Invertible and Non-invertible Topological Orders by Prof. Yang Qi

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Classification of topological phases in condensed matter physics:

- Long-range entanglement: intrinsic topological order
- Short-range entanglement:

– SPT

– Invertible topological phases

The concept of **invertible topological phases** needs some clarification. When Xiaogang Wen started to study topological phases, he called topological phases with ground state degeneracy and anyons *intrinsic* topological phases, and in contrary were SPTs without ground state degeneracy and anyons and whose topological properties come from the quantum phase protected by certain symmetry. Some topological phases also lack anyons and ground state degeneracy, but they are not protected by any symmetry, making them hard to classify. Examples of such phases include the integer quantum Hall effect and one dimensional Kitaev chain. Note that though Kitaev chains have particle-hole duality, it arises from technical reasons and does not protect the topological phase. These phases are obviously closer to SPTs as they have no anyons and cannot be classified by tensor categories, but they are not SPTs.

It should be noted that fermion systems have a hidden symmetry - **fermion-parity symmetry**, that if we add a minus symbol for each species of fermions, the Hamiltonian remains the same. The symmetry group is

$$\mathbb{Z}_2^f = \{1, (-1)^{P_f}\}, \quad P_f = \pm 1. \tag{1}$$

Therefore the interplay between two kinds of symmetric elements that occurs in the double group construction in the band theory of electrons happens again here: If the "bosonic" symmetry group of a system is G_b, there is no guarantee that the complete symmetry group G_f is $G_b \times \mathbb{Z}_2^f$. For example, if $G_b = \mathbb{Z}_2^T$, the compatibility conditions requires

$$T^2 = (-1)^{P_f}, \tag{2}$$

so G_f is either isomorphism to $\mathbb{Z}_2 \times \mathbb{Z}_2$ or isomorphism to $\mathbb{Z}_4$.