Overview: Analytical Roadmap

Objective:
Summarize five connected stages in quantitative risk modeling.

Key Idea:
D29–D33 form a continuous pipeline — from recognizing market irregularities to developing, forecasting, and validating dynamic volatility models.

Flow:
Stylized Facts → ARCH/GARCH → EGARCH → Volatility Forecasting → VaR Backtesting.

Stylized Facts of Financial Returns

Goal: Empirically verify that market returns are not Normal.

Findings:

- Returns show **fat tails** and **skewness** (Kurtosis 8–28).
- Volatility is **clustered**, not constant.
- **Negative skewness** dominates indices; some stocks show **positive skew**.

Implication:
Traditional Gaussian models underestimate extreme risks — models must capture **non-normality** and **time-varying volatility**.

Introduction to ARCH/GARCH

Purpose:
Model volatility clustering and persistence seen in stylized facts.

Key Findings:

- GARCH(1,1) outperforms ARCH(1).
- Volatility persistence: $\alpha_1 + \beta_1 \approx 0.97$.
- **t-distribution errors** capture fat tails better than Normal.

Outcome:
GARCH makes volatility **forecastable**, not fixed — marking the first dynamic modeling step.

Advanced GARCH (EGARCH)

Focus:
Incorporate **asymmetry (leverage effects)** — negative shocks increase volatility more than positive ones.

Model: EGARCH(1,1)-t

Results:

- **Lowest AIC/BIC:** (6697.87 / 6727.28)
- Persistence $\beta_1 \approx 0.97$, tail parameter $\nu \approx 6.3$.
- Captures **asymmetric risk reactions** effectively.

Conclusion:
EGARCH(1,1)-t becomes the **optimal volatility model** for NIFTY 50.

Volatility Forecasting

Goal:
Use EGARCH(1,1)-t to forecast future volatility and risk metrics.

Results (30-day horizon):

- **Mean σ:** 0.7332% (≈11.6% annualized)
- **95% return range:** ±1.47%
- **VaR_{95}%:** −1.465%

Takeaway:
Volatility forecasts provide actionable **forward risk estimates**, guiding portfolio exposure and leverage control.

VaR Backtesting

Purpose:
Validate model reliability via statistical tests.

Models Tested: Historical, GARCH-norm, GARCH-t, EGARCH-t.
Tests: Kupiec (POF), Christoffersen Independence, Conditional Coverage.

Result @99% VaR:
Only **EGARCH-t passes all 3 tests**, confirming it captures dynamic tail behavior accurately.

Conclusion:
EGARCH-t offers statistically sound downside risk prediction.

Analytical Continuity

Framework Progression:

1. **Day 29:** Identify problem → Markets are non-Gaussian.
2. **Day 30–Day 31:** Build solution → GARCH & EGARCH model volatility dynamics.
3. **Day 32:** Apply solution → Forecast risk metrics.
4. **Day 33:** Validate solution → Backtest reliability.

Outcome:
A **closed analytical loop** linking theory, modeling, forecasting, and validation.

Key Insights

1. Normality Fails:
Real-world returns are fat-tailed and asymmetric.

2. EGARCH(1,1)-t is Superior:

- Captures persistence ($\alpha_1 + \beta_1 \approx 0.97$)
- Models heavy tails ($\nu \approx 6.3$)
- Handles asymmetry effectively

3. Strategic Use:
Transforms volatility into a **forecastable, controllable variable** for tactical risk management.

Strategic Conclusion

Core Insight:
Volatility is **not static risk**, but a **dynamic, forecastable strategic variable**.

Value for Investors:

- Anticipate risk regimes
- Optimize hedging and capital allocation
- Strengthen portfolio resilience

Validated Model:
EGARCH(1,1)-t — the **gold standard** for capturing real-world volatility, persistence, and asymmetry.