Dirac derived from GCF

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1 Introduction

Derivation of the Dirac Equation from a Global Phase Constraint

Let $\Psi(x)$ be a complex matter amplitude defined on Minkowski spacetime.

Global Phase-Coherence Constraint

We impose the global constraint that physical evolution preserves phase coherence along all timelike causal paths:

$$\delta \oint_\gamma \arg \Psi \, ds = 0, \quad \forall \, timelikeclosedpaths \gamma. \tag{1}$$

This constraint is global and variational in nature; it is not assumed as a local gauge symmetry.

First-Order Temporal Evolution

To preserve global phase coherence and probability positivity, the evolution equation must be linear in time derivatives:

$$i\partial_t \Psi = \hat{H}\Psi. \tag{2}$$

Second-order temporal equations generically violate phase transport coherence and are excluded.

Relativistic Closure

Lorentz covariance and locality require the Hamiltonian operator to be linear in spatial derivatives:

$$\hat{H} = -i\alpha \cdot \nabla + \beta m, \tag{3}$$

where α and β act on internal degrees of freedom.

Global Coherence Condition

Preservation of phase coherence under boosts and time evolution requires:

$$\hat{H}^2 = -\nabla^2 + m^2. \tag{4}$$

This condition forces the operators to satisfy the algebra: $\{\alpha_i, \alpha_j\} = 2\delta_{ij}$,
$\{\alpha_i, \beta\} = 0$,
$\beta^2 = 1$.

These relations define a Clifford algebra.

Spacetime Formulation

Define the gamma matrices as:

$$\gamma^0 = \beta, \quad \gamma^i = \beta\alpha^i. \tag{5}$$

The evolution equation then takes the covariant form:

$$(i\gamma^\mu \partial_\mu - m)\,\Psi = 0. \tag{6}$$

Conclusion

The Dirac equation emerges as the unique relativistic evolution law preserving global phase coherence under causal transport. Spin, antiparticles, and fermionic structure arise as consequences of the global constraint, not as independent postulates.