A Brief Introduction to Percolation Theory

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Abstract

Consider a cube of water-permeable material. What is the probability that that if water is poured on top of the cube it may drain all the way through the cube and out the opposite face? Initially developed by Paul Flory and Walter Stockmayer in 1944, percolation theory attempts to answer such questions by rephrasing them in terms of vertices (sites) and edges (bonds) of graphs and examining the connectedness of such graphs. The connectedness of these graphs—in the infinite case—is determined by a threshold probability, p_c, describing whether the water may pass through each site or bond. This essay will introduce the ideas of site and bond percolation as well as the notion of clusters and critical (threshold) probabilities. We will also analyse the one dimensional case to garner a basic understanding before exploring higher dimensional cases. After discussing percolation in the abstract, we will move on and look at the many applications of the theory discussed in the earlier parts of the essay.

Contents

1 Introduction **3**

2 The one dimensional case **3**

3 Higher dimensional cases **3**

4 Applications **3**

1 Introduction

When introducing the idea of percolation, the most reasonable place to start is by considering the lattice on $\mathbb{Z}^2$.

2 The one dimensional case

3 Higher dimensional cases

4 Applications