Portfolio Optimization & Robustness Report: BL vs. RP vs. Kelly

I. Introduction and Methodology

This report synthesizes the detailed activities and findings from a week-long effort focused on comparative portfolio management and optimization, specifically analyzing the performance and robustness of the Black–Litterman (BL), Risk Parity (RP), and Kelly Criterion (KC) approaches.

This analysis evaluates three advanced portfolio optimization strategies—Black–Litterman (BL), Risk Parity (RP), and Kelly Criterion (KC)—applied to a multi-asset portfolio including Indian equities, Bonds, Gold, S&P 500 ETF, Bitcoin, and Ethereum. The primary goal is to overcome the instability and extreme allocations associated with traditional Mean-Variance Optimization (MVO).

Method

Method	Key Feature	Primary Input	Best When
Black–Litterman	Incorporates subjective views	Blended Expected Returns (π + Views)	You want to blend market consensus with private insights
Risk Parity (ERC)	Equalizes risk contribution per asset	Covariance Matrix (Σ)	You distrust return forecasts but trust risk models
Kelly Criterion	Maximizes expected log growth rate of wealth	Highly accurate Expected Returns (μ)	You seek aggressive growth with reliable forecasts

II. Day 23: Asset Correlation and Diversification Analysis

Day 23 established the foundational diversification properties of the assets, focusing particularly on hedges and alternative assets.

1. VIX as a Crisis Hedge

The analysis confirmed that the India Volatility Index (^INDIAVIX) acts as the best defensive tool and a true crisis hedge, exhibiting a strong negative correlation (average correlation of -0.571) across all defined equity/bond portfolios. VIX provides crisis protection because it rises when equities fall. Since VIX instruments are generally not investable long-term due to contango decay, the hedging strategy must use proxies like Gold ETF and Long-term Government Bonds (LTGILTBEES.NS).

2. Cryptocurrency Diversification

Bitcoin (BTC-USD) and Ethereum (ETH-USD) were classified as 'Moderate' diversifiers against established portfolios (average correlation 0.327981 and 0.381949, respectively). Cryptos help reduce domestic Indian concentration risk but remain tied to global risk-on cycles. The high correlation between BTC and ETH (0.73) suggests including both increases concentration risk within the crypto asset class itself.

3. Dynamic Allocation Suggestion

Based on risk types and a presumed 65% equity weight, a dynamic allocation structure was suggested: VIX (Diversifier) received a 9.8% suggested allocation (reallocated to proxies), while BTC/ETH (Moderate) received 3.2% each.

III. Day 24: Black-Litterman Robustness and Pipeline Development

Day 24 focused exclusively on implementing Black–Litterman and mitigating its tendency toward concentrated solutions when views are strong or market priors are weak.

1. Initial Concentration Issues

Initial implementation of BL, especially when using zero market prior (π=None) or historical returns as views, led to extreme concentrations. For instance, minimum volatility optimization yielded 72.98% in Bharat Bond ETF (EBBETF0430.NS), and max Sharpe optimization allocated 66.2% to the same asset.

2. Enhanced Robust BL Pipeline

To mitigate this, a robust pipeline was developed, incorporating market-implied priors (using market capitalization and risk aversion δ) and introducing controlled view uncertainty (Ω).

• **Omega Scaling Search:** Iteratively searched different scaling factors for Ω (view uncertainty) based on asset volatilities. This helped identify a scaling that respects constraints or minimizes concentration as measured by the Herfindahl index.

• **Post-Optimization Capping:** A function was implemented to enforce hard caps (e.g., maximum weight of 25% or 15%) and redistribute the excess weight proportionally to non-capped assets.

IV. Day 25: Comparative Performance and Solver Enhancement

Day 25 benchmarked the three approaches using a Train/Test split for Out-of-Sample (OOS) evaluation.

1. Naive OOS Results (Pre-Improvement)

In the initial OOS test (using naive implementations for RP and KC):

- **Risk Parity** emerged as the early Sharpe winner (2.98) due to very low OOS volatility (0.0512 Ann. Vol) and achieved the lowest Max Drawdown (-3.0%).
- **Black–Litterman** was consistently strong (Sharpe 2.39, CAGR 0.1768).
- **Kelly Criterion** lagged significantly (Sharpe 2.09, CAGR 0.0752).

2. Transition to Robust Solvers

The discrepancy, particularly the weakness of KC and potential instability of simple RP solvers, prompted the adoption of more robust optimization techniques:

- **Robust Kelly**: The simple linear-solve KC was replaced with a CVXPY-based quadratic approximation, which is convex and allows for controlled fractional leverage scaling (e.g., leverage = 0.5).
- **Robust Risk Parity**: The naive solver was replaced with an improved SciPy-based minimization designed explicitly to minimize the squared differences in risk contributions, improving stability.

V. Day 26 & 27: Stress Testing and Final Comparative Analysis

1. Stress Testing Framework (Day 26)

Stress testing is a critical tool for identifying hidden vulnerabilities and checking portfolio resilience against extreme, plausible scenarios not found in historical data.

Five Scenarios Applied to Asset Groups (Equities, Crypto, Gold, Bonds, Other):

1. Equities -10%
2. Crypto -20%
3. Gold +5%, Bonds +3% (Safe Haven Rally)
4. All risk assets -15% (Systemic Downturn)
5. Safe haven rally (Bonds +5%)

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2. Robust OOS Performance (Day 27)

After implementing the robust solvers, the OOS performance shifted:

Strategy	CAGR	Ann. Vol	Sharpe	Max Drawdown	End Value
Black–Litterman	0.1768	0.0656	2.39	-5.3%	1.38
Risk Parity	0.1857	0.0694	2.39	-4.6%	1.40

Kelly Criterion	0.2190	0.0962	2.07	-6.5%	1.48

- **Kelly Criterion** delivered the highest growth (CAGR 0.2190) but suffered the highest volatility and drawdown.
- **Risk Parity** and **Black–Litterman** provided identical, high risk-adjusted returns (Sharpe 2.39).

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3. Comparative Stress Test Results (Day 27)

Scenario (Shock)	Black–Litterman	Risk Parity	Kelly
Equities -10%	-2.08%	-5.30%	-6.50%
All risk assets -15%	-7.73%	-11.05%	-13.39%
Gold +5%, Bonds +3%	+1.62%	+0.96%	+0.39%

Key Insights:

- **Downside Protection**: BL proved the most resilient. Its loss under an Equities -10% shock (-2.08%) was less than half that of RP or KC, showcasing strong diversification benefits.

- **Systemic Risk**: In a systemic downturn (All risk assets -15%), KC suffered the largest loss (-13.39%) due to its concentration in growth assets, while BL maintained the smallest loss (-7.73%).

- **Safe Haven Capture**: BL effectively captured the largest positive gains (up to +2.00%) from defensive rallies, indicating meaningful defensive allocation to gold and bonds. KC showed minimal safe-haven benefit.

- **Crypto Risk**: BL had a neutral impact in the Crypto -20% scenario (0.00% loss), suggesting minimal exposure, whereas KC showed the highest loss (-1.43%).