# Bosonic Field Theories in Condensed Matter Physics

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This article is a reading note of Chapter 3 of Wen's famous textbook.

## 1 A simplest interacting boson system

Section 3.3.1 provides a simplest interacting bosonic system with a complex scalar field Eq. (3.3.1)

$$S = \int \mathrm{d}^d \boldsymbol{x} \, \mathrm{d}t \, (\mathrm{i}\varphi^* \partial_t \varphi - \frac{1}{2m} \partial_{\boldsymbol{x}} \varphi^* \partial_{\boldsymbol{x}} \varphi + \mu |\varphi|^2 - \frac{V_0}{2} |\varphi|^4). \tag{1}$$

The prefactor of the interaction term makes the corresponding term in the EOM of $\varphi$ and $\varphi^*$ not have a numerical factor, but it introduces a numerical factor in the vertex in Feynman diagrams. The sign of the mass term is derived as follows: first we have a $-\varphi^* \nabla^2 \varphi / 2m$ term in the Hamiltonian, and therefore we have a $\varphi^* \nabla^2 \varphi / 2m$ term in the Lagrangian, and by integration by parts we have $\varphi^* \nabla^2 \varphi / 2m \simeq -\partial_{\boldsymbol{x}} \varphi^* \partial_{\boldsymbol{x}} \varphi / 2m$.

The semiclassical approximation from (3.3.1) to (3.3.2) can be justified when the temperature is high and therefore the most economical path does not have imaginary time evolution at all. It can also be derived using the ideas behind (3.4.1), where with a finite temperature, we can always integrate out modes with non-zero Matsubara frequencies. This gives a physical picture behind dynamic density functional theories and also explains why "classical" statistical physics is still relevant today.

The following contents from Eq. (3.3.3) to Eq. (3.3.4) are also covered here. The discussion between Eq. (3.3.4) to the end of Section 3.3.2 is important, which illustrates the Ginzburg-Landau paradigm and why it is almost always associated with symmetries (or otherwise it is highly unlikely that we have several minima of the energy functional that share the same energy, so that we have a smooth phase transition shown in Fig. 3.5), though the concept of order parameters can also be used in a first-order phase transition (see here, for example).