Condensed Matter Physics 2023
Formula Sheet

$$m = 9.11 \times 10^{-31}\ \text{kg};\ e = 1.602 \times 10^{-19}\ \text{C};\ \hbar = 1.05457 \times 10^{-34}\ \text{J s};$$
$$k_B = 1.38 \times 10^{-23}\ \text{J/K} = 8.617 \times 10^{-5}\ \text{eV/K}$$
$$\mu_0 = 4\pi \times 10^{-7}\ \text{H/m};\ \epsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{J m})$$

$$\langle v \rangle = \sqrt{\frac{3k_BT}{m}}; \qquad v_d = -\frac{e\tau}{m}E; \qquad \mu = \frac{e\tau}{m}$$
$$j = \frac{ne^2\tau}{m}E; \qquad \sigma = \frac{ne^2\tau}{m}; \qquad \rho = \frac{1}{\sigma}$$
$$R_H = \frac{\rho_{xy}}{|B|} = -\frac{1}{ne}$$
$$\kappa = \frac{1}{3}nc_v\langle v \rangle \lambda; \qquad L = \frac{\kappa}{\sigma T} = \frac{3k_B^2}{2e^2}$$

$$E(\mathbf{k}) = \frac{\hbar^2 k^2}{2m}; \qquad \mathbf{k} = (k_x, k_y, k_z) = \left(\frac{2\pi n_x}{L}, \frac{2\pi n_y}{L}, \frac{2\pi n_z}{L}\right); \qquad \psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{V}}e^{i\mathbf{k}\cdot\mathbf{r}}$$
$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}; \qquad T_F = \frac{E_F}{k_B}; \qquad k_F = (3\pi^2 n)^{1/3}; \qquad v_F = \sqrt{\frac{2E_F}{m}} = \frac{\hbar k_F}{m}$$
$$g(\epsilon) = \frac{3n}{2E_F}\left(\frac{\epsilon}{E_F}\right)^{1/2}$$
$$C_v = \frac{\pi^2}{3}k_B^2 T V g(E_F) = \frac{\pi^2}{3}\frac{3Nk_B}{2}\frac{T}{T_F}$$

$$\mathbf{R} = m\mathbf{a}_1 + n\mathbf{a}_2 + o\mathbf{a}_3; \qquad \mathbf{G} = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3; \qquad e^{i\mathbf{G}\cdot\mathbf{R}} = 1$$
$$\mathbf{b}_1 = \frac{2\pi}{V}\,\mathbf{a}_2 \times \mathbf{a}_3; \quad \mathbf{b}_2 = \frac{2\pi}{V}\,\mathbf{a}_3 \times \mathbf{a}_1; \quad \mathbf{b}_3 = \frac{2\pi}{V}\,\mathbf{a}_1 \times \mathbf{a}_2; \quad V = \mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3); \quad \mathbf{a}_i \cdot \mathbf{b}_j = 2\pi\delta_{ij}$$

$$S_{(hkl)} = \sum_j f_j\, e^{2\pi i(hx_j + ky_j + lz_j)}$$
$$n\lambda = 2d\sin\theta; \qquad d = 2\pi/|\mathbf{G}|$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}}u_{\mathbf{k}}(\mathbf{r}); \qquad u_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r} + \mathbf{R})$$
$$v_i(\mathbf{k}) = \frac{1}{\hbar}\frac{\partial E(\mathbf{k})}{\partial k_i}; \qquad m^*_{ij} = \hbar^2\left(\frac{\partial^2 E}{\partial k_i \partial k_j}\right)^{-1}$$

$$\omega = 2\sqrt{\frac{\kappa}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|; \qquad k = \frac{2\pi}{aN}m = \frac{2\pi}{\lambda}$$
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$n_{\text{intrinsic}} = p_{\text{intrinsic}} = \sqrt{np} = \frac{1}{\sqrt{2}}\left(\frac{k_BT}{\pi\hbar^2}\right)^{3/2}(m_e^* m_h^*)^{3/4}\, e^{-E_{\text{gap}}/(2k_BT)}$$
$$\mu = \frac{1}{2}(\epsilon_c + \epsilon_v) + \frac{3}{4}(k_BT)\log\left(\frac{m_h^*}{m_e^*}\right)$$

$$\mathbf{M} = \chi\mathbf{H}; \qquad \chi_{\text{Larmor}} = -\frac{Zne^2\mu_0\langle r^2\rangle}{6m}$$
$$\chi_{\text{Landau}} = -\frac{1}{3}\chi_{\text{Pauli}}; \qquad \chi_{\text{Pauli}} = \mu_0\mu_B^2 g(E_F); \qquad \chi_{\text{Curie}} = \frac{n\mu_0(g\mu_B)^2}{3}\frac{J(J+1)}{k_BT} = \frac{C}{T}$$

Final Exam PHYS3936: Condensed Matter Physics 2023

1. Explain what is meant by the Fermi energy, Fermi temperature and Fermi surface of a metal. Calculate the values of the Fermi energy, Fermi temperature and Fermi wavevector for copper, knowing that its atomic density is 8.45×10^{28} m^{-3} and that copper is monovalent (one free electron per atom).

(4 points)

2. Describe qualitatively five types of chemical bonds, explaining the atomic scale mechanisms responsible for each of them and giving an example of materials that exhibit each type of bond.

(4 points)

3. Consider a monoatomic solid that crystallises in a body-centered tetragonal structure. The conventional unit cell is described by the primitive vectors $(a, 0, 0)$, $(0, a, 0)$ and $(0, 0, c)$, with $a = 4.2$ Å and $c = 3a/2$, and a basis consisting of two atoms at positions $(0, 0, 0)$ and $(a/2, a/2, c/2)$.

   (a) Find the primitive vectors of the reciprocal lattice.

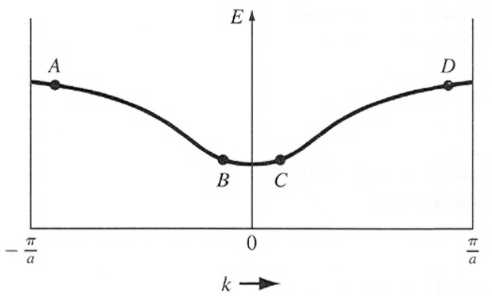
   (1 point)

   (b) A powder sample is analysed by X-ray diffraction using a monochromatic X-ray of wavelength $\lambda = 1.5$ Å. Calculate the diffraction angles 2θ of the first three diffraction peaks (i.e. those with the lowest 2θ values).

   (3 points)

4. The energy E vs. k diagram for a particular energy band is shown below. Determine the sign of the effective mass and the direction of (group) velocity for an electron at each of the four positions A, B, C, D shown. Approximately at which k is the effective mass largest and why?

(3 points)



5. (a) Describe which experiments can be used to determine the following properties of a semiconductor: sign of the majority carriers, carrier concentration (assuming that one carrier type is dominant), band gap, and mobility of the majority carriers.

   (b) What is the effect on the conductivity of increasing temperature on an intrinsic semiconductor and on a metallic conductor?

(5 points)

6. (a) Briefly explain the origin of Larmor diamagnetism.

   (b) Argon (Ar) is a noble gas with atomic number 18 and an atomic radius of 1.88 Å. At low temperature it is solid, forming an fcc crystal structure with lattice parameter $a = 5.26$ Å. What is the magnetic susceptibility of solid argon?

(4 points)

7. Choose one topic from the list below. From the perspective of condensed matter physics, briefly explain the essential concepts and principles that underlie our understanding the topic.

   - Scattering and diffraction
   - Magnetism in solids
   - Superconductivity

(6 points)