Phenomena That Can Be Explained Solely by Band Theory

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This article is a reading note of Xiaogang Wen's Quantum Field Theories of Many-body Systems, Chapter 4.

1 The shape of the Fermi surface and equal-time Green function

In this section we explicitly evaluate the equal-time Green function. An important fact is that it is highly affected by the shape of the Fermi surface. When $T = 0$, we have (when not explicitly mentioned, when there is no spin polarization mentioned, we are working with only one spin polarization) Sec. 4.2.4

$$\begin{aligned} \mathrm{i}G(-0^+, \boldsymbol{x}) &= \mathcal{T}\,\langle c(\boldsymbol{x}, -0^+)c^\dagger(0,0)\rangle = -\langle c^\dagger(0,0)c(\boldsymbol{x},0)\rangle \\ &= -\int \frac{\mathrm{d}^d\boldsymbol{k}}{(2\pi)^d} n_{\mathrm{F}}(\xi_{\boldsymbol{k}})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} = -\int \frac{\mathrm{d}^d\boldsymbol{k}}{(2\pi)^d}\Theta(-\xi_{\boldsymbol{k}})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}. \end{aligned} \tag{1}$$

We define

$$\tilde{N}(k, \hat{\boldsymbol{x}}) = \int \frac{\mathrm{d}^d\boldsymbol{k}}{(2\pi)^d}\Theta(-\xi_{\boldsymbol{k}})\delta(k - \boldsymbol{k}\cdot\hat{\boldsymbol{x}}), \tag{2}$$

and since when $k = \boldsymbol{k}\cdot\hat{\boldsymbol{x}}$, we have $k|\boldsymbol{x}| = \boldsymbol{k}\cdot\boldsymbol{x}$, we have

$$\mathrm{i}G(-0^+, \boldsymbol{x}) = -\int_{-\infty}^{\infty} \mathrm{d}k\, \tilde{N}(k, \hat{\boldsymbol{x}})\mathrm{e}^{\mathrm{i}k|\boldsymbol{x}|}. \tag{3}$$

Now the most important task is to evaluate (2). The δ-function is non-zero on the plane $\boldsymbol{k}\cdot\hat{\boldsymbol{x}} = k$ in the momentum space.

2 Density-density correlation function

3 Linear response and effective theory

Chern-Simons

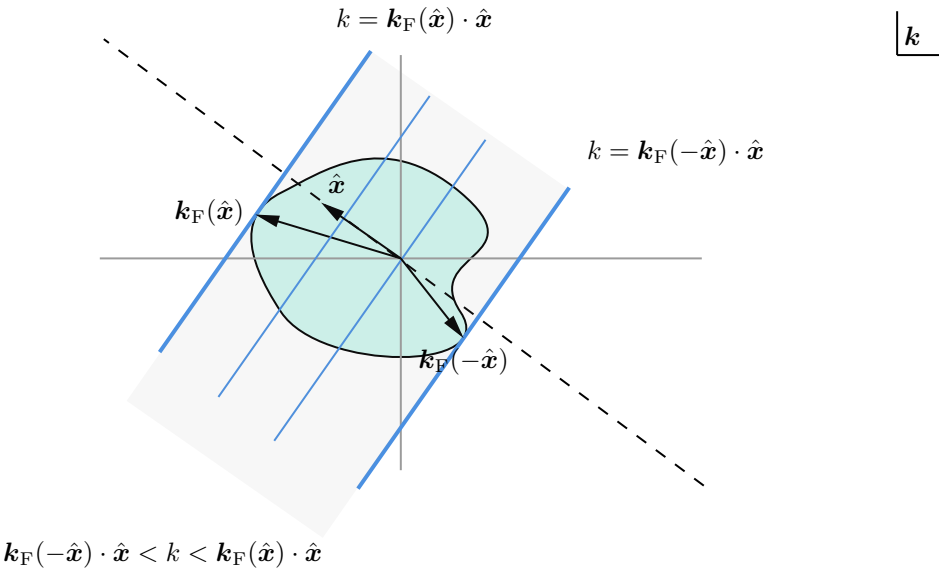

Figure 1: The shape of the Fermi surface and (2)