Quantitative Finance Progression Report

Days D29–D33: From Stylized Facts to Validated Risk Forecasting

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1. Overview

This report summarizes activities across **Days D29 through D33**, outlines their analytical connection, provides a detailed interpretation of results, and concludes with key insights.

The five-day sequence represents a coherent **analytical pipeline in quantitative finance** — from identifying empirical market anomalies to building, applying, and validating dynamic risk models.

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2. Summary of Each Day (D29–D33)

Day	Focus	Purpose	Key Insight / Finding
D29	Stylized Facts of Returns	Examine statistical patterns (heavy tails, clustering, skewness) to understand true market risk beyond averages.	Financial returns violate Normality assumptions, exhibiting heavy tails and skewness — necessitating non-Gaussian models.
D30	Introduction to ARCH/GARCH	Model and forecast time-varying volatility reflecting clustering and persistence.	GARCH(1,1) outperforms ARCH(1). For NIFTY 50, volatility persistence ($\alpha_1 + \beta_1 \approx 0.97$) is key; Student's t errors improve fit.
D31	Advanced GARCH (EGARCH)	Capture asymmetry (leverage effects) where negative shocks raise volatility more than positive ones.	EGARCH(1,1)-t provides best fit for NIFTY 50, balancing persistence ($\beta_1 \approx 0.97$) and tail realism ($v \approx 6.3$).

D32	Forecasting Volatility	Translate EGARCH-t results into volatility forecasts and risk intervals.	30-day EGARCH-t forecasts imply stable volatility but realistic asymmetric, fat-tailed risk (e.g., $\text{VaR}_{95}\%$ = −1.465%).
D33	VaR Backtesting	Validate forecast accuracy using Kupiec and Christoffersen tests.	At 99% VaR, EGARCH(1,1)-t is the only model passing all statistical backtests, confirming its reliability.

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3. Analytical Connection Across the Five Days

The days form a **structured methodological sequence**, connecting theory, modeling, and validation:

• D29 – Establishing the Problem:
Confirms that **financial markets are non-Gaussian** with volatility clustering. Classical constant-volatility models are insufficient.

• D30–D31 – Building the Solution:
ARCH/GARCH capture volatility persistence; **EGARCH** adds asymmetry. Empirically, **EGARCH(1,1)-t** emerges as the optimal specification (lowest AIC/BIC).

• D32 – Applying Forecasts:
The **EGARCH-t model** projects forward volatility and VaR, making risk **forecastable** rather than static.

• D33 – Validating the Model:
Backtesting confirms EGARCH-t's predictive validity — a necessary step for practical institutional risk management.

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4. Detailed Report

I. D29 — Necessity of Non-Normal Modeling

Financial return distributions exhibit **heavy tails, volatility clustering, and skewness**, defying Normal assumptions. Key findings from 2015–2025 data confirm that risk cannot be accurately modeled using constant-variance Gaussian methods.

- **Heavy Tails (High Kurtosis):** NIFTY 50 (8.6), NIFTYBANK (17.3), TITAN (28.4); Student's t v between 2.5–3.6.
- **Negative Skew:** Market indices show downside bias; select equities (INFY, RELIANCE) show mild positive skew.
- **Indices vs. Equities:** Indices offer diversification but remain crash-prone; select equities show better convexity.

Implication:
Stylized facts demonstrate the need for **non-normal, dynamic volatility models** to accurately measure and manage risk.

II. D30–D31 — Dynamic Volatility Modeling(Nifty 50 10Yrs data)

A. D30: GARCH vs. ARCH Fit

- **GARCH(1,1)** provides a stronger fit (LLF Δ +91.68) than ARCH(1).
- Persistence ($\alpha_1 + \beta_1 \approx 0.97$) shows long memory in volatility.
- Fat tails modeled effectively with Student's t ($v \approx 6.4$).

B. D31: Introducing EGARCH

- **EGARCH(1,1)-t** achieves lowest AIC/BIC (6697.87 / 6727.28).
- Captures **leverage effects**: volatility reacts more to negative shocks.
- **Model parameters:** Persistence $\beta_1 \approx 0.9742$; Tail thickness $v \approx 6.332$.

Outcome:
Transition from static to adaptive volatility modeling completed — a robust foundation for forecasting and risk control.

III. D32 — Volatility Forecasting and Risk Metrics

Applying **EGARCH(1,1)-t** for 30-day forecasts:

Metric	Result	Interpretation
Mean daily σ	0.7332% (≈11.6% annualized)	Stable short-term volatility regime
95% return range	±1.47%	Daily fluctuations typically within this bound
VaR_{95}%	−1.4653%	Expected daily loss exceeded once every ~20 days

Strategic Implication:
Forecasts enable **proactive risk management** — adjusting leverage or hedging based on anticipated volatility regimes.

IV. D33 — VaR Backtesting and Model Validation

Backtesting compared **Historical, GARCH-norm, GARCH-t, and EGARCH-t** models using **Kupiec (POF)**, **Christoffersen Independence**, and **Conditional Coverage** tests at **99% confidence**.

Model	Failure Rate (Expected 1%)	Kupiec	Independence	Conditional Coverage
Historical	1.46%	REJECT	REJECT	REJECT
GARCH-norm	1.96%	REJECT	PASS	REJECT
GARCH-t	0.04%	REJECT	PASS	REJECT
EGARCH-t	**1.40%**	**PASS**	**PASS**	**PASS**

Conclusion:
Only **EGARCH(1,1)-t** passes all three tests at 99% confidence — proving its superior dynamic calibration and tail accuracy.

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5. Key Insights and Strategic Implications

1. **Normality Assumption Fails:**
   Real markets exhibit fat tails and asymmetry. Normal-based models underestimate extreme risks.

2. **EGARCH(1,1)-t is Empirically Superior:**

   ○ Heavy tails: $v \approx 6.3$

   ○ Persistence: $\alpha_1 + \beta_1 \approx 0.97$

   ○ Asymmetry: Leverage effect properly modeled

3. **Forecastable Volatility = Strategic Advantage:**
   Treating volatility as a **dynamic, forecastable variable** allows portfolio managers to:

   ○ Anticipate regime shifts

   ○ Adjust exposure proactively

   ○ Enhance tail-risk resilience

Final Note

The Day 29–Day 33 analytical progression transforms volatility from a static measure of uncertainty into a **predictive and actionable dimension of strategic risk management**, enabling informed, adaptive, and resilient investment decision-making.