Quantum Optics by Prof. Saijun Wu

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We consider the following non-Hermitian effective Hamiltonian:

$$\tilde{H}_{\text{eff}} = \left(H_{\text{SE}} - \sum_e \mathrm{i}\hbar \frac{\Gamma_e}{2} |e\rangle\langle e| \right) |0\rangle\langle 0| + \hbar \sum_e g^*_{\text{eg,k}} \mathrm{e}^{-\mathrm{i}\Delta_k t} a^\dagger_k |g, 0\rangle\langle e, 0| . \tag{1}$$

We find

1 Stochastic wave function approach to a three-level system

Now we consider a Λ-type three-level system shown in Figure 1 on page 1. We try to use the technique in Section 3 in the last lecture to describe this system. It should be noted that this approach does not always work. For example, for a V-type system, two excited states may jump to one ground state, which may create some subtlety.

For the system in Figure 1 on page 1, suppose there is no quantum jump before $t = t_0$, and we have

$$|\psi_{\text{s}}(t)\rangle = \frac{C}{\sqrt{2}} (\mathrm{e}^{-\mathrm{i}\omega_e t - \Gamma_e t/2} |e\rangle + \mathrm{e}^{-\mathrm{i}\omega_a t - \Gamma_a t/2} |a\rangle), \tag{2}$$

and the probability of quantum jump (i.e. spontaneous radiation) is

$$\delta P_{\text{jump}} = \gamma(t)\, \delta t\,, \quad \gamma(t) = 1 + \mathrm{e}^{-\Delta\Gamma t} \cos(\Delta\omega t). \tag{3}$$

The oscillating form of γ is called **quantum beat**.

We can also calculate the survival ratio until $t = t_0$, i.e.

$$\prod_{j=1}^N (1 - \delta P_{\text{j}}\,(t_j)) \to \mathrm{e}^{-\int_0^t \gamma(\tau) \mathrm{d}\tau}. \tag{4}$$

2 Linear response in a two-level system

We consider a two level system with an external field, the Hamiltonian of which is

$$H = H_0 + \frac{\hbar}{2} \Omega\, |e\rangle\langle g| + \text{h.c.}, \tag{5}$$

and we take spontaneous radiation into account, so we work with

$$H_{\text{eff}} = H_0 - \frac{\mathrm{i}\Gamma_e}{2} |e\rangle\langle e| + \frac{\hbar\Omega}{2} |e\rangle\langle g| + \text{h.c.}. \tag{6}$$

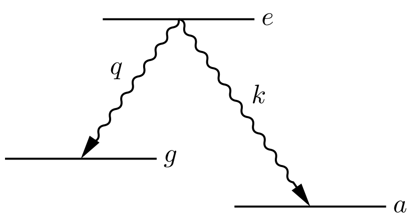

Figure 1: The energy level diagram of a Λ-type three-level system

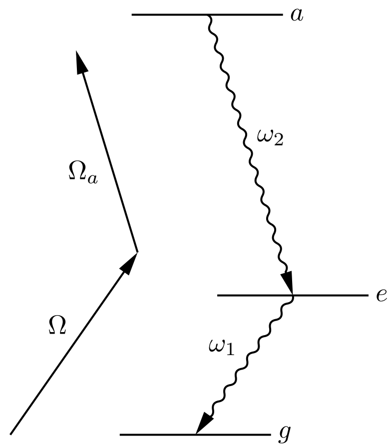

Figure 2: A ladder-type three-level system

We again use the stochastic wave function approach, and we have

$$\mathrm{i}\dot{c}_e = \left(\Delta - \frac{\mathrm{i}\Gamma}{2}\right) c_e + \frac{\Omega}{2} c_g, \quad \mathrm{i}\dot{c}_g = \frac{\Omega^*}{2} c_e. \tag{7}$$

We find a stable solution

$$c_g \approx 1, \quad c_e = -\frac{\Omega/2}{\Delta - \mathrm{i}\Gamma/2}. \tag{8}$$

We therefore find the decay rate is

$$\gamma = \langle \psi_{\mathrm{s}} | C^\dagger C | \psi_{\mathrm{s}} \rangle = \frac{|\Omega|^2}{4\Delta^2 + \Gamma^2} \Gamma. \tag{9}$$

The response of the electric dipole can now be found to be

$$\langle \psi_{\mathrm{s}} | \boldsymbol{d} | \psi_{\mathrm{s}} \rangle = \frac{\boldsymbol{d}_{eg} E \mathrm{e}^{-\mathrm{i}\omega t}}{\hbar(\Delta - \mathrm{i}\Gamma/2)} + \text{h.c.}. \tag{10}$$

From the definition of

$$E_{\text{ext}} = E_{\text{in}} \mathrm{e}^{-\rho\sigma L/2}, \tag{11}$$

we have

$$\sigma = k\alpha = \frac{3\lambda^2}{2\pi} \frac{\Gamma/2}{\Delta - \mathrm{i}\Gamma/2}. \tag{12}$$

It can be found that $\operatorname{Im} \alpha$ is just the scattering rate, which is a result of the optical theorem.

3 Spontaneous radiation as energy correction

We already know that a stochastic wave function theory is fully described by both the Hamiltonian and a set of collapse operators. Sometimes, for example (21) in the last lecture, wave function collapse caused by spontaneous radiation and the subsequent observation of emitted photon is weak, and the main effect of spontaneous radiation is a correction of energy levels.

In this section we consider a ladder-type three-level system shown in Figure 2 on page 2. The Hamiltonian is

$$\dot{\rho}(t) = \frac{1}{\mathrm{i}\hbar} H_{\text{eff}} \rho - \frac{1}{\mathrm{i}\hbar} \rho H_{\text{eff}} + \sum_j C_j \rho C_j^\dagger. \tag{13}$$