# Project

Jinyuan Wu

January 10, 2022

**Problem 1** Consider a one dimensional infinite chain on the $z$ direction consisting of metallic balls, each of which have radius $a$ and is made of a metal with permittivity

$$\epsilon_\mathrm{r} = 1 - \frac{\omega_\mathrm{p}^2}{\omega(\omega + \mathrm{i}\gamma)}. \tag{1}$$

When $a \to 0$, we have

$$\alpha(\omega) = 4\pi\epsilon_0 a^3 \frac{\epsilon_\mathrm{r}(\omega) - 1}{\epsilon_\mathrm{r}(\omega) + 2}, \tag{2}$$

We use Mathematica to plot the real and the imaginary part of $\alpha(\omega)$ in Figure 1 on page 1. TODO: features

**Problem 2** We need to solve

$$\boldsymbol{p}_m = \alpha(\boldsymbol{E}_\mathrm{ext}(\boldsymbol{r}_m) + \omega^2 \mu_0 \sum_{n \neq m} \overleftrightarrow{\boldsymbol{G}} (\boldsymbol{r}_m - \boldsymbol{r}_n) \cdot \boldsymbol{p}_n), \tag{3}$$

and when there is no external field, by the Bloch condition

$$\boldsymbol{p}_m = \boldsymbol{u} \mathrm{e}^{\mathrm{i}kz_m}, \tag{4}$$

we have

$$\boldsymbol{u} \mathrm{e}^{\mathrm{i}kz_m} = \alpha \omega^2 \mu_0 \sum_{n \neq m} \overleftrightarrow{\boldsymbol{G}} (\boldsymbol{r}_m - \boldsymbol{r}_n) \cdot \boldsymbol{u} \mathrm{e}^{\mathrm{i}kz_n},$$

$$\left( \overleftrightarrow{\boldsymbol{I}} - \alpha \omega^2 \mu_0 \sum_{n \neq m} \overleftrightarrow{\boldsymbol{G}} (\boldsymbol{r}_m - \boldsymbol{r}_n) \mathrm{e}^{\mathrm{i}kz_n} \mathrm{e}^{-\mathrm{i}kz_m} \right) \boldsymbol{u} = 0,$$

and we have

$$\overleftrightarrow{\boldsymbol{M}} = \alpha^{-1} \overleftrightarrow{\boldsymbol{I}} - \omega^2 \mu_0 \sum_{n \neq m} \overleftrightarrow{\boldsymbol{G}} (\boldsymbol{r}_m - \boldsymbol{r}_n) \mathrm{e}^{\mathrm{i}k(z_n - z_m)}, \quad \overleftrightarrow{\boldsymbol{M}} \boldsymbol{u} = 0, \tag{5}$$

and we need to evaluate

$$\overleftrightarrow{\boldsymbol{W}} = \omega^2 \mu_0 \sum_{n \neq m} \overleftrightarrow{\boldsymbol{G}} (\boldsymbol{r}_m - \boldsymbol{r}_n) \mathrm{e}^{\mathrm{i}k(z_n - z_m)}. \tag{6}$$

The eigenvalues are actually "eigen polarization":

$$\overleftrightarrow{\boldsymbol{M}} \cdot \boldsymbol{u} = \frac{1}{\lambda^\mathrm{eigen}} \boldsymbol{u}. \tag{7}$$

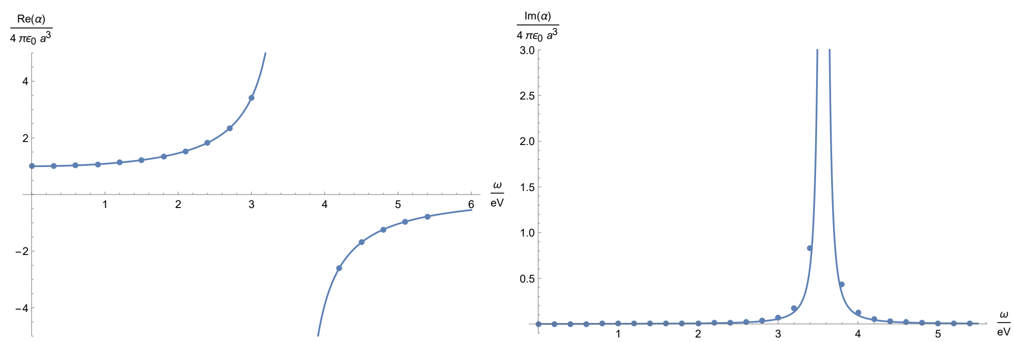


Figure 1: The real and the imaginary part of $\alpha(\omega)$. The lines are plotted by definition, and the scattered points are obtained by K-K relations. (a) The real part. (b) The imaginary part.

In the $\gamma \to 0$ limit, (5) can be written as

$$H\psi = \frac{\omega^2}{\omega_{\mathrm{p}}^2}\psi, \tag{8}$$

where