Quantum Optics, Homework 5

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January 5, 2022

Stochastic wavefunction of a leaky cavity photon field The effective Halmitonian for the photon field in a leaky cavity is given by: $H_{\text{eff}} = \hbar\left(-\mathrm{i}\frac{\kappa}{2}\right)a^\dagger a$, with the associated quantum jump operator: $C = \sqrt{\kappa}a$. Discuss the non-Hermitian evolution of stochastic wavefunction $|\psi(t)\rangle$ without quantum jump, and provide wavefunction after a quantum jump at time t.

(a) For $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|3\rangle + |1\rangle)$

(b) For $|\psi(t=0)\rangle = |\alpha\rangle$

(c) For $|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle)$ (here we consider $|\alpha|^2 \gg 1$)

Solution The time evolution operator is now

$$U(t,0) = \mathrm{e}^{-\kappa n/2}. \tag{1}$$

(a) We have

$$U(t,0)\,|\psi(0)\rangle = \frac{1}{\sqrt{2}}(\mathrm{e}^{-3\kappa t/2}\,|3\rangle + \mathrm{e}^{-\kappa t/2}\,|1\rangle),$$

and after normalization we have

$$|\psi(t)\rangle_{\text{no jump}} = \frac{|3\rangle + \mathrm{e}^{\kappa t}\,|1\rangle}{\sqrt{1+\mathrm{e}^{2\kappa t}}}. \tag{2}$$

After a quantum jump, we have

$$C\,|\psi(t)\rangle_{\text{no jump}} = \kappa\frac{\sqrt{3}\,|2\rangle + \mathrm{e}^{\kappa t}\,|0\rangle}{\sqrt{1+\mathrm{e}^{2\kappa t}}},$$

and after normalization we have

$$|\psi(t)\rangle_{\text{jump}} = \frac{\sqrt{3}\,|2\rangle + \mathrm{e}^{\kappa t}\,|0\rangle}{\sqrt{3+\mathrm{e}^{2\kappa t}}}. \tag{3}$$

(b) We have

$$\begin{aligned}
U(t,0)\,|\alpha\rangle &= \mathrm{e}^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\mathrm{e}^{-\kappa n t/2}\,|n\rangle \\
&= \mathrm{e}^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{1}{\sqrt{n!}}(\alpha\mathrm{e}^{-\kappa t/2})^n\,|n\rangle \\
&= \mathrm{e}^{-|\alpha|^2/2}\mathrm{e}^{|\alpha|^2\mathrm{e}^{-\kappa t}/2}\,|\alpha\mathrm{e}^{-\kappa t/2}\rangle \\
&= \mathrm{e}^{|\alpha|^2(\mathrm{e}^{-\kappa t}-1)/2}\,|\alpha\mathrm{e}^{-\kappa t/2}\rangle,
\end{aligned}$$

and therefore

$$|\psi(t)\rangle_{\text{no jump}} = |\alpha\mathrm{e}^{-\kappa t/2}\rangle. \tag{4}$$

After a quantum jump, we have

$$C\,|\psi(t)\rangle_{\text{no jump}} = \kappa a\,|\alpha\mathrm{e}^{-\kappa t/2}\rangle = \kappa\alpha^{-\kappa t/2}\,|\alpha\mathrm{e}^{-\kappa t/2}\rangle,$$

and therefore

$$|\psi(t)\rangle_{\text{jump}} = |\alpha\mathrm{e}^{-\kappa t/2}\rangle. \tag{5}$$

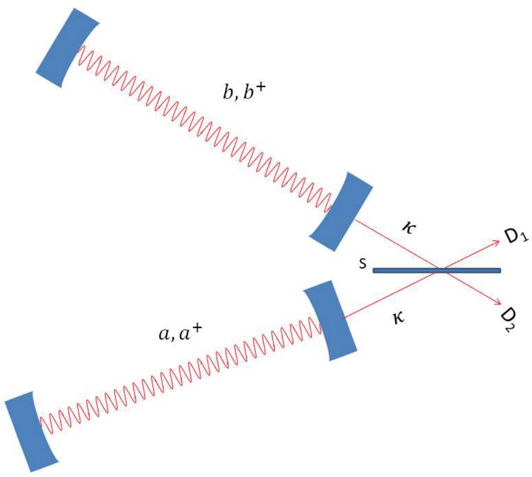

Figure 1: Output of two cavities are mixed by splitter S and detected by photon-counter D_1 and D_2

(c) We have

$$U(t,0)\left|\alpha\right\rangle = \mathrm{e}^{|\alpha|^2(\mathrm{e}^{-\kappa t}-1)/2}\left|\alpha\mathrm{e}^{-\kappa t/2}\right\rangle, \quad U(t,0)\left|-\alpha\right\rangle = \mathrm{e}^{|\alpha|^2(\mathrm{e}^{-\kappa t}-1)/2}\left|-\alpha\mathrm{e}^{-\kappa t/2}\right\rangle,$$

and therefore after normalization we have

$$\left|\psi(t)\right\rangle_{\text{no jump}} = \frac{1}{\sqrt{2}}(\left|\alpha\mathrm{e}^{-\kappa t/2}\right\rangle + \left|-\alpha\mathrm{e}^{-\kappa t/2}\right\rangle). \tag{6}$$

Here we use the approximation that α is large and therefore $\left|\alpha\right\rangle$ and $\left|-\alpha\right\rangle$ almost have no intersection. After a quantum jump, we have

$$C\left|\psi(t)\right\rangle_{\text{no jump}} = \frac{1}{\sqrt{2}}(\alpha\mathrm{e}^{-\kappa t/2}\left|\alpha\mathrm{e}^{-\kappa t/2}\right\rangle - \alpha\mathrm{e}^{-\kappa t/2}\left|-\alpha\mathrm{e}^{-\kappa t/2}\right\rangle),$$

so we have

$$\left|\psi(t)\right\rangle_{\text{jump}} = \frac{1}{\sqrt{2}}(\left|\alpha\mathrm{e}^{-\kappa t/2}\right\rangle - \left|-\alpha\mathrm{e}^{-\kappa t/2}\right\rangle). \tag{7}$$

Stochastic wavefunction of two cavity fields We consider the setup in Figure 1 on page 2, the leak fields of two identical cavities are mixed by a beamsplitter S and then detected by the photon counter D1 and D2. Following the stochastic wavefunction method, the effective Hamiltonian for the photon field of the twocavity system can be written as $H_{\text{eff}} = \hbar\left(-i\frac{\kappa}{2}\right)(a^+a + b^+b)$. While normally we would have quantum jump operators of $C_a = \sqrt{\kappa}a$ and $C_b = \sqrt{\kappa}b$, here it is more convenient to introduce collective jump operators $C_1 = \sqrt{\kappa}(ta + rb)$ and $C_2 = \sqrt{\kappa}\,(-r^*a + tb)$, with r, t to be the reflective and transmission coefficients of the beamsplitter S.

2.1 Consider the initial state to be a product coherent state: $\left|\psi(0)\right\rangle = \left|\alpha, \beta\right\rangle$. Evaluate the stochastic wavefunction of the photon field, $\left|\psi_{\mathrm{S}}(\mathrm{t})\right\rangle$, for the non-Hermitian evolution without any quantum jump, and after a quantum jump by C_1 operator (D1 "click")

2.2 Consider the simple situation of a 50% beamspitter, $r = t = 1/\sqrt{2}$, continue with the first problem to derive the photon detection rate $\gamma_1(t) = \left\langle\psi(t)\left|C_1^+C_1\right|\psi(t)\right\rangle$ and $\gamma_2(t) = \left\langle\psi(t)\left|C_2^+C_2\right|\psi(t)\right\rangle$. Is it possible to properly choose non-zero α and β values, so as to have $\gamma_1 \equiv 0$?

2.3 Repeat 2.1 with the Fock initial state $\left|\psi(0)\right\rangle = \left|N, N\right\rangle$.

2.4 Continue with 2.2, again assuming $r = t = 1/\sqrt{2}$, evaluate $\gamma_1(t) = \left\langle\psi(t)\left|C_1^+C_1\right|\psi(t)\right\rangle$ and $\gamma_2(t) = \left\langle\psi(t)\left|C_2^+C_2\right|\psi(t)\right\rangle$ before there is any quantum jump, and after a quantum jump with a D_1 "click". For $N = 1$. Discuss your results in terms of the HongOu-Mandel effect.

Solution

2.1 Following the same procedure in the last problem, we have

$$\mathrm{e}^{-\kappa(n_a+n_b)t/2}\,|\alpha,\beta\rangle = \mathrm{e}^{|\alpha|^2(\mathrm{e}^{-\kappa t}-1)/2}\mathrm{e}^{|\beta|^2(\mathrm{e}^{-\kappa t}-1)/2}\,|\alpha\mathrm{e}^{-\kappa t/2},\beta\mathrm{e}^{-\kappa t/2}\rangle\,,$$

and after normalization we get

$$|\psi(t)\rangle_{\text{no jump}} = |\alpha\mathrm{e}^{-\kappa t/2},\beta\mathrm{e}^{-\kappa t/2}\rangle\,. \tag{8}$$

After a click, since a coherent state is a eigenstate of the annihilation operator, noting happens and we have

$$|\psi(t)\rangle_{\text{jump}} = |\alpha\mathrm{e}^{-\kappa t/2},\beta\mathrm{e}^{-\kappa t/2}\rangle\,. \tag{9}$$

2.2 We have

$$\begin{aligned} C_1\,|\psi(t)\rangle_{\text{no jump}} &= \sqrt{\frac{\kappa}{2}}(a+b)\,|\alpha\mathrm{e}^{-\kappa t/2},\beta\mathrm{e}^{-\kappa t/2}\rangle \\ &= \sqrt{\frac{\kappa}{2}}(\alpha\mathrm{e}^{-\kappa t/2}+\beta\mathrm{e}^{-\kappa t/2})\,|\alpha\mathrm{e}^{-\kappa t/2},\beta\mathrm{e}^{-\kappa t/2}\rangle\,, \end{aligned}$$

and therefore before a click, we have

$$\gamma_1 = \langle\psi(t)|C_1^\dagger C_1|\psi(t)\rangle = \frac{\kappa}{2}\mathrm{e}^{-\kappa t}|\alpha+\beta|^2. \tag{10}$$

Similarly,

$$\begin{aligned} C_2\,|\psi(t)\rangle_{\text{no jump}} &= \sqrt{\frac{\kappa}{2}}(-a+b)\,|\alpha\mathrm{e}^{-\kappa t/2},\beta\mathrm{e}^{-\kappa t/2}\rangle \\ &= \sqrt{\frac{\kappa}{2}}(-\alpha\mathrm{e}^{-\kappa t/2}+\beta\mathrm{e}^{-\kappa t/2})\,|\alpha\mathrm{e}^{-\kappa t/2},\beta\mathrm{e}^{-\kappa t/2}\rangle\,, \end{aligned}$$

and therefore

$$\gamma_2 = \langle\psi(t)|C_2^\dagger C_2|\psi(t)\rangle = \frac{\kappa}{2}\mathrm{e}^{-\kappa t}|\alpha-\beta|^2. \tag{11}$$

We see it is possible to always keep $\gamma_1 = 0$, as long as $\alpha+\beta = 0$.

2.3 We have

$$\mathrm{e}^{-\kappa(n_a+n_b)t/2}\,|N,N\rangle = \mathrm{e}^{-\kappa(N+N)t/2}\,|N,N\rangle\,,$$

and therefore after normalization we have

$$|\psi(t)\rangle_{\text{jump}} = |N,N\rangle\,. \tag{12}$$

After a click, we have

$$\begin{aligned} C_1\,|\psi(t)\rangle_{\text{no jump}} &= \sqrt{\frac{\kappa}{2}}(ta+rb)\,|N,N\rangle \\ &= \sqrt{\frac{\kappa}{2}}(t\sqrt{N}\,|N-1,N\rangle + r\sqrt{N}\,|N,N-1\rangle), \end{aligned}$$

and since $|t|^2+|r|^2 = 1$, we have

$$|\psi(t)\rangle_{\text{jump}} = t\,|N-1,N\rangle + r\,|N,N-1\rangle\,. \tag{13}$$

2.4 We can evaluate γ_1 and γ_2 as

$$\begin{aligned} \gamma_1 &= \langle\psi(t)|C_1^\dagger C_1|\psi(t)\rangle \\ &= \frac{\kappa}{2}\,\langle N,N|(a^\dagger+b^\dagger)(a+b)|N,N\rangle \\ &= \frac{\kappa}{2}\times 2N = \kappa N, \end{aligned}$$

and

$$\begin{aligned}\gamma_2 &= \langle\psi(t)|C_2^\dagger C_2|\psi(t)\rangle \\ &= \frac{\kappa}{2}\langle N,N|(-a^\dagger+b^\dagger)(-a+b)|N,N\rangle \\ &= \frac{\kappa}{2}\times 2N = \kappa N,\end{aligned}$$

so

$$\gamma_1 = \gamma_2 = \kappa N. \tag{14}$$

After a quantum jump, by (13), we have

$$|\psi(t)\rangle_{\text{jump}} = \frac{1}{\sqrt{2}}(\,|N-1,N\rangle + \,|N,N-1\rangle). \tag{15}$$

We find a measurement entangles two cavities together, and after the jump, we have

$$\begin{aligned}\gamma_1 &= \langle\psi(t)|C_1^\dagger C_1|\psi(t)\rangle \\ &= \frac{\kappa}{2}\langle\psi(t)|(a^\dagger+b^\dagger)(a+b)|\psi(t)\rangle \\ &= \frac{\kappa}{4}|(a+b)(\,|N-1,N\rangle + \,|N,N-1\rangle)|^2,\end{aligned}$$

and since

$$\begin{aligned}&(a+b)(\,|N-1,N\rangle + \,|N,N-1\rangle) \\ &= \sqrt{N-1}\,|N-1,N\rangle + 2\sqrt{N}\,|N-1,N-1\rangle + \sqrt{N-1}\,|N,N-2\rangle\,,\end{aligned}$$

we have

$$\gamma_1 = \frac{\kappa}{4}\times(N-1+4N+N-1) = \frac{\kappa}{2}(3N-1). \tag{16}$$

Similarly, we have

$$\begin{aligned}\gamma_2 &= \langle\psi(t)|C_2^\dagger C_2|\psi(t)\rangle \\ &= \frac{\kappa}{2}\langle\psi(t)|(-a^\dagger+b^\dagger)(-a+b)|\psi(t)\rangle \\ &= \frac{\kappa}{4}|(-a+b)(\,|N-1,N\rangle + \,|N,N-1\rangle)|^2,\end{aligned}$$

and since

$$\begin{aligned}&(-a+b)(\,|N-1,N\rangle + \,|N,N-1\rangle) \\ &= -\sqrt{N-1}\,|N-1,N\rangle + \sqrt{N-1}\,|N,N-2\rangle\,,\end{aligned}$$

we have

$$\gamma_2 = \frac{\kappa}{4}\times(N-1+N-1) = \frac{\kappa}{2}(N-1). \tag{17}$$

Note that we do not need to single out the case of $N=1$: terms like $|N-2\rangle$ automatically vanish because of the $\sqrt{N-1}$ factor. When $N=1$, we have

$$|\psi(t)\rangle_{\text{jump}} = \frac{1}{\sqrt{2}}(\,|0,1\rangle + \,|1,0\rangle), \tag{18}$$

and after a D_1 click,

$$\gamma_1 = \kappa, \quad \gamma_2 = 2. \tag{19}$$

This means if we wait and detect another photon, it can only occur at D_1. This is an example of observation induced localization and can be viewed as the asynchronous version of Hong-Ou-Mandel effect.

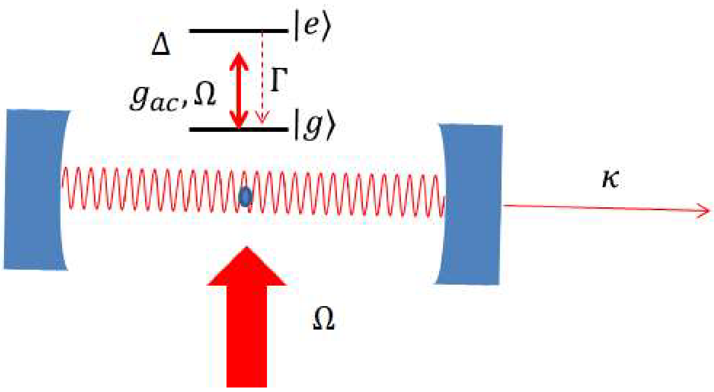

Figure 2: The device in the third problem

Cavity QED and single photon source As in the figure above, consider a 2-level atom coupled to a cavity field with single-photon Rabi frequency g_{ac}. The atom is in addition subjected to a laser field excitation from the side with a Rabi frequency $\Omega(\mathrm{t})$. Taking into account the radiative decay by the atom and the leak of the cavity, the effective Hamiltonian of the controlled and coupled system is given by

$$H_{\text{eff}} = \hbar\left(-\mathrm{i}\frac{\Gamma}{2}\right)|e\rangle\langle e| + \hbar\left(-\Delta - \mathrm{i}\frac{\kappa}{2}\right)a^{+}a + \left[\hbar\left(\frac{\Omega(\mathrm{t})}{2} + g_{ac}a\right)|e\rangle\langle g| + \text{ h.c. }\right], \tag{20}$$

where $\Delta = \omega - \omega_{eg}$ is the detuning of the cavity mode frequency from the atomic resonant frequency. The collapse operators are given by $C_1 = \sqrt{\Gamma}|g\rangle\langle e|$ and $C_2 = \sqrt{\kappa}a$. 3a) Consider $\Omega(\mathrm{t}) = 0$ and with system initially in $|\psi_S(0)\rangle = |g, n = 1\rangle$, that is, the atom is in the ground state and the cavity mode is in $\mathrm{n} = 1$ Fock state. Consider good cavity ($\kappa \ll g, \Gamma$) and weak coupling ($g \ll \Delta, \Gamma$) limits. Expand $|\psi_S(t)\rangle$ in proper basis of choice, and to derive the Schrodinger equation for the coefficients of the stochastic wavefunction, without quantum jump. Perturbatively derive the system decay rate $\gamma_1(t) = \left\langle \psi_S \left| C_1^{+} C_1 \right| \psi_S \right\rangle$ and $\gamma_2(t) = \left\langle \psi_S \left| C_2^{+} C_2 \right| \psi_S \right\rangle$ (i.e., using the adiabatic elimination method which assumes $|\psi_S(t)\rangle \approx \left|\widetilde{\psi_S}\right\rangle$, with $H_{\text{eff}}\left|\widetilde{\psi_S}\right\rangle \approx -\Delta - \frac{i\kappa}{2}\left|\widetilde{\psi_S}\right\rangle\Big)$. 3b) Repeat Question 3a, but with system initially in $|\psi\rangle = |e, n = 0\rangle$ and in the bad cavity ($\kappa >> g, \Gamma$) and weak coupling ($g \ll \Delta, \Gamma$) limit. You should arrive at a total decay rate $\gamma = \gamma_1 + \gamma_2$ that describes the Purcell effect as in the class. Discuss the condition under which $\gamma_2 \gg \gamma_1$, that is, the decay of the system more likely leading to a single photon emission into the cavity leak mode. 3c) With the system initially in $|\psi\rangle = |g, n = 0\rangle$ and with a resonant pulse $\Omega(t) = \Omega_0 \sin\left(\frac{\pi t}{\tau}\right)$ switched on and off smoothly for $0 < t < \tau$. Assuming $|\psi(t)\rangle$ to be driven by the 2-level Hamiltonian $H_a = H_{eff}(t; \Gamma, \kappa, g \to 0)$ [note: this happens effectively when $\Gamma, \kappa, g \ll 1/\tau$]. Now, putting back all the parameters into H_{eff}, Calculate $\gamma_1(t) = \left\langle \psi \left| C_1^{\dagger} C_1 \right| \psi \right\rangle$ and $\gamma_2(t) = \left\langle \psi \left| C_2^{\dagger} C_2 \right| \psi \right\rangle$ for stochastic wavefunction without quantum jump during $0 < \mathrm{t} < \mathrm{T}$, with $T \gg \frac{1}{\kappa}, \frac{1}{\Gamma}$ 4d) Discuss $\Omega(t)$ and other parameters in Eq. (2), so that a single photon can be deterministically generated into the cavity leaking mode with high efficiency. Discuss the form of the single-photon wavefunction, and the fidelity of the single-photon source (how likely there is exactly one photon in the time-dependent leaky mode).

Solution

(a) It is easy to find that only $|e, n = 0\rangle$ and $|g, n = 1\rangle$ have coupling. With the basis $\{|g, n = 1\rangle, |e, n = 0\rangle\}$, we have

$$H_{\text{eff}} = \begin{pmatrix} -\hbar(\Delta + \mathrm{i}\kappa/2) & \hbar g_{ac}^{*} \\ \hbar g_{ac} & -\mathrm{i}\hbar\Gamma/2 \end{pmatrix}. \tag{21}$$

The correction of the energy of $|e, n = 0\rangle$ is of $\mathcal{O}(g_{ac}^2/\Gamma)$ order, and we can just throw it away. On the other hand, the coupling between $|g, n = 1\rangle$ and $|e, n = 0\rangle$ has a first order correction to $|g, n=1\rangle$, which possibly contributes a non-zero term to γ_1. So what we need to do is to find the eigenstate correction.

Suppose

$$|\psi_{\mathrm{S}}(t)\rangle = c_g\,|g, n=1\rangle + c_e\,|e, n=0\rangle\,.$$

Since there is no energy correction, we have

$$\mathrm{i}\dot{c}_g = (-\Delta - \mathrm{i}\kappa/2)c_g, \quad \mathrm{i}\dot{c}_e = (-\Delta - \mathrm{i}\kappa/2)c_e.$$

On the other hand, (21) gives

$$\mathrm{i}\hbar\dot{c}_e = \hbar g_{ac}c_g - \mathrm{i}\hbar\Gamma/2 \times c_e,$$

and therefore we have

$$\frac{\mathrm{i}\Gamma - 2\Delta - \mathrm{i}\kappa}{2}c_e = g_{ac}c_g \approx g_{ac},$$

where we have omitted the time evolution factor of c_g, which is fine since both c_e and c_g evolve in the same pace, and the normalization after each time step can wipe away factors like $\mathrm{e}^{-\Gamma t/2}$, and we have

$$c_e = \frac{2g_{ac}}{\mathrm{i}\Gamma - 2\Delta - \mathrm{i}\kappa},$$

$$|\psi(t)\rangle \approx |g, n=1\rangle + \frac{2g_{ac}}{\mathrm{i}\Gamma - 2\Delta - \mathrm{i}\kappa}\,|e, n=0\rangle\,. \tag{22}$$

This is a (quasi)stationary state, and no further normalization is required, since there is no quantum jump and therefore no total probability loss. We have

$$\begin{aligned}\gamma_1 &= \Gamma\,\langle\psi|e\rangle\,\langle e|\psi\rangle = \Gamma|c_e|^2 \\ &= \Gamma\left|\frac{2g_{ac}}{\mathrm{i}\Gamma - 2\Delta - \mathrm{i}\kappa}\right|^2 \\ &= \Gamma\frac{4|g_{ac}|^2}{(\Gamma-\kappa)^2 + 4\Delta^2},\end{aligned}$$

so

$$\gamma_1 = \Gamma\frac{4|g_{ac}|^2}{(\Gamma-\kappa)^2 + 4\Delta^2} \approx \Gamma\frac{4|g_{ac}|^2}{\Gamma^2 + 4\Delta^2}. \tag{23}$$

Also,

$$\gamma_2 = \kappa\,\langle\psi|a^\dagger a|\psi\rangle = \kappa. \tag{24}$$

So we can see the leading order contribution of the coupling between $|g, n=1\rangle$ and $|e, n=0\rangle$ is (23).

(b) Now we repeat the argument in (a) and make no energetic correction to $|e, n=0\rangle$. Suppose $|\psi(t)\rangle = c_e\,|e, n=0\rangle + c_g\,|g, n=1\rangle$, and the approximate time evolution equations are

$$\mathrm{i}\hbar\dot{c}_e = -\mathrm{i}\hbar\Gamma/2c_e, \quad \mathrm{i}\hbar\dot{c}_g = -\mathrm{i}\hbar\Gamma/2c_g.$$

On the other hand we have the exact evolution equation of c_g, which is

$$\mathrm{i}\hbar\cdot c_g = -\hbar(\Delta + \mathrm{i}\kappa/2)c_g + \hbar g_{ac}^* c_a,$$

and we have

$$c_g = \frac{g_{ac}^*}{\Delta + \mathrm{i}\kappa/2 - \mathrm{i}\Gamma/2}c_e \approx \frac{g_{ac}^*}{\Delta + \mathrm{i}\kappa/2 - \mathrm{i}\Gamma/2}.$$

Therefore we have

$$|\psi(t)\rangle = |e, n=0\rangle + \frac{g_{ac}^*}{\Delta + \mathrm{i}\kappa/2 - \mathrm{i}\Gamma/2}\,|g, n=1\rangle\,, \tag{25}$$

again a quasi-stationary state. The damping rates are

$$\gamma_1 = \Gamma|\,\langle\psi|e\rangle|^2 \approx \Gamma, \tag{26}$$

and

$$\gamma_2 = \kappa \langle \psi(t)|a^\dagger a|\psi(t)\rangle = \kappa |c_g|^{\,2} = \kappa \frac{4|g_{ac}|^{\,2}}{4\Delta^2 + (\kappa - \Gamma)^2} \approx \kappa \frac{4|g_{ac}|^{\,2}}{4\Delta^2 + \kappa^2}. \tag{27}$$

Therefore we find

$$\gamma = \Gamma + \kappa \frac{4|g_{ac}|^{\,2}}{4\Delta^2 + \kappa^2}. \tag{28}$$

We see that the existence of the cavity gives rise to the a and $a^\dagger$ modes, which in turn give rise to the $a\,|e\rangle\langle g|$ term in the Hamiltonian, which dresses the excited state, and it is exactly the correction term $c_g\,|g, n = 1\rangle$ in $|\psi(t)\rangle$ (which is the dressed excited state) that contributes a non-zero value to γ_2. Therefore, we see the cavity increases the probability of quantum jump, which is an instance of Purcell effect.

(c)