Logistics Engineer and Supply Chain Design Problem

Chapter 1 : Capacity Planning

6 Step in capacity planning :

- Step 1. Estimate future capacity requirements.
- Step 2. Find the the capacity available in present facilities.
- Step 3. Identify gaps between capacity needed and available.
- Step 4. Develop alternative plans for overcoming any gaps.
- Step 5. Evaluate the capacity plans and find the best.
- Step 6. Implement the best and monitor performance

- Demand forecast is converted to a number that can be compared directly with the capacity measure being used.

Number of machines required for one product/service	Number of machines required for multiple products/services → setup time
$M = \frac{D * p}{N\left[1 - \left(\frac{C}{100}\right)\right]}$	$M = \frac{\left[D * p + \left(\frac{D}{Q}\right)s\right]_{product\ 1} + \cdots + \left[D * p + \left(\frac{D}{Q}\right)s\right]_{product\ n}}{N\left[1 - \left(\frac{C}{100}\right)\right]}$

D= number of units (customers) forecast per year
p = processing time (hrs/unit or hrs/customer)
N= total number of hours per year of the operating process
Q = number of units in each lot
s = setup time (hrs) for each lot
C= desired capacity cushion - the amount of reserve capacity that a firm maintains to handle sudden increases in demand.

Problem 1

- A vehicle repair department built up a repair facility designed to fix up to 50 trucks per day.
- However, the repair facility can produce only 40 trucks per day in reality, despite working at its maximum effort.

→ What is the efficiency and the utilization of the vehicle repair department if the actual output is 36 trucks per day?

$$\text{Effeciency} = \frac{\text{Actual output}}{\text{Effective capacity}} * 100\% = \frac{36}{40} * 100\% = 90\%$$

$$\text{Utilization} = \frac{\text{Actual output}}{\text{Design capacity}} * 100\% = \frac{36}{50} * 100\% = 72\%$$

→ The utilization rates show that the **current output is lower than both its design capacity and effective capacity** → The vehicle repair department is not using capacity to its fullest extent.

Problem 2 :

A&B Coaches of Blackpool plan their capacity in terms of 'coach-days'.

• Forecasts show expected annual demands for the next five years to average 400,000 full-day passengers and 750,000 half-day passengers.

• A&B have 61 coaches, each with an effective capacity of 40 passengers a day for 300 days a year. Breakdowns and other unexpected problems reduce efficiency to 90%.

• They employ 86 drivers who work an average of 220 days a year, but illness and other absences reduce their efficiency to 85%. If there is a shortage of coaches the company can buy extra ones for $110,000 or hire them for $100 a day. If there is a shortage of drivers they can recruit extra ones at a cost of $20,000 a year, or hire them from an agency for $110 a day.

→ How can the company plan for the required demand?

Step 1. Estimate future capacity requirements

- 400,000 full-day passengers: 400,000/40 = 10,000 coach days a year, or 10,000/300 = 33.33 coaches.
- 750,000 half-day passengers: 750,000 / (40 × 300 × 2) = 31.25 coaches.
- The total demand: 33.33 + 31.25 = 64.58 coaches.
- Each coach needs 300/220 drivers → company needs atotal of 88.06 drivers.

Step 2. Find the capacity available

- There are 61 coaches with an efficiency of 90%: 61 ×0.9 = 54.9 coaches.
- There are 86 drivers with an efficiency of 85%: 86 × 0.85 = 73.1 drivers

Step 3. Identify gaps between capacity needed and available

There is a total shortage of: 64.58 – 54.9 = 9.68 coaches and 88.06 – 73.1 = 14.96 drivers.

Step 4. Develop alternative plans for overcoming any gaps

To buy 10 coaches would cost \$1,100,000. To hire coaches to make up the shortage would cost $9.68 \times 300 \times 100 = \$290,400$/year. There is the alternative of buying some coaches and hiring others.

To hire 15 drivers would cost \$300,000/year, while using temporary drivers from an agency would cost $14.96 \times 220 \times 110 = \$362,032$/year. There is the option of hiring some drivers and using the agency for shortage

Step 5. Evaluate the capacity plans and find the best

We do not have enough information to make the final decisions, and we have only outlined some of the alternatives. This very limited analysis might suggest a reasonable solution of buying 8 coaches and making up any shortages by hiring, and hiring 12 drivers and making up the shortage from the ageny.

Problem 3 :

A copy center in an office building prepares bound reports for two clients. The center makes multiple copies (the lot size) of each report. The processing time to run, collate, and bind each copy depends on, among other factors, the number of pages.

• The center operates 250 days per year, with one eight-hour shift. Management believes that a capacity cushion of 15% is best. It currently has three copy machines. Based on the following table of information, determine how many machines are needed at the copy center

Item	X	Y
Annual demand forecast (copies)	2000	6000
Standard processing time (hrs/copy)	0.5`	0.7
Average lot size (copies/report)	20	30
Standard setup time (hrs)	0.25	0.4

$$M = \frac{\left[Dp + \left(\frac{D}{Q}\right)s\right]_{item\ 1} + \left[Dp + \left(\frac{D}{Q}\right)s\right]_{item\ 2}}{N[1-(C/100)]}$$

$$= \frac{\left[2000(0.5)+(\frac{2000}{20})(0.25)\right]+\left[6000(0.7)+(\frac{6000}{30})(0.4)\right]}{250\times1\times8(1-\frac{15}{100})}$$

$$= 3.12$$

→ The number of machines needed at the copy center is 4

Problem 4 :

Company A has a supply chain system for its powdered milk products as follows:

- The processing plant has a capacity of 4,000 kg/day for 7 days/week. It fills standard boxes of 850g.
- The boxes are passed to two packing areas, each of which can form up to 220 cases/day with 15 boxes/case. The packing areas works a 7-day week.
- The cases are then taken to a warehouse by a transport company whose 5 trucks can each carry 70 cases and make up to 2 trips/day for 5 days/week.
- The warehouse can handle up to 3,000 cases/week.
- The delivery vans can handle only half of the warehouse's capacity passed to them.

→ What is the capacity of each member of the supply chain? What is the overall capacity of the current supply chain system? (Unit: boxes/week)

- The processing plant has a capacity = $7\times4{,}000/0.85= 32{,}941$ boxes/week.
- The packing area has a capacity = $2x7\times15\times220 = 46{,}200$ boxes/week.
- The transport company has a capacity = $5\times2\times5\times70\times15 = 52{,}500$ boxes/week.
- Warehouses have a capacity = $3{,}000\times15 = 45{,}000$ boxes/week.
- The capacity of the delivery vans is half of that of the warehouses (22,500 boxes/week).

→ The overall capacity of the current supply chain system is capacity of the delivery vans which is the smallest among other parts' capacities in the supply chain.

Problem 5 :

The owner of Old-Fashioned Berry Pies, S. Simon, is contemplating adding a new line of pies, which will require leasing new equipment for a monthly payment of $6,000. Variable costs would be $2 per pie, and pies would retail for $7 each.

a. How many pies must be sold in order to break even?

b. What would the profit (loss) be if 1,000 pies are made and sold in a month?

c. How many pies must be sold to realize a profit of $4,000?

d. If 2,000 can be sold, and a profit target is $5,000, what price should be charged per pie?

FC = \$6,000, VC = \$2 per pie, R = \$7 per pie

a. $Q_{BEP} = \dfrac{FC}{R - VC} = \dfrac{\$6{,}000}{\$7 - \$2} = 1{,}200$ pies/month

b. For $Q = 1{,}000$, $P = Q(R - v) - FC = 1{,}000\ (\$7 - \$2) - \$6{,}000 = -\$1{,}000$

c. $P = \$4{,}000$; solve for Q using Formula 5–8:

$$Q = \frac{\$4{,}000 + \$6{,}000}{\$7 - \$2} = 2{,}000 \text{ pies}$$

d. $\text{Profit} = Q(R - v) - FC$

$$\$5{,}000 = 2{,}000(R - \$2) - \$6{,}000$$

$$R = \$7.50$$

Problem 7 :

Customers arrive at a bakery at an average rate of 18 per hour on weekday mornings. The arrival distribution can be described by a Poisson distribution with a mean of 18. Each clerk can serve a customer in an average of three minutes. This time can be described by an exponential distribution with a mean of 3.0 minutes.

a. What are the arrival and service rates?

b. Compute the average number of customers being served at any time.

c. Suppose it has been determined that the average number of customers waiting in line is 8.1. Compute the average number of customers in the system (i.e., waiting in line or being served), the average time customers wait in line and the average time in the system.

d. Determine the system utilization for M = 1, 2, and 3 servers.

a. The arrival rate is given in the problem: $\lambda = 18$ customers per hour. Change the service *time* to a comparable hourly rate. Thus,

60 minutes per hour / 3 minutes per customer = $\mu = 20$ customers per hour

b. $r = \frac{\lambda}{\mu} = \frac{18}{20} = .90$ customer

c. Given: $L_q = 8.1$ customers

$L_s = L_q + r = 8.1 + .90 = 9.0$ customers

$W_q = \frac{L_q}{\lambda} = \frac{8.1}{18} = .45$ hour

W_s = Waiting in line plus service

$= W_q + \frac{1}{\mu} = .45 + \frac{1}{20} = .50$ hour

d. System utilization is $\rho = \frac{\lambda}{M\mu}$.

For $M = 1, \rho = \frac{18}{1(20)} = .90$

For $M = 2, \rho = \frac{18}{2(20)} = .45$

For $M = 3, \rho = \frac{18}{3(20)} = .30$

Problem 8 :

An airline is planning to open a satellite ticket desk in a new shopping plaza, staffed by one ticket agent. It is estimated that requests for tickets and information will average 15 per hour, and requests will have a Poisson distribution. Service time is assumed to be exponentially distributed. Previous experience with similar satellite operations suggests that mean service time should average about three minutes per request. Determine each of the following:

a. System utilization

b. Percentage of time the server (agent) will be idle

c. The expected number of customers waiting to be served

d. The average time customers will spend in the system

e. The probability of zero customers in the system and the probability of four customers in the system.

$$\lambda = 15 \text{ customers per hour}$$

$$\mu = \frac{1}{\text{Service time}} = \frac{1 \text{ customer}}{3 \text{ minutes}} \times 60 \text{ minutes per hour}$$
$$= 20 \text{ customers per hour}$$

a. $\rho = \frac{\lambda}{M\mu} = \frac{15}{1(20)} = .75$

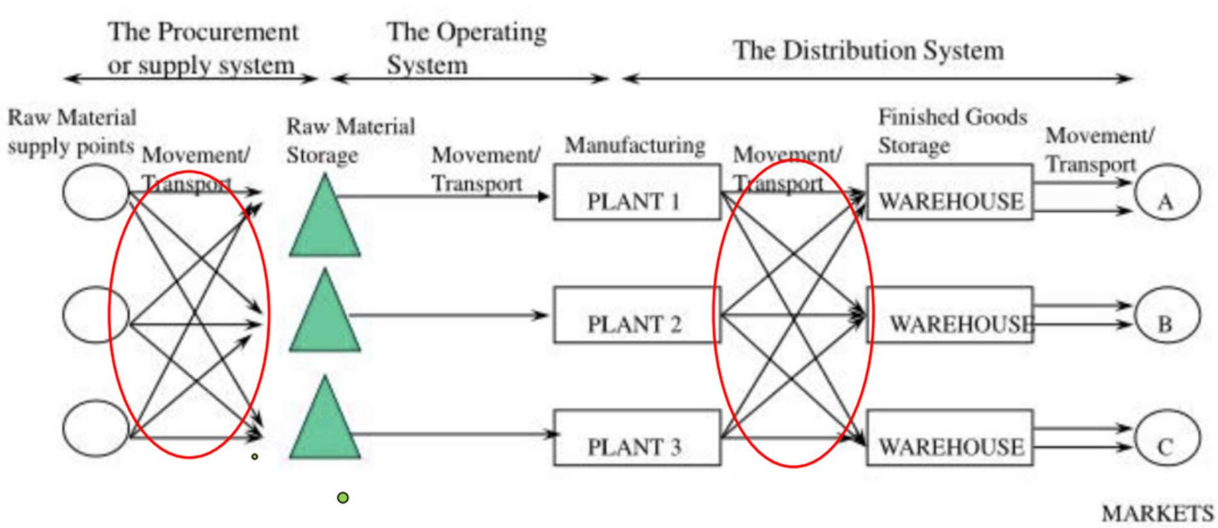
b. Percentage idle time $= 1 - \rho = 1 - .75 = .25$, or 25 percent

c. $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{15^2}{20(20 - 15)} = 2.25$ customers

d. $W_s = \frac{L_q}{\lambda} + \frac{1}{\mu} = \frac{2.25}{15} + \frac{1}{20} = .20$ hour, or 12 minutes

e. $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{15}{20} = .25$ and $P_4 = P_0\left(\frac{\lambda}{\mu}\right)^4 = .25\left(\frac{15}{20}\right)^4 = .079$

Chapter 2 : Aggregation in Supply Chain



Problem 1 :

Best Buy sells three models of computers, the Litepro, the Medpro, and the Heavypro. Annual demands for the three products are DL = 12,000 units for the Litepro, DM = 1,200 units for the Medpro, and DH = 120 units for the Heavypro.

- Each model costs Best Buy \$500. A fixed transportation cost of \$4,000 is incurred each time an order is delivered. For each model ordered and delivered on the same truck, an additional fixed cost of \$1,000 per model is incurred for receiving and storage. Best Buy incurs a holding cost of 20%.
- Evaluate the lot sizes that the Best Buy manager should order if lots for each product are ordered and delivered independently. Also evaluate the annual cost of such a policy.\

Demand, DL = 12,000/year, DM = 1,200/year, DH = 120/year

- Fixed Ordering cost: a separate truck delivers each model
- Common order cost, S = \$4,000
- Product-specific order cost, sL = \$1,000, sM = \$1,000, sH = \$1,000
- Holding cost, h = 0.2
- Unit cost, CL = \$500, CM = \$500, CH = \$500

Case 1 : Each product is ordered independently of the other

Calculate by EOQ and order individually, similar to previous questions.

	Litepro	Medpro	Heavypro
Demand per year	12,000	1,200	120
Fixed cost/order	\$5,000	\$5,000	\$5,000
Optimal order size	1,095	346	110
Cycle inventory	548	173	55
Annual holding cost	\$54,772	\$17,321	\$5,477
Order frequency	11.0/year	3.5/year	1.1/year
Annual ordering cost	\$54,772	\$17,321	\$5,477
Average flow time	2.4 weeks	7.5 weeks	23.7 weeks
Annual cost	\$109,544	\$34,642	\$10,954

Note: Although these figures are correct, some may differ from calculations due to rounding.

Case 2 : Lots are ordered and delivered jointly for all products

- All products are ordered and delivered on the same truck each time an order is placed.
- The combined fixed order cost per order:

$$S^* = S + \sum_i s_i$$

- Let n be the number of orders placed per year

→ Total annual cost: $TC = \sum_i (\frac{D_i h C_i}{2n}) + nS^*$

- The optimal order frequency: $n^* = \sqrt{\frac{\sum_i D_i h C_i}{2S^*}}$
- If the capacity of each truck is considered, then:

$$n^* = \frac{D_i}{\textit{Capacity for each product}}$$

- The combined order cost from four suppliers is given by

$$S^* = S + s_1 + s_2 + s_3 + s_4 = \$900 \text{ per order}$$

- The optimal order frequency is

$$n^* = \sqrt{\frac{\sum_{i=1}^{4} D_i h C_i}{2S^*}} = \sqrt{\frac{4 \times 10{,}000 \times 0.2 \times 50}{2 \times 900}} = 14.91$$

- The quantity ordered from each supplier is

$$Q = 10{,}000/14.91 = 671 \text{ units per order}$$

- If the capacity of truck is 2,500 units, then the capacity for each supplier/product is 2,500/4 = 625 < Q
- The optimal order frequency is n* = 10,000/625 = 16

→ Thus, the limited truck capacity results in an optimal order frequency of 16 orders per year instead of 14.91 orders per year

Case 3 : Lots are ordered and delivered jointly for selected subset of the products

Step 1: Identify the frequency of the most frequently ordered product i^*, assuming each product is ordered independently

$$\bar{n} = \bar{n}_{i*} = max\left\{\bar{n}_i = \sqrt{\frac{hC_iD_i}{2(S+s_i)}}\right\}$$

→ The most frequently ordered product is i^* included each time an order is placed.

Step 2: For all products $i \neq i^*$, evaluate the ordering frequency:

$$\bar{\bar{n}}_i = \sqrt{\frac{hC_iD_i}{2s_i}}$$

Step 3: For all $i \neq i^*$, evaluate the frequency of product i relative to the most frequently ordered product i^* to be:

$$m_i = \lceil \bar{n}/\bar{\bar{n}}_i \rceil$$

Step 4: Having decided the ordering frequency of each product i, recalculate the ordering frequency of the most frequently ordered product i^* to be:

$$n = \sqrt{\frac{\sum_i hC_im_iD_i}{2(S+\sum_i s_i/m_i)}}$$

Step 5: Evaluate an order frequency of $n_i = n/m_i$ and the total cost of such an ordering policy

$$TC = nS + \sum_i n_is_i + \sum_i \left(\frac{D_i}{2n_i}\right)hC_i$$

- Demand, DL = 12,000/year, DM = 1,200/year, DH = 120/year
- Fixed Ordering cost: a separate truck delivers each model
  - Common order cost, S = \$4,000
  - Product-specific order cost, sL = \$1,000, sM = \$1,000, sH = \$1,000
- Holding cost, h = 0.2
- Unit cost, CL = \$500, CM = \$500, CH = \$500

Step 1: We obtain

$$\bar{n}_L = \sqrt{\frac{hC_LD_L}{2(S+s_L)}} = 11.0,\ \bar{n}_M = \sqrt{\frac{hC_MD_M}{2(S+s_M)}} = 3.5,\ \bar{n}_H = \sqrt{\frac{hC_HD_H}{2(S+s_H)}} = 1.1$$

Clearly, Litepro is the most frequently ordered model. Thus, we set $\bar{n} = 11.0$.

Step 2: We first obtain:

$$\bar{\bar{n}}_M = \sqrt{\frac{hC_MD_M}{2s_M}} = 7.7 \quad \text{and} \quad \bar{\bar{n}}_H = \sqrt{\frac{hC_HD_H}{2s_H}} = 2.4$$

Step 3: We have:

$$m_M = \left\lceil \frac{\bar{n}}{\bar{\bar{n}}_M} \right\rceil = \left\lceil \frac{11.0}{7.7} \right\rceil = 2 \text{ and } m_H = \left\lceil \frac{\bar{n}}{\bar{\bar{n}}_H} \right\rceil = \left\lceil \frac{11.0}{2.4} \right\rceil = 5$$

Step 4: Recalculate the ordering frequency of the most frequently ordered model as:

$$n = \sqrt{\frac{hC_Lm_LD_L + hC_Mm_MD_M + hC_Hm_MD_H}{2\left(S + s_L/m_L + s_{M/m_M + s_H/m_H}\right)}} = 11.47$$

Step 5: Ordering frequency for each product will be:

$$n_L = 11.47/\text{year},\ n_M = 11.47/2 = 5.74/\text{year, and } n_H = 11.47/5 = 2.29/\text{year}$$

- The annual holding cost of this policy is \$65,383.50.
- The annual order cost is

$$nS + n_L s_L + n_M s_M + n_H s_H = \$65{,}383.50$$

- The total annual cost is thus equal to \$130,767 → **cost reduction of \$5,761 (4%) compared with the joint ordering of all models.** The cost reduction results because each model-specific fixed cost of \$1,000 is not incurred with every order.

	Litepro	Medpro	Heavypro
Demand per year (*D*)	12,000	1,200	120
Order frequency (*n*)	11.47/year	5.74/year	2.29/year
Order size (*D*/*n*)	1,046	209	52
Cycle inventory	523	104.5	26
Annual holding cost	\$52,307	\$10,461	\$2,615
Average flow time	2.27 weeks	4.53 weeks	11.35 weeks

Problem 2 : Impact of Supply Uncertainty on Safety Inventory

- ❖ If demand during the lead time > ROP

→ **Stock-out**

→ Thus, we need to identify the distribution of customer demand during the uncertain lead time.

- ❖ Assume demand during the lead time is normally distributed with:

$$D_L = D * L$$

$$\sigma_L = \sqrt{L\sigma_D^2 + D^2 s_L^2}$$

- ❖ With a given desired **CSL**, we have:

$$ss = NORMSINV(CSL) * \sigma_L$$

- Daily demand for tablets at Amazon is normally distributed, with a mean of 2,500 and a standard deviation of 500. The tablet supplier takes an average of *L* = 7 days to replenish inventory at Amazon.
- Amazon is targeting a CSL of 90 percent (providing a fill rate close to 100 percent) for its tablet inventory. Evaluate the safety inventory of tablets that Amazon must carry if the standard deviation of the lead time is seven days. Amazon is working with the supplier to reduce the standard deviation to zero.

→ Evaluate the reduction in safety inventory that Amazon can expect as a result of this initiative.

❖ We have:

- Average demand per period, D = 2,500
- Standard deviation of demand per period, σ_D = 500
- Average lead time for replenishment, L = 7 days
- Standard deviation of lead time, s_L = 7 days

❖ First, evaluating the distribution of demand during the lead time

- Mean demand during lead time:

$$D_L = D \times L = 2,500 \times 7 = 17,500$$

- Standard deviation of demand during lead time

$$\begin{aligned} \sigma_L &= \sqrt{L\sigma_D^2 + D^2 s_L^2} \\ &= \sqrt{7 \times 500^2 + 2,500^2 \times 7^2} = 17,550 \end{aligned}$$

❖ The required safety inventory is

$ss = NORMSINV(CSL) \times \sigma_L = NORMSINV(0.90) \times 17{,}550 = 22{,}491$ tablets

TABLE [redacted] Required Safety Inventory as a Function of Lead Time Uncertainty

s_L	σ_L	*ss* (units)	*ss* (days)
6	15,058	19,298	7.72
5	12,570	16,109	6.44
4	10,087	12,927	5.17
3	7,616	9,760	3.90
2	5,172	6,628	2.65
1	2,828	3,625	1.45
0	1,323	1,695	0.68

❖ The reduction in lead time uncertainty allows Amazon to reduce its safety inventory of tablets by a significant amount.

❖ As the standard deviation of lead time declines from 7 days to 0 and the amount of safety inventory also declines.

→ A reduction in supply uncertainty can help to **reduce the required safety inventory without hurting product availability**.

Problem 3 : Aggregation on Safety Inventory

Consider k regions, with demand in each region normally

distributed with the inputs:

Di : mean periodic demand in region i, $i = 1, \ldots, k$

σi : standard deviation of periodic demand in region i, $i = 1, \ldots, k$

ρij: correlation of periodic demand in region i, j, $1 \leq i \neq j \leq k$

L: replenishment lead time

CSL: desired cycle service level

There are two ways to serve demand in the k regions.

• Case 1: Have local inventories in each region.

• Case 2: Aggregate all inventories into one centralized facility.

→ Our goal is to compare safety inventories in the two cases.

Total safety inventory in decentralized option:

$$ss = \sum_{i=1}^{k} F_S^{-1}(CSL) \text{ x } \sigma_i \text{ x } \sqrt{L}$$

Case 1 : Inventory Centralization

	Aggregate demand is **normally distributed**	All regions demand is **identically distributed**	All regions demand is **independent and identically distributed**
Mean **D^C**	$D^C = \sum_{i=1}^{k} D_i$	$D^C = kD$	$D^C = kD$
Standard deviation **σ_D^C**	$\sigma_D^C = \sqrt{\sum_{i=1}^{k} \sigma_i^2 + 2\sum_{i>j} \rho_{ij}\sigma_i\sigma_j}$	$\sigma_D^C = \sqrt{k\,\sigma_D^2 + k(k-1)\,\sigma_D^2\rho}$	$\sigma_D^C = \sqrt{k}\sigma_D$ $(\rho = 0)$

❖ Total **safety inventory** on aggregation:

$$ss = F_S^{-1}(CSL) \text{ x } \sigma_D^C \text{ x } \sqrt{L}$$

❖ Holding cost **savings on aggregation** per unit sold:

$$\frac{F_S^{-1}(CSL)H\sqrt{L}}{D^C} \times \left(\sum_{i=1}^{k} \sigma_i - \sigma_D^C\right)$$

❖ Conclusions regarding the value of aggregation:

- The safety inventory savings on aggregation increase with the desired ***cycle service level CSL***.
- The safety inventory savings on aggregation increase with the **replenishment *lead time L***.
- The safety inventory savings on aggregation increase with the ***holding cost H***.
- The safety inventory savings on aggregation increase with the ***coefficient of variation (σ_D/D) of demand***.
- The safety inventory savings on aggregation decrease as the ***correlation coefficients ρ_{ij}*** increase.

Problem 4 :

A BMW dealership has $k = 4$ retail outlets serving the entire Chicago area (disaggregate option). Weekly demand at each outlet is normally distributed, with a mean of $D = 25$ cars and a standard deviation of □D = 5. The lead time for replenishment from the manufacturer is $L = 2$ weeks. Each outlet covers a separate geographic area, and the correlation of demand across any pair of areas is ρ . The dealership is considering the possibility of replacing the four outlets with a single large outlet (aggregate option). Assume that the demand in the central outlet is the sum of the demand across all four areas. The dealership is targeting a CSL of 0.90.

→ Compare the level of safety inventory needed in the two options as the correlation coefficient ρ varies between 0 and 1.

We have:

- Average demand per period, D = 25 cars
- Standard deviation of weekly demand, σ_D = 5
- Lead time for replenishment, L = 2 weeks
- $k = 4$, $CSL = 0.9$

First, we analyse for the case when **demand in each area is independent (i.e., $\rho = 0$)**

The required safety inventory in ***decentralized option***:

$$ss = k \times F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D = 4 \times NORMSINV(0.9) \times \sqrt{2} \times 5$$
$$= 36.25\ cars$$

The required safety inventory in ***centralized option***:

$$ss = F_S^{-1}(CSL) \times \sqrt{L} \times {\sigma_D}^C$$
$$= NORMSINV(0.9) \times \sqrt{2} \times 10 = 18.12\ cars$$

with ${\sigma_D}^C = \sqrt{k} \text{ x } \sigma_D = \sqrt{4} \text{ x } 5 = 10$

TABLE [redacted] Safety Inventory in the Disaggregate and Aggregate Options

ρ	Disaggregate Safety Inventory	Aggregate Safety Inventory
0	36.25	18.12
0.2	36.25	22.93
0.4	36.25	26.88
0.6	36.25	30.33
0.8	36.25	33.42
1.0	36.25	36.25

Most of cases, the **safety inventory for the disaggregate option is higher than for the aggregate option,** except when all demands are perfectly positively correlated.

The benefit of aggregation decreases as **demand in different areas is more positively correlated**.

Problem 5 : - Trade-Offs of Physical Centralization

An online retailer is debating whether to serve the United States through four regional distribution centers or one national distribution center. Weekly demand in each region is normally distributed, with a mean of 1,000 and a standard deviation of 300. Demand experienced in each region is independent, and supply lead time is four weeks. The online retailer has a holding cost of 20 percent and the cost of each product is $1,000. The retailer promises its customers next-day delivery. With four regional distribution centers, the retailer can provide next-day delivery using ground transportation at a cost of $10/unit. With a single national distribution center, the retailer will have to use a more expensive mode of transport that will cost $13/unit for next-day service. Building and operating four regional DCs costs $150,000 per year more than building and operating one national distribution center.

→ What distribution network do you recommend? Assume a desired CSL of 0.95

Each region, we have:

- Average demand per period, D = 1,000pcs/week
- Standard deviation of weekly demand, σ_D = 300
- Lead time for replenishment, L = 4 weeks
- $k = 4$, $CSL = 0.95$

The required safety inventory in ***across all four regional distribution centers***:

$$ss = k \times F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D$$
$$= 4 \times NORMSINV(0.95) \times \sqrt{4} \times 300 = 3.948$$

Because demand in all four areas is independent, $\boldsymbol{\rho = 0}$. The required safety inventory in centralized option:

$$ss = F_S^{-1}(CSL) \times \sqrt{L} \times {\sigma_D}^C$$
$$= NORMSINV(0.95) \times \sqrt{4} \times 300 = 1.974$$

with ${\sigma_D}^C = \sqrt{k} \text{ x } \sigma_D = \sqrt{4} \text{ x } 300 = 10$

We can now evaluate the effects of the changes in inventory, transportation, and facility costs on aggregation as follows:

- Decrease in annual inventory holding cost on aggregation

= (3,948 - 1,974) x $1,000 x 0.2 = $394,765

- Decrease in annual facility costs on aggregation = $150,000
- Increase in annual transportation costs on aggregation

= 4 x 52 x 1,000 x (13 - 10) = $624,000

- The annual costs for the online retailer increase by

= $624,000 - $394,765 - $150,000 = $79,235

→ The online retailer is better off with four regional distribution centers.

Non-physical Aggregations:

- Information Centralization
- Specialization
- Product Substitution
- Component Commonality
- Postponement

Specialization of inventory based on Product Type: The higher the coefficient of variation of an item, the greater is the reduction in safety inventories as result of centralization

Items with low demand (slow-moving items) with high coefficient of variation → stocked at a centralized location → reduce the safety inventory without hurting customer response time or adding to transportation costs.

• Items with high demand (fast-moving items) with low coefficient of variation → stocked at decentralized locations close to the customer.

Problem 6 : - Impact of Coefficient of Variation on Aggregation

Assume that W.W. Grainger, a supplier of MRO products, has 1,600 stores distributed throughout the United States. Consider two products - large electric motors and industrial cleaner. Large electric motors are high-value items with low demand, whereas the industrial cleaner is a low-value item with high demand. Each motor costs $500 and each can of cleaner costs $30. Weekly demand for motors at each store is normally distributed, with a mean of 20 and a standard deviation of 40. Weekly demand for cleaner at each store is normally distributed, with a mean of1,000 and a standard deviation of 100. Demand experienced by each store is independent, and supply lead time for both motors and cleaner is four weeks. W.W. Grainger has a holding cost of 25 percent. → For each of the two products, evaluate the reduction in safety inventories that will result if they are removed from retail stores and carried only in a centralized DC. Assume a desired CSL of 0.95.

	Motors	Cleaner
Inventory is stocked in each store		
Mean weekly demand per store	20	1,000
Standard deviation	40	100
Coefficient of variation	2.0	0.1
Safety inventory per store	132	329
Total safety inventory	211,200	526,400
Value of safety inventory	$105,600,000	$15,792,000
Inventory is aggregated at the DC		
Mean weekly aggregate demand	32,000	1,600,000
Standard deviation of aggregate demand	1,600	4,000
Coefficient of variation	0.05	0.0025
Aggregate safety inventory	5,264	13,159
Value of safety inventory	$2,632,000	$394,770
Savings		
Total inventory saving on aggregation	$102,968,000	$15,397,230
Total holding cost saving on aggregation	$25,742,000	$3,849,308
Holding cost saving per unit sold	$15.47	$0.046
Savings as a percentage of product cost	3.09%	0.15%

Strategy 03 : Product Substitution

Substitution → the use of one product to satisfy demand for a different product. Substitution may occur in two situations:

Manufacturer-driven one-way substitution: The manufacturer or supplier makes the decision to substitute.

Typically, the manufacturer substitutes a higher-value product for a lower-value product that is not in inventory.

→ Increases profitability for the manufacturer by allowing aggregation of demand → reduces the inventory requirements for the same level of availability.

• Customer-driven two-way substitution: Customers make the decision to substitute.

→ Recognizes of customer-driven substitution and joint management of inventories across substitutable products → allows to reduce the required inventory while ensuring high level of product availability.

Strategy 04 : Component Commonality

Given the large number of components in each finished product → demand uncertainty will be high → resulting in high levels of safety inventory.

❖ When products with common components are designed → the demand for each component is an aggregation of the demand for all the finished products using the component.

❖ Component demand is thus more predictable → reduces the component inventories carried in the supply chain.

❖ With increasing product variety, component commonality is a key to reducing supply chain inventories required without hurting product availability → a key factor for success in the electronics industry.

Problem 07 : Value of Component Commonality

Assume that Dell is to manufacture 27 servers with three distinct components: processor, memory, and hard drive. Under the disaggregate option, Dell designs specific components for each server, resulting in 3 * 27 = 81 distinct components. Under the common-component option, Dell designs servers such that three distinct processors, three distinct memory units, and

three distinct hard drives can be combined to create 27 servers. Each component is thus used in nine servers. Monthly demand for each of the 27 servers is independent and normally distributed, with a mean of 5,000 and a standard deviation of 3,000. The replenishment lead time for each component is one month. Dell is targeting a CSL of 95 percent for component inventory.

→ Evaluate the safety inventory requirements with and without the use of component commonality. Also evaluate the change in safety inventory requirements as the number of finished products of which a component is a part varies from one to nine.

We first evaluate the disaggregate option (Case 1), in which components are specific to a server. For each component, we have:

Standard deviation of monthly demand = 3,000

Given a lead time of one month and a total of 81 components across 27 servers, we obtain:

Total safety inventory required = 81 * *NORMSINV*(0.95)* $\sqrt{1}$ * 3,000 = 399,699 units

In the case of component commonality (Case 2), each component ends up in 9 finished products. Therefore, the demand at the component level is the sum of demand across nine products. The safety inventory required for each component is thus

Safety inventory per common component = *NORMSINV*(0.95)*$\sqrt{1}$*$\sqrt{9}$*3,000~ 14,804 units

With component commonality, there are a total of nine distinct components. The total safety inventory across all nine components is thus

Total safety inventory required = 9 * 14,803.68 = 133,233

Thus, having each component common to 9 products results in a reduction in safety inventory for Dell from 399,699 to 133,233 units.

Number of Finished Products per Component	Safety Inventory	Marginal Reduction in Safety Inventory	Total Reduction in Safety Inventory
1	399,699		
2	282,630	117,069	117,069
3	230,766	51,864	168,933
4	199,849	30,917	199,850
5	178,751	21,098	220,948
6	163,176	15,575	236,523
7	151,072	12,104	248,627
8	141,315	9,757	258,384
9	133,233	8,082	266,466

Strategy 05 : Postponement

Postponement is the ability of a supply chain to delay product differentiation or customization until closer to the time the product is sold.

Ex: the final mixing of paint is done at the retail store after the customer has selected the color he or she wants → Thus, paint variety is produced only when demand is known with certainty. Postponement allows paint retailers to carry significantly low safety inventories.

❖ Without component commonality postponement, product differentiation occurs → most of the supply chain inventories are disaggregate.

❖ Postponement allows to delay product differentiation → most of the inventories in the supply chain are aggregate.

❖ Postponement thus allows a supply chain to exploit aggregation to reduce safety inventories without hurting product availability.

Problem 8 : Value of Postponement

Consider a paint retailer that sells 100 different colors of paint.Assume that weekly demand for each color is independent and is normally distributed with a mean of 30 and a standard deviation of 10. The replenishment lead time from the paint factory is two weeks and the retailer aims for a CSL = 0.95.

→ How much safety stock will the retailer have to hold if paint is mixed at the factory and held in inventory at the retailer as individual colors?

→ How does the safety stock requirement change if the retailer holds base paint (supplied by the paint factory) and mixes colors on demand?

We first evaluate the disaggregate option without postponement, in which the retailer holds safety inventory for each color sold. For each color, we have:

$$D = 30\text{/week}, \sigma_D = 10, L = 2 \text{ weeks}$$

Given the desired CSL = 0.95, the required safety inventory across all 100 colors is obtained using Decentralized equation (Case 1) to be

$$ss = F_S^{-1}(CSL)\sqrt{L}\sigma_D = 100 \text{ x } NORMSINV(0.95) \text{ x } \sqrt{2} \text{ x } 10 = 2{,}326$$

Now, consider the option whereby mixing is postponed until after the customer orders (Case 2). Safety inventory is held in the form of base paint, whose demand is an aggregate of demand of the 100 colors. Because demand in all 100 colors is independent, $\rho = 0$. The standard deviation of aggregate weekly demand of base paint is

$$\sigma_D{}^C = \sqrt{k} \text{ x } \sigma_D = \sqrt{100} \text{ x } 10 = 100$$

For a CSL of 0.95, safety inventory required for the aggregate option:

$$ss = F_S^{-1}(CSL)\sqrt{L}\ \sigma_D{}^C = NORMSINV(0.95) \text{ x } \sqrt{2} \text{ x } 100 = 233$$

Observe that postponement reduces the required safety inventory at the paint retailer from 2,326 units to 233 units.

Chapter 3 : Transportation Design

Problem 1 : Selecting a Transportation Network

A retail chain has eight stores in a region supplied from four supply sources. Trucks have a capacity of 40,000 units and cost \$1,000 per load plus \$100 per delivery. Thus, a truck making two deliveries charges \$1,200. The cost of holding one unit in inventory at retail for a year is \$0.20. The vice president of supply chain is considering whether to use direct shipping from suppliers to retail stores or setting up milk runs from suppliers to retail stores.

→ What network do you recommend if annual sales for each product at each retail store are 960,000 units? What network do you recommend if sales for each **product at each retail store are 120,000 units?**

We have:

• Eight stores, four supply sources

• Truck capacity = 40,000 units

• Cost $1,000 per load and $100 per delivery

• Holding cost = $0.20/year

• Two options:

o Direct shipping from suppliers to retail stores.

o Milk runs from suppliers to retail stores.

→ Case 1: If annual sales = 960,000 units and suppliers run milk runs to two stores on each truck. What network do you recommend?

→ Case 2: If annual sales = 120,000 units and suppliers run milk runs to four stores on each truck. What network do you recommend?

When annual sales of each product at each retail store are 960,000 units

	Direct shipping	**Milk runs**
Batch size shipped from each supplier to each store	40,000 units	40,000/2 = 20,000 units
Number of shipments/year from each supplier to each store	960,000/40,000 = 24	960,000/20,000 = 48
Transportation cost per shipment	1,000 + 100 =$1,100 (one store/truck)	1,000/2 + 100 =$600 (two stores/truck)
Annual trucking cost	24 x 1,100 x 4 x 8 = $844,800	48 x 600 x 4 x 8 = $921,600
Average inventory at each store for each product	40,000/2 = 20,000 units	20,000/2 = 10,000 units
Annual holding inventory cost	20,000 x 0.2 x 4 x 8 = $128,000	10,000 x 0.2 x 4 x 8 = $64,000
Total annual cost	$844,800 + $128,000 = $972,800	$921,600 + $64,000 = $985,600

When annual sales of each product at each retail store are 120,000 units

	Direct shipping	Milk runs
Batch size shipped from each supplier to each store	40,000 units	40,000/4 = 10,000 units
Number of shipments/year from each supplier to each store	120,000/40,000 = 3	120,000/10,000 = 12
Transportation cost per shipment	1,000 + 100 =$1,100 (one store/truck)	1,000/4 + 100 =$350 (four stores/truck)
Annual trucking cost	3 x 1,100 x 4 x 8 = $105,600	12 x 350 x 4 x 8 = $134,400
Average inventory at each store for each product	40,000/2 = 20,000 units	10,000/2 = 5,000 units
Annual holding inventory cost	20,000 x 0.2 x 4 x 8 = $128,000	5,000 x 0.2 x 4 x 8 = $32,000
Total annual cost	$105,600 + $128,000 = $233,600	$134,400 + $32,000 = $166,400

Problem 2 : Trade-Offs when Selecting Transportation Mode

Eastern Electric (EE) is a major appliance manufacturer with a large plant in the Chicago area. EE purchases all the motors for its appliances from Westvie Motors near Dallas. EE currently purchases 120,000 motors each year from Westview at $120/motor. Demand has been relatively constant for several years and is expected to stay that way. Each motor averages about 10 pounds in weight, and EE has traditionally purchased lots of 3,000 motors. Westview ships each EE order within a day of receiving it (lead time is one day more than transit time). Transit time using truck is three days and transit time for rail is five days. EE carries a safety inventory equal to 50% of the average demand for motors during the replenishment lead time. EE's annual holding inventory cost is 25%.

❖ The plant manager at EE has received several proposals for transportation and must decide on the one to accept. The details of various proposals are provided in the Table (next slide), where 1 cwt = 100 pounds.

❖ Golden's pricing represents a marginal unit quantity discount. Golden's representative has proposed lowering the marginal rate for the quantity over 250 cwt in a shipment from $4/cwt to $3/cwt and suggested that EE increase its batch size to 4,000 motors to take advantage of the lower transportation cost.What should the plant manager do ?

We have:

- D = 120,000 motors, P = $120/motor, 1 motor = 10 pounds
- W = 10 lbs/motor, h = 25%
- Safety stock = 50% demand during RLT. 1cwt = 100pounds
- Transit time: 5 days by rail and 3 days by truck.
- Leadtime = 1 day more than transit time.

Carrier	Range of Quantity Shipped (cwt)	Shipping Cost ($/cwt)
AM Railroad	200+	6.50
Northeast Trucking	100+	7.50
Golden Freightways	50 – 150	8.00
Golden Freightways	151 – 250	6.00
Golden Freightways	251+	4.00

- Golden Freightways suggests new proposal to
  - Increase the batch size to 4,000 motors from the lot of 3,000 motors.
  - Lower shipping cost from $4/cwt to $3/cwt for the quantity over 250 cwt.

Alternative 1: Using AM Rail, we have

- Shipping cost = $6.5/cwt = $0.65/motor (1 motor = 10 pounds = 0.1 cwt)

→ **Annual transportation cost = D* 0.65 = 120,000*0.65 = $78,000**

- Lot size Q = 2,000 motors, Replenishment lead time L = 5+1=6 days.
- Holding cost = h*P= 25% * 120 = $30/motor
- Total inventory = Cycle inventory + Safety inventory + In-transit inventory

$$= \frac{Q}{2} + (\frac{D}{365} * 6) * 50\% + (\frac{D}{365} * 5)$$

$$= \frac{2,000}{2} + (\frac{120,000}{365} * 6) * 50\% + (\frac{120,000}{365} * 5)$$

= 3,630 motors

→ **Annual holding inventory cost = 3,630*30 = $108,900**

Total annual cost for inventory and transportation using AM Rail

= $78,000 + $108,900 = $186,900

Alternative 2: Using Northeast Trucking, we have

- Shipping cost = $7.5/cwt = $0.75/motor (1 motor = 10 pounds = 0.1 cwt)

→ **Annual transportation cost = D* 0.75 = 120,000*0.75 = $90,000**

- Lot size Q = 1,000 motors, Replenishment lead time L = 3+1= 4 days.
- Holding cost = h*P= 25% * 120 = $30/motor
- Total inventory = Cycle inventory + Safety inventory + In-transit inventory

$$= \frac{Q}{2} + (\frac{D}{365} * 4) * 50\% + (\frac{D}{365} * 3)$$

$$= \frac{1,000}{2} + (\frac{120,000}{365} * 4) * 50\% + (\frac{120,000}{365} * 3)$$

= 2,144 motors

→ **Annual holding inventory cost = 2,144*30 = $64,315**

Total annual cost for inventory and transportation using Northeast Trucking

= $90,000 + $64,315 = $154,315

Alternative 3: Using Golden Freightways (Q=500), we have

- Shipping cost = $8.0/cwt = $0.80/motor (1 motor = 10 pounds = 0.1 cwt)

→ **Annual transportation cost = D* 0.80 = 120,000*0.80= $96,000**

- Lot size Q = 500 motors, Replenishment lead time L = 3+1= 4 days.
- Holding cost = h*P= 25% * 120 = $30/motor
- Total inventory = Cycle inventory + Safety inventory + In-transit inventory

$$= \frac{Q}{2} + (\frac{D}{365} * 4) * 50\% + (\frac{D}{365} * 3)$$

$$= \frac{500}{2} + (\frac{120,000}{365} * 4) * 50\% + (\frac{120,000}{365} * 3)$$

= 1,894 motors

→ **Annual holding inventory cost = 1,894*30 = $56,815**

Total annual cost for inventory and transportation using Golden Freightways

= $96,000 + $56,815 = $152,815

Alternative 7: Using Golden Freightways (Q=4,000), we have

- Shipping cost:
  - ≤ 1,500 motors: $0.80/motor;
  - 2,500 motors: $0.60/motor;
  - > 2,500 motors: $0.30/motor.
- The number of shipments: D/Q = 120,000/4,000 = 30 shipments
- Transportation cost per shipment:

  1,500*$0.80 + (2,500-1,500)*$0.60 + (4,000-2,500)*$0.30 = $2,250

→ **Annual transportation cost = 30* $2,250 = $67,500**

- Lot size Q = 4,000 motors, Replenishment lead time L = 3+1= 4 days.
- Holding cost = h*P= 25% * 120 = $30/motor
- Total inventory = Cycle inventory + Safety inventory + In-transit inventory

$$= \frac{4,000}{2} + (\frac{120,000}{365} * 4) * 50\% + (\frac{120,000}{365} * 3)$$

= 3,644 motors

→ **Annual holding inventory cost = 3,644*30 = $109,315**

Total annual cost for inventory and transportation using Golden Freightways

= $67,500 + $109,315= $176,815

Alternative	Lot Size (Motors)	Transportation Cost	Cycle Inventory	Safety Inventory	In-Transit Inventory	Inventory Cost	Total Cost
1. AM Rail	2,000	\$78,000	1,000	986	1,644	\$108,900	\$186,900
2. Northeast Trucking	1,000	\$90,000	500	658	986	\$64,320	\$154,320
3. Golden	500	\$96,000	250	658	986	\$56,820	\$152,820
4. Golden	1,500	\$96,000	750	658	986	\$71,820	\$167,820
5. Golden	2,500	\$86,400	1,250	658	986	\$86,820	\$173,220
6. Golden	3,000	\$80,000	1,500	658	986	\$94,320	\$174,320
7. Golden (new proposal)	4,000	\$67,500	2,000	658	986	\$109,320	\$176,820

→ The plant manager should decide to sign a contract with Golden Freightways and order motors in lots of 500. This option has the highest transportation cost, but the lowest overall cost.

Problem 3 : Trade-Offs when Aggregating Inventory

- HighMed, a manufacturer of medical equipment, is located in Madison, Wisconsin. HighMed currently divides the United States into **twenty-four territories**, each with its own sales force. All product inventories are maintained locally in each territory and replenished from Madison **every four weeks** using UPS. The average replenishment lead time using UPS is **one week**. UPS charges at a rate of **\$0.66 + 0.26x**, where **x is the quantity shipped in lbs**. The products sold fall into two categories - **HighVal and LowVal**.
- HighVal products weigh 0.1 lbs and cost \$200 each. Weekly demand for HighVal products in each territory is normally distributed, with a mean of $\mu_H = 2$ and a standard deviation of $\sigma_H = 5$. LowVal products weigh 0.04 pounds and cost \$30 each. Weekly demand for LowVal products in each territory is normally distributed, with $\mu_L = 20$ and $\sigma_L = 5$.
- HighMed maintains sufficient safety inventories in each territory to provide a **CSL of 0.997** for each product. Annual holding cost at HighMed is **25%**. In addition to the current approach, the management team at HighMed is considering two other options:

Option A: Keep the current structure but **replenish inventory once a week** rather than once every four weeks.

Option B: Eliminate inventories in the territories, **aggregate all inventories in a finished-goods warehouse at Madison**, and replenish the warehouse once a week.

- If inventories are aggregated at Madison, orders will be shipped using FedEx, which charges **\$5.53 + 0.53x per shipment**, where **x is the quantity shipped in lbs**. The factory requires a one-week lead time to replenish finished-goods inventories at the Madison warehouse. An average customer order is for 1 unit of HighVal and 10 units of LowVal. **What should HighMed do?**

We have:

- 24 regions
- Highval – weekly demand μ_H = 2, σ_H = 5, weight = 0.1 lbs, cost = $200
- Lowval – weekly demand μ_L = 20, σ_L = 5, weight = 0.04 lbs, cost = $30
- *CSL* = 0.997, h = 25%, *L* = 1 week, *T* = 4 weeks (reorder interval)
- UPS lead time = 1 week, $0.66 + 0.26x*;* x is quantity shipped
- FedEx lead time = overnight, $5.53 + 0.53x

→ Calculate the total cost (Inventory cost + Transportation cost) in three scenarios and choose the best:

- ***Current scenario:*** Disaggregate with T = 4 weeks.
- ***Option A:*** Keep the current structure disaggregate with T = 1 weeks.
- ***Option B:*** *A*ggregate all inventories in a finished-goods warehouse with *T* = 1 weeks.

Current scenario: HighMed Inventory costs

	HighVal	**LowVal**
Average lot size = expected demand during T weeks	$Q_H = T\mu_H = 4 * 2 = 8$ units	$Q_L = T\mu_L = 4 * 20 = 80$ units
Cycle inventory	$\frac{Q_H}{2} = 4$ units	$\frac{Q_L}{2} = 40$ units
Safety inventory	$ss_H = F_S^{-1}(CSL) \times \sqrt{T+L} \times \sigma_H = NORMSINV(0.997) \times \sqrt{4+1} \times 5 = 30.7$ units	$ss_L = F_S^{-1}(CSL) \times \sqrt{T+L} \times \sigma_L = NORMSINV(0.997) \times \sqrt{4+1} \times 5 = 30.7$ units
Average inventory across 24 regions for each product category	24 x (4 + 30.7) = 832.8 units (833.3 units without rounding)	24 x (40 + 30.7) = 1,696.8 units (1,697.3 units without rounding)
Annual holding inventory cost for HighMed	(Average HighVal inventory x $200 + Average LowVal inventory x $30) x h = (832.8 x $200 + 1,696.8 x $30) x $0.25 = **$54,366 ($54,395 without rounding)**	

Current scenario: HighMed Transportation costs

	HighVal	LowVal
Average lot size = Average replenishment order quantity	$Q_H = T\mu_H = 4 * 2 = 8$ units	$Q_L = T\mu_L = 4 * 20 = 80$ units
Average weight of each replenishment order	$8 * 0.1 = 0.8$ lbs	$80 * 0.04 = 3.2$ lbs
Shipping cost per replenishment order by UPS	$0.66 + 0.26 * (0.8 + 3.2) = $1.70	
Number of replenishment order per year	52/ 4 = 13 orders	
Annual transportation cost for HighMed acrsoss 24 regions	$1.70 * 13 * 24 = **$530**	

Current scenario: Total annual inventory and transportation costs

= $54,366 + $530 = $54,896 ($54,926 without rounding)

TABLE [redacted] HighMed Costs Under Different Network Options

	Current Scenario	Option A	Option B
Number of stocking locations	24	24	1
Reorder interval	4 weeks	1 week	1 week
HighVal cycle inventory	96 units	24 units	24 units
HighVal safety inventory	737.3 units	466.3 units	95.2 units
HighVal inventory	833.3 units	490.3 units	119.2 units
LowVal cycle inventory	960 units	240 units	240 units
LowVal safety inventory	737.3 units	466.3 units	95.2 units
LowVal inventory	1,697.3 units	706.3 units	335.2 units
Annual inventory cost	$54,395	$29,813	$8,473
Shipment type	Replenishment	Replenishment	Customer order
Shipment size	8 HighVal + 80 LowVal	2 HighVal + 20 LowVal	1 HighVal + 10 LowVal
Shipment weight	4 lbs.	1 lb.	0.5 lb.
Annual transport cost	$530	$1,148	$14,464
Total annual cost	$54, 926	$30,961	$22,938

→ **Increasing the replenishment frequency** (Option A) **decreases total cost**.

→ HighMed can also reduce total cost by **aggregating all inventories and using FedEx for transportation**, because the decrease in inventories upon aggregation is larger than the increase in transportation cost.

Problem 4 : Trade-Offs between Transportation Cost and Responsiveness.

- Alloy Steel is a steel service center in the Cleveland area. All orders are shipped to customers using an LTL carrier that charges **$100 + 0.01x, where x is the number of steel shipped in lbs on the truck**. Currently, Alloy Steel ships orders on the day they are received. Allowing for two days in transit, this policy allows Alloy to achieve a response time of two days. Daily demand at Alloy Steel over a two-week period is shown in the below Table.

TABLE [redacted] Daily Demand at Alloy Steel over Two-Week Period

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Week 1	19,970	17,470	11,316	26,192	20,263	8,381	25,377
Week 2	39,171	2,158	20,633	23,370	24,100	19,603	18,442

- The general manager at Alloy Steel believes that customers do not really value the **two-day response time** and would be satisfied with a **four-day response time**. **What are the cost advantages of increasing the response time?**

TABLE [redacted] Quantity Shipped and Transportation Cost as a Function of Response Time

		Two-Day Response		Three-Day Response		Four-Day Response	
Day	Demand	Quantity Shipped	Cost ($)	Quantity Shipped	Cost ($)	Quantity Shipped	Cost ($)
1	19,970	19,970	299.70	0	—	0	—
2	17,470	17,470	274.70	37,440	474.40	0	—
3	11,316	11,316	213.16	0	—	48,756	587.56
4	26,192	26,192	361.92	37,508	475.08	0	—
5	20,263	20,263	302.63	0	—	0	—
6	8,381	8,381	183.81	28,644	386.44	54,836	648.36
7	25,377	25,377	353.77	0	—	0	—
8	39,171	39,171	491.71	64,548	745.48	0	—
9	2,158	2,158	121.58	0	—	66,706	767.06
10	20,633	20,633	306.33	22,791	327.91	0	—
11	23,370	23,370	333.70	0	—	0	—
12	24,100	24,100	341.00	47,470	574.70	68,103	781.03
13	19,603	19,603	296.03	0	—	0	—
14	18,442	18,442	284.42	38,045	480.45	38,045	480.45
			$4,164.46		$3,464.46		$3,264.46

❖ As the response time increases, Alloy Steel has the opportunity to aggregate demand over multiple days for shipping. For **a response time of three days**, Alloy Steel can *aggregate demand over two successive days before shipping*. For a **response time of four days**, Alloy Steel can *aggregate demand over three days before shipping*.

❖ The manager evaluates the quantity shipped and transportation costs for different response times over the two-week period. **The transportation cost for Alloy Steel decreases as the response time increases**.

- As the response time increases **from two to three days**, **transportation cost** over the two-week window **decreases by $700**.
- Increasing the response time **from three to four days** **reduces the transportation** cost by **only $200**.

❖ Thus, Alloy Steel may be better off offering a three-day response, because the marginal benefit from further increasing the response time is small.

Chapter 4 : Smart Pricing

Reconsider a retailer with the demand function:

$$D = 1,000 - 0.5p$$

Since $1,000 is the price that maximizes revenue → this seems like the *best pricing strategy*. However, according to the demand-price curve, the retailer charges only $1,000 from 500 customers who are willing to pay a higher price.

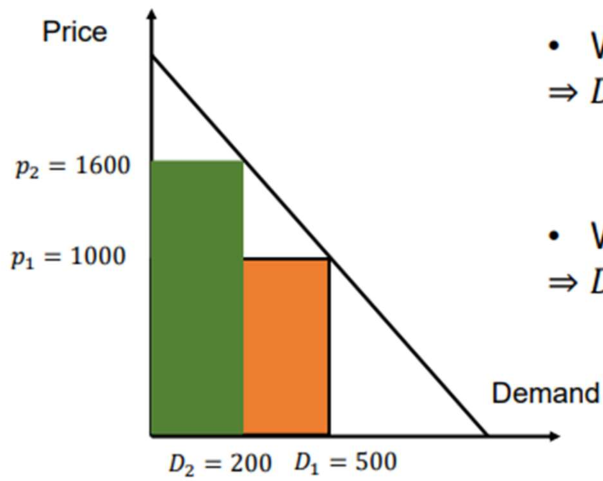
In fact, there are about 375 customers among the 500 who are willing to pay $1,250 per item. And 250 out of these 375 customers are willing to pay $1,500. But now all of these customers are charged the same price, $1,000.

P	Demand	Revenue $
$ 250	875	$218,750
500	750	375,000
750	625	468,750
1,000	500	500,000
1,250	375	468,750
1,500	250	375,000

→ The retailer should consider different pricing strategies, which tailor pricing to different market segments.

- Case 1: If the retailer employs the **two-tier pricing strategy**, in which *two prices $1,600 and $1,000* are introduced. What's his new revenue?

- Case 2: If the retailer employs the **three-tier pricing strategy**, in which *three prices $1,800; $1,600 and $1,000* are introduced. What's his new revenue?



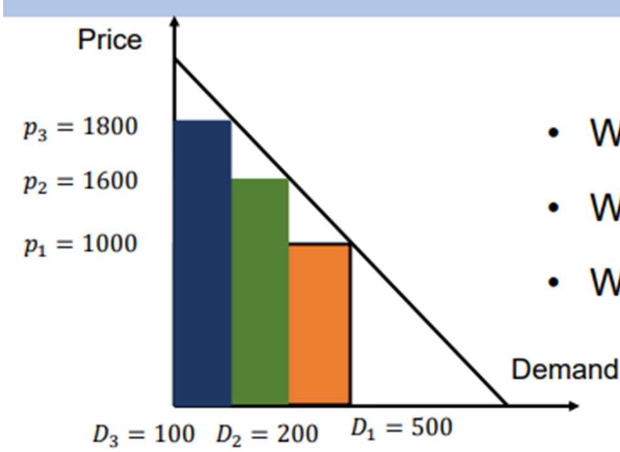
- When $p_1 = \$1{,}000$
$\Rightarrow D_1 = 1{,}000 - 0.5 * 1{,}000 = 500$

- When $p_2 = \$1{,}600$
$\Rightarrow D_2 = 1{,}000 - 0.5 * 1{,}600 = 200$

- **The revenue of company in Case 1:**
$$\boldsymbol{TR_1 = \$1,600 * 200 + \$1,000 * (500 - 200) = \$620,000}$$

→ Total revenue increases by \$120,000 (24%) in comparision with the single-tier pricing strategy (only one price $\$\mathbf{1,000}$)

Example 3 - Three-tier pricing strategy (4/4)



- When $p_1 = \$1{,}000 \Rightarrow D_1 = 500$
- When $p_2 = \$1{,}600 \Rightarrow D_2 = 200$
- When $p_3 = \$1{,}800 \Rightarrow D_3 = 100$

- **The revenue of company in Case 2:**
$$\boldsymbol{TR_2 = \$1,800 * 100 + \$1,600 * (200 - 100) + \$1,000 * (500 - 200)}$$
$$\boldsymbol{= \$640,000}$$

→ Total revenue increases by \$140,000 (28%) and by \$20,000 (3.22%) in comparision with the single-tier strategy and the two-tier strategy, respectively.

Problem 2 : Revenue Management

Notation	Meaning
Y	Demand
x^*	Optimal order quantity
c_u	Unit understocking cost
c_o	Unit overstocking cost

Critical Fractile - the ideal point for demand distribution to balance c_u and c_o - Overstocking probability

$$F(x^*) = \Pr(Y \leq x^*) = \frac{c_u}{c_u + c_o}$$

Understocking probability

$$1 - F(x^*) = \Pr(Y \geq x^*) = \frac{c_o}{c_u + c_o}$$

Let *D* be uncertain demand for high fare class.

- There is an **over-protecting cost**:
  - If **D < Q**, then you **protected too many seats** and earn nothing on **Q - D** seats.
  - We could have **sold those empty seats at the low fare (P_L)**, so $c_o = P_L$.
- There is an **under-protecting cost**:
  - If **D > Q** then you **protected too few seats.**
  - **D - Q** seats could have been **sold at the high fare (P_H)**, instead of the low fare, so $c_u = P_H - P_L$.

→ **Choose Q to balance the over-protecting and under-protecting costs.**

- **Choosing the optimal high fare protection level is a Newsvendor problem** with properly chosen underage and overage costs.
- *Optimal high fare protection level*:

$$F(Q^*) = \frac{c_u}{c_u + c_o} = \frac{P_H - P_L}{P_H} \rightarrow Q^*$$

- *Optimal low fare booking limit* = $C - Q^*$

Problem 3 :

❖ A hotel in NYC has capcacity of 118 King rooms

- Early bird discounted fare $P_L = \$150$ (low fare) targeting leisure travelers.
- Full fare $P_{\mathrm{H}} = \$225$ (high fare) targeting business travelers.

❖ We assume the demand for discounted low fare rooms is abundant, while the **demand *D* for full-fare rooms follows a normal distribution *N*(80, 20^2)**.

→ What are the unit overstocking cost and the unit understocking cost?

→ What are the probability of overstocking and the probability of understocking?

→ How many seats should be protected for full fares to minimize expected cost (maximize expected total revenue)?

> We have:
>
> - $P_L = \$150$; $P_{\mathrm{H}} = \$225$
> - Let Q be the protection level (number of full-fare rooms)
>
> → 118 – Q is the booking limit on the discounted-fare rooms
>
> - $\mathrm{D} \sim N(80, 20^2)$

What is the unit overstocking cost?

- If $\mathrm{D} < \mathrm{Q}$, the hotel protects too many full-fare rooms
- The number of empty room, $\mathrm{Q} - \mathrm{D}$, could have sold at a low fare P_L

→ $c_o = P_L$ = $150

What is the unit understocking cost?

- If $\mathrm{D} > \mathrm{Q}$, the hotel protects too few full-fare rooms
- The number of room, $\mathrm{D} - \mathrm{Q}$, could have been sold at the high fare P_{H}, instead of low fare P_L

→ $c_u = P_{\mathrm{H}} - P_L$ = $225 - $150 = $75

→ The penalty of overstocking is greater than the penalty of understocking

$$\mathrm{D} \sim N(\mu, \sigma^2) = N\,(80, 20^2)$$

The cumulative distribution function is

$$F(\mathrm{Q}^*) = \Pr(\mathrm{D} \le \mathrm{Q}^*) = \phi\left(\frac{Q^* - \mu}{\sigma}\right) = 33.33\%$$

$$\Rightarrow \phi\left(\frac{\mathrm{Q}^* - 80}{20}\right) = 33.33\%$$

Using inverse of the standard normal cumulative distribution calculator:

$$\frac{\mathrm{Q}^* - 80}{20} = -0.43 \Rightarrow Q^* = 71.4 \approx 71 \text{ rooms}$$