Defect String Operator in CFT and Topological Orders by Prof. Yang Qi

Jinyuan Wu

December 25, 2021

The idea originated in an article by Liang Kong and Kitaev, and Wen-Jie Li and Xiao-Gang Wen, PR Research 2, 033417 2020. We consider an (1+1)-dimensional transverse field Ising model. When we adjust the transverse field h, there is a well known quantum phase transition, and there is a CFT on the critical point. We can, then, consider the system as the boundary of a (2+1)-dimensional $\mathbb{Z}_2$ toric-code model. The ferromagnetic phase can then be viewed as an anyon condensation phase where $\langle e \rangle \neq 0$, while the paramagnetic phase is an anyon condensation phase where $\langle m \rangle \neq 0$.

Info

Haldane conjecture says spin-1/2 chains are gapless, and the conjecture can be generalized into higher dimensions. Now we consider a spin-1/2 chain as the boundary state of a (2+1)-dimensional spin system (just like the spin-1/2 edge states of a AKLT chain). This idea means **Lieb-Schultz-Mattis (LSM) theorem**, which states that a spin system with translation and spin rotation symmetry and half-integer spin per unit cell does not admit a gapped symmetric ground state lacking fractionalized excitations.
This can even be done for Fermi surface. Now people tend to consider an arbitrary gapless system as the boundary of a gapped system.

Here we consider some edge states of a toric-code mode. The **smooth** boundary is shown in Figure 1 on page 1, where the boundary Hamiltonian is the sum of terms like $\sigma_1^x \sigma_2^x \sigma_3^x$. We can see that an e-particle on the boundary cost energy, while an m-particle does not cost energy. Therefore, there is a m-particle condensation in the smooth boundary. The same works for e-particles. Consider the **rough** boundary in Figure 2 on page 2, where the boundary Hamiltonian is the sum of $\sigma_1^z \sigma_2^z \sigma_3^z$, and we find an m-particle on the boundary costs energy, while an e-particle does not, so there is an e-particle condensation.

We are still unable to determine the correspondence between the two any condensation phases and the two quantum phases of transverse Ising chain. This can be more easily found in Wen-Plaquette model, which can be mapped precisely into the toric code model.

Now we consider the generalized case in conformal field theories. We can impose *defects* into the theory, or in other words we can impose certain kind of external potential field. It is natural to impose a *string defect* in an (1+1)-dimensional theory. For example, in a spin chain, we can flip the J coefficient on a certain bond, or in other words turning it from antiferromagnetic into ferromagnetic. At different time steps we can flip different bonds, so what we are doing is to impose a string defect in the spacetime (see Figure 3 on page 2). Note that, however, that a closed string defect is trivial, because we can just flip all spins inside the closed string, and we find in this way the defect does not effect the outer world. Therefore, only open string defects

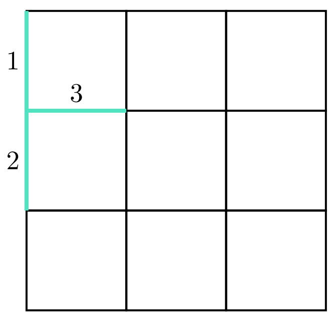

Figure 1: Smooth boundary

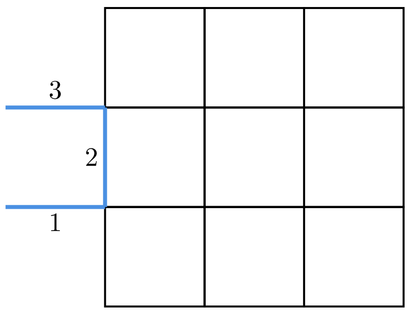

Figure 2: Rough boundary

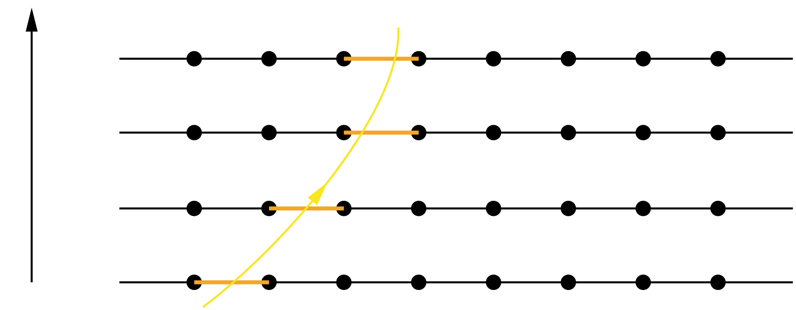

Figure 3: A string defect in an (1+1)-dimensional system. The orange bonds are flipped, and the yellow line is the defect in the spacetime.

matter, and open strings defects with the same edges are equivalent. So what really matters are the edges of open strings, and in other words in (1+1)-dimensional CFTs, what matters are pairs of point excitations.

This approach to identify low energy degrees of freedom can be generalized into a (2+1)-dimensional CFT, where we can impose defect *surfaces*, and again closed defect surfaces are trivial, and open surfaces with the same edge are equivalent (because they only differ with a trivial closed surface), so what matter are the edge of open surfaces, and we have *line excitations*. People usually call this **defect string operators**. Note that they are objects in a (2+1)-dimensional CFT and do not have direct relation with the string defects in an (1+1)-dimensional CFT.

The properties of these excitations are unknown, but note that we can always put an CFT at the boundary of a gapped system - usually a topological order - by this duality we can already know a lot about the gapless state on the boundary.

This is precisely the case in a two-dimensional transverse field Ising model, which is considered as the boundary of a (3+1)-dimensional toric code model, and the two phases in the TFIM correspond to one anyon condensation phase and another line excitation condensation phase.