Scattering in Relativistic Quantum Field Theories by Prof. Dingyu Shao

Jinyuan Wu

November 8, 2021

1 Demonstration of the relation between interaction Feynman propagators, free Feynman propagators, S-matrices using scalar field

Suppose the vacuum state $|\Omega\rangle$ of a field theory with interaction is not orthogonal to the vacuum state $|0\rangle$ of the free theory. In order to build connection between the two states, we imaginarily set $|0\rangle$ to be the initial state and turn on the interaction, and make the excited components in $|0\rangle$ "relaxed" back to $|\Omega\rangle$. We add an imaginary part to the time to make this happen and we have

$$|\Omega\rangle = \lim_{T\to\infty(1-\mathrm{i}\epsilon)} \left(e^{-iE_0 T}\langle\Omega \mid 0\rangle\right)^{-1} e^{-\mathrm{i}HT}|0\rangle.$$

$$|\Omega\rangle = \lim_{T\to\infty(1-\mathrm{i}\epsilon)} (\mathrm{e}^{-\mathrm{i}E_0(t-(-T))}\,\langle\Omega|0\rangle)^{-1} U(t_0, -T)\,|0\rangle\,, \tag{1}$$

Suppose $x^0 > y^0 > t_0$, we have

$$\langle\Omega|\mathcal{T}[\phi(x)\phi(y)]|\Omega\rangle$$

See Peskin 4.2 for details

So in the end we have

$$\langle\Omega|\mathcal{T}[\phi(x)\phi(y)]|\Omega\rangle = \lim_{T\to\infty(1-\mathrm{i}\epsilon)} \frac{\langle 0|\mathcal{T}\,\phi_{\mathrm{I}}(x)\phi_{\mathrm{I}}(y)\exp\left(-\mathrm{i}\int_{-T}^{T}\mathrm{d}t\,H_{\mathrm{I}}(t)\right)|0\rangle}{\langle 0|\mathcal{T}\exp\left(-\mathrm{i}\int_{-T}^{T}\mathrm{d}t\,H_{\mathrm{I}}(t)\right)|0\rangle}. \tag{2}$$

$$S = \mathbb{1} + \mathrm{i}T, \tag{3}$$

and we define

$$_0\langle p_1, p_2, \ldots, p_m|T|q_1, q_2, \ldots, q_n\rangle_0 = (2\pi)^{m+n}\delta^{(4)}(\sum p - \sum q)\mathcal{M}(p_1, p_2, \ldots, p_m \to q_1, q_2, \ldots, q_n). \tag{4}$$

2 Scattering

A scattering experiment involves processes like the following:

$$p_1 + p_2 \longrightarrow \sum_i q_i. \tag{5}$$

The

$$\mathrm{d}P = \tag{6}$$