Excitons in Band Metal and Insulators

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1 Underlying particles

In the first several chapters in the solid state physics note we discussed the underlying particles in condensed matters and how to treat quasiparticles in condensed matters as irreducible representations of groups. Here we briefly review the underlying degrees of freedom, or in other words, review the first principles.

In QED, by computing the low energy scattering amplitude we find the Coulomb interaction between electrons, which is an *effective* interaction channel in high energy physics, but we usually just accept it as the first principle interaction between electrons in condensed matter physics.

Electrons in condensed matter physics are also *not* the Dirac fermion in the sense of QED. Actually, we need to integrate out the anti-particle modes of the Dirac field to get a $\boldsymbol{p}^2/(\mathrm{i}\partial_t - e\varphi + m)$ term in the Hamiltonian, separate a $\mathrm{e}^{\mathrm{i}mt}$ factor from the electron field and replace $\mathrm{i}\partial_t$ with m, and throw away the $e\varphi$ to get a non-relativistic approximation, in order to get **condensed matter physics electrons**. The *electrons* in condensed matter physics are highly dressed compared to electrons in QED. After the whole procedure, the quadratic Hamiltonian for electrons is

$$H = \frac{(\boldsymbol{p} - e\boldsymbol{A})^2}{2m} + \boldsymbol{\mu} \cdot \boldsymbol{B}, \tag{1}$$

where $e = -|e|$.

So in the end, in condensed matter physics, electrons are non-relativistic massive particles and are minimally coupled to an external electromagnetic field $\boldsymbol{A}$, and their spins are also coupled to an external magnetic field, and electrons scatter with each other according to Coulomb law.

We also have atomic nuclei, which may be viewed as point particles. Light nuclei's quantum fluctuation may be significant. Heavy nuclei's low orbitals may have relativistic effects, but in condensed matters we are usually only interested in valence electrons, which are almost always non-relativistic.

In solid state physics, the role of atoms can be concluded into several aspects. First they regulate the motion of electrons, and free electrons (in the sense of condensed matter physics) are regulated into **band electrons**. The vibration of atoms gives rise to **phonons**. Nonlinearity of atom motion is equivalent to **phonon-phonon scattering**, and the presence of phonons may cause charge density change, which leads to **electron-phonon scattering**. Deviation from ideal lattice is taken into account as **defects** or **disorder** (defects with certain random distribution).

2 Excitation in a free electron gas

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