Quantifying, Modeling, and Validating Downside Financial Risk using VaR, CVaR, and EVT

Why Quantify Downside Risk?

- Traditional volatility (σ) fails to capture *extreme losses* (tail risk).
- Accurate tail-risk modeling is crucial for:
  - Portfolio protection
  - Regulatory compliance (Basel III FRTB)
  - Crisis resilience
- **Objective:** Quantify and validate downside financial risk using:
  - Value at Risk (VaR)
  - Conditional VaR (CVaR)
  - Extreme Value Theory (EVT)

From Volatility to Coherent Risk Measures

- **VaR:** Max loss at confidence level (e.g., 99%).
- **CVaR (Expected Shortfall):** Avg. loss *beyond* VaR → tail-sensitive.
- **Basel III FRTB:** Replaced 99% VaR with 97.5% CVaR → ensures coherence (sub-additivity).
- **EVT:** Statistical framework for *rare, extreme* losses (black swans).

Visual:
Timeline showing evolution: *Standard Deviation → VaR → CVaR → EVT (Basel III)*

Comparing NIFTY 50 and Bitcoin VaR

- Methods used:
  - Historical VaR (empirical quantiles)
  - Normal VaR (Gaussian)
  - Student-t VaR (fat tails)
- BTC's tail risk **4–5× higher** than NIFTY.
- **Normal VaR** underestimates BTC losses.
- **Student-t VaR** (df ≈ 3) better fits BTC's heavy tails.

Measuring Losses Beyond VaR

- CVaR = Expected loss given VaR is breached.
- Tail Severity Ratio (CVaR / VaR) quantifies tail fatness.
- Both NIFTY and BTC show non-Gaussian tails:
    - NIFTY ≈ 1.45×
    - BTC ≈ 1.60×
- BTC’s losses escalate more sharply once past VaR threshold.

Visual:
Side-by-side bar graph of Tail Severity Ratios (CVaR/VaR).

Modeling Rare and Catastrophic Losses

- **Peaks Over Threshold (POT)** approach using **Generalized Pareto Distribution (GPD)**.
- Captures extreme losses > 95th percentile.
- **Findings:**
  - NIFTY: EVT-VaR ≈ Historical VaR → moderate tail risk.
  - BTC: EVT-VaR (99% ≈ 9.20%) > Historical (8.59%) > Normal (7.87%).
- EVT delivers *most realistic estimates* for heavy-tailed assets.

Visual:
GPD fit plot showing fat tails and EVT extrapolation region.

Backtesting Historical VaR with Kupiec Test

- **Kupiec Proportion of Failures (POF)** test checks calibration.
- Data: 2005–2025 (≈4,325 obs) for NIFTY & SPX.
- **Results:**
    - Long-term: Passed POF test (expected breach rate ≈ observed).
    - Indicates good calibration in normal markets.

Visual:
Table summarizing Expected vs Observed breaches for NIFTY & SPX (e.g., 5% expected → 5.1% observed).

Model Performance Under Turbulence

- Tested during:
  - **2008 GFC**
  - **2020 COVID Crash**
- 95% VaR breached on **≈30–31% of days** (vs 5% expected).
- VaR fails catastrophically in non-stationary, crisis regimes.

Visual:
Line chart of daily losses vs 95% VaR limit (highlight breaches during GFC/COVID).

Risk Measure Comparison — Strengths & Weaknesses

Metric	Strength	Weakness	Best Use
VaR	Simple, intuitive	Ignores tail severity	Normal markets
CVaR	Coherent, tail-sensitive	Computationally heavier	Regulatory & portfolio risk
EVT	Models unseen extremes	Data-intensive	Crisis risk & stress testing

Key Observation:
BTC exhibits extreme fat tails — traditional VaR inadequate; **CVaR + EVT combination** most effective.

Visual:
Radar or heatmap comparing methods across criteria (Accuracy, Tail Sensitivity, Robustness).

Conclusions & Practical Takeaways

Main Insights:

1. **VaR underestimates** extreme risk—fails during crises.
2. **CVaR (Expected Shortfall)** aligns with modern regulatory standards (Basel III).
3. **EVT provides statistical rigor** for modeling black swan events.
4. **Hybrid modeling (CVaR + EVT)** offers comprehensive tail-risk insight.
5. Effective **stress testing and dynamic calibration** essential for risk resilience.

Visual:
Infographic summarizing framework: *VaR → CVaR → EVT → Integrated Risk Model.*