FINAL SIMULATION THEORY – BT

Problem 1: ***(Chi-square for continuous)***

The time required for the transmission of a message (in milliseconds) is sampled electronically at a communication center. The last 30 values in the sample are as follows:

8.47	7.17	10.00	7.30	10.94
6.70	9.13	8.86	10.27	8.12
7.20	7.65	7.87	8.82	11.29
9.17	11.70	8.68	6.94	9.19
10.53	10.85	9.61	11.44	8.31
10.40	10.53	10.61	8.23	9.52

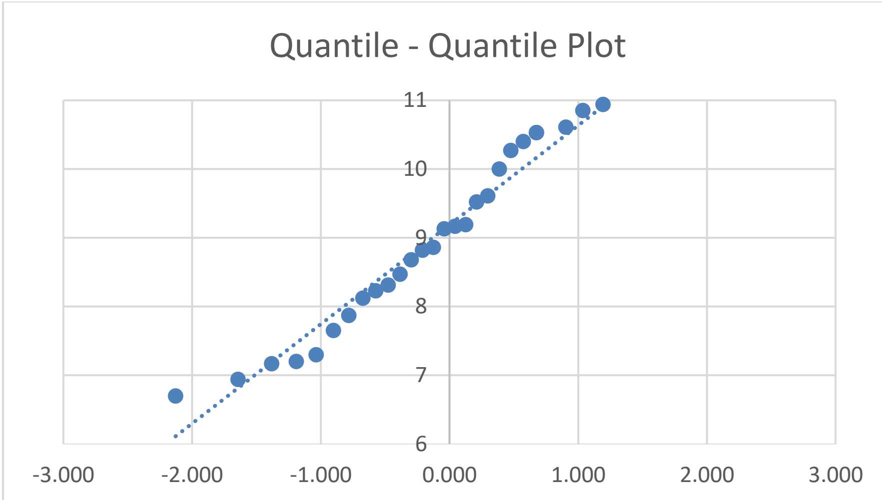
a. Construct a Quantile-Quantile Plot to justify the exponential distribution of the data.

b. Use the Chi-square test to test the hypothesis that data are exponential distributed. Let the number of class intervals be k = 6. Use the level of significance = 0.05.

Answer:

a. Construct a Quantile-Quantile Plot to justify the exponential distribution of the data.

Data	Rank	Percentile	Z-score
6.7	1	0.017	-2.128
6.94	2	0.050	-1.645
7.17	3	0.083	-1.383
7.2	4	0.117	-1.192
7.3	5	0.150	-1.036
7.65	6	0.183	-0.903
7.87	7	0.217	-0.784
8.12	8	0.250	-0.674
8.23	9	0.283	-0.573
8.31	10	0.317	-0.477
8.47	11	0.350	-0.385
8.68	12	0.383	-0.297
8.82	13	0.417	-0.210
8.86	14	0.450	-0.126
9.13	15	0.483	-0.042
9.17	16	0.517	0.042
9.19	17	0.550	0.126
9.52	18	0.583	0.210
9.61	19	0.617	0.297
10	20	0.650	0.385
10.27	21	0.683	0.477
10.4	22	0.717	0.573
10.53	23	0.750	0.674
10.53	23	0.750	0.674
10.61	25	0.817	0.903
10.85	26	0.850	1.036
10.94	27	0.883	1.192
11.29	28	0.917	1.383
11.44	29	0.950	1.645
11.7	30	0.983	2.128



<u>Conclusion:</u> The Q-Q plot does not show that the data follows exponential distribution.

b. Use the Chi-square test to test the hypothesis that data are exponential distributed. Let the number of class intervals be k = 6. Use the level of significance = 0.05.

The average value of the data is 9.183, with reciprocal rate value l = 0.109

With k = 6, the probability of an observation falling into any one of them will be $p = 0.167$

$F(a_i) = 1 - exp\ (l\ a_i\ )$

$\rightarrow a_i = -\frac{1}{\lambda} ln\ (1 - ip) \qquad i\ =\ 0, 1, 2, \ldots., k$

$a_1 = -\frac{1}{\lambda}\ln(1 - ip) = -\frac{1}{0.109}\ln(1 - 0.167) = 1.676$

$a_2 = -\frac{1}{0.109}\ln(1 - 2 \times 0.167) = 3.729$

$a_3 = -\frac{1}{0.109}\ln(1 - 3 \times 0.167) = 6.378$

$a_4 = -\frac{1}{0.109}\ln(1 - 4 \times 0.167) = 10.116$

$a_5 = -\frac{1}{0.109}\ln(1 - 5 \times 0.167) = 16.53$

Interval	$\boldsymbol{O_i}$	$\boldsymbol{E_i}$	$\boldsymbol{\frac{(O_i - E_i)^2}{E_i}}$
[0,1.676]	0	5	5
[1.676,3.729]	0	5	5
[3.729,6.378]	0	5	5
[6.378,10.116]	20	5	45
[10.116,16.53]	10	5	5
[16.53,∞]	0	5	5

To use a Chi-square goodness-of-fit test, form a hypotheses as follows:
H_0: the random variable follows the exponential distribution
H_1: the random variable does not follow the exponential distribution

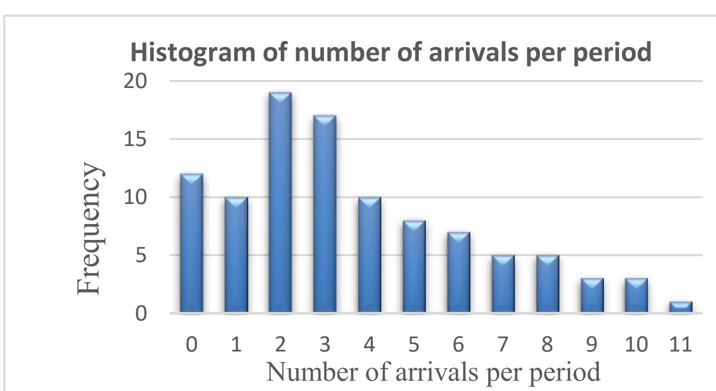
The chi-square value:
$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 70$
$\chi^2_{1-\alpha, r-p-1} = \chi^2_{0.05,4} = 9.49$

Conclusion: The null hypothesis is rejected

Problem 2: ***(Chi-square for discrete)*** Use data below:
(BÀI NÀY LÀ BÀI TRONG SLIDE CHAP 7 TỜ 7)
a. Find the best fit of distributions
b. Estimate parameters
c. Comment on your results and conclusions

Arrivals per Period	Frequency
0	12
1	10
2	19
3	17
4	10
5	8
6	7
7	5
8	5
9	3
10	3
11	1

Answer:
a.
A frequency distribution and histogram is useful in identifying the shape of a distribution.
From the data table, ue can analyze the below histogram. The data table is the result of vehicles arriving test, it provides a test about the number of vehicles arriving at northwest corner of an intersection in a 5-minute period between 7:00 AM and 7:05 AM was monitored for five workdays over a 20-week period.
The table illustrates that the first entry indicates that there were 12 times of 5-minutes periods during, which no vehicle arrived. Next, one vehicle arrived the corner in 10 periods during. Additionally, when the number of vehicles arriving were directly inversely to the frequency.
This example is discrete event,' thus the number of vehicles are a discrete variable, and there are ample data. The result histogram is shown below.



Based on the basic physical of each distribution, we can select the distribution of Example 9.5. It is Possion Distribution. Possion Distribution describes the number of independent events that occur in a fixed amount of time or space. Comparing on our example, it is the number vehicles arriving in 5 minutes.

b. Parameter Estimation:

After we choose the family of distribution, the next step that need to be done is estimate the parameter of the distribution.

The data are discrete and have been grouped in a frequency distribution:

$$\bar{X} = \frac{\sum_{j=1}^{n} f_j X_j}{n} \qquad S^2 = \frac{\sum_{j=1}^{n} f_j X_j^2 - n\bar{X}^2}{n-1}$$

where f_j is the observed frequency of value X_j

From the table, we can define:

frequency	12	10	19	17	10	8	7	5	5	3	3	1
X_j	0	1	2	3	4	5	6	7	8	9	10	11

The sample mean:

$$\bar{X} = \frac{\sum_{j=1}^{n} f_i x_i}{n} = \frac{364}{100} = 3.64$$

The sample variance:

$$S^2 = \frac{\sum_{j=1}^{n} f_i {x_i}^2 - n\bar{x}^2}{n-1} = \frac{2080 - 100 \times 3.64^2}{100-1} = 7.6266$$

The result shows that the sample mean is not equal to sample variance. However, according to Possion Distribution which is selected in Selecting a Family Distribution, the sample mean and the sample variance must equal. This may occur because each estimator is a random variable, is not perfect.

c.

Now we conduct hypothesis testing on input data to check assumption in two previous parts: Are those Possion Distribution.

We will use Chi-square Test:

$H_0: The\ random\ variables\ are\ Possion\ Distribution$

$H_1: The\ random\ variables\ are\ not\ Possion\ Distribution$

x	Observation Frequency	p(x)	Expected Frequency	$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$
0	12	0.026	2.6	7.87
1	10	0.096	9.6	
2	19	0.174	17.4	0.15
3	17	0.211	21.1	0.8

4	10	0.192	19.2	4.41
5	8	0.14	14	2.57
6	7	0.085	8.5	0.26
7	5	0.044	4.4	
8	5	0.02	2	
9	3	0.008	0.8	11.63
10	3	0.003	0.3	
11	1	0.001	0.1	

The pmf for Possion distribution:

$$p(x) = \begin{cases} \dfrac{e^{-\alpha}\alpha^{x}}{x!}, & x = 0, 1, 2, \ldots \\ 0, & \text{otherwise} \end{cases}$$

The value of E_1 is given by $np_0 = 100 \times 0.026 = 2.6$ with $\alpha = 3.64$

In a similar manner, the remaining E_i values are computed.

Since $E_1 = 2.6 < 5$, E_1 and E_2 are combined.

The last five class intervals are also combined, for the same reason, and k is further reduced by four.

The calculated χ_0^2 is 27.68.

The degrees of freedom for the tabulated value of χ_0^2 is k - s - 1 = 7 - 1 - 1 = 5. Here, s = 1 since one parameter that was estimated from the data.

At the 0.05 level of significance, the critical value $\chi_{0.05,5}^2$ is 11.1. Thus, H_0 would be rejected at level of significance 0.05

$\chi_0^2 = 27.68 > \chi_{0.05,5}^2 = 11.1$

Therefore, we might search for a better-fitting model or use the empirical distribution of the data.

In addition, we also calculate p-value for this test statistics. The p-value is the significance level a which one just rejects H_0 for the given data. The p-value is 0.00004. Its meaning that we rejected the data are Possion at the 0.00004 significant level instead of 0.05.

<u>Conclusion:</u>

- Possion Distribution is a poor fit.
- Empirical Distribution can be another suitable distribution because it resamples from the actual data collected; often used when no theoretical distribution seems appropriate.

Problem 3: ***(Chi-square for discrete)***
Records pertaining to the number of defects per hour for the past 50 hours were as follows:

0	1	1	2	0	1	2	2	0	1
1	1	1	0	1	2	1	0	1	3
1	1	0	1	2	0	2	2	1	1
0	4	4	1	1	1	1	1	3	2
0	3	5	0	1	1	0	0	1	1

a. Which goodness-of-fit test should you apply to test frequency of this data? Explain

b. Apply Chi-square test to these data to test the hypothesis that the distribution is Poisson with mean 1. Use level of significance 0.05

Answer:

a.

- *Kolmogorov-Smimov test*: The Kolmogorov-Smimov test is particularly useful when sample sizes are small and when no parameters have been estimated from the data.
- *Chi-square test*: Procedure for testing the hypothesis that a random sample of size n of the random variable X follows a specific distributional form is the chi-square goodness-of-fit test. This test formalizes the intuitive idea of comparing the histogram of the data to the shape of the candidate density or mass function. The test is valid for large sample sizes and for both discrete and continuous distributional assumptions when parameters are estimated by maximum likelihood.
- It is known that the Kolmogorov-Smirnov test can be applied to small sample sizes, whereas the chi-square is valid only for large samples, say N ≥50.
  → In this case, the number of input is 50 so it should be tested by ***Chi-square test***

b.

H0: the random variable is Poisson distributed.
H1: the random variable is not Poisson distributed.

	x_j	$frequency$	$x_j \times fj$
	0	12	0
	1	24	24
	2	8	16
	3	3	9
	4	2	8
	5	1	5
Total	15	50	62

The formula for mean frequency distribution is: $\boldsymbol{\lambda} = \boldsymbol{\mu} = \frac{\sum_{j=1}^{n} x_j \times fj}{\boldsymbol{n}} = \frac{62}{50}$ =1.24

The pmf for the Poisson distribution was given in Equation:

$$\boldsymbol{P(X = x)} = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\boldsymbol{P(X = 0)} = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1.24}1.24^0}{0!} = 0.289384$$

$$\boldsymbol{P(X = 1)} = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1.24}1.24^1}{1!} = 0.358836$$

$$\boldsymbol{P(X = 2)} = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1.24}1.24^2}{2!} = 0.222479$$

$$\boldsymbol{P(X = 3)} = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1.24}1.24^3}{3!} = 0.091958$$

$$\boldsymbol{P(X = 4)} = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1.24}1.24^4}{4!} = 0.028507$$

$$\boldsymbol{P(X = 5)} = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-1.24}1.24^5}{5!} = 0.00707$$

	x	Observed Frequency	Expected frequency	$\frac{(O_i - E_i)^2}{E_i}$
	0	12	14.4692	0.421374274
	1	24	17.9418	2.045602294
	2	8	11.12395	0.877302002
	3	3	4.5979	1.492261244
	4	2	1.42535	
	5	1	0.4418	
Total	15	50	50	4.836539814

Since E(3), E(4), E(5) < 5 , so we combine it into 1 class

The calculated x_0^2 =4.84.

The degrees of freedom for the tabulated value of x^2 is $k - s - 1 = 4 - 1 - 1 = 2.$

At the 0.05 level of significance, the critical value $x_{0.05,4}^2$ is 5.99.

Thus, H0 would not be rejected at level of significance 0.05.

So, the random variable is Poisson distributed.

<u>Problem 4</u>: Re-calculate Confidence Interval with Specified Precision.

<u>Answer</u>:

Recall that *R₀=10 replications* and complete synchronization of random number yield the 95% confidence interval for the difference in expected response time. The interval is rewritten as 0.4 $\pm 0.9\, minutes$. Suppose that a difference larger than ± 0.5 is considered to be practically significant. We therefore want to make enough replications to obtain a $H \leq \in = 0.5$

Therefore a confidence interval was $\bar{D} \pm t_{\alpha/2, Ro-1}\frac{S_D}{\sqrt{R_0}}$ with $\bar{D} = 0.4$, $R_0 = 10$, $t_{0.025,9} = 2.26$ and $S_D^2 = 1.7$. To obtain the desired precision, we find R such that: $R \geq \left(\frac{t_{\alpha/2, R_0-1} S_D}{\epsilon}\right)^2$

Substituting $t_{0.025,9} = 2.26$ and $S_D^2 = 1.7$, we obtain: $R \geq \frac{2.26^2 \times 1.7}{0.5^2}$ =34.73

➔ Implying that 35 replications are needed, 25 more than in the initial experiment

We have: $t_{\frac{\alpha}{2}, R-1} = t_{0.025,34} = 2.032$

The confidence interval with specific precision of $\pm 0.5\ mins$ is given as follows:

$$\bar{D} \pm t_{\frac{\alpha}{2}, R-1} \times \frac{S_D}{\sqrt{R}} = 0.4 \pm 2.032 \times \frac{\sqrt{1.7}}{\sqrt{35}} = 0.4 \pm 0.448\ mins$$

Thus, the new confidence interval is $[-0.048; 0.848]\ minutes$, which is narrower than the original confidence interval of $0.4 \pm 0.9\ mins = [-0.5; 1.3]\ mins$.

In conclusion: The new confidence interval to be more precise in predicting the response time difference than the first one.

Problem 5: (Minimization Problem) Consider K = 4 different design:

Suppose that we would like 0.95% confidence of selecting the best (smallest expected response time) system design when the best differs from the second best by at least 2 minutes. (*Note: refer to Example 12.4, text book* ***Discrete Event Simulation 4th e***)

Answer:

Recall the example 12.4, there are 4 alternative system designs

- existing system (parallel stations)
- no space between stations in series
- one space between brake and headlight inspection only
- one space between headlight and steering inspection only

In this example, we make *comparison* of each performance measure, θi to a control $\theta 1$, (where $\theta 1$ could represent the mean performance of *an existing system*)

For example, we make a comparison between system 1 and system 2

63.72	32.24	40.28	36.94	36.29	56.94	34.1	63.36	49.29	87.2	Mean	STD
63.06	31.78	40.32	37.71	36.79	57.93	33.39	62.92	47.67	80.79		
0.66	0.46	-0.04	-0.77	-0.5	-0.99	0.71	0.44	1.62	6.41	0.8	2.120849

$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i$ =0.8

$S_D^2 = \frac{1}{n-1}\sum_{i=1}^{n}(D - \bar{D}_i)^2$ = 2.120849^2 = 4.498

$S_D = 2.120849$

$$Standard\ error = \frac{S_D}{\sqrt{10}} = \frac{2.120849}{\sqrt{10}} = 0.671$$

And we continue to do similarly to get the result for another system with existing system 1 and the result is reflected based on below table.
Since the overall error probability is aE = 0.05 and C = 3 confidence intervals are to be constructed, let a_i = 0.05/3 = 0.0167 for I = 2,3,4.

Replication,	Average Response Time for System Design				Observed Difference with System Design 1		
	1,	2,	3,	4,			
r	Y_{r1}	Y_{r2}	Y_{r3}	Y_{r4}	D_{r2}	D_{r3}	D_{r4}
1	63.72	63.06	57.74	62.63	0.66	5.98	1.09
2	32.24	31.78	29.65	31.56	0.46	2.59	0.68
3	40.28	40.32	36.52	39.87	−0.04	3.76	0.41
4	36.94	37.71	35.71	37.35	−0.77	1.23	−0.41
5	36.29	36.79	33.81	36.65	−0.50	2.48	−0.36
6	56.94	57.93	51.54	57.15	−0.99	5.40	−0.21
7	34.10	33.39	31.39	33.30	0.71	2.71	0.80
8	63.36	62.92	57.24	62.21	0.44	6.12	1.15
9	49.29	47.67	42.63	47.46	1.62	6.66	1.83
10	87.20	80.79	67.27	79.60	6.41	19.93	7.60
Sample mean, $\bar{D}_i$					0.80	5.686	1.258
Sample standard deviation, S_{D_i}					2.12	5.338	2.340
Sample variance, $S^2_{D_i}$					4.498	28.498	5.489
Standard error, $S_{D_i}/\sqrt{R}$					0.671	1.688	0.741

This is a minimization problem. so we focus on the differences θ_i - min θ_j $(i \neq j)$ for i = 1, 2, 3, 4. Then we can apply the Two- Stage Bonferroni Procedure as follows:

1. $\in = 2\ , 1 - \alpha = 0.95, R_0 = 10, t = t\alpha_{/_{K-1}, R_0 - 1} = t = t_{0.0167,9} = 2.508$
2. The data in Table above, which was obtained by using CRN, is employed.
3. From the table above, we get $s_{12}^2 = 4.498, s_{13}^2 = 28.498, s_{14}^2 = 5.489$. With similar calculation, we get $s_{23}^2 = 11.875, s_{24}^2 = 0.119, s_{34}^2 = 9.849$
4. We see that $s^2 = s_{13}^2 = 28.498$ is the largest sample variance
   $R = max\left\{10, [\frac{2.508^2 \times 28.498}{2^2}]\right\}$ = max{10,[44.8]}=45
5. Make 45 - 10 = 35 additional replications of each system.
6. Calculate the overall sample means: $\bar{\bar{Y_i}} = \frac{1}{45}\sum_{r=l}^{45} Y_n\ for\ i = 1,2,3,4$
7. Select the system with smallest $\bar{\bar{Y_i}}$ as the best.
   Also, form the confidence intervals:
   min $\{0, \bar{\bar{Y_i}} - min_{j\neq i}\ \bar{\bar{Y_j}} - 2\}$ ≤ θ_i - $min_{j\neq i}\ \theta_j$ ≤ max $\{0, \bar{\bar{Y_i}} - min_{j\neq i}\bar{\bar{Y_j}} + 2\}$; for i = 1,2,3,4.

Problem 6: (ABSOLUTE PERFORMANCE – BÀI CÓ LQUAN SLIDE THẦY PHÚC)

Average waiting times of 2 systems from simulation using CRN technique are shown in the table below.

	Replication 1	Replication 2	Replication 3	Replication 4	Replication 5	Replication 6	Replication 7
System 1	8.2	11.6	10.1	7.7	13.6	10.8	9.5
System 2	6.6	9.5	7.5	5.9	11.4	7	8.1

a. Use hypothesis testing ($\alpha = 0.05$)to test the difference of average waiting time of the 2 systems
b. Conclude the difference of average waiting time of the 2 systems using 95% C.I.
c. Suppose that we want to validate the simulation model of system 1 and the actual waiting time of the "real system" is 11 minutes. Use hypothesis testing with Alpha =0.05 to draw conclusions.
d. If the difference between the simulation result and the actual value is expected to be at least 3 minutes to be considered practically different, what is the power of the test in question a.

Answer:

a.

The first assumption

Assume that to have the average time for each system, we just need one source of input so we use pair observation

In order to test the difference of average waiting time of the 2 systems by using CRN technique, we have hypothesis below:

$$\begin{cases} H0: u_D = 0 \\ H1: u_D \neq 0 \end{cases}$$

Dof = $n_{replication}$ - 1 = 7 - 1 = 6

$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} D_i = 2.214286$

$S_D^2 = \frac{1}{n-1}\sum_{i=1}^{s}(D - \bar{D_\iota})^2 = 0.805044^2$

Test statistics $t_t = \frac{\bar{D}-D}{S_D/\sqrt{n}} = \frac{2.214286-0}{0.805044/\sqrt{6}} = 6.73736$

$t\alpha_{/2,n-1} = t_{0.025,9} = 2.447$

Reject Ho if:

$t_t > t\alpha_{/2,s-1}$ or t_t<-$t\alpha_{/2,n-1}$

We see that: 6.73736 > 2.447 $\Rightarrow$ reject Ho

We can conclude that the model output is not consistent with system behavior.

The second assumption

The second assumption is if we don't use the same source of input. So in this case, we use independent sampling test with equal variance assumption

	Replication 1	Replication 2	Replication 3	Replication 4	Replication 5	Replication 6	Replication 7
System 1	8.2	11.6	10.1	7.7	13.6	10.8	9.5
System 2	6.6	9.5	7.5	5.9	11.4	7	8.1
System 1 minus System 2	1.6	2.1	2.6	1.8	2.2	3.8	1.4

And the hypothesis is:

$$\begin{cases} H0: \mu1 - \mu2 = 0 \\ H1: \mu1 - \mu2 \neq 0 \end{cases}$$

Systems	Sample mean	Sample variance
1	10.21	3.52
2	8.00	3.06

The pooled estimate of the population variance:

$${s_p}^2 = \frac{(7)\times(3.52)+(7)\times(3.06)}{14} = 3.29$$

$$t = \frac{(10.21-8)}{\sqrt{3.29\times\left(2\times\frac{1}{7}\right)}} = 2.279$$

Dof = $n_1 + n_2 - 2 = 7 + 7 - 2 = 12$

$t\alpha_{/2,12}$ = 2.179

Reject Ho if:

$t_t > t\alpha_{/2,s-1}$ or $t_t <$ -$t\alpha_{/2,n-1}$

We see that 2.279 > 2.179 $\Rightarrow$ reject H_0

We can conclude that the model output is not consistent with system behavior

b. (*use first assumption, pair observation test)*

The sample variance was 0.805044^2 (with v = 9 degrees of freedom), and the standard error was s.e.(D) = 0.805044. A 95% c.i. for the true mean difference is given by

$\overline{D} \pm t\alpha_{/2,n-1}\frac{s}{\sqrt{n}}$ = $2.214286 \pm 2.447 \times \frac{0.805044}{\sqrt{6}}$ = [1.41006; 3.018512]

Denote

$\theta1$ *be mean of the system* 1

$\theta2$ *be mean of the system* 2

We see that $1.41006 \leq \theta1 - \theta2 \leq 3.018512$

Since the c.i. for $\theta1 - \theta2$ is totally to the right of zero then there is strong evidence that $\theta1 - \theta2$ > 0, or equivalently $\theta1 > \theta2$.

In conclusion: *the first system design is better than the other.*

c.

In order to validate the simulation model of system 1 and the actual waiting time of the "real system" is 11 minutes, so we have a hypothesis testing like this:

$H0: u = 11$
$H1: u \neq 11$

Dof = 7 – 1 = 6

Test statistics $t_t = \frac{\bar{x}-u_0}{s/\sqrt{n}} = \frac{10.21429-11}{2.026021/\sqrt{6}}$ = -1.22636

$t\alpha_{/2,n-1} = t_{0.025,6} = 2.447$

	Rep. 1	Rep. 2	Rep. 3	Rep. 4	Rep. 5	Rep. 6	Rep. 7	Mean	STD
System 1	8.2	11.6	10.1	7.7	13.6	10.8	9.5	10.21429	2.026021

$\bar{y}\ system\ 1 = \frac{1}{n}\sum_{i=1}^{n} Yi = 10.21429$

$S\ system\ 1 = \frac{1}{n-1}\sum_{i=1}^{s}(Yi - \bar{y})^2 = 2.026021^2$

Reject Ho if:

$t_t > t\alpha_{/2,s-1}$ or $t_t < -t\alpha_{/2,n-1}$

$We\ see\ -1.22636 > -2.447 \Rightarrow$ not reject Ho

<u>In conclusion</u> *:Thus, we can conclude that there is enough evident to prove design system 1 be consistent with real model 11 minutes*

d. (*use standard deviation of the first assumption)*

We have the formular for true difference is: $\delta = \frac{|E(Y)-\mu|}{\sigma}$

$\Rightarrow \boldsymbol{\delta} = \frac{|3|}{0.805044} = 3.726504 \approx 3.8$

Since 3.8 is not included in Table A.10 and operating Characteristics Curves for The Two-sided t test. In this case we use z- distribution to calculate for the power. And we based on, the formular here: $n = \frac{\left(z_{\frac{\alpha}{2}}+z_b\right)^2}{(ES)^2}$

$n = \frac{(z_{\frac{\alpha}{2}}+z_b)^2}{\boldsymbol{\delta}} \Rightarrow 10 = \frac{(z_{0.025}+z_b)^2}{\mathbf{3.8}} \Rightarrow z_b = \sqrt{10 \times 3.8 - 1.96} = 6 \sim \beta = 0.999999$

<u>In conclusion:</u> the power of the test in question is 1 - 0.999999 = 0.000001

Problem 7:

Review and place order based on Inventory POSITION

The Halfwidth is 77.52, which is larger than the required precion of 5 units

User Specified

Output

Output	Average	Half Width	Minimum Average	Maximum Average
Mean Monthly Total Cost	303.55	77.52	208.16	512.11

H<=5 thì mới satisfied

Nếu không có Halfwwidth thì collect data từ 10 replication và tính toán như dưới đây để ra H như trong slide

Calculate Standard deviation from the 95% C.I. Halfwidth

R	10	R là number of replication
alpha	0.05	
t(0.025,9)	2.26	
H	77.52	chạy ra từ arena
S	108.46892	Tính S = H*sqrt(R)/t

$$H = t\left(\frac{\alpha}{2}, R-1\right)\frac{S}{\sqrt{R}}$$

Calculate 90% C.I. Halfwidth

R	10	
alpha	0.1	
t(0.05,9)	1.83	
H	62.770619	< 95%C.I HW

Calculate number of replications to have the 90% C.I. Halfwidth wthin 5 units

exeron	5
H0	62.770619
R0	10
R	1576.0603

$$\frac{R_1}{R_0} = \left(\frac{H_0}{H_1}\right)^2$$

Calculate number of replications to have the 95% C.I. Halfwidth wthin 5 units

exeron	5
H0	77.52
R0	10
R	2403.7402

Replication	
1	567.9
2	507.78
3	355.75
4	291.31
5	279.44
6	372.06
7	512.11
8	326.45
9	451.78
10	259.49
Mean	392.407
Std	109.90896 tầm trung trung phần này
t critical	-2.262157
	2.2621572
H	78.624136

Conclusion: 90% confidence interval for mean monthly total cost

392.407	+-	78.624136
313.78286	**471.03114**	