Portfolio Management and Optimization: Advanced Option Pricing (Days 50–54)

Transitioning from Classical BSM to Modern Stochastic Models

Key Points:

- Focused on advanced stochastic and numerical option pricing models.
- Moved beyond constant-volatility Black–Scholes–Merton (BSM) assumptions.
- Applied Monte Carlo, Finite Difference, and Jump Diffusion frameworks.
  Visual Suggestion: Abstract timeline or schematic of model evolution (BSM → Heston → Jump Diffusion).

Project Objectives

Key Points:

- Identify BSM limitations in reflecting real market dynamics.
- Implement stochastic-volatility (SV) and jump-diffusion (JD) models.
- Validate models numerically using convergence and stability tests.
- Integrate all approaches into a unified analytical framework.
  Visual Suggestion: 4-pillar infographic (Accuracy, Volatility, Tail Risk, Integration).

Day 50 – Limitations of BSM

Key Points:

- BSM assumes constant volatility and lognormal returns.
- **Key Failures:**
  - Volatility smile/skew
  - Fat-tailed return distributions
  - Volatility clustering & leverage effect
  - No jump or shock capture
- Motivation: need for stochastic volatility representation.
  Visual Suggestion: Chart comparing implied volatility smile vs. BSM flat line.

Day 51 – Monte Carlo Heston Model

Key Points:

- Volatility modeled as a mean-reverting random process.
- Parameters: κ (mean reversion), θ (long-term variance), ξ (vol of vol), ρ (correlation).
- Monte Carlo simulations validated against analytical BSM values.
- Convergence achieved at finer grid resolutions (M = 800).
  Visual Suggestion: Diagram of Heston process or SDE equations with arrows.

Day 52 – Crank–Nicolson Finite Difference Implementation

Key Points:

- Used Crank–Nicolson with Rannacher smoothing to solve BSM PDE.
- Tested on M&M ticker for call / put options.
- Observed numerical **instability**: errors increased with refinement.
- Diagnosis: boundary conditions, domain truncation, and Δt / ΔS ratio issues.
  Visual Suggestion: Flowchart of FD grid with instability trend graph.

Day 53 – Merton Jump Diffusion (MJD) Framework

Key Points:

- Adds Poisson jump process to standard diffusion.
- Captures fat tails and abrupt price moves.
- Parameters calibrated via MSE minimization.
- MJD prices > BSM prices → jump risk premium.
- Example: +2.16 difference for 3700 call.
  Visual Suggestion: Graphic showing occasional price “jumps” on time-series path.

Day 54 – Integrated Stochastic Models Framework

Key Points:

- Comparative analysis of 5 methods: BSM, FD, MC GBM, MC Heston, MC Jump Diffusion.
- **Findings:**
  - FD (when stable): ≈ BSM (< 0.3 % deviation).
  - MC GBM: Slightly lower due to path variance.
  - MC Heston: Lower due to mean-reverting volatility.
  - MC Jump: Higher call prices (+14.44 for 3600 call).
    Visual Suggestion: Comparison bar chart across models.

Strategic Insights

Key Points:

- BSM/FD: serve as calibration benchmarks.
- Heston: effective for volatility stress testing.
- Jump Diffusion: models event-driven tail risks.
- Unified approach → more resilient portfolio valuation.
  Visual Suggestion: Table linking model type → market condition → best use.

Conclusions and Roadmap

Key Points:

- Each model offers unique perspective on pricing and risk.
- Stochastic and jump models capture real-market behaviors better.
- Next steps:
  - Refine FD solver for stability.
  - Extend calibration to multi-asset options.
  - Automate model selection via error minimization.

Visual Suggestion: Roadmap timeline for next-phase development.