# A Global Constraint Framework as a Constrained Variational Principle

## 1. Ontological Postulate (Global Primacy)

Let $\Omega \subset \mathbb{R}^4$ be a spacetime region with boundary $\partial\Omega$.

**Axiom 1 (Global Constraint):** Physical states are not defined by local initial data alone, but by *global admissibility* under constraints imposed on $\partial\Omega$.

Formally, the physically realizable fields $\phi(x)$ belong to a constrained function space

$$\phi \in \mathcal{A}(\Omega) \subset \mathcal{F}(\Omega),$$

where admissibility is determined by global constraints

$$\mathcal{C}_i[\phi] = 0.$$

## 2. Constrained Action Functional

Define the action

$$S[\phi] = \int_\Omega \mathcal{L}(\phi, \partial_\mu \phi)\, d^4x$$

subject to *global constraints*

$$\mathcal{C}i[\phi] \equiv \int \Omega G_i(\phi, \partial\phi)\, d^4x + \int_{\partial\Omega} H_i(\phi)\, d^3\sigma = 0.$$

Introduce Lagrange multipliers $\lambda_i$ and define the *effective constrained action*

$$S_{\text{eff}}[\phi] = S[\phi] + \sum_i \lambda_i \mathcal{C}_i[\phi].$$

## 3. Variational Principle

**Axiom 2 (Constrained Extremality):**

$$\delta S_{\text{eff}}[\phi] = 0 \quad \text{for all } \delta\phi \in T\mathcal{A}.$$

This yields modified Euler–Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} + \sum_i \lambda_i \frac{\delta G_i}{\delta \phi} = 0.$$

Crucially:

- $\lambda_i$ are *global*, not local fields.
- Local dynamics depend parametrically on global constraint satisfaction.

## 4. Emergence of Conservation Laws

Let $\mathcal{C}[\phi]$ be invariant under a continuous transformation

$$\phi \to \phi + \epsilon \Delta \phi.$$

Then stationarity of $S_{\text{eff}}$ implies

$$\partial_\mu J^\mu = 0.$$

*Note:* Conservation laws arise from *constraint invariance*, not assumed symmetry of $\mathcal{L}$. Noether's theorem is recovered as a corollary.

## 5. Quantization as Constraint Eigenstructure

Admissible solutions satisfy

$$\mathcal{C}[\phi_n] = 0,$$

defining an eigenvalue problem:

$$\mathcal{O}\phi_n = \lambda_n \phi_n.$$

Dimensionless physical constants (e.g., coupling strengths) appear as

$$\alpha_n = f(\lambda_n),$$

so constants are global eigenvalues, not free parameters.

## 6. Coherence and Decoherence

Define a coherence functional

$$\mathcal{K}[\phi] = \int_\Omega \phi^* \phi \, d^4x.$$

**Prediction:**

- Decoherence corresponds to loss of admissibility: $\phi \notin \mathcal{A}(\Omega)$.
- There exists a minimum decoherence rate set by constraint incompatibility:

$$\Gamma \geq \Gamma_{\text{constraint}}.$$

## 7. Locality as an Approximation

If constraints are weak or on large scales,

$$\lambda_i \to 0 \quad \Rightarrow \quad \text{Standard local field theory recovered.}$$

Thus:

- Local physics is an *effective theory*.
- Global consistency is fundamental.

## 8. Falsifiability Condition

The framework is falsifiable if any of the following are observed:

1. Arbitrary continuous variation of dimensionless constants.
2. Complete elimination of decoherence via local isolation.
3. Absence of boundary-condition sensitivity in coherent systems.

## Summary

Physical law arises from global consistency conditions imposed on admissible histories; local dynamics, conservation laws, and constants emerge as constrained extrema of a global variational principle.