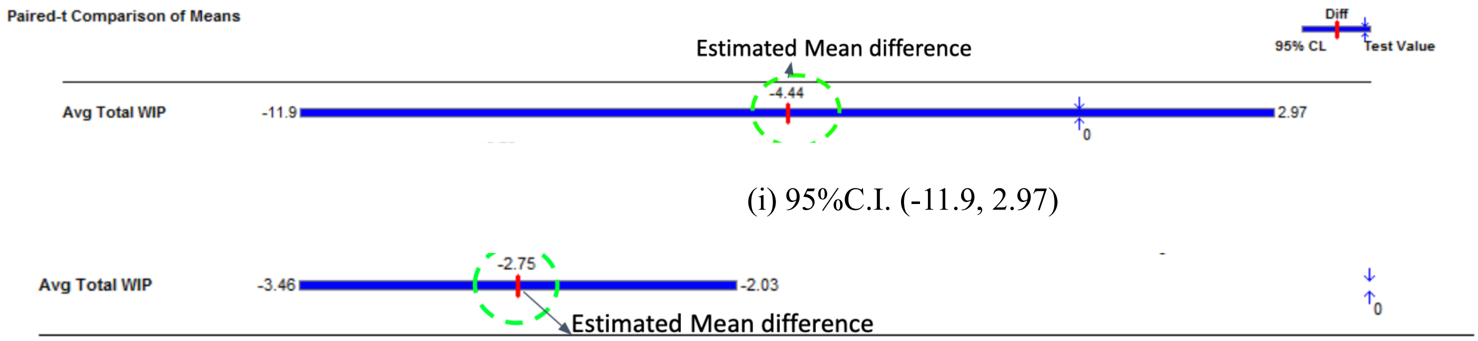
Answer

N2/n1 = (HW1/HW2)^2 -→ n2=5x(2)^2=20

Question 5 (20 pts)

Two systems are compared based on the average Total WIP obtained from simulation. 95% Confidence Interval of the difference of the average Total WIP of the 2 systems is calculated and plotted in Figure 3.

a. Give conclusion on the difference of the 2 systems in the case of Figure 3(i) and the case of Figure 3(ii) **(10 pts)**



(i) 95%C.I. (-11.9, 2.97)

(ii) 95%C.I. (-3.46, -2.03)

Figure 3

Answer

Figure 3(i) insignificant difference

Figure 3(ii) significant difference (system 1<system 2)

b. For the case of Figure 3(i), calculate the 90% confidence interval (C.I.) with the current number of replications 10. **(10 pts)**

95% CI: (-11.9, 2.97), n = 10

-11.9 = $\bar{Y} - t_{0.025,9} * \frac{S}{\sqrt{10}}$

$$S = 10.4383$$

To calculate the 90% CI:

$$\bar{Y} - t_{0.05,9} * \frac{S}{\sqrt{10}}$$

LB: $-4.44 - 1.83 * \frac{10.4383}{\sqrt{10}} = -10.48$

UB: $-4.44 + 1.83 * \frac{10.4383}{\sqrt{10}} = 1.6$

So, 90% CI (-10.48, 1.6)

THE END