Quiz 2.4

The experimental specific heat of potassium metal at low temperature has the form $C = \gamma T + \alpha T^3$. Its Fermi temperature is 37368K

- Explain the origin of each of the two terms in this expression.

  In the low-temperature specific heat expression for potassium metal, $C = \gamma T + \alpha T^3$, the two terms correspond to the contributions of electrons and lattice vibrations (phonons) to the specific heat in the metal.

  - γT term (linear term): This term originates from the free electrons in the metal. The density of states of free electrons near the Fermi energy in metals has significant variations, and these electrons play an important role in heat conduction. At low temperatures (much lower than the Fermi temperature), only electrons near the Fermi energy can be excited. The heat capacity of these electrons is proportional to the temperature, thus giving rise to the linear term γT. Here, γ is a constant, which is related to the density of electronic states and effective mass in the metal.

  - αT^3 term (cubic term): This term originates from the phonons (lattice vibrations) in the metal lattice. Phonons are quantized excitations of lattice vibrations and can be considered as a kind of "particle" in the lattice. At low temperatures, the excitation probability of lattice vibrations is low, and the heat capacity is proportional to the cube of the temperature. This relationship is derived based on the Debye model, which is a classical theory for describing the phonon heat capacity in solids. In this expression, α is a constant, which is related to the Debye temperature of the lattice.

- Estimate what fraction of electrons can contribute to the heat capacity of potassium at room temperature.

  The Fermi temperature of potassium is given as $T_F = 37368\text{K}$

  Room temperature $T = 300\text{K}$

  We have: $T/T_F \approx 008028$

  We got, at room temperature, roughly 0.8% of the electrons can contribute to the heat capacity of potassium. This fraction represents the electrons near the Fermi energy that can be thermally excited at room temperature.