Condensed Matter Physics 2023
Quiz 4 (Week 11)

1. In a powder diffraction experiment using a collimated beam of monochromatic X-rays with wavelength $\lambda = 1.62$ Å, diffraction peaks are observed at angles $2\theta = 42.3$, 49.2, 72.2 and 87.4°.
   (a) Identify the lattice type.
   (b) Calculate the lattice constant.

2. Mainstream only. Given $E(k) = 3A\sin^2(\frac{ka}{2})$ is the energy of an electron band in a one-dimensional material, calculate the group velocity $v_g(k)$ and show that at the Brillouin zone boundary v_g is zero.

3. Advanced only. Show that the effective mass of an electron in a one-dimensional crystal that travels with group velocity $v_g(k)$ and energy $E = \hbar\omega(k)$ is given by $m^* = \hbar^2(\partial^2 E/\partial k^2)^{-1}$. Hint: consider the effective mass m^* as the quantity that satisfies Newton's second law; the force on the electron is given by $F = dp/dt = \hbar dk/dt$.

4. (a) Consider a simple square lattice (two dimensions). The kinetic energy of a free electron at a corner of the first Brillouin zone is higher than that of an electron at midpoint of a side face of the zone by a factor b. What is the value of b?
   (b) What is the corresponding factor for a simple cubic lattice in three dimensions?
   (c) Consider now a two- or three-dimensional crystal formed by a divalent element, described within the nearly-free electron model. Given the results above, can you explain a scenario where this solid is a metal?