QFT I, Homework 4

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December 2, 2021

Scalar QED Consider the theory of a complex scalar field ϕ interacting with the electromagnetic field A^μ. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi - m^2 \phi^* \phi. \tag{1}$$

where $D_\mu = \partial_\mu + \mathrm{i} e A_\mu$ is the usual gauge covaraint derivative.

(a) Show the Lagrangian is invariant under the gauge transformations

$$\phi(x) \to \mathrm{e}^{-\mathrm{i}\alpha(x)} \phi(x), \quad A_\mu(x) \to A_\mu(x) + \frac{1}{e} \partial_\mu \alpha(x). \tag{2}$$

(b) Derive the Feynman rules for the interaction between photons and scalar particles.

(c) Draw all the leading-order Feynman diagrams and compute the amplitude for the process $\gamma\gamma \to \phi\phi^*$.

(d) Compute the differential cross section $\mathrm{d}\sigma / \mathrm{d}\cos\theta$. You can take an average over all initial state polarizations. For simplicity, you can restrict your calculation in the limit $m = 0$.

(e) Draw all leading order Feynman diagrams, that contribute to the Compton scattering process $\gamma\phi \to \gamma\phi$ and compute the differential cross section $\mathrm{d}\sigma / \mathrm{d}\cos\theta$ with $m = 0$.

Solution

(a) Under the gauge transformation (2), we have

$$F_{\mu\nu} \to F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu = \partial_\mu \left(A_\nu + \frac{1}{e} \partial_\nu \alpha \right) - \partial_\nu \left(A_\mu + \frac{1}{e} \partial_\mu \alpha \right) = \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu},$$

so the first term in (1) remains the same. It is obvious that under (2)

$$\phi^* \phi \to \phi'^* \phi' = \mathrm{e}^{\mathrm{i}\alpha} \phi^* \mathrm{e}^{-\mathrm{i}\alpha} \phi = \phi^* \phi,$$

so the third term in (1) is also invariant. Also

$$D^\mu \phi \to (\partial^\mu + \mathrm{i} e A'^\mu) \phi' = (\partial^\mu + \mathrm{i} e A^\mu + \mathrm{i} \partial^\mu \alpha) \, \mathrm{e}^{-\mathrm{i}\alpha} \phi$$