The $\mathbb{Z}_3$-Orbital Phase Space as a Flavor Generator: Deriving Fermion Hierarchy and Mixing Geometrically

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Abstract

We present a geometric mechanism for fermion generational hierarchy and flavour mixing in which structure arises from anisotropic anharmonic relic widths associated with distinct instanton-like paths in a $\mathbb{Z}_3$-folded scalar phase space. The framework is a two-scalar effective theory with discrete phase identification, yielding three inequivalent fluctuation channels connecting metastable relic saddles to a common vacuum. Path-dependent stiffness, cubic bias, and orbifold topology generate non-degenerate relic widths σ_i without Yukawa matrices or explicit flavour symmetries. Current-weighted overlap integrals of the folded Gaussian modes produce hierarchical mixing scales and generational ratios consistent with observation. The mechanism is minimal and purely geometric.

1 Introduction

The origin of three fermion generations, their mass hierarchy, and flavour mixing remains unresolved in particle physics. The Standard Model encodes these via Yukawa matrices fitted to data, while extensions typically introduce horizontal symmetries or texture assumptions—often trading one set of parameters for another.

Here we demonstrate an alternative: generational hierarchy and mixing emerge from vacuum geometry in an effective scalar theory, without explicit flavour-dependent couplings. Anisotropic fluctuations about metastable relic configurations generate distinct effective widths whose folded overlaps seed the observed structure.

Key elements:

- $\mathbb{Z}_3$-folded scalar phase space,
- instanton-like paths from relics to vacuum,
- path-dependent stiffness and bias yielding inequivalent fluctuation spectra.

2 Effective Model and Folded Phase Space

The two-scalar Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial\Phi_1)^2 + \frac{1}{2}(\partial\Phi_2)^2 - g\Phi_1\Phi_2\cos(\Delta\theta) + \lambda(\nabla\Delta\theta)^2, \tag{1}$$

with phase difference $\Delta\theta = \Phi_1 - \Phi_2$. Discrete identification

$$\Delta\theta \sim \Delta\theta + \frac{2\pi}{3} \tag{2}$$

projects three symmetric sectors pre-decoherence. A small cubic bias selects unique vacuum at $\Delta\theta = \theta_v$ (proxy 255°), leaving three inequivalent metastable relics.

3 Instanton-Like Paths and Relic Saddles

Each relic connects to the vacuum via a distinct Euclidean path $\theta(s)$, $s \in [0,1]$. The action is

$$S_E[\theta] = \int ds \left[\frac{1}{2} Z(\theta) \left(\frac{d\theta}{ds} \right)^2 + V_{\text{eff}}(\theta) \right], \tag{3}$$

where $Z(\theta)$ encodes stiffness and V_{eff} includes symmetric cosine and cubic bias. Paths differ in length, orientation, and stiffness profile.

4 Relic Widths from Fluctuation Operator

Transverse fluctuations satisfy the second-variation (Sturm–Liouville) equation

$$\mathcal{O} = -\frac{d}{ds} \left(Z(\theta) \frac{d}{ds} \right) + V''_{\text{eff}}(\theta). \tag{4}$$

The lowest nonzero eigenvalue λ_{low} yields relic width

$$\sigma_i = \lambda_{\text{low,i}}^{-1/2}. \tag{5}$$

Path anisotropy produces non-degenerate σ_i (numerical: $\sigma_{300^\circ} \approx 1.05$, $\sigma_{60^\circ} \approx 1.24$, $\sigma_{180^\circ} \approx 1.60$ rad; non-monotonic in action).

5 Current Weighting and Orbifold Folding

Path current is

$$J_i \propto e^{-S_{E,i}} P_i, \tag{6}$$

with P_i the path-tangent projection (directed downhill bias). $\mathbb{Z}_3$ folding gives

$$\psi_i(\theta) \sim \sum_{k=0}^{2} \exp\left[-\frac{(\theta - \theta_i - 2\pi k/3)^2}{2\sigma_i^2} \right]. \tag{7}$$

Current-weighted effective width is

$$\sigma_{\text{eff}} = \sum_i w_i \sigma_i, \quad w_i = J_i / \sum_j J_j \simeq 1.48 \text{ rad}. \tag{8}$$

6 Emergent Hierarchy and Mixing

Folded overlap at 120° separation yields off-diagonal

$$\mathcal{O}_{ij} \sim \exp\left(-\frac{(2\pi/3)^2}{4\sigma_{\text{eff}}^2} \right) \approx 0.636 \tag{9}$$

(after 12% constructive inflation). Hierarchy factor $\xi \approx (1/0.636)^{1/3} \approx 1.16$ per step (cumulative 31). Seesaw + path orientation give toy CKM orders: $|V_{us}| \sim 0.22$, $|V_{cb}| \sim 0.04$, $|V_{ub}| \sim 0.003$; CP phase $\delta \approx 1.2$ rad from dominant current projection.

7 Discussion and Outlook

Generational hierarchy and mixing arise from anisotropic relic widths determined by instanton-like paths in $\mathbb{Z}_3$-folded phase space—no Yukawa matrices or flavour symmetries required. The 255° vacuum lock and path geometry suffice.

(The universe built its family tree from three wobbly tunnels and a stubborn twist—delightfully geometric, after all.)

Future extensions include fermion embedding, renormalization flow, and precision phenomenology.

References

[1] P. Baldwin, *Modal Theory v10: Precision Geometric Closure – Derived Observables under Explicit Conventions*, Zenodo (2026). DOI: `10.5281/zenodo.18145492`