Path Generation for Articulated Steering Type Vehicle Using Symmetrical Clothoid

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Abstract

Autonomous trajectory following of a wheel loader is closely related to v-shape path planning. Given the desired initial position and orientation of the wheel loader as $(x_o,\ y_o,\ \theta_o)$ and the desired final position and orientation as $(x_{truck},\ y_{truck},\ \theta_{truck})$, successful v-shape path planning can be achieved by the derivations of 2 important parameters of the clothoid curve, k (sharpness) and s_m (total distance of 1 clothoid curve) and determinations of the summit point of v-shape path, (x_e, y_e) and direction, θ_e.

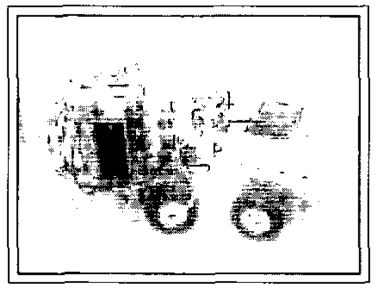

Figure 1: Miniature Wheel Loader at Intelligent Robot Laboratory, University of Tsukuba

1. Introduction

We have been interested in autonomous control of an articulated steering type vehicle like a wheel loader(WL)[1]. During the operation at the construction site, a typical motion of this machine such as shovelling the material from the gravel pile and then unloading to the bed of dump truck, forms a v-shaped path as in Figure 2. If we take this path with a combination of line segments and circular arcs, WL has to experience the discontinuity of curvatures at the junction between line and arc segment which results in the step transition of steering angle(Figure 3) because curvature of the vehicle's trajectory and steering angle is proportional if the steering angle is small. However, due to the inertia of wheel loader, it hardly tracks the line-arc combination trajectory during the transition from B to SE or from SF to C in Figure 3 and tracking error may remain. On the other hand, it seems to be a good idea if curvature profile ought to be changed to a profile of A to D via B, CC ,and C. With profile from B to C via CC, WL will move on a symmetrical clothoid(Figure 2)[1] which has the initial position at B, and the trajectory curvature, i.e., steering angle will be proportionally increased from zero as function of the distance s until the maximum value at CC. Then the steering angle will be linearly decreased as a function of distance s to zero at position C in Figure 2 and 3. The same situation is applied to the course from A to gravel pile position as well. The symmetrical clothoid has better property than line-arc combinations because there is no discontinuity of the trajectory curvature[4], which means that angular velocity of the body of WL is continuous and its angular acceleration is also bounded. Usually, at the construction site, the suitable position for shovelling gravel at the gravel pile, $(x_o,\ y_o)$ and the direction, θ_o with which a wheel loader will leave it, as well as the position of dump truck, $(x_{truck},\ y_{truck})$ and the direction, θ_{truck} with which the wheel loader will approach it, are initially specified. The problem discussed here is how to plan the v-shaped path with combinations of line segments and symmetrical clothoids under the initial and termination constraints given by $(x_o,\ y_o,\ \theta_o)$ and $(x_{truck},\ y_{truck},\ \theta_{truck})$. This problem can be resolved into two parts:

1. Finding a location of summit of v-shape, and
2. Calculation of clothoid curve parameters.

This paper is organized as follows; in Section 2., an algorithm to find v-shape summit location is presented. Section 3. illustrates how to obtain clothoid parameters. Due to some constraints such as physical limitation of maximum steering angle of wheel loader, Section 3..1,then, illustrates the minimum value of existing distance[3], r_1, of a symmetrical clothoid. Finally, Section 4. demonstrates some examples to plan the desired path.

2. Location algorithm to find the summit of v-shape

Typical motions of a wheel loader is illustrated with series of actions: shovelling gravel, transporting gravel and loading it onto the truck bed. These series runs on the v-shape trajectory which can be depicted in Figure 2.

After gravel shovelling position (x_o,y_o) with the proper body alignment of θ_o, and the truck bed position (x_{truck},y_{truck}) with the right posture of θ_{truck} are given in advance, the path planner has first to figure out the appropriate position (x_e,y_e) and orientation θ_e of a summit of the v-shape. To consider the algorithm for finding a location of the summit of the v-shape, let us consider such a local coordinate system A that the origin of A is located at $(x_o,\ y_o)$

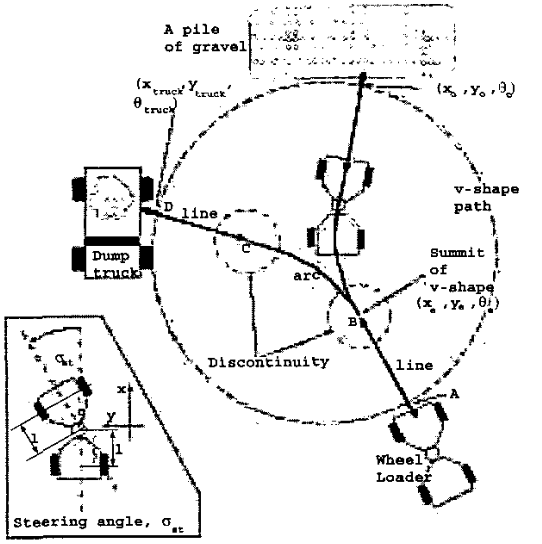

Figure 2: Typical motion of Wheel Loader as V-shape path

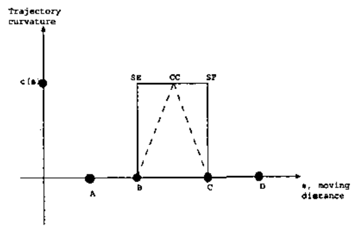

Figure 3: Steering angle profile of Wheel Loader

and the local x-axis of A is in the direction of θ_o. Therefore, the gravel point position on A can be denoted as $(x_o^l, y_o^l) = (0, 0)$, orientation of the WL, $\theta_o^l = 0$, and initial steering angle, $\sigma_o = 0$. Accordingly, the truck bed position and posture, $(x_{truck}, y_{truck}, \theta_{truck})$ is translated into $(x_{truck}^l, y_{truck}^l, \theta_{truck}^l)$ with respect to A, a particular point $P(x_p, y_p)$ on the local x-axis of A classifies the algorithm into 3 cases. The point P is a cross point of the local x-axis and a line passing through $(x_{truck}^l, y_{truck}^l)$ whose angle is θ_{truck}^l. Then classified cases are:

1. $x_p^l > 0$ and $\theta_{truck}^l \neq 0$
2. $x_p^l \leq 0$ and $\theta_{truck}^l \neq 0$
3. $\theta_{truck}^l = 0$.

Case1: $x_p^l > 0$ and $\theta_{truck}^l \neq 0$

Algorithm for locating the summit of v-shape $S(x_e^l, y_e^l)$ and orientation θ_e^l can be concluded as follows:

1. After the local coordinate system A is set up, we draw line from $T(x_{truck}^l, y_{truck}^l)$ with slope of tan θ_{truck}^l to local x-axis. This step will create point $P(x_p^l, y_p^l)$ at the intersection point on local x-axis. r_2 is the distance from T to P(Figure 5).

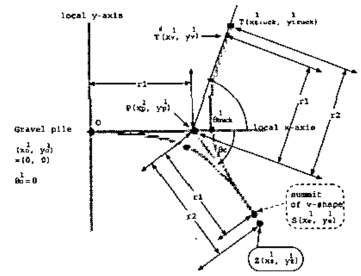

Figure 4: v-shape path

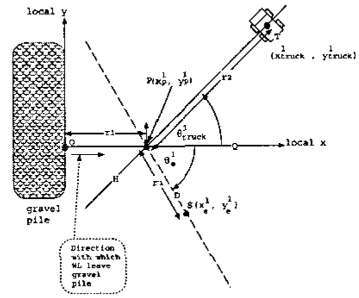

Figure 5: Locating point P on local x-axis and θ_e^l determination $(x_p > 0)$

2. Drawing dotted line passing through point $P(x_p^l, y_p^l)$ which divides $\widehat{HPQ}$ into 2 equal half angles, $\widehat{HPD}$ and $\widehat{DPQ}$. This step will produce θ_e^l(Figure 5).
3. $S(x_e^l, y_e^l)$ is on the dotted line at distance r_1, where $r_1 = \overline{OP}$, and in different quadrant from $(x_{truck}^l, y_{truck}^l)$(Figure 5).

$$x_p^l = x_{truck}^l - \frac{y_{truck}^l}{\tan\theta_{truck}^l} \quad (1)$$

$$\theta_e^l = \begin{cases} -\frac{\pi - \theta_{truck}^l}{2} & if\ \theta_{truck}^l \geq 0 \\ \frac{\pi + \theta_{truck}^l}{2} & if\ \theta_{truck}^l < 0 \end{cases} \quad (2)$$

$$r_1 = x_{truck}^l - \frac{y_{truck}^l}{\tan\theta_{truck}^l} \quad (3)$$

$$r_2 = \frac{y_{truck}^l}{\sin\theta_{truck}^l} \quad (4)$$

$$x_e = r_1(1+\cos\theta_e^l) \quad (5)$$

$$y_e = r_1 \sin\theta_e^l \quad (6)$$

After a summit of the v-shape, $S(x_e^l, y_e^l)$ and orientation θ_e^l, are located, two different paths on which WL is able to move can be demonstrated depend on the following two criteria:

1. $r_1 \leq r_2$, then, the symmetrical clothoids, $O \frown S$, and $S \frown T'$ will be defined (see Section 3.). A section

between T′ and T will be filled with a line segment(Figure 4).

2. $r_1 > r_2$, then, the symmetrical clothoid, O⌒S will be defined. However, a section between S and Z will be filled with a line segment, then the symmetrical clothoid, Z⌒T will be defined subsequently.

Case2: $x_p^l \leq 0$ and $\theta_{truck}^l \neq 0$

Algorithm for locating the summit of v-shape $S(x_e^l, y_e^l)$ and orientation θ_e^l can be concluded as follows:

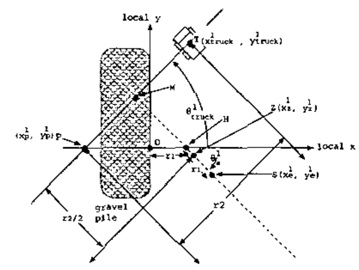

Figure 6: Locating point P on local x-axis and θ_e^l determination ($x_p < 0$)

1. After the local coordinate system A is set up, we draw line from $T(x_{truck}^l, y_{truck}^l)$ with slope of $\tan\theta_{truck}^l$ to local x-axis. This step will create point $P(x_p^l, y_p^l)$ at the intersection point on local x-axis(Figure 6), where $r_2 = \overline{TP}$.

2. Dividing $\overline{PT}$ into 2 equal half distance, point M. Drawing perpendicular dotted line to $\overline{PT}$ from point M to local x-axis at point H. This will generate θ_e^l(Figure 6).

3. $S(x_e^l, y_e^l)$ is on the dotted line extending $\overline{MH}$ by distance r_1, where $r_1 = \overline{OH}$ and in different quadrant from $(x_{truck}^l, y_{truck}^l)$(Figure 6).

$$r_2 = \frac{y_{truck}^l}{\sin\theta_{truck}^l} \tag{7}$$

$$x_p^l = x_{truck}^l - \frac{y_{truck}^l}{\tan\theta_{truck}^l} \tag{8}$$

$$\theta_{tr}^l = |\,\theta_{truck}^l\,| \tag{9}$$

$$\theta_e^l = \begin{cases} -(\frac{\pi}{2} - \theta_{tr}^l) & if\ \theta_{truck}^l \geq 0 \\ (\frac{\pi}{2} - \theta_{tr}^l) & if\ \theta_{truck}^l < 0 \end{cases} \tag{10}$$

$$r_1 = \frac{r_2}{2\sin[\frac{\pi}{2} - \theta_{tr}^l]} + x_p^l \tag{11}$$

$$x_e^l = r_1(1 + \cos\theta_e^l) \tag{12}$$

$$y_e^l = r_1 \sin\theta_e^l \tag{13}$$

After a summit of the v-shape, $S(x_e^l, y_e^l)$, and orientation, θ_e^l are located, path on which WL is able to move can be demonstrated as follows:
The symmetrical clothoid, O⌒S will be defined. A section between S and Z will be filled with a line segment, where $\overline{MZ} = r_2/2$. Consequently, the symmetrical clothoid, Z⌒T will be defined(Figure 6).

Case3: $\theta_{truck} = 0$

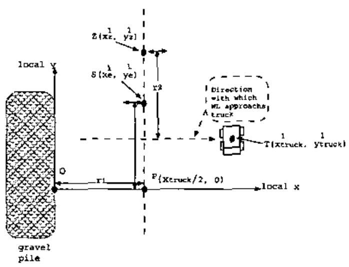

Figure 7: Locating point P on local x-axis and θ_e^l determination

Algorithm for locating the summit of v-shape $S(x_e^l, y_e^l)$ and orientation θ_e^l can be concluded as follows:

1. After the local coordinate system A is set up, we draw dotted line passing through point $P(x_{truck}/2, 0)$ and parallel to local y-axis(Figure 7).

2. $S(x_e, y_e)$ is on the dotted line at the distance $r1$ from P, where $r_1 = \overline{OP}$(Figure 7).

$$r_2 = \frac{x_{truck}^l}{2} \tag{14}$$

$$\theta_e^l = \begin{cases} \frac{\pi}{2} & if\ y_{truck}^l \geq 0 \\ -\frac{\pi}{2} & if\ y_{truck}^l < 0 \end{cases} \tag{15}$$

$$r_1 = \frac{x_{truck}^l}{2} \tag{16}$$

$$x_e^l = r_1 \tag{17}$$

$$y_e^l = \begin{cases} r_1 & if\ y_{truck}^l \geq 0 \\ -r_1 & if\ y_{truck}^l < 0 \end{cases} \tag{18}$$

(19)

After a summit of the v-shape, $S(x_e^l, y_e^l)$, and orientation, θ_e^l are located, path on which WL is able to move can be demonstrated as follows:
The symmetrical clothoid, O⌒S will be defined. A section between S and Z will be filled with a line segment, where $\overline{SZ} = y_{truck}^l$. Consequently, the symmetrical clothoid, Z⌒T will be defined(Figure 7).

3. Symmetrical clothoid parameter determinations

As we see from the previous section, V-shape path partially consists of 2 symmetrical clothoids. The characteristics of 1 symmetrical clothoid can be illustrated in Figure 8. We define such a symmetrical clothoid curve C that has following properties:

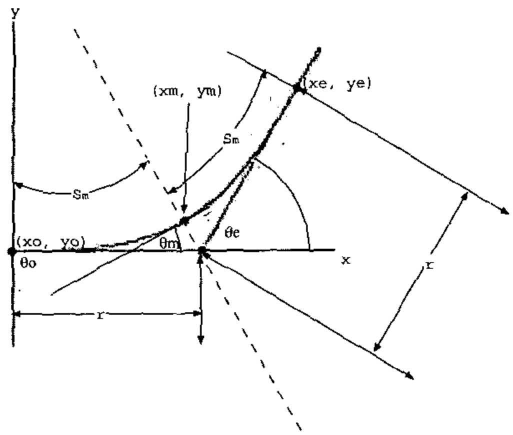

Figure 8: Symmetrical Clothoid

1. At (x_o, y_o), and (x_e, y_e), curvature c of C is zero.
2. At the middle of (x_m, y_m) *i.e.* equidistance point from (x_o, y_o) and from (x_e, y_e) along C, whose distance is denoted as s_m, the symmetrical clothoid C has maximum curvature c_m.
3. Curvature c along C from (x_o, y_o) to (x_m, y_m) is proportional to s of distance from (x_o, y_o) along C: $c = ks$, which yields $c_m = ks_m$, where k is so called as "sharpness".
4. Curvature c along C from (x_m, y_m) to (x_e, y_e) is denoted as :
$$c = -k(s - s_m) + c_m. \tag{20}$$
Here, s is distance along C from (x_o, y_o).

According to the properties of the symmetrical clothoid, parameters of curvature at s, $c(s)$, $\theta(s)$ of orientation of WL at s, and WL position, $(x(k, s), y(k, s))$ at s with k, are derived as follows:

$$c(s) = \begin{cases} ks & ;(0 \leq s \leq s_m) \\ -ks + ks_m & ;(s_m \leq s \leq 2s_m) \end{cases} \tag{21}$$
$$\theta(s) = \int_0^s c(s)ds, \tag{22}$$
$$x(k, s) = \int_0^s \cos[c(s)]ds, \tag{23}$$
$$y(k, s) = \int_0^s \sin[c(s)]ds. \tag{24}$$

We can easily derive following properties:

$$s_m = \sqrt{\frac{\theta_e}{k}} \tag{25}$$
$$\theta_m = \frac{\theta_e}{2} \tag{26}$$

Let us consider k first. We can see that k is a function of (y_e, θ_e) or (x_e, θ_e), assuming if $-\pi/2 \leq \theta_e \leq \pi/2$. Considering Figure 8, x_e and y_e are denoted as:

$$x_e = r + r\cos\theta_e \tag{27}$$
$$y_e = r\sin\theta_e. \tag{28}$$

From the properties of $x(k, s)$ and $y(k, s)$,

$$x_m = x_o + \int_0^{s_m} cos[\frac{ks^2}{2}]ds \tag{29}$$
$$y_m = y_o + \int_0^{s_m} sin[\frac{ks^2}{2}]ds \tag{30}$$

However, (x_m, y_m) is simultaneously denoted as:

$$x_m = x_e - \int_0^{s_m} cos[\frac{-ks^2}{2} + ks_m s + \frac{ks_m^2}{2}]ds \tag{31}$$
$$y_m = y_e - \int_0^{s_m} sin[\frac{-ks^2}{2} + ks_m s + \frac{ks_m^2}{2}]ds \tag{32}$$

Therefore, we can obtain following equations:

$$x_e - x_o = \int_0^{s_m} \{\cos[\frac{ks^2}{2}] + \cos[\frac{-ks^2}{2} + ks_m s + \frac{ks_m^2}{2}]\}ds \tag{33}$$
$$y_e - y_o = \int_0^{s_m} \{\sin[\frac{ks^2}{2}] + \sin[\frac{-ks^2}{2} + ks_m s + \frac{ks_m^2}{2}]\}ds \tag{34}$$

In the mathematical formulation, k and s_m could be derived if we could solve the simultaneous equations (33) and (34), which is very difficult to be solved analytically. Therefore, we will solve k and s_m by numerical way. For the time being, we expand $\cos[f(x)]$ into Taylor series of $f(x)$ to 6^{th} order and $\sin[f(x)]$ into the series of $f(x)$ to 5^{th} order. As s_m is already denoted by $s_m = \sqrt{\theta_e/k}$, substitution of s_m into Equation (33) or (34) yields equation of k, which can be solved numerically. After k is obtained, s_m is immediately obtained. Then, the symmetrical clothoid is derived with parameter s.

We solved (33) and (34) by means of Mathematica to obtain numerical solution. See[5] for analytical form to solve this problem to obtain k and s_m. However, even if we take a form in [5], we have to obtain the solution by numerical integration in practice.

3..1 Feasible region of (x_e, y_e)

Due to assumption of $-\pi/2 \leq \theta_e \leq \pi/2$ and physical limitation of steering angle $-\sigma_{st}^{max} \leq \sigma_{st} \leq \sigma_{st}^{max}$ (for our model, $\sigma_{st}^{max} = \pi/9$), these affect the symmetrical clothoid destination(x_e, y_e) of WL to be within the possible region.

Calculating the realizable area has to refer to the following equations,

$$k = \left(\frac{\sigma_{st}^{max}}{2\,l}\right)^2 \frac{1}{\theta_e} \qquad (35)$$

$$s_m = \sqrt{\frac{\theta_e}{k}} \qquad (36)$$

where l is distance between front(or rear) axle and center joint.

By varying θ_e, and σ_{st} from the minimum boundary to the maximum one, the feasible (x_e,y_e) region can be illustrated in Figure 9. From Figure 9, the feasible region of (x_e,y_e) is the area within the perimeter beginning from point a, moving as a symmetrical clothoid with $\sigma_{st} = \sigma_{st}^{max}(=\pi/9$ for our model Figure 1) to point b and as a symmetrical clothoid with $\sigma_{st}^{max} = -\sigma_{st}^{max}$ to point i, and then translating from b to c and to ∞ as a straight line with the equation of $y = x$, also translating from i to h and to $-\infty$ as a straight line with the equation of $y = -x$.

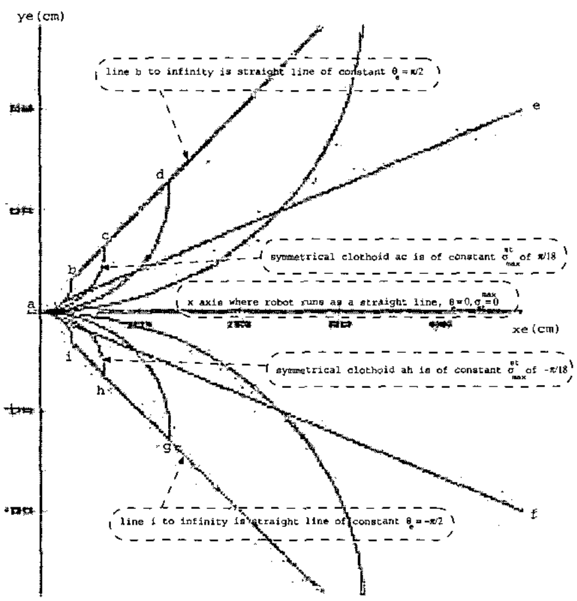

Figure 9: Feasible $(x_e,\ y_e)$ region

4. Path generation results

By using location algorithm of the summit of v-shape from section 2., path generation results are illustrated in Figure 10, 11 and 12:

Figure 10 is an example of path as a result of applying location algorithm from Case 1 of Section 2.. A symmetrical clothoid O⌒S of which k of 3.97302e^{-6} rad/cm^2 and distance($2s_m$) of 2× 534.361 cm is defined first. After that symmetrical clothoid S⌒T′ of which k of 3.97302e^{-6}

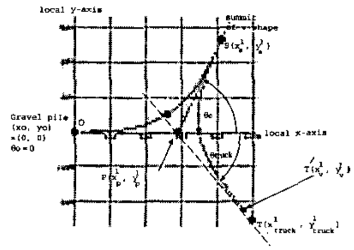

Figure 10: $T(x^l_{truck},y^l_{truck})$ = (1000.0, -500.0) cm, $\theta^l_{truck} = \frac{-50\pi}{180}\ rad$, $x^l_p \approx 600.0$ cm

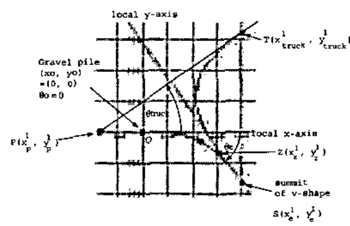

Figure 11: $T(x^l_{truck},y^l_{truck})$ = (800.0, 800.0) cm, $\theta^l_{truck} = \frac{35\pi}{180}\ rad$, $x^l_p = -342.518$ cm

rad/cm^2 and distance($2s_m$) of 2×534.361 cm is defined. Finally, a section between $T'(x^l_v,y^l_v)$ and $T(x^l_{truck},y^l_{truck})$, will be filled with a line segment.

Figure 11 is an example of path as a result of applying location algorithm from Case 2 of Section 2.. A symmetrical clothoid O⌒S of which k of -4.15979e^{-6} rad/cm^2 and distance($2s_m$) of 2×480.379 cm is defined first. Then, a section between $S(x^l_z,y^l_z)$ and $Z(x^l_z,y^l_z)$ is filled with line segment. Finally a symmetrical clothoid, Z⌒T of which k of -4.59368e^{-6} rad/cm^2 and distance($2s_m$) of 2×584.763 cm is defined.

Figure 12 is an example of path as a result of applying location algorithm from Case 3 of Section 2.. A symmetrical clothoid, O⌒S of which k of 8.93631e^{-6} rad/cm^2 and distance($2s_m$) of 2×419.258 cm is defined first. Next, a section between $S(x^l_e,y^l_e)$ and $Z(x^l_z,y^l_z)$ is filled with a line segment. Finally, a symmetrical clothoid, Z⌒T of which k of 8.93631e^{-6} rad/cm^2 and distance($2s_m$) of 2×419.258 cm is defined.

5. Conclusion and Future Work

For a symmetrical clothoid, given the desired initial position and orientation as (x_o,y_o,θ_o) respectively which are always locally $(0,0,0)$ and 2 of 3 parameters of the desired destination and orientation which are (x_e,y_e,θ_e), on the condition that all of 3 parameters of the desired destination and orientation must comply with the constraints of eq(27), eq(28) and $\frac{-\pi}{2} \leq \theta_e \leq \frac{\pi}{2}$, this paper presents the way to draw a symmetrical clothoid beginning from the desired initial po-

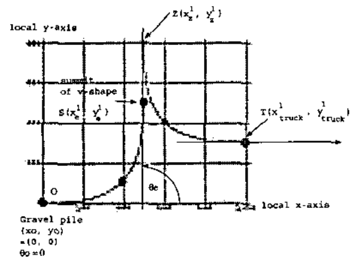

Figure 12: $T(x^l_{truck}, y^l_{truck}) = (1000.0,\ 300.0)$ cm, $\theta^l_{truck} = 0\ rad, x^l_p = 500$ cm

sition and orientation, and terminating at a destination and an orientation with the acceptable error. Two importants parameters, k(sharpness) and s_m (total length of a clothoid curve) are symbolically derived. With the numerially calculated value of k and s_m, the allowable error between the desired position and orientation and the actual ones can be attained. On the ground that we introduced Taylor series approximations of 6^{th} order around point $f(x) = 0$ for trigonometric functions during k and s_m derivation, for each calculated k and s_m, the maximum error of thedestination will arise when the desired parameter, $|\ \theta_e\ | = \frac{\pi}{2}$. Figure 13 shows the maximum error of the destination when $\theta_e = \frac{\pi}{2}$.

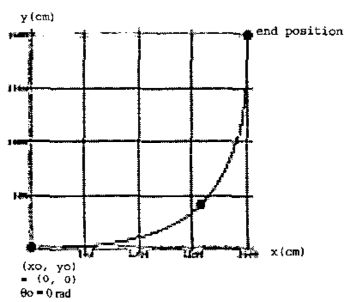

Figure 13: desired destination is $(2000, 2000)$ cm with orientation of $\frac{\pi}{2}$ rad, actual end position is $(1996.57, 1996.57)$ cm with orientation of $\frac{\pi}{2}$ rad

Extending the fundamental concept of symmetrical clothoid to v-shape path planning, with the same initial position and orientation of symmetrical clothoid, however, the destination becomes (x_{truck}, y_{truck}) and θ_{truck}, v-shape path can be created by primarily locating the summit of v-shape(x_e, y_e) and orientation θ_e, then traversing from the intial position to the summit and from the summit to the truck position is the courses of 2 symmetrical clothoids. Locating summit of v-shape(x_e, y_e) together with orientation θe depends on (x_{truck}, y_{truck}) and θ_{truck}, which can be classified into 3 cases, yielding to different v-shape configurations. Nevertheless, wheel loader has physically limitation of maximum steering angle $|\ \sigma^{max}_{st}\ | = \frac{20\pi}{180}$, and the introduction of Taylor series approximation for trigonometric functions brings up the constraint condition of $\frac{-\pi}{2} \leq \theta_e \leq \frac{\pi}{2}$, therefore, for a symmetrical clothoid, the feasible region of the destination which can be expressed as the graphical representation of r_1 against θ_e must come into consideration, which means as well for v-shape path planning. After the derivation of v-shape path planning, future work will implement the concepts into the real robot and controller with the main kinematic relationship among k $\frac{d\sigma_{st}}{dt} = \omega_{st}$, steering angular velocity and travelling velocity of robot on the symmetrical clothoid, $\frac{ds}{dt}$.

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