Quantum Optics by Prof. Saijun Wu

Jinyuan Wu

November 4, 2021

1 The rotational wave approximation and the Bloch sphere

Under the rotational wave approximation and the corresponding RWA transformation, the Hamiltonian of a two-level system is

$$H = \frac{\hbar}{2}\boldsymbol{\Omega}\cdot\boldsymbol{\sigma}, \tag{1}$$

where

$$|\boldsymbol{\Omega}| = \sqrt{\Omega^2+\Delta^2}, \tag{2}$$

$$\Omega = \frac{\boldsymbol{E}_0\cdot\boldsymbol{d}_{eg}}{\hbar} \tag{3}$$

is the **Rabi frequency** and Δ the **detuning**. The wave function is always in the form of

$$|\psi\rangle = \cos\frac{\theta}{2}\mathrm{e}^{\mathrm{i}\varphi/2}|g\rangle + \sin\frac{\theta}{2}\mathrm{e}^{-\mathrm{i}\varphi/2}|e\rangle, \tag{4}$$

and the density matrix is

$$\rho = \frac{1}{2}(1+\boldsymbol{n}\cdot\boldsymbol{\sigma}), \tag{5}$$

where

$$\boldsymbol{n} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta). \tag{6}$$

It is natural to put $\boldsymbol{n}$ on a sphere. The constructions are standard for a qubit and can be found in Section 1.1 in the quantum information note. The equation of motion is

$$\frac{\mathrm{i}\hbar}{2}\boldsymbol{n}\cdot\boldsymbol{\sigma} = \mathrm{i}\hbar\rho = [H,\rho] = [\boldsymbol{\Omega}\cdot\boldsymbol{\sigma}, \frac{1}{2}\boldsymbol{n}\cdot\boldsymbol{\sigma}] = \frac{1}{2}(\boldsymbol{\Omega}\times\boldsymbol{n})\cdot\boldsymbol{\sigma},$$

and therefore we have

$$\dot{\boldsymbol{n}} = \boldsymbol{\Omega}\times\boldsymbol{n}. \tag{7}$$

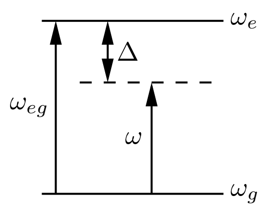

Figure 1: The energy diagram of a two-level system in an external optical field