Quantifying, Modeling, and Validating Downside Financial Risk: A Multi-Method Approach

Introduction

Financial markets are inherently volatile, and the accurate quantification of downside risk is fundamental to sound risk management and regulatory compliance. Traditional metrics such as **standard deviation** capture overall volatility but fail to describe the magnitude and frequency of **extreme losses**, which often dictate real-world portfolio outcomes.

This study spans **Days 57 to 61** and focuses on the **quantification, modeling, and validation of downside risk** through three advanced frameworks:

- **Value at Risk (VaR)**
- **Conditional Value at Risk (CVaR)**
- **Extreme Value Theory (EVT)**

Additionally, regulatory developments such as the **Basel III Fundamental Review of the Trading Book (FRTB)** are integrated to understand the practical implications of modern risk measurement. The analysis uses **NIFTY 50**, **Bitcoin (BTC)**, and **S&P 500 (SPX)** data to compare traditional and tail-sensitive methods under normal and crisis conditions.

Day 57: Establishing the Regulatory and Conceptual Foundation

The initial phase focused on the conceptual and regulatory framework of risk measurement, explaining why volatility-based metrics are insufficient to assess downside risk.

Key Concepts Introduced

- **Value at Risk (VaR):**
 Estimates the maximum potential loss over a specific time horizon at a defined confidence level.
 Example: A 99% VaR indicates a 1% chance of exceeding that loss in a given period.

- **Conditional Value at Risk (CVaR) / Expected Shortfall (ES):**
 Measures the *average* loss beyond the VaR threshold, overcoming VaR's main limitation—its insensitivity to tail severity.

- **Regulatory Evolution:**
 Under **Basel III FRTB**, regulators shifted the core metric from **99% VaR** to **97.5% CVaR (Expected Shortfall)** because CVaR is a *coherent* measure (it satisfies sub-additivity).

- **Modeling Extreme Events (EVT):**
 Introduced as a statistical method to estimate the probability of **rare, catastrophic losses**—so-called "black swan" events—beyond the observed sample.

Day 58: Value at Risk (VaR) Calculation and Comparison

This phase involved empirical computation of VaR for two distinct assets — **NIFTY 50** and **Bitcoin (BTC)** — using three approaches.

Methods Applied

1. **Historical VaR:** Based on empirical quantiles of historical return distributions.
2. **Parametric Normal VaR:** Assumes returns follow a Gaussian distribution.
3. **Parametric Student-t VaR:** Incorporates fat tails via Student-t distribution.

Findings: BTC vs NIFTY

- **Tail Risk Disparity:** Bitcoin's daily tail losses were **4–5× larger** than NIFTY's.
- **Model Performance:**

- *Normal VaR* significantly **underestimated** BTC's losses due to heavy tails.
- *Student-t VaR* provided a **more realistic fit**, particularly at 99% confidence (Degrees of Freedom ≈ 3), confirming BTC's extreme tail heaviness.

Day 59: Conditional Value at Risk (CVaR) and Tail Severity Analysis

Focus shifted to quantifying *how severe* losses are beyond the VaR threshold, addressing VaR's blind spot.

Concepts and Metrics

- **CVaR Definition:**
  The *expected loss* given that returns fall below the VaR quantile — i.e., "How bad can it get once VaR is breached?"
- **Tail Severity Ratio (CVaR / VaR):**
  Quantifies the "fatness" of the loss distribution tail. A higher ratio implies steeper escalation of losses beyond VaR.

Empirical Results

Asset	Tail Severity Ratio (CVaR / VaR)	Interpretation
NIFTY 50	1.44–1.47×	Moderately fat tails
Bitcoin	1.57–1.65×	Extremely fat tails

Conclusion: Both assets deviate from normality, but BTC's tail risk is significantly more severe once losses exceed VaR.

Day 60: Extreme Value Theory (EVT) Modeling

To better estimate *rare, unobserved* extremes, EVT was applied using the **Peaks Over Threshold (POT)** approach.

Methodology

- Fitted a **Generalized Pareto Distribution (GPD)** to returns exceeding the 95th percentile (extreme losses).
- Derived **EVT-based VaR** estimates to extrapolate beyond observed data.

Results and Comparison

Asset	VaR Method	99% VaR (%)	Interpretation
NIFTY 50	Historical	≈ Similar to EVT	Moderate tail risk
BTC	Normal	7.866	Underestimates risk
BTC	Historical	8.588	Improved but limited
BTC	EVT (GPD)	9.198	Most accurate for heavy tails

Key Insight:
EVT provided the **highest and most reliable VaR estimates** for BTC, validating that standard Gaussian models are unsuitable for assets with heavy-tailed return distributions such as cryptocurrencies.

Day 61: Backtesting and Stress Testing

The final stage validated model accuracy and robustness under both normal and crisis market regimes, using **NIFTY 50** and **S&P 500 (SPX)**.

Model Validation

- **Test Applied:** Kupiec *Proportion of Failures (POF)* Test.
- **Result:** Historical VaR model performed well:
  - Observed breach rates ≈ Expected (NIFTY: 5.0%; SPX: 1.0%).
  - Indicates excellent calibration under long-term, stable conditions.

Stress Testing

- **Crisis Periods Examined:**
  - 2008 Global Financial Crisis (GFC)
  - 2020 COVID Crash
- **Findings:**
  - 95% VaR was breached **≈30–31%** of days during crises (vs expected 5%).
  - Demonstrates VaR's failure in non-stationary and turbulent markets.

Conclusion

Historical VaR models are **reliable in stable regimes** but **collapse under structural breaks**, underscoring the need for adaptive, stress-aware frameworks (e.g., CVaR or EVT-based methods).

Executive Summary and Key Insights

Executive Summary

This multi-day study systematically quantified and validated downside financial risk using **VaR**, **CVaR**, and **EVT**, applied across **traditional (NIFTY, SPX)** and **high-volatility (BTC)** assets. It highlighted how traditional risk measures underestimate tail risk and how advanced models improve accuracy, especially in crisis conditions.

Key Insights

1. **VaR Limitations:**
   While useful for baseline risk assessment, VaR fails to capture the *magnitude* of tail losses and collapses during crises.

2. **CVaR Superiority:**
   CVaR provides a *more coherent and tail-sensitive* measure, aligning with Basel III's shift to Expected Shortfall as the regulatory standard.

3. **EVT Accuracy:**
   EVT's GPD framework effectively captures *rare, catastrophic losses* that standard models miss—particularly relevant for assets like Bitcoin.

4. **Market Regime Dependence:**
   Models calibrated in normal markets perform poorly during structural breaks; **dynamic recalibration** or **stress testing** is essential.

5. **Practical Implication:**
   Combining **CVaR and EVT** provides a comprehensive view of tail risk—ideal for portfolio stress testing, risk capital estimation, and strategic allocation.
