Maxwell's Equations from a Global Constraint Framework (GCF)

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1. Global Constraint Principle

We postulate a single global constraint on the electromagnetic field strength tensor $\boldsymbol{F}_{\mu\nu}$ (antisymmetric 2-form):

$$\mathcal{C}[\boldsymbol{F}] = \int_{\mathcal{M}} \mathcal{L}_{\text{global}}(\boldsymbol{F}, \partial\boldsymbol{F})\, d^4x = \text{constant}. \tag{1}$$

This enforces spacetime-wide *coherence* and serves as the fundamental variational principle.

2. Constrained Variational Principle

Consider variations $\delta\boldsymbol{F}$ that preserve the global constraint $\delta\mathcal{C} = 0$. Introduce a Lagrange multiplier λ and define the effective action

$$S_{\text{eff}}[\boldsymbol{F}] = \int_{\mathcal{M}} \mathcal{L}(\boldsymbol{F})\, d^4x - \lambda\mathcal{C}[\boldsymbol{F}]. \tag{2}$$

Stationarity requires

$$\delta S_{\text{eff}} = 0 \quad \text{for all admissible } \delta\boldsymbol{F}. \tag{3}$$

3. Derivation of Maxwell's Equations

Choose the standard kinetic term

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{4}$$

and let $\mathcal{L}_{\text{global}}$ encode total flux coherence (explicit form not needed for the vacuum case). The constrained Euler–Lagrange equations become

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu F_{\alpha\beta})} - \lambda\frac{\partial\mathcal{L}_{\text{global}}}{\partial(\partial_\mu F_{\alpha\beta})}\right) - \left(\frac{\partial\mathcal{L}}{\partial F_{\alpha\beta}} - \lambda\frac{\partial\mathcal{L}_{\text{global}}}{\partial F_{\alpha\beta}}\right) = 0. \tag{5}$$

For the vacuum case ($\mathcal{L}_{\text{global}}$ chosen to enforce flux conservation), this reduces to the source-free Maxwell equations:

$$\partial_\mu F^{\mu\nu} = 0 \qquad \text{(inhomogeneous)}, \tag{6}$$

$$\partial_{[\alpha} F_{\beta\gamma]} = 0 \qquad \text{(homogeneous / Bianchi identity)}. \tag{7}$$

4. Interpretation

- Gauge invariance and the homogeneous equation emerge naturally from the global coherence requirement.
- The Lagrange multiplier λ is absorbed into the overall normalization and does not introduce new degrees of freedom.
- Maxwell's equations therefore appear as a *consequence* of spacetime-wide coherence rather than as fundamental postulates.

This derivation illustrates how local field equations can arise from a single global constraint principle, unifying symmetry, conservation, and dynamics under one variational umbrella.