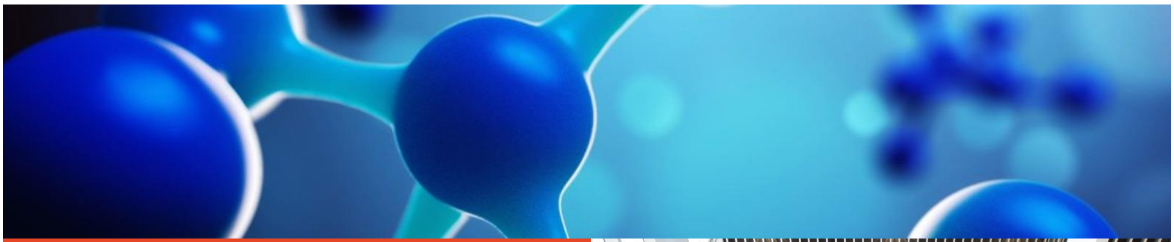
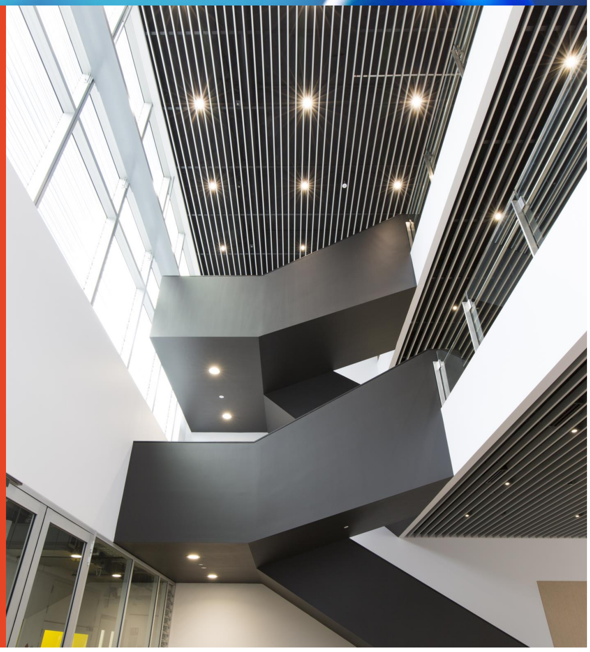


CMP Lecture 1

Carla Verdi

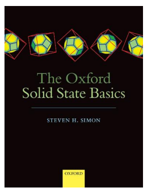
School of Physics
The University of Sydney

THE UNIVERSITY OF SYDNEY



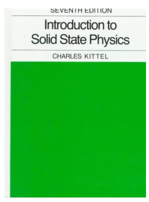
Condensed Matter Physics: info

- 19 **lectures** (Tue 1pm, LT1; Wed 9am, LT2; Fri 12pm, LT1)
- 1 **Assignment** (5% weight)
- 6 **Quizzes** (1% each for good attempt; take best 5) (5% weight)
  Due Friday 11:59pm, solutions Monday, no submissions after Sunday.
  1: solid attempt to answer all but 1 question at least; 0.5: fewer questions attempted or severe mistakes; 0: only 1 question attempted
- **Exam** (PP+CMP) (55%)
- Use **ED** for questions, or email: carla.verdi@sydney.edu.au
- **Weekly** reading + lecture notes (slides) on Canvas
- Zoom "**open-office** style tutes" on Thursdays at 1pm



- Main **book**:
  The Oxford Solid State Basics 1st Edition by S. H. Simon (2013); eBook in Library (relevant chapters posted with Lecture Notes weekly)
- Other good books:

i. *Solid State Physics*, N. Ashcroft and N. D. Mermin
ii. *Introduction to Solid State Physics*, C. Kittel



Condensed Matter Physics: info

- Assignment will be available on Canvas from Friday 5 May (latest), due Friday 19 May
- What is examinable? – material provided in Lectures, Assignments, Quizzes
- The last 3 lectures will be separate for M & A.
- Prof. David McKenzie will give Mainstream lectures in week 13: Tue 1pm LT2, Wed 9am LT5, Fri 12pm LT5

Lectures

- **1-3** Physics of solids without considering microscopic structure: Drude Theory, Free Electron Theory; types of matter
- **4** Chemical bonding
- **5-6** Crystal structure and reciprocal lattice
- **7-8** Electron scattering and diffraction
- **9** Bloch's Theorem
- **10** Nearly free electron model
- **11** Tight binding model
- **12-13** Lattice vibrations and phonons
- **14-15** Semiconductors
- **16-17** Magnetism in solids
- **18** CMP in research
- **19** Overview, revision

Condensed Matter Physics

- Condensed Matter Physics (also known as Solid State Physics) explains the properties of solid and liquid materials. It is the study of the behaviour of atoms when they are placed in close proximity to one another.
- The properties are expected to follow from Schrödinger's equation for a collection of atomic nuclei and electrons interacting with electrostatic forces.
- The fundamental laws governing the behaviour of solids are known and well tested.
- *"Condensed Matter" being more general than just solid state was coined by Nobel-Laureate Philip W. Anderson.*

Crystalline solids

- We will deal with crystalline solids, that is solids with an atomic structure based on a regular repeated pattern.
- Many important solids are crystalline.
- More progress has been made in understanding the behaviour of crystalline solids than that of non-crystalline materials since the calculation are easier in crystalline materials.

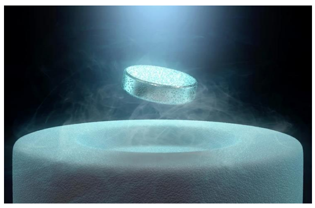
CMP & future device technologies

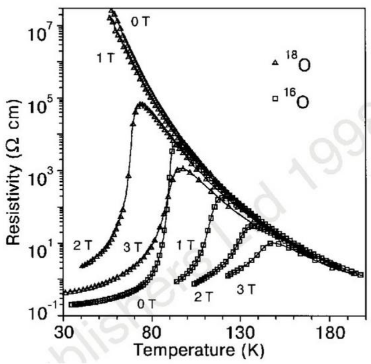
- Understanding the electrical properties of solids is right at the heart of modern society and technology.
- For example, the entire computer and electronics industry relies on the tuning of a special class of material, the semiconductor, which lies right at the metal-insulator boundary. Condensed matter physics provides a background to understand what goes on in semiconductors.
- New technology for the future will inevitably involve developing and understanding new classes of materials (e.g. superconductors, spintronic, topological insulators, catalytic etc)

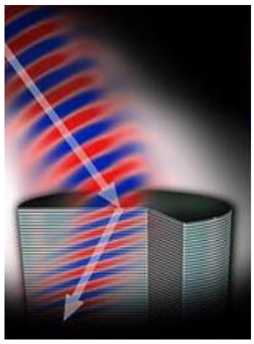
Frontiers of Condensed Matter Physics

New 'functional materials'

- Photovoltaics
- Superconductors
- Magnetoelectrics/ magnetoresistors
- Thermoelectrics
- Spintronics
- Metamaterials
- ...



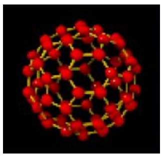


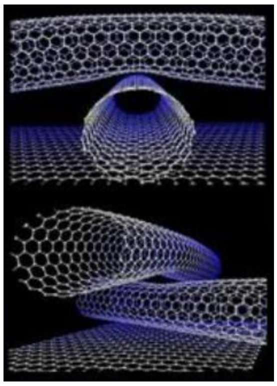


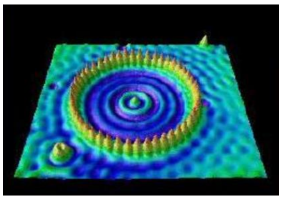
Frontiers of Condensed Matter Physics

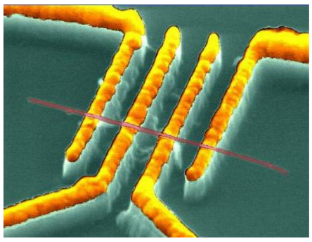
Physics of nanostructures

- Quantum dots
- Nanowires
- Heterostructures
- Molecular self-assembly
- Graphene / carbon nanotubes
- Nano-biology
- ...





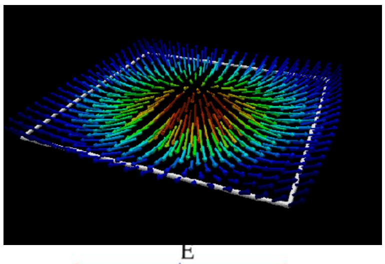


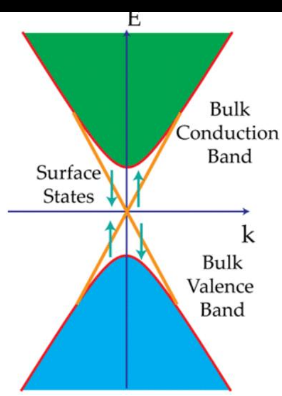


Frontiers of Condensed Matter Physics

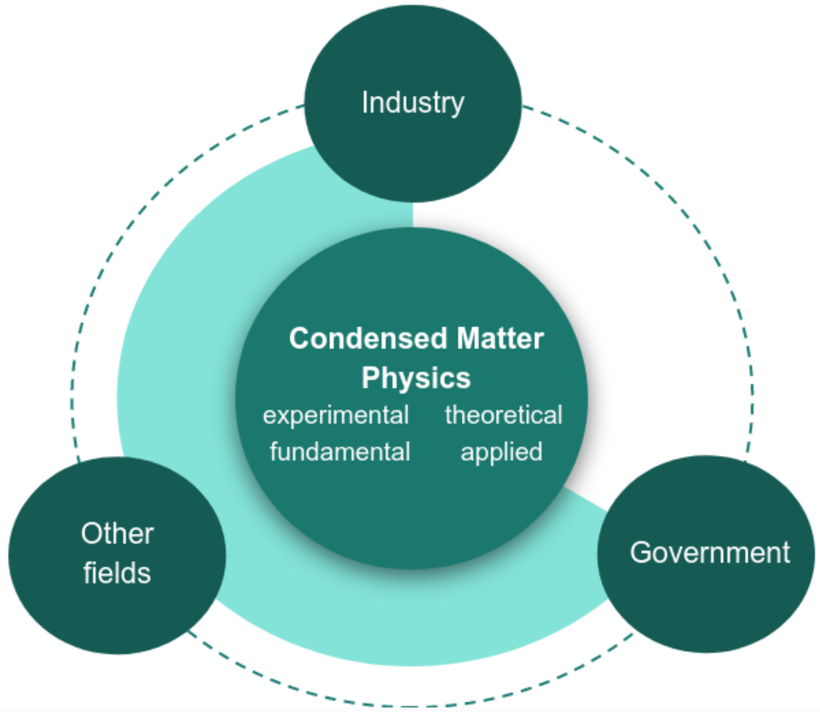
New emergent states of matter

- Fractional quantum Hall effect
- Unconventional superconductivity
- Skyrmions
- Polarons
- Quantum critical phases
- Topological insulators
- ...





Frontiers of Condensed Matter Physics



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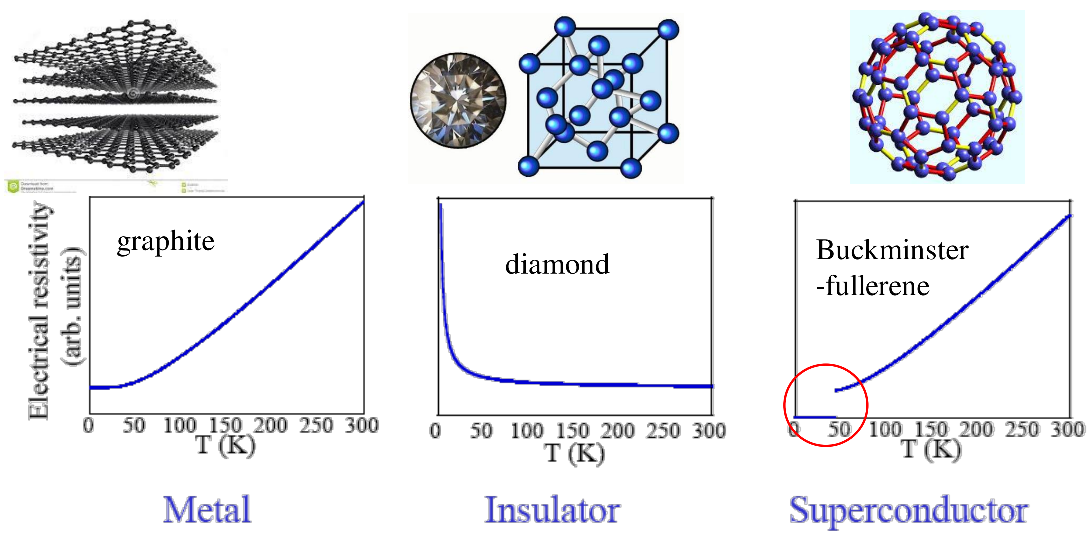
Properties of matter depend upon how the atoms are put together

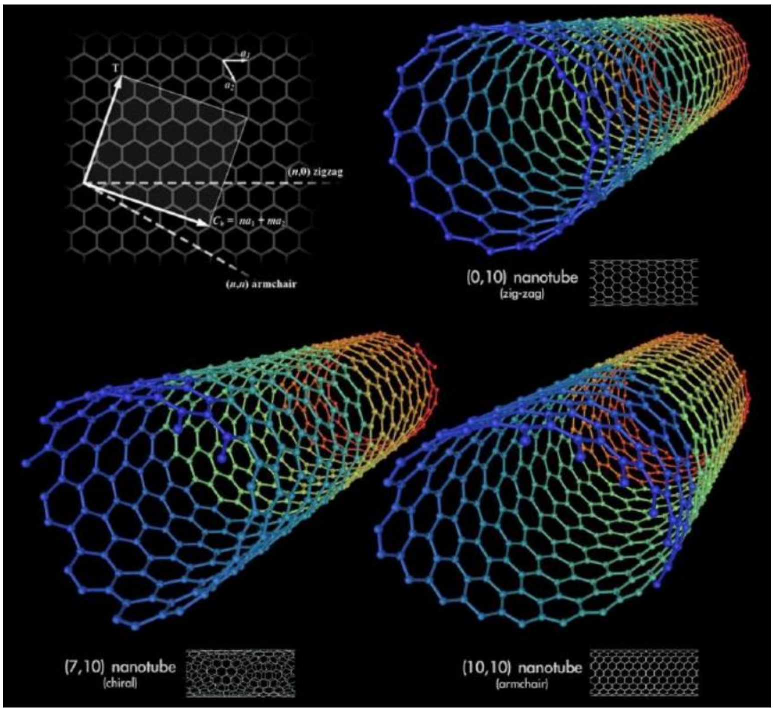
e.g. Electrical resistivity of three states of solid matter:



They are all just carbon!

12

Why are some carbon nanotubes metals and others semiconductors?



Electrons in metals: Drude theory (classical approach)

The defining characteristic of a metal is that it conducts electricity. At some level the reason for this conduction is due to the fact that electrons are mobile in these materials.

In 1900 Paul Drude realized that he could apply Boltzmann's kinetic theory of gases to understanding electron motion within metals. This theory was remarkably successful, providing a first understanding of metallic conduction.

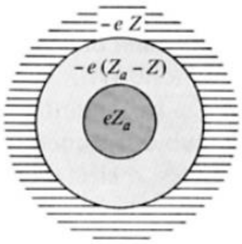
Three assumptions about the motion of electrons:
(1) Electrons have a scattering time τ. The probability of scattering within a time interval dt is dt/τ.
(2) Once a scattering event occurs, we assume the electron returns to momentum $p = 0$.
(3) In between scattering events, the electrons (which are negatively charged), respond to an externally applied electric field E and magnetic field B.

The first two of these assumptions are those made in the kinetic theory of gases. The third assumption is a logical generalization to account for the fact that, unlike gas molecules, electrons are charged and must therefore respond to electromagnetic fields.

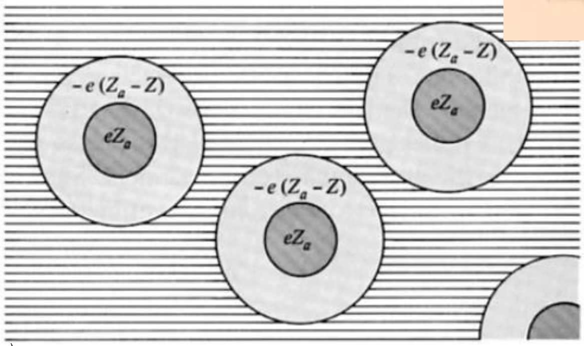
Chapter 3, Oxford Basics
A&M Ch1,pg 2-15;20-25

Drude picture of atom

Isolated atom



Atoms in metal



Z_a is the atomic number
Z is the number of valence electrons
Z_a - Z is the number of core electrons (ions)

Nucleus
Core electrons
Valence electrons

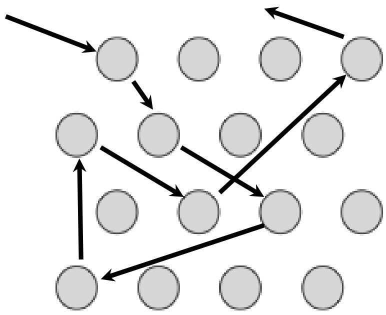
Ion: Nucleus, Core
Conduction electrons

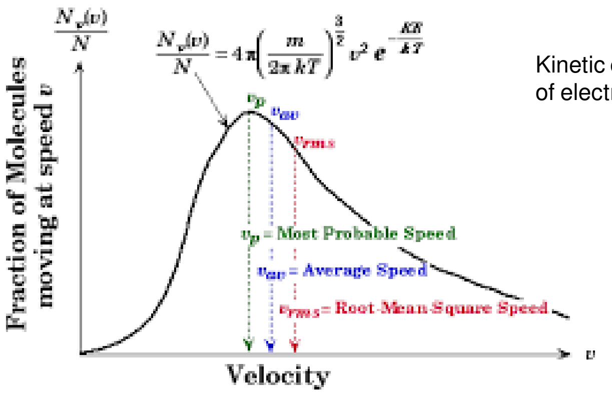
Fig. 1.1 A&M

Valence electrons move freely through metal
Ions remain intact, act as immobile positive particles

Drude's classical theory

- Drude treated the (free) electrons as a **classical ideal gas** but the electrons collide with the stationary ions, not with each other.
- No interaction with each other, independent electron approximation
- Interaction between electrons and core instantaneous and short range





$v_p = \sqrt{2\frac{kT}{m}}$ $v_{av} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$ $v_{rms} = \sqrt{3\frac{kT}{m}}$

Kinetic energy of electron

$$\frac{1}{2}mv_t^2 = \frac{3}{2}k_BT$$

energy of electron at temp, T

$$v_t = \sqrt{\frac{3k_BT}{m}}$$

so at room temp.

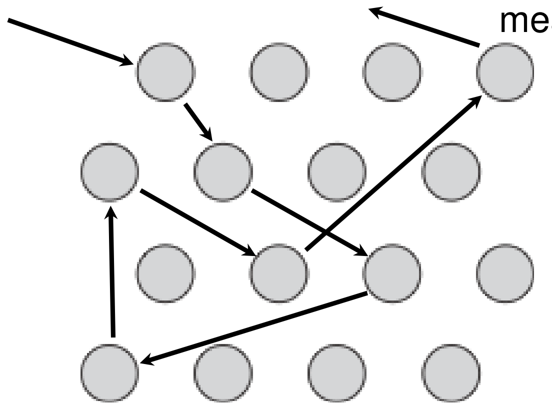
$$v_t \approx 10^5 \mathrm{ms}^{-1}$$

rms velocity

Drude's classical theory

Three assumptions:

1) Electrons have a scattering/relaxation time τ (average time between scattering events)
2) Scattered, electron returns to momentum = 0
3) In between scattering events the electrons respond to **E** and **B**-fields



mean free path $\lambda = \tau v_t$

$$\lambda \approx 1\mathrm{nm}$$
$$v_t \approx 10^5 \mathrm{ms}^{-1}$$
$$\tau \approx 1 \times 10^{-14}\ \mathrm{s}$$

Drude theory

We consider an electron with momentum p at time t and ask what momentum it will have at time t+dt. There are two terms in the answer.
There is a probability dt/τ that it will scatter to momentum zero. If it does not scatter to momentum zero (with probability $1 - dt/\tau$) it simply accelerates as dictated by its usual equations of motion $dp/dt = F$.
Putting the two terms together we have:

$$\langle p(t+dt)\rangle = \left(1 - \frac{dt}{\tau}\right)(p(t)+Fdt)+0\ dt/\tau \longrightarrow \frac{d\mathbf{p}}{dt} = \mathbf{F} - \frac{\mathbf{p}}{\tau}$$

where here the force F on the electron is just the Lorentz force

$$F = -e(E + v \times B).$$

One can think of the scattering term $-p/\tau$ as just a drag force on the electron.
In the absence of any externally applied field the solution to this differential equation is an exponentially decaying momentum

$$p(t)= p_{initial}\, e^{-t/\tau}$$

Consistent with particles that lose momentum by scattering.

Drude theory

Electrons in an Electric Field

Take the electric field to be non-zero but the magnetic field is zero. Then,

$$\frac{dp}{dt} = -eE - \frac{p}{\tau}$$

In steady state, $dp/dt = 0$ so we have

$$mv = p = -e\tau E$$

with m the mass of the electron and v its velocity.
If there is a density n of electrons in the metal each with charge −e, and they are all moving at velocity v, then the electrical current is given by,

$$j = -env = \frac{e^2 \tau n}{m} E$$

And the conductivity of the metal, defined via $\mathbf{j} = \sigma \mathbf{E}$ is,

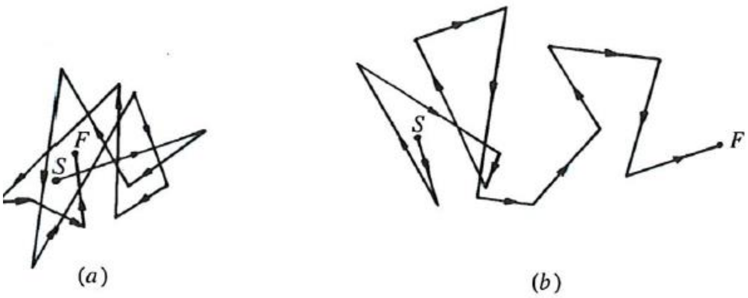
$$\sigma = \frac{e^2 \tau n}{m}$$

So by measuring the conductivity of the metal we can determine the product of the electron density and scattering time of the electron.

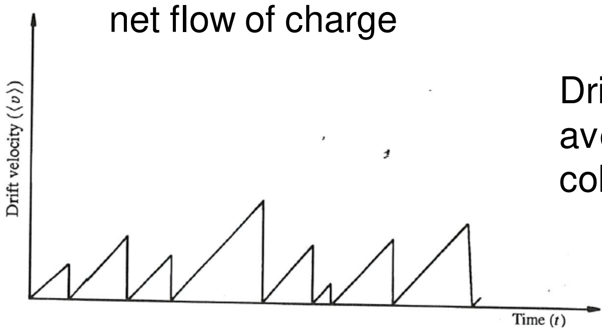
The (drift) velocity is then

$$v = -\frac{e\tau}{m} E$$ and for $E = 10$ V/m, $v = 10^{-2}$ m/s

Concept of drift velocity



Zero (a) and non-zero (b) electric field
In E-field the start S and finish F positions differ: net flow of charge



Drift velocity versus time; average time between collisions is the relaxation time

Drude theory: electrical conductivity

Conductivity: $$\sigma = \frac{e^2 \tau n}{m}$$

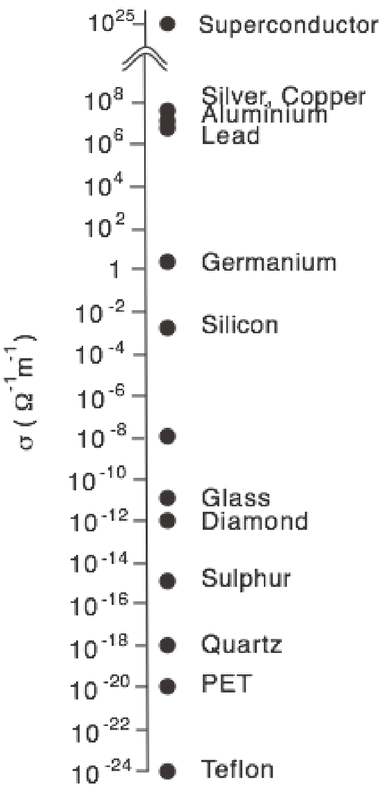
Resistivity: $$\rho = \frac{m}{n e^2 \tau} = \frac{1}{\sigma}$$

Mobility: $$\mu = e\tau / m$$

Drift velocity: $$\mathbf{v} = \frac{-e\tau}{m}\mathbf{E}$$

$$\mathbf{v} = -\mu \mathbf{E}$$

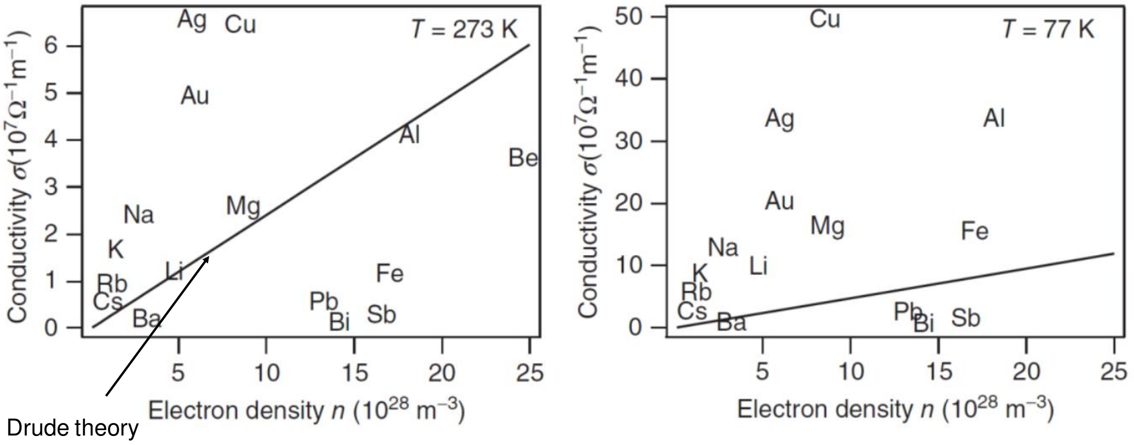
Electrical conductivity of materials



Varies by 32 orders of magnitude (not including superconductivity)

From size of atom to earth-sun distance is only 22 orders of magnitude...

Drude theory: electrical conductivity



Drude theory prediction

$$\sigma = \frac{ne^2\tau}{m_e} \qquad \frac{1}{2}mv_t^2 = \frac{3}{2}k_BT$$

$$\lambda \approx 1\text{nm} \qquad v_t = \sqrt{\frac{3k_BT}{m}}$$

$$\tau = \lambda/v_t$$

$$\boxed{\sigma \propto \frac{1}{\sqrt{T}}}$$

Experimental values increase by a factor of 10 for the low temperature; BUT calculated values by Drude theory, less than a factor of 2.
The lower the temperature the worse this gets.

Questions

A uniform silver wire has a resistivity of 1.54×10^{-8} Ωm at room temperature. For an electric field along the wire of 1 volt cm^{-1}, compute the average drift velocity of electron assuming that there is 5.8×10^{28} conduction electrons $/m^3$. Also calculate the mobility.

v_d = 0.6997 m/s
$\mu = 6.997 \times 10^{-3}\ m^2/(Vs)$

Drude theory

Electrons in Electric and Magnetic Fields

Consider the transport equation for a system in both an electric and a magnetic field,

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \mathbf{p}/\tau.$$

Again, setting this to zero in the steady state, and using $\mathbf{p}=m\mathbf{v}$ and $\mathbf{j}=-ne\mathbf{v}$, we obtain,

$$0 = -e\mathbf{E} + \frac{\mathbf{j} \times \mathbf{B}}{n} + \frac{m}{ne\tau}\mathbf{j}$$

$$\mathbf{E} = \left(\frac{1}{ne}\mathbf{j} \times \mathbf{B} + \frac{m}{ne^2\tau}\mathbf{j}\right).$$

Now defining a 3x3 resistivity matrix $\underset{\sim}{\rho}$ which relates the current vector to the electric field,

$$\mathbf{E} = \underset{\sim}{\rho}\mathbf{j}$$

where the components are given by, $\rho_{xx} = \rho_{yy} = \rho_{zz} = \dfrac{m}{ne^2\tau}$

Drude theory: Hall coefficient

Taking $\mathbf{B}$ in the $\hat{z}$ direction then,

$$\rho_{xy} = -\rho_{yx} = \frac{B}{ne}$$

and all other components are zero. This off diagonal term is called the *Hall resistivity*.

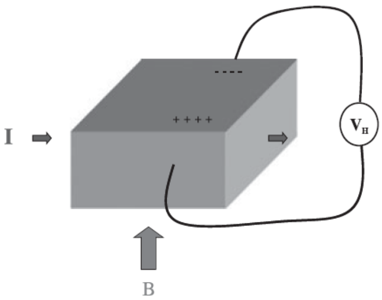
The Hall coefficient R_H is defined as

$$R_H = \frac{\rho_{yx}}{|B|}$$

and in Drude theory as

$$R_H = \frac{-1}{ne}$$

This allows to measure the density of electrons in a metal.



Hall's 1879 experiment
The voltage measured perpendicular to both the magnetic field and current is known as the Hall voltage, which is proportional to B and inversely proportional to the electron density

Drude theory: Hall coefficient

Calculating $n = -1/(eR_H)$ for various metals and dividing by the density of atoms should give the number of free electrons per atom. This is done in the below table. Middle column obtained from Drude theory with measured Hall Coefficient, seems in good agreement for valence of Li, Na, K being one. Also the effective valence of Cu is one.

Material	$\dfrac{1}{-e\,R_H\,n_{atomic}}$	Valence
Li	.8	1
Na	1.2	1
K	1.1	1
Cu	1.5	1
Be	-0.2*	2
Mg	-0.4	2
Ca	1.5	2

But for divalent atoms can be wrong; the sign of the Hall coefficient can be incorrect (due to Be and Mg having opposite sign charge carriers to electrons, namely “holes”).

Drude theory: Lorenz number

Thermal transport

Drude also attempted to calculate the thermal conductivity κ due to mobile electrons using Boltzmann’s kinetic theory obtaining,

$$\kappa = \frac{1}{3} n c_v \langle v \rangle \lambda$$

where c_v is the heat capacity per particle, $\langle v \rangle$ is the average thermal velocity and $\lambda = \langle v \rangle \tau$ is the scattering length. For a monoatomic gas the heat capacity per particle is,

$$c_v = \frac{3}{2} k_B \quad \text{and} \quad \langle v \rangle = \sqrt{\frac{8 k_B T}{\pi m}}$$

We then obtain,

$$\kappa = \frac{4}{\pi} \frac{n \tau k_B^2 T}{m}$$

Then, taking the ratio of the thermal conductivity to electrical conductivity,

$$L = \frac{\kappa}{T\sigma} = \frac{4}{\pi} \left(\frac{k_B}{e} \right)^2 \approx 0.94 \times 10^{-8}\ \mathrm{WattOhm/K^2}\,.$$

Called the Lorenz number

Drude theory: Lorenz number

If instead of using $\langle v \rangle^2$ we use $\langle v^2 \rangle$
a slightly different prediction is obtained:

$$L = \frac{\kappa}{T\sigma} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 \approx 1.11 \times 10^{-8} \ \mathrm{WattOhm/K^2}$$

Viewed at the time as huge success since almost all metals have roughly the same value of this ratio – known as the Wiedemann-Franz law.
Drude's predicted Lorenz number is close to the measured one (below) – off by a factor of about 2.

Lorenz numbers $\kappa/(T\sigma)$

Material	L
Lithium (Li)	2.22
Sodium (Na)	2.12
Copper (Cu)	2.20
Iron (Fe)	2.61
Bismuth (Bi)	3.53
Magnesium (Mg)	2.14

10^{-8} WattOhm/K^2

BUT – was a coincidence since there are two mistakes that roughly cancel out: the specific heat is too large and the velocity is too small!
Both mistakes are due to Fermi statistics of the electron (ignored so far) and the Pauli exclusion principle.

End