Project

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December 8, 2021

Consider a one dimensional infinite chain on the z direction consisting of metallic balls, each of which have radius a and is made of a metal with permittivity

$$\epsilon_{\mathrm{r}} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega + \mathrm{i}\gamma)}. \tag{1}$$

When $a \to 0$, we have

$$\alpha(\omega) = 4\pi\epsilon_0 a^3 \frac{\epsilon_{\mathrm{r}}(\omega) - 1}{\epsilon_{\mathrm{r}}(\omega) + 2}, \tag{2}$$

$$\boldsymbol{p}_m = \alpha(\boldsymbol{E}_{\mathrm{ext}} + \omega^2 \mu_0 \sum_{n \neq m} \overleftrightarrow{\boldsymbol{G}} \cdot \boldsymbol{p}_n), \tag{3}$$

and by the Bloch condition

$$\boldsymbol{p}_m = \boldsymbol{u} \mathrm{e}^{\mathrm{i}kz_m}, \tag{4}$$

we have

$$\overleftrightarrow{\boldsymbol{M}} \cdot \boldsymbol{u} \mathrm{e}^{\mathrm{i}kz_m} = \left(\overleftrightarrow{\boldsymbol{I}} - \alpha\omega^2\mu_0 \sum_{n \neq m} \overleftrightarrow{\boldsymbol{G}} (\boldsymbol{r}_m - \boldsymbol{r}_n) \mathrm{e}^{\mathrm{i}kz_n} \mathrm{e}^{-\mathrm{i}kz_m} \right) \boldsymbol{u} \mathrm{e}^{\mathrm{i}kz_m} = 0. \tag{5}$$

The eigenvalues are actually "eigen polarization":

$$\overleftrightarrow{\boldsymbol{M}} \cdot \boldsymbol{u} = \frac{1}{\lambda^{\mathrm{eigen}}} \boldsymbol{u}. \tag{6}$$

In the $\gamma \to 0$ limit, (5) can be written as

$$H\psi = \frac{\omega^2}{\omega_{\mathrm{p}}^2} \psi, \tag{7}$$

where