Proceedings of the 2022 6th CAA International Conference on Vehicular Control
and Intelligence (CVCI), Nanjing, China, October 28-30, 2022

Time Optimal Trajectory Optimization for Vertical Parking of Heavy Duty Vehicle based on Chebyshev Pseudospectral Method

1st Mingzhuo Zhao
The School of Cyber Science and Engineering
Southeast University
Nanjing, China
zhao_mz@seu.edu.cn

2nd Guodong Yin*
The School of Mechanical Engineering
Southeast University
Nanjing, China
ygd@seu.edu.cn

3rd Tong Shen
The School of Mechanical Engineering
Southeast University
Nanjing, China
kylo_sen@163.com

4th Yanjun Ren
The School of Mechanical Engineering
Southeast University
Nanjing, China
ryj19940918@163.com

5th Fanxun Wang
The School of Mechanical Engineering
Southeast University
Nanjing, China
fxwseu@163.com

6th Yang Zhang
The School of Automotive and Transportation Engineering
Hefei University of Technology
Hefei, China
2020010090@mail.hfut.edu.cn

2022 6th CAA International Conference on Vehicular Control and Intelligence (CVCI) | 978-1-6654-5374-5/22/$31.00 ©2022 IEEE | DOI: 10.1109/CVCI56766.2022.9964800

***Abstract*—Heavy duty vehicle are open-loop unstable while reversing owing to their unusual articulated construction, while vertical parking in tighter surroundings is sometimes quite problematic due to their lengthy wheelbase and significant nonholonomic dynamics. This paper theoretically analyzes the difference brought by the nonholonomic constraints for the heavy duty vehicle model, and precisely portrays the physical and related environmental constraints of the vehicle; Considering the jackknife phenomenon brought by the unique articulation characteristics of the heavy duty vehicle, the hitch angle of the vehicle in reverse is restricted with the objective of time optimality, and the state space is carried out without compromising the quality of the local optimal solution substantial pruning; A finite-dimensional nonlinear programming(NLP) is constructed for vertical parking of heavy duty vehicles with an accurate obstacle avoidance strategy and an optimal nonholonomic dynamics, boundary and path constrained discretization scheme based on Chebyshev's pseudospectral method; Matlab is used to solve the NLP problem, and the final simulation results show that: the vertical parking trajectory planning for heavy duty vehicle in narrower aisles is done accurately and less time-consumingly in a unified framework without relying on search and sampling methods.**

***Index Terms*—Heavy Duty Vehicle, Nonholonomic Dynamics, Trajectory Optimization, Vertical Parking, Chebyshev Pseudospectral**

I. INTRODUCTION

Heavy duty vehicle have long been the dominant force of road transportation, and their transit efficiency significantly inhibits the efficiency of logistical transportation. And heavy duty vehicle are open-loop unstable while reversing owing to their unusual articulated construction, while vertical parking in tighter surroundings is sometimes quite problematic due to their lengthy wheelbase and significant nonholonomic dynamics. Vertical parking planning of heavy duty vehicle belongs to the category of motion planning of vehicles. Previous literature has more often considered the motion planning of passenger cars, and less literature has made an in-depth study on the motion planning of heavy duty vehicle.

Li et al. [1] proposed an adaptively homotopic warm-starting approach to solve the problem of heavy duty vehicle easily falling into local optima in optimization-based motion planning, after which they proposed an asymptotic constraint strategy for iterative optimization to avoid reliance on search algorithms [2]. Oliveira et al. [3] suggested a Geometric Approach to On-road Motion Planning for Long and Multi-Body Heavy-Duty Vehicles. Based on geometric derivations, they attempt to identify the ideal trade-off between the competing aims of centering various axles of the vehicle in the lane. Bergman et al. [4] integrate a sampling-based path planner with a numerical optimum control technique in a structured fashion that enables the former to solve the combinatorial component of the issue and the latter to get a locally optimal solution that is not bound by a discrete search space. Manav et al. [5]provide kinematically viable and deterministic parking maneuvers and obstacle avoidance, the iterative analysis method (IAM), a realistic and deterministic model of parking behavior, was paired with the closed-loop rapid exploration tree (CL-RRT) approach for cascaded route planning.

Most of the previous literature did not consider the nonholonomic constraint of heavy duty vehicle, and thus only considered the trajectory planning at the kinematic level, and also hardly considered the open-loop instability characteristic of heavy duty vehicle when reversing, which makes reversing

This work is supported by National Natural Science Foundation of China (52025121,51975118).

more difficult to track than forward at the control level, and requires higher quality of the planned trajectory.At the same time, in narrow vertical parking conditions, the search-based and sampling-based planners are very likely to fail or make the incremental search and sampling process difficult, and the subsequent search and sampling time grows.

In this paper, on the basis of analyzing the nonholonomic dynamic characteristics and off-axis connection of heavy duty vehicle, more accurate dynamic characteristics are considered in the trajectory optimization process, and the open-loop instability characteristics of heavy duty vehicle reversing are also considered to avoid the jackknifing situation in the trajectory optimization process, so that the planned trajectory can be tracked and controlled more easily.

II. Analysis of vertical parking constraint of heavy duty vehicle

A. Dynamics model of heavy duty vehicle

Heavy duty vehicle may normally be separated into tractor and semi-trailer.Tractor and semi-trailer are articulated by the traction pin. Heavy duty vehicle susceptible to holonomic restrictions, but also to nonholonomic constraints. Holonomic restrictions are imposed by the articulation between the truck and the semitrailer. By contrast, nonholonomic limitations are created by the wheeled movable mechanism of the heavy duty vehicle. The entire heavy duty vehicle system has complex nonlinear characteristics. The theoretical model [6]–[8] is shown in Fig. 1. In this paper, we simplify the conventional five-axle heavy duty vehicle into an equivalent three-axle model [9], and impose nonholonomic constraints on the non-steering axes to model its dynamics as (1).

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{pmatrix} = \begin{pmatrix} -\frac{\mathrm{L_c} v\cos(\phi)\cos(\psi)\sin(\theta) - \mathrm{L_{oH}} v\sin(\phi)\sin(\psi)\sin(\theta)}{\mathrm{L_C}} \\ \frac{\mathrm{L_c} v\cos(\phi)\cos(\psi)\cos(\theta) - \mathrm{L_{oH}} v\cos(\theta)\sin(\phi)\sin(\psi)}{\mathrm{L_c}} \\ \frac{\sigma_2+\sigma_1}{\mathrm{L_c L_t}} \\ -\frac{\sigma_2 - \mathrm{L_t} v\sin(\psi)+\sigma_1}{\mathrm{L_c L_t}} \end{pmatrix}, \tag{1}$$

where

$$\begin{aligned} \sigma_1 &= \mathrm{L_{oH}} v\cos(\phi)\sin(\psi) \\ \sigma_2 &= \mathrm{L_c} v\cos(\psi)\sin(\phi) \end{aligned} . \tag{2}$$

L_c is the wheelbase of the tractor; L_t is the wheelbase of the semi-trailer; L_{oH} is the distance from the hitch point to the rear axle of the tractor; x, y are the horizontal and vertical coordinates of the center point of the rear axle of the semi-trailer in the Cartesian coordinate system; θ is the hitch angle between the tractor and the semi-trailer; ϕ is the heading angle of the semi-trailer; ψ is the steering angle of the tractor.

B. Bounds on state and control

For any optimal control problem, the feasible sets of the state space and control space of the system are not infinite. For the optimization of the vertical parking trajectory of the heavy duty vehicle in this paper, the constraints for the system

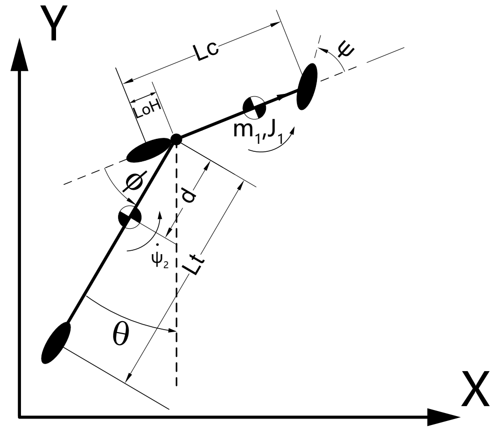

Fig. 1. heavy duty vehicle model.

state space are shown as (3) , and the control space constraints for the system are shown as (4).

$$\begin{cases} x_{\min} \le x(t) \le x_{\max} \\ y_{\min} \le y(t) \le y_{\max} \\ \theta_{\min} \le \theta(t) \le \theta_{\max} \\ \phi_{\min} \le \phi(t) \le \phi_{\max} \end{cases} \quad \text{for any } 0 \le t \le t_f, \tag{3}$$

$$\begin{cases} v_{\min} \le v(t) \le v_{\max} \\ \psi_{\min} \le \psi(t) \le \psi_{\max} \end{cases} \quad \text{for any } 0 \le t \le t_f, \tag{4}$$

where the maximum and minimum values of x and y depend on the corresponding parking environment(Fig. 3). $t_f$is the final time of the end of parking. It will be used as a decision variable for the final optimization to achieve the shortest time parking trajectory optimization. The range of θ is set at $(-\pi, \pi)$ to prevent collision between tractor and semi-trailer. The range of ϕ is set at $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ to prevent the heavy duty vehicle from jackknifing [10]. And the execution constraints of the control variables are restricted to $\left(-\frac{5}{3.6}, \frac{5}{3.6}\right)$ and $\left(-\frac{2}{9}\pi, \frac{2}{9}\pi\right)$, respectively.

C. Path and Boundary Constraints

This part mainly includes the obstacle avoidance constraint of heavy duty vehicle and parking space and lane line.In this paper, the tractor and semi-trailer are considered as two rectangles, the parking space is also considered as composed of two rectangular obstacles, and the vertices of the rectangles are A, B, C, D in clockwise order. As shown in Fig. 2, the subscripts t, s, O_A, O_B represent tractor, semi-trailer, obstacle A and obstacle B [1]. Then there are path constraints as shown in (5) - (6) . Where, t in the symbol denotes time. t_0 and t_f represents the start time and end time, respectively.

Boundary constraints place restrictions on the state variables of the heavy duty vehicle at the start and end moments of vertical parking. It as shown in (7).

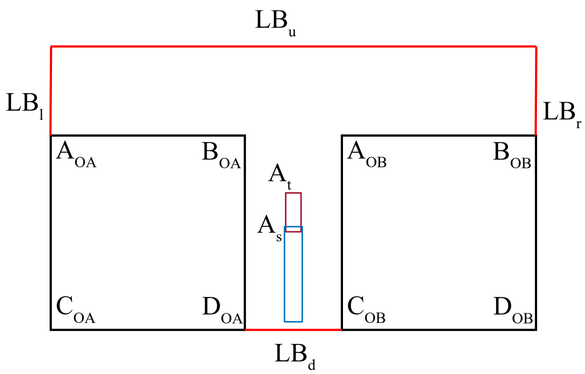

Fig. 2. collision avoidance diagram.

$$
\begin{gathered}
S_{\Delta \mathrm{XA_iB_i}} + S_{\Delta \mathrm{XB_iC_i}} + S_{\Delta \mathrm{XC_iD_i}} + S_{\Delta \mathrm{XD_iA_i}} > S_{\mathrm{A_jB_jC_jD_j}}, \\
S_{\Delta \mathrm{YA_jB_j}} + S_{\Delta \mathrm{YB_jC_j}} + S_{\Delta \mathrm{YC_jD_j}} + S_{\Delta \mathrm{YD_jA_j}} > S_{\mathrm{A_iB_iC_iD_i}}, \\
X \in \{\mathrm{A_j}, \mathrm{B_j}, \mathrm{C_j}, \mathrm{D_j}\}, \\
Y \in \{\mathrm{A_i}, \mathrm{B_i}, \mathrm{C_i}, \mathrm{D_i}\}, \\
\mathrm{i}, \mathrm{j} \in \{t, s, O_{\mathrm{A}}, O_{\mathrm{B}}\}.
\end{gathered} \tag{5}
$$

$$
\begin{gathered}
\mathrm{LB_l} \leqslant \zeta_1 \leqslant \mathrm{LB_r}, \\
\mathrm{LB_d} \leq \zeta_2 \leq \mathrm{LB_u}, \\
\zeta_1 \in \{X_{\mathrm{A_i}}(t), X_{\mathrm{B_i}}(t), X_{\mathrm{C_i}}(t), X_{\mathrm{D_i}}(t)\}, \\
\zeta_2 \in \{Y_{\mathrm{A_i}}(t), Y_{\mathrm{B_i}}(t), Y_{\mathrm{C_i}}(t), Y_{\mathrm{D_i}}(t)\}, \\
\mathrm{i} \in \{t, s\}.
\end{gathered} \tag{6}
$$

$$
\begin{gathered}
X_{\mathrm{t_0}} = (x_{\mathrm{t_0}}, y_{\mathrm{t_0}}, \theta_{\mathrm{t_0}}, \phi_{\mathrm{t_0}}), \\
X_{\mathrm{t_F}} = (x_{\mathrm{t_F}}, y_{\mathrm{t_F}}, \theta_{\mathrm{t_F}}, \phi_{\mathrm{t_F}}).
\end{gathered} \tag{7}
$$

III. Trajectory Optimization Problem Formulation

A. Continuous Trajectory Optimization

Minimize the objective function:

$$
\begin{aligned}
\min_{t_0, t_F, \boldsymbol{x}(t), \boldsymbol{u}(t)} & J_B\left(t_0, t_F, \boldsymbol{x}\left(t_0\right), \boldsymbol{x}\left(t_F\right)\right) \\
& + \int_{t_0}^{t_F} J_P(\tau, x(\tau), \boldsymbol{u}(\tau)) d\tau,
\end{aligned} \tag{8}
$$

where $J_B = t_F - t_0$ and $J_{\mathrm{P}} = u^T R u$.
Subject to the constraints:

$\dot{x}(t) = f(t, x(t), u(t))$	system dynamics (1)
$x^- \leq x(t) \leq x^+$	bounds on state (3)
$u^- \leq u(t) \leq u^+$	bounds on control (4)
$C_P(t, x(t), u(t)) \leq 0$	path constraints (5), (6)
$C_B\left(t_0, t_F, x\left(t_0\right), x\left(t_F\right)\right) = 0$	boundary constraints (7)
$t^- \leq t_0 < t_F \leq t^+$	bounds on initial and final time

Assuming that the user-defined objective, dynamics, and constraint functions (J_P, J_B, f, C_P, C_B) are smooth [11]. The user must provide an initial guess for the decision variables $t_0, t_F, x(t)$, and $u(t)$, and ensure that a feasible solution exists.

B. Chebyshev Pseudospectral Method

In general, the methods for solving the above continuous trajectory optimization problem can be divided into direct and indirect methods. Generally speaking the direct method is more advantageous than the indirect method for solving optimal control problems when the system dynamics constraints are highly nonlinear and the path constraints are more complex (e.g., covering obstacle avoidance). The key feature of the direct method is that it discretizes the trajectory optimization problem itself, usually transforming the original trajectory optimization problem into a nonlinear programming(NLP), the transformation process known as transcription.

Almost all transcription methods use polynomial splines to approximate the target function. In general, polynomials of higher order can better fit the target function, but in fact polynomials of too high an order are more likely to make the numerical computation unstable, and the use of orthogonal collocation can well avoid such a situation. Chebyshev pseudo-spectral (CPS) method chooses node points as the Chebyshev-Gauss-Lobatto (CGL) points so that the interpolating polynomial is closest to the optimal polynomial in the maximum parametric approximation of any function [12].

Chebyshev pseudospectral method use polynomial curves based on the interpolation at the quadrature nodes to approximate the state variables of the system, $x(t)$. Using the CPS method to discretize the optimal control problem as shown in Eq. (8), it is first necessary to select Chebyshev-Gauss-Lobatto (CGL) nodes, which are determined to be discrete quadrature nodes [13], and CGL nodes are defined as shown in Eq. (9).

$$
t_k = \cos(\pi(N-k)/N), \quad k = 0, 1, \ldots, N, \tag{9}
$$

In all accounts of this paper, it is assumed that the function to be approximated has been mapped to the interval $[-1, 1]$, because all functions(The interval $\tau \in [\tau_A, \tau_B]$) can be remapped to the interval $t \in [-1, 1]$. Where:

$$
t = 2\frac{\tau - \tau_A}{\tau_B - \tau_A} - 1, \tag{10}
$$

These points lie in the interval $[-1, 1]$ and are the extrema of the N th-order Chebyshev polynomial $T_N(t) = \cos\left(N \cos^{-1} t\right)$

Throughout the discretization process, it is first necessary to determine the discrete form of the control system state space (state variables, state variable derivatives, control variables), and the state variables and control variables are approximated at each quadrature node as $\bar{x}^k \in \mathbb{R}^{N_{\mathbf{x}}}$ and $\bar{u}^k \in \mathbb{R}^{N_{\mathbf{u}}}$.Where:

$$
x\left(t_k\right) \approx \bar{x}^k = \left[\begin{array}{llll} \bar{x}_1^k, & \bar{x}_2^k, & \cdots, & \bar{x}_{N_x}^k \end{array}\right]^T, \tag{11}
$$

$$
u\left(t_{\mathrm{k}}\right) \approx \bar{u}^k = \left[\bar{u}_1^k, \quad \bar{u}_2^k, \quad \cdots, \quad \bar{u}_{\mathrm{N_x}}^k\right]^T, \tag{12}
$$

The discrete sequence of all state variables at all quadrature nodes is shown in Eq. (13).The same applies to control variables $\bar{u}$.

$$\bar{x}=\left[\begin{array}{cccc} \bar{x}_1^0 & \bar{x}_1^1 & \cdots & \bar{x}_1^N \\ \bar{x}_2^0 & \bar{x}_2^1 & \cdots & \bar{x}_2^N \\ \vdots & \vdots & \vdots & \vdots \\ \bar{x}_{N_x}^0 & \bar{x}_{N_x}^1 & \cdots & \bar{x}_{N_x}^N \end{array}\right], \tag{13}$$

$\bar{x}_i$ is the i th row of $\bar{x}$, which represents the discrete approximation of the i th component, $x_i(t)$, at all nodes; and $\bar{x}^i$ is the i th column of $\bar{x}$, which represents the approximation of the state, $x(t)$, at i th node [14].

In summary, the continuous state variables will be approximated by the orthogonal polynomial from Eq. (14):

$$x_i(t) \approx x_i^N(t)=\sum_{k=0}^N \bar{x}_i^k \phi_k(t), \tag{14}$$

$$u_i(t) \approx u_i^N(t)=\sum_{k=0}^N \bar{u}_i^k \phi_k(t), \tag{15}$$

where $\phi_k(t)$ is the Lagrange interpolating polynomial

$$\phi_k(t)=\frac{(-1)^{k+1}}{N^2 c_k} \frac{\left(1-t^2\right) \dot{T}_N(t)}{t-t_k}, \tag{16}$$

$$c_k=\begin{cases}2, & \text{if } k=0, N \\ 1, & \text{if } 1 \leq k \leq N-1\end{cases}, \tag{17}$$

$T_N(t)=\cos \left(N \cos ^{-1} t\right)$, it is the extrema of the N th-order Chebyshev polynomial.

The relationship between $\dot{x}_{\mathrm{i}}^N(t)$ and $x_i^N(t)$ at the quadrature nodes t_k can be obtained by differentiating (14),i.e.

$$\left[\begin{array}{llll} \dot{x}_i^N\left(t_0\right), & \dot{x}_i^N\left(t_1\right), & \cdots & \dot{x}_i^N\left(t_N\right) \end{array}\right]=\bar{x}_i \cdot D^T, \tag{18}$$

The differentiation matrix D is defined by $D_{kj}=\dot{\phi}_k\left(t_j\right)$, where

$$\dot{\phi}_k\left(t_j\right)=\left\{\begin{array}{ll} \left(c_k / c_j\right)\left[(-1)^{j+k} /\left(t_j-t_k\right)\right], & \text{if } j \neq k \\ t_k /\left(2-2 t_k^2\right), & \text{if } 1 \leq j=k \leq N-1 \\ -\left(2 N^2+1\right) / 6, & \text{if } j=k=0 \\ \left(2 N^2+1\right) / 6, & \text{if } j=k=N \end{array}\right. . \tag{19}$$

So far, the Chebyshev orthogonal collocation with dynamical constraints (20) can be completed by substituting (14),(15),(18),(19) into (1).

$$f\left(x_{\mathrm{i}}(t), \dot{x}_{\mathrm{i}}(t), u_{\mathrm{i}}(t)\right)=0, \tag{20}$$

After the discrete expression of the state space is determined, the discrete form of the objective function is shown in (21).

$$J[x(\cdot), u(\cdot)] \approx \bar{J}^N\left(\bar{x}^N, \bar{u}^N\right)=\sum_{k=0}^N\left(\bar{u}^T R \bar{u}\right) \omega_{\mathrm{k}}+2. \tag{21}$$

It is worth noting that t_0 and t_F are remapped here to -1 and 1, respectively, where ω_k is are the quadrature weights. For N even, the weights are

$$\begin{aligned} & w_0=w_N=1 /\left(N^2-1\right), \\ & w_s=w_{N-s}=\frac{4}{N} \sum_{j=0}^{N / 2^{\prime \prime}} \frac{1}{1-4 j^2} \cos \left(\frac{2 \pi j s}{N}\right), \\ & s=1,2 \ldots, N / 2 . \end{aligned} \tag{22}$$

For N odd, the weights are given by

$$\begin{aligned} & w_0=w_N=1 / N^2, \\ & w_s=w_{N-s}=\frac{4}{N} \sum_{j=0}^{(N-1) / 2^{\prime \prime}} \frac{1}{1-4 j^2} \cos \left(\frac{2 \pi j s}{N}\right), \\ & s=1, \ldots,(N-1) / 2 . \end{aligned} \tag{23}$$

Similarly, other constraints (path constraints, etc.) in the continuous trajectory optimization problem can be discretized by substituting (14),(15),(18),(19) for the orthogonal collocation.The specific form is shown below [15]:

$$\begin{gathered} x^{-} \leqslant x_{\mathrm{i}}(0) \leqslant x^{+} \\ u^{-} \leqslant u_{\mathrm{i}}(0) \leqslant u^{+} \\ p\left(x_{\mathrm{i}}(t), \dot{x}_{\mathrm{i}}(t), u_{\mathrm{i}}(t)\right) \leqslant 0 \\ b\left(x_{\mathrm{i}}(0), x_{\mathrm{i}}(N)\right)=0 \end{gathered} \tag{24}$$

The final nonlinear procedure after discretization by Chebyshev Pseudospectral Method is as follows:

$$\begin{array}{ll} & min J[x(\cdot), u(\cdot)] \\ s.t. & \\ & Dynamics\ constraint\ (20) \\ & bounds\ on\ state \\ & bounds\ on\ control \quad (24) \\ & Path\ constraint \\ & Boundary\ constraints \end{array} \tag{25}$$

IV. Simulation Settings

A. NLP solver settings

After going through the above discretization of the objective function, equation constraints, and inequality constraints using the chebyshev pseudospectral method. In order to prevent trajectory nonlinear optimization from falling into local optima as well as to speed up convergence, this paper chooses to utilize a two-layer optimization strategy, with the interior-point method chosen as the NLP solution algorithm for each layer of optimization. The upper-layer optimization algorithm has lower solution accuracy and exit requirements than the lower-layer optimization algorithm, because the upper-layer optimization is only to provide the lower-layer algorithm with a better initialization strategy to avoid the single-layer optimization algorithm from falling into the convergence domain of locally optimal solutions and having difficulty reaching the set optimization exit conditions for a long time. The specific parameters of the two-layer optimization algorithm are set as Table I.

TABLE I
NLP SOLVER SETTINGS

Parameter classification	NLP 1	NLP 2
Solving method	fmincon:interior-point	fmincon:interior-point
number of collocation points in chebyshev	9	15
Function Tolerance	1e-3	1e-6
Maximum number of iterations	1e3	1e4
Maximum number of function evaluations	1e6	5e6

TABLE II
PARAMETERS OF THE HEAVY DUTY VEHICLE

Parameters	Description	Value (m)
L_c	Tractor wheelbase	4.8
L_t	Semi-trailer wheelbase	11.75
L_{oH}	Distance from the off-axis point to the rear axle of the tractor	-0.635
Wb_t	Tractor wheelbase	1.9
Wb_s	Semi-trailer wheelbase	2.2
e_t	Distance from the front wheels of the tractor to the front bumper	0.38
e_s	Distance from the rear wheels of the to the rear bumper	0.44

B. Simulation conditions

In this paper, a nonlinear two-level trajectory optimization algorithm based on Chebyshev orthogonal collocation is proposed for the vertical parking problem of heavy duty vehicle.The parameters of the heavy duty vehicle used for the simulation are shown in Table II, Where, The subscripts t and s represent the parameters of tractor and semi-trailer respectively, Wb denotes the wheelbase, e is the distance from the tire to the bumper. And the parking and obstacle environment is shown in Fig. 3. The coordinates of the four vertices of obstacle A in clockwise order from the top left are $A_1(-30, 24), A_2(-6, 24), A_3(-6, 0), A_4(-30, 0)$, the coordinates of the vertex of the obstacle B are $B_1(6, 24), B_2(30, 24), B_3(30, 0), B_4(0, 0)$. The initial state of the heavy duty vehicle is $\left(27, 28, \frac{\pi}{2}, 0\right)$ and the final state is $(0, 1, 0, 0)$.

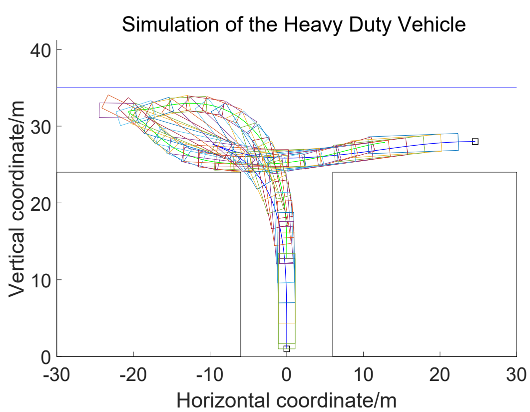

Fig. 3. heavy duty vehicle vertical parking simulation results

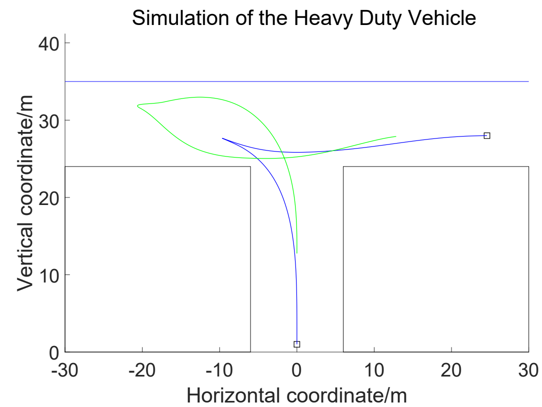

Fig. 4. heavy duty vehicle vertical parking simulation trajectory.

V. Simulation Results

It can be seen that the Chebyshev pseudospectral-based vertical parking trajectory planner for heavy duty vehicle can complete the vertical parking operation smoothly in narrow passages (Fig. 3), and the planned trajectory in Fig. 4 is also quite smooth, because the nonholonomic constraints of the system and the minimization of the control variables are fully considered in the trajectory optimization process. Fig. 5 shows how the state variables change during the parking process. It can be seen that the speed and steering wheel angle are in a small rate of change, so that the planned open-loop control output is also more informative for the subsequent controller design. The final planner completes the optimization of the time-optimal vertical parking trajectory, and the final time spent for parking is $t_f = 62.3999s$. The upper-level optimizer takes $13.2966s$, the lower-level optimizer takes $29.8818s$, and the total time is $43.1784s$.

VI. Conclusion

The time-optimal heavy duty vehicle vertical parking trajectory optimization method based on Chebyshev pseudospectral method proposed in this paper can fully consider the nonholonomic constraints, jackknifing constraints, control constraints, obstacle avoidance constraints, time-optimal and other characteristics of the system. The vertical parking trajectory planning for heavy duty vehicle in narrower aisles is done accurately and less time-consumingly in a unified framework without relying on search and sampling methods,

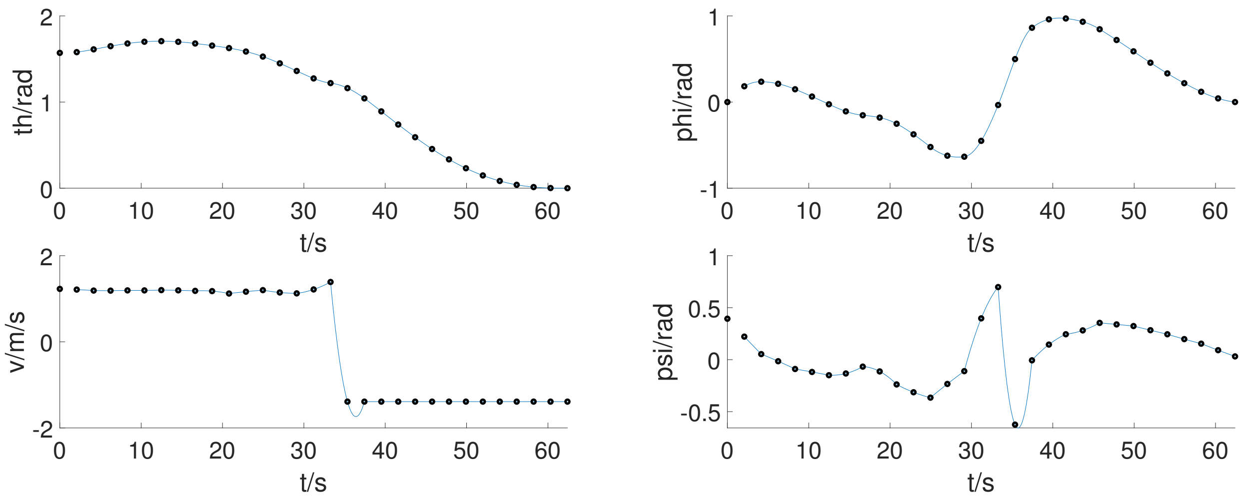

Fig. 5. Changes in state variables and control variables.

and the corresponding open-loop control rate is solved for subsequent interface interaction with the control module.

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