IT-42016: Digital Signal Processing II

Chapter-16
Windowed-Sinc Filters

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Windowed-sinc filters

- In signal processing, a sinc filter is an idealized filter that removes all frequency components above a given cutoff frequency, without affecting lower frequencies, and has linear phase response.

- The filter's impulse response is a sinc function in the time domain, and its frequency response is a rectangular function in the frequency domain.

- It can be considered as a brick-wall filter because it allows perfectly passing low frequencies and perfectly cutting high frequencies

Windowed-sinc filters

- Windowed-sinc filters are used to separate one band of frequencies from another
- Poor performance in the time domain, including excessive ripple and overshoot in the step response
- By standard convolution, windowed-sinc filters are easy to program, but slow to execute
- The ***ideal*** filter kernel (impulse response) of the general form: sin(x)/x is called the **sinc function**, given by $$h[i] = \frac{\sin(2\pi f_C i)}{i\pi}$$
- Convolving an input signal with this filter kernel provides a perfect low-pass filter

Windowed-sinc filters

- Windowed-sinc filters are primarily used for designing finite impulse response (FIR) filters in digital signal processing (DSP).
- They are especially popular for creating low-pass, high-pass, band-pass, and band-stop filters.

Low-Pass Filtering

- ✓ Application: Removing high-frequency noise from signals such as audio, biomedical signals (e.g., ECG, EEG), and sensor data.
- ✓ Purpose: To allow low-frequency components to pass through while attenuating high-frequency components. This is essential in applications where the desired signal is in the low-frequency range, and high-frequency noise needs to be removed.

Windowed-sinc filters

High-Pass Filtering

- ✓ Application: Eliminating low-frequency noise or drift in signals, such as removing the DC component from audio signals or enhancing the detection of high-frequency events.

- ✓ Purpose: To allow high-frequency components to pass through while attenuating low-frequency components. This is useful in scenarios where high-frequency information is of interest, and low-frequency noise needs to be suppressed.

Band-Pass Filtering

- ✓ Application: Isolating a specific frequency range in communication systems, audio processing, and biomedical signal analysis.

- ✓ Purpose: To allow a specific range of frequencies to pass through while attenuating frequencies outside this range. This is critical for applications that require focusing on a particular frequency band, such as detecting specific tones or frequencies in a signal.

Windowed-sinc filters

Band-Stop Filtering

- ✓ Application: Eliminating specific frequency interference, such as powerline noise (50/60 Hz) in biomedical signals or removing certain harmonic frequencies in audio processing.
- ✓ Purpose: To attenuate a specific range of frequencies while allowing frequencies outside this range to pass through. This is particularly useful for removing narrowband interference without affecting the surrounding frequencies significantly.

Mention about Windowed-Sinc Filters.

- ✓ Windowed-sinc filters are used to separate one band of frequencies from another
- ✓ Poor performance in the time domain, including excessive ripple and overshoot in the step response
- ✓ By standard convolution, windowed-sinc filters are easy to program, but slow to execute
- ✓ The ideal filter kernel (impulse response) of the general form: sin(x)/x is called the sinc function.
- ✓ Convolving an input signal with this filter kernel provides a perfect low-pass filter

Windowed-sinc filters

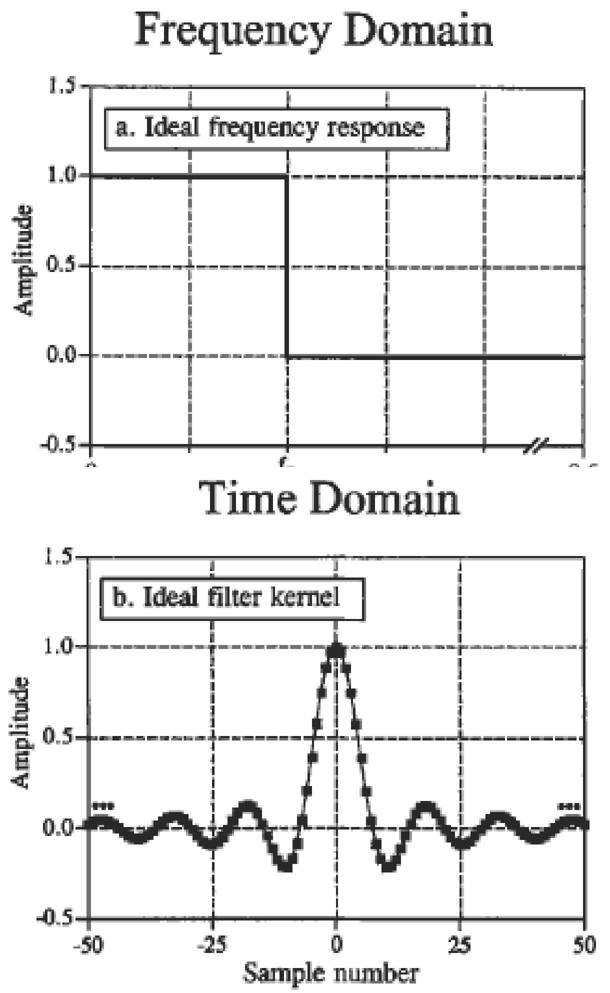
Steps to Design a Windowed-sinc Filter

- Define the Sinc Function:
  - ✓ Choose the cutoff frequency and create the ideal sinc function.
- Apply the Window Function:
  - ✓ Select a window function and multiply it by the sinc function.
- Normalize the Filter Coefficients:
  - ✓ Ensure the filter coefficients sum to one to maintain the signal's amplitude.

Derivation of the windowed-sinc filter kernel

- The frequency response of the ideal low-pass filter is shown in (a).

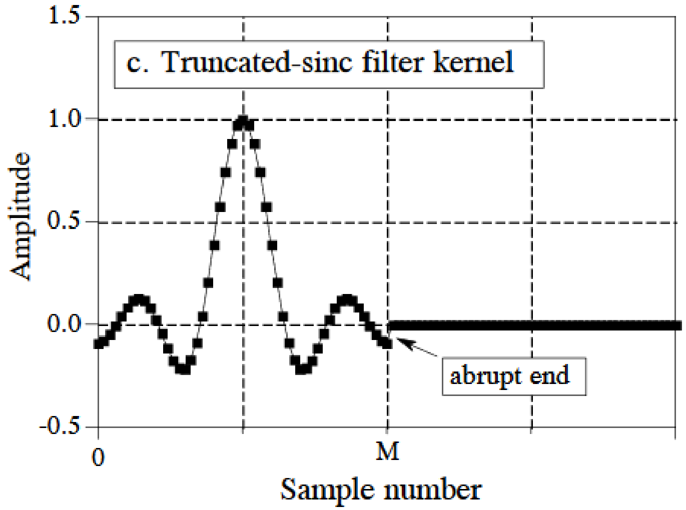
- Taking the inverse Fourier transform of this ideal frequency response produces the ideal filter kernel (impulse response) in (b), a **sinc function**.



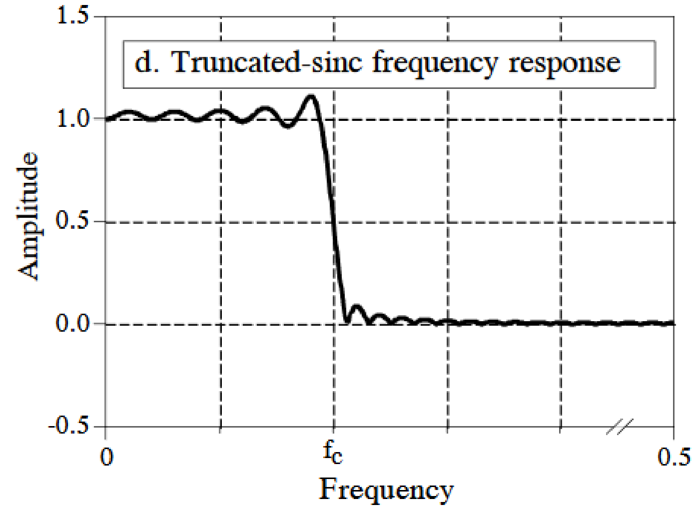
Derivation of the windowed-sinc filter kernel

- Since the sinc is infinitely long, it must be **truncated** to be used in a computer, as shown in (c).
  1. Truncated to $M + 1$ points, symmetrically chosen around the main lobe, where M is an even number. All samples outside these $M + 1$ points are set to zero.
  2. The entire sequence is shifted to the right

Time Domain

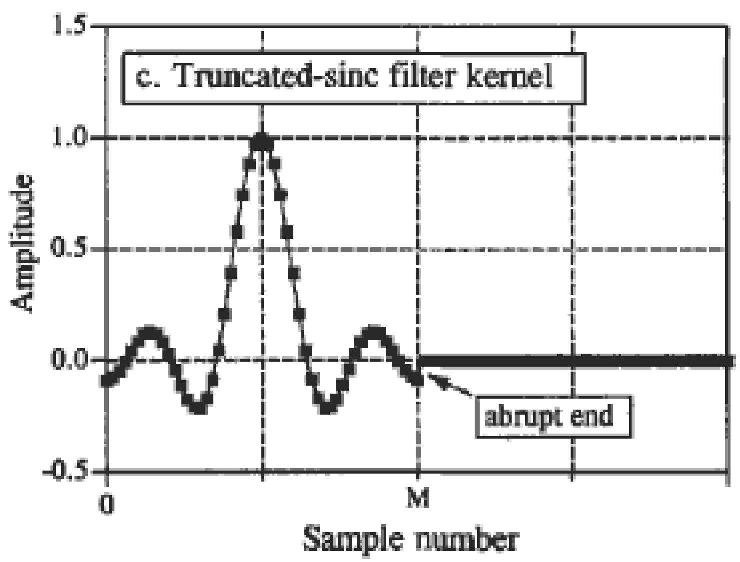


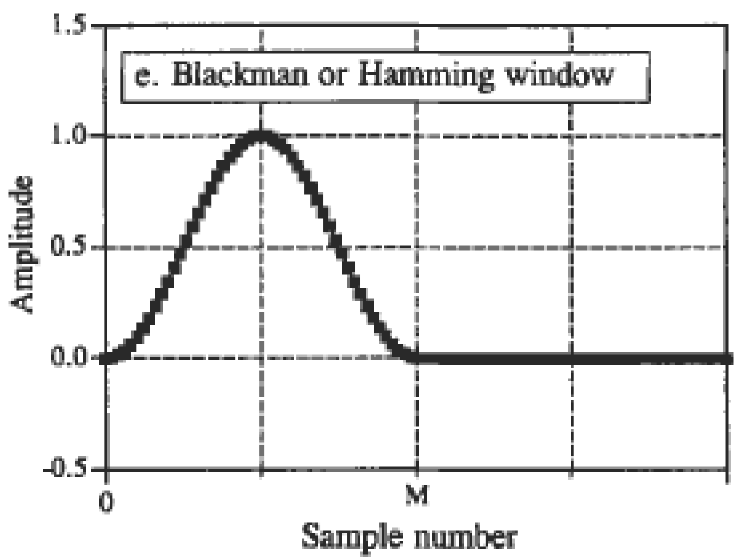
Frequency Domain



Derivation of the windowed-sinc filter kernel

- The discontinuity is significant no matter how long *M* is made.
- Fortunately, there is a simple method of improving this situation. Figure (e) shows a smoothly tapered curve called a Blackman window.

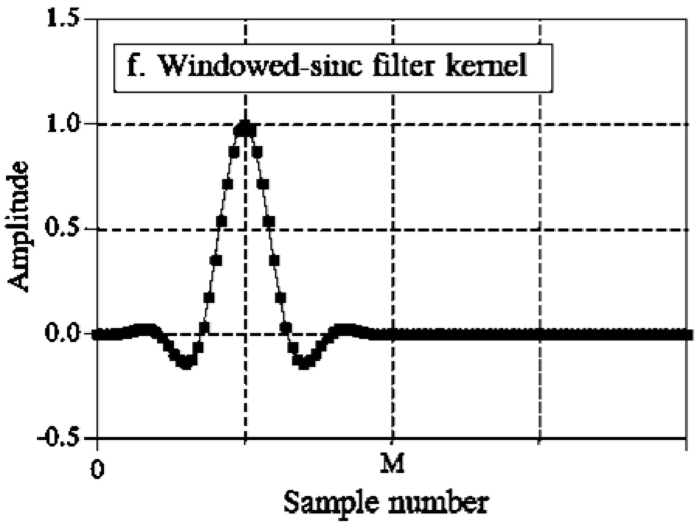




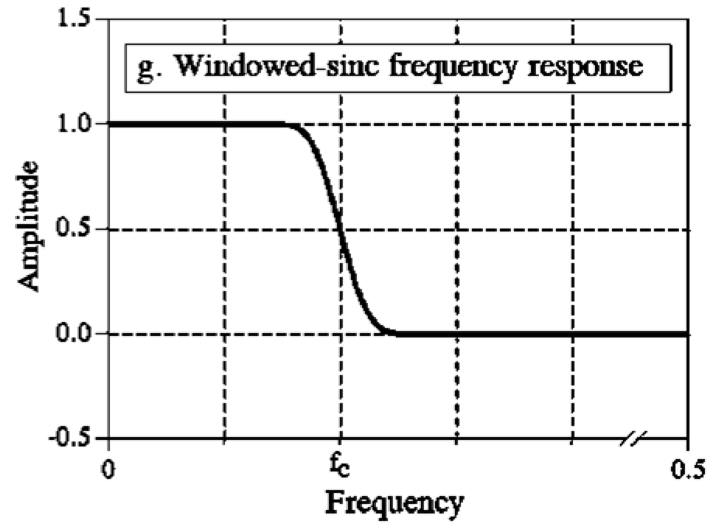
Derivation of the windowed-sinc filter kernel

- Multiplying the truncated-sinc, (c), by the Blackman window, (e), results in the windowed-sinc filter kernel shown in (f).

Time Domain



Frequency Domain



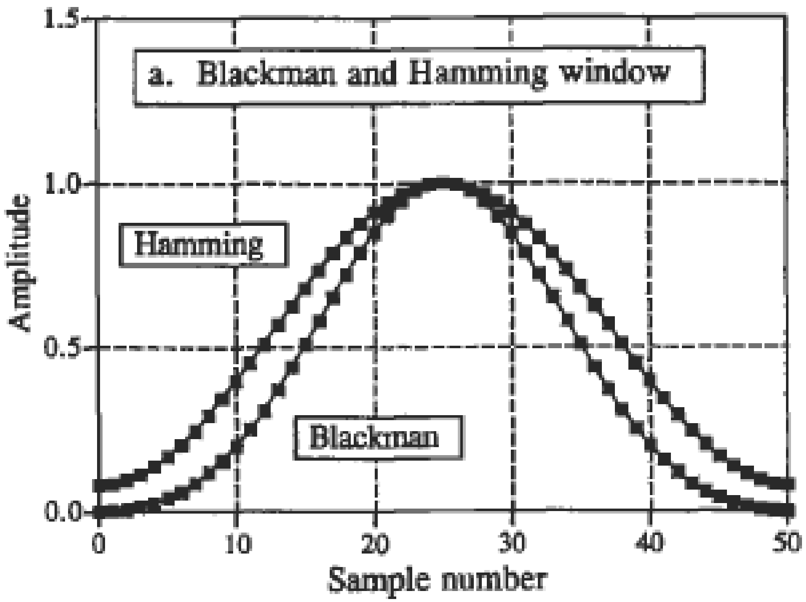
Different Windows

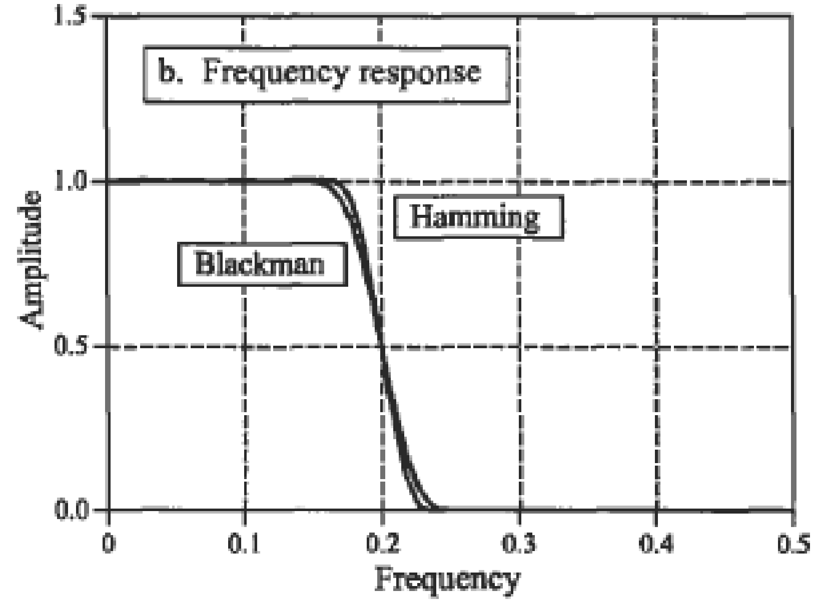
The table below gives the equations for different window types.

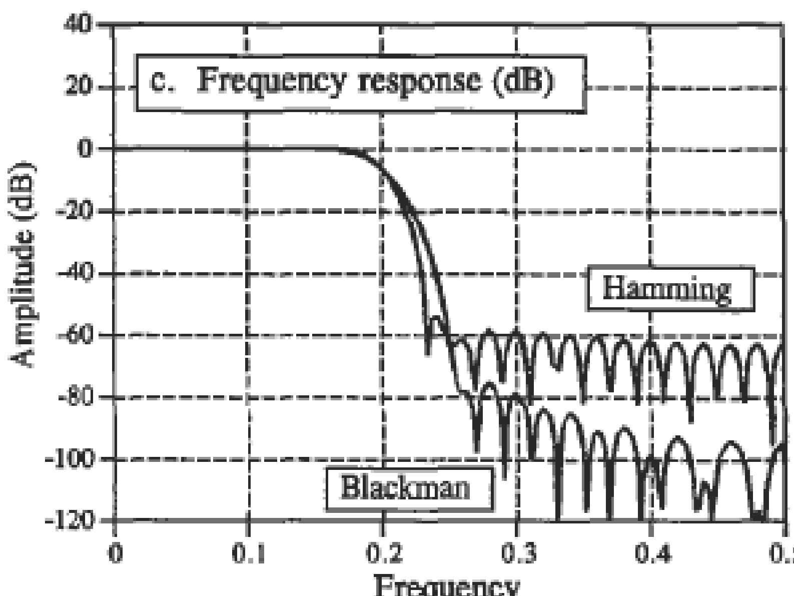
Window Type	**Weight Equation**
Rectangular	$w(n) = 1$
Bartlett	$w(n) = 1 - \frac{2\left\|n - \frac{M}{2}\right\|}{M}$
Hanning	$w(n) = 0.5 - 0.5\ cos\left(\frac{2\pi n}{M}\right)$
Hamming	$w(n) = 0.54 - 0.46\ cos\left(\frac{2\pi n}{M}\right)$
Blackman	$w(n) = 0.42 - 0.5\ cos\left(\frac{2\pi n}{M}\right) + 0.08\ cos\left(\frac{4\pi n}{M}\right)$

Blackman and Hamming windows

- The shapes of these two windows are shown in (a).
- As shown in (b), the Hamming window results in about 20% faster roll-off than the Blackman window.
- However, the Blackman window has better stopband attenuation (Blackman: 0.02%, Hamming: 0.2%), and a lower passband ripple (Blackman: 0.02% Hamming: 0.2%).


a. Blackman and Hamming window
Hamming
Blackman
Amplitude
1.5
1.0
0.5
0.0
0
10
20
30
40
50
Sample number


b. Frequency response
Hamming
Blackman
Amplitude
1.5
1.0
0.5
0.0
0
0.1
0.2
0.3
0.4
0.5
Frequency


c. Frequency response (dB)
Hamming
Blackman
Amplitude (dB)
40
20
0
-20
-40
-60
-80
-100
-120
0
0.1
0.2
0.3
0.4
0.5
Frequency

Designing the Windowed-sinc Filter

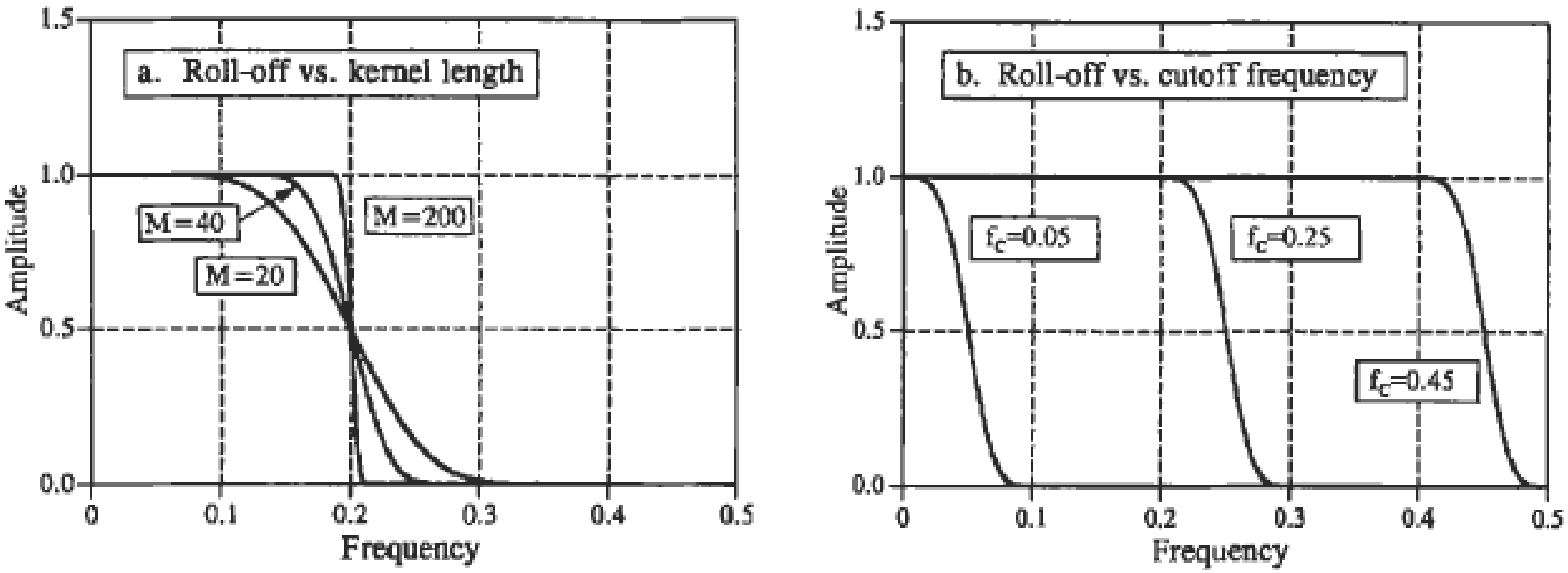
- To design a windowed-sinc, two parameters must be selected: the cutoff frequency, f_c, and the length of the filter kernel, **M**.
- The cutoff frequency is expressed as a fraction of the sampling rate, and therefore must be between 0 and 0.5.
- The value for M sets the roll-off according to the approximation:

$$M \approx \frac{4}{BW}$$

- B W is the width of the transition band. The transition bandwidth is also expressed as a fraction of the sampling frequency, and must between 0 and 0.5.
- After f_c and M have been selected, the filter kernel is calculated from the relation:

$$h[i] = K \frac{\sin(2\pi f_c (i - M/2))}{i - M/2} \left[0.42 - 0.5\cos\left(\frac{2\pi i}{M}\right) + 0.08\cos\left(\frac{4\pi i}{M}\right)\right]$$

Designing the Filter



Filter length vs. roll-off of the windowed-sinc filter. As shown in (a), for M = 20,40, and 200, the transition bandwidths are BW = 0.2,0. 1, and 0.02 of the sampling rate, respectively. As shown in (b), the shape of the frequency response does not change with different cutoff frequencies. In (b), M= 60.

Examples of Windowed-Sinc Filters

- An electroencephalogram, or EEG, is a measurement of the electrical activity of the brain.
- If you close your eyes and relax, the predominant EEG pattern will be a slow oscillation between about 7 and 12 hertz. This waveform is called the alpha rhythm.
- Opening your eyes and looking around causes the EEG to change to the beta rhythm, occurring between about 17 and 20 hertz.
- In this example, we will assume that the EEG signal has been digitized at a sampling rate of 100 samples per second. Acquiring data for 50 seconds produces a signal of 5,000 points.
- Our goal is to separate the alpha from the beta rhythms. To do this, we will design a digital low-pass filter with a cutoff frequency of 14 hertz, or 0.14 of the sampling rate.

Example of windowed-sinc filters

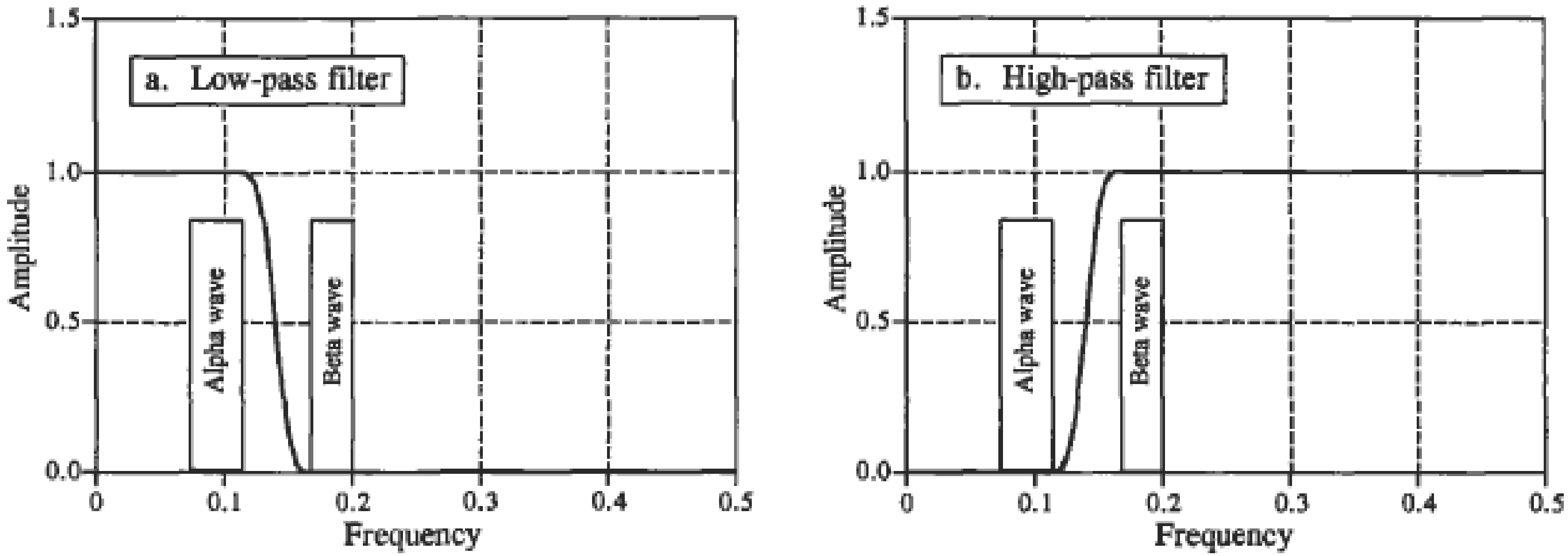


FIGURE 16-5
Example of windowed-sinc filters. The alpha and beta rhythms in an EEG are separated by low-pass and highpass filters with M = 100

https://tomroelandts.com/articles/how-to-create-a-simple-low-pass-filter

Thank You!

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