Condensed Matter Physics 2023
Quiz 3 (Week 10)

1. (a) Find the primitive reciprocal lattice vectors of a face-centered-cubic (fcc) lattice. What type of lattice is the reciprocal lattice?

   (b) The primitive vectors of a hexagonal lattice can be taken as $\mathbf{a}_1 = \frac{a\sqrt{3}}{2}\hat{\mathbf{x}} + \frac{a}{2}\hat{\mathbf{y}}$, $\mathbf{a}_2 = -\frac{a\sqrt{3}}{2}\hat{\mathbf{x}} + \frac{a}{2}\hat{\mathbf{y}}$ and $\mathbf{a}_3 = c\hat{\mathbf{z}}$. Determine the volume of the primitive cell, the primitive reciprocal lattice vectors and what type of lattice is the reciprocal lattice.

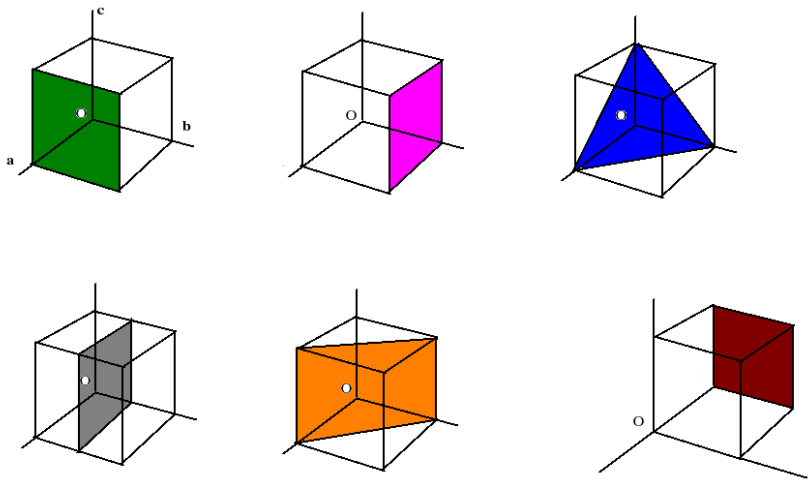
2. Consider a simple cubic crystal of side a.

   (a) Give the Miller indices of the planes shown in the figure below.

   (b) Draw the [100], [110], [111], [120] and $[\bar{1}00]$ directions.

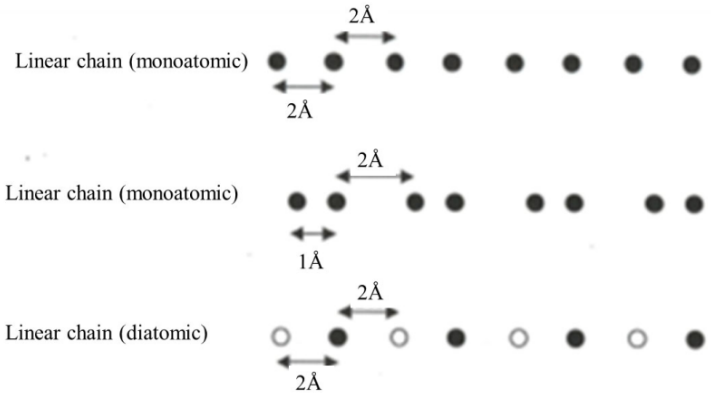
   (c) Write all possible $\langle 001\rangle$, $\langle 110\rangle$ and $\langle 111\rangle$ directions.

   (d) The first Brillouin zone boundary in the [111] reciprocal space direction occurs at $\frac{2\pi}{a}(0.5, 0.5, 0.5)$. Is this correct, and why?



3. Define the Wigner-Seitz cell and explain how to construct it. What is the Wigner-Seitz cell of the reciprocal lattice?

4. <u>Mainstream only.</u> Find the reciprocal lattice vectors for the three lattices below (include units in your answers).



5. <u>Advanced only.</u> Consider a two-dimensional (2D) monoatomic solid measuring $L \times L$ in which the atoms are arranged in a square lattice with lattice parameter a (L is an integer multiple of a). Each atom in the solid contributes one electron to the electron (Fermi) sea.

(a) Compute the Fermi wavevector k_F. (Hint: find the number of k points contained in the Fermi circle, by knowing that the area per k point is $(2\pi/L)^2$, then consider the total number of electrons in the solid.)

(b) Make a 2D sketch of the reciprocal lattice and indicate the Fermi circle. In particular, compare the size of the Fermi circle with the size of the first Brilloin zone.