General Relativity as an Effective Field Theory

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This article is a reading note of the chapters in Schwartz about how general Relativity (henceforth GR) can be thought as an effective field theory.

1 The free spin-2 theory

We first construct a spin-2 field theory with the approach of Schwartz Section 8.7. Consider a symmetric rank 2 tensor $h_{\mu\nu}$. We try to write down a free theory of the theory, which must be quadratic in $h_{\mu\nu}$, and either quadratic or zeroth order in ∂_μ. From $h_{\mu\nu}$ we can construct a list of objects that are first order in $h_{\mu\nu}$ and do not contain $\mathcal{O}(\partial^3)$: Sec. 8.7

$$h_{\mu\nu},\ h_{\alpha\alpha},\ \Box h_{\mu\nu},\ \partial_\mu\partial_\nu h_{\mu\nu},\ \partial_\mu\partial_\nu h_{\mu\alpha},\ \Box h_{\alpha\alpha},$$

and we can construct terms that are possible to appear in the free Lagrangian:

$$h_{\mu\nu}^2,\ h_{\mu\nu}\Box h_{\mu\nu},\ h_{\nu\alpha}\partial_\mu\partial_\nu h_{\mu\alpha},\ h_{\alpha\alpha}^2,\ h_{\alpha\alpha}\Box h_{\beta\beta},\ h_{\alpha\alpha}\partial_\mu\partial_\nu h_{\mu\nu},$$

and hence we get (8.126) (8.126)

$$\mathcal{L} = a h_{\mu\nu}\Box h_{\mu\nu} + b h_{\mu\nu}\partial_\mu\partial_\alpha h_{\nu\alpha} + c h\Box h + d h\partial_\mu\partial_\nu h_{\mu\nu} + m^2\left(x h_{\mu\nu}^2 + y h^2\right), \tag{1}$$

where we define $h = h_{\alpha\alpha}$.

Now we consider the "inner structure" of $h_{\mu\nu}$. We first do the decomposition (8.124) to (8.125)

$$h_{\mu\nu} = h_{\mu\nu}^{\mathrm{T}} + \partial_\mu\pi_\nu + \partial_\nu\pi_\mu, \tag{2}$$

where we require

$$h_{\mu\nu}^{\mathrm{T}} = h_{\nu\mu}^{\mathrm{T}}, \quad \partial^\mu h_{\mu\nu}^{\mathrm{T}} = 0. \tag{3}$$

Again we can decompose π_μ into

$$\pi_\mu = \pi_\mu^{\mathrm{T}} + \partial_\mu\pi^{\mathrm{L}}, \quad \partial^\mu\pi_\mu^{\mathrm{T}} = 0. \tag{4}$$

Now we find the decomposition actually imposes strong constraints on (1). First, since

$$x h_{\mu\nu}^2 + y h^2 = 2x(\Box\pi^{\mathrm{L}})^2 + 2y(\partial_\mu\partial_\nu\pi^{\mathrm{L}})^2 \simeq -2x\pi^{\mathrm{L}}\Box^2\pi^{\mathrm{L}} - 2y\partial_\mu\partial_\nu\partial^\mu\partial^\nu\pi^{\mathrm{L}} = -2(x+y)\pi^{\mathrm{L}}\Box^2\pi^{\mathrm{L}},$$

and there should be no $\mathcal{O}(\partial^4)$ terms in (1), we find $x + y = 0$.

2 Coupling with another field and the self interaction

The conclusion is that there is, actually, *no* conflict between general relativity and quantum mechanics. What is really concerning is that GR, in the context of QFT, is not Sec. 22.4