1 Problem 5

5.3 $\dfrac{2\lambda}{d} \times 50\text{cm} = 1\text{cm} \quad \Rightarrow \quad d = 63\mu\text{m}.$

5.4 $D = 2 \times \dfrac{1.22\lambda}{d} \times 3.76 \times 10^8\text{m}, \quad \lambda = 632.8\text{nm}.$

Table 1: Diameters

on Earth d	on Moon D
2mm	290.3km
2m	290.3m
5m	116.1m

5.6 $\theta_0 = 1.22\lambda/D = 1.342 \times 10^{-7}\text{rad} = 0.02768''$. The angular limit of resolution of a human eye is $1'$, $1'/0.02768'' = 2167$.

5.9 **(1)** $\delta y = 0.61\lambda/0.85 = 287\text{nm}$. **(2)** $\delta y = 0.61\lambda/1.45 = 168\text{nm}$, the limit of resolution of a human eye is $\delta y' = 25\text{cm} \times 1' = 72.7\mu\text{m}$. $\delta y'/\delta y = 432$.

5.13 $\theta = 2\lambda/d$, thus $\delta\theta = 2\delta\lambda/d = 5 \times 10^{-6}\text{rad} = 1.03''$. While half the linewidth is $\Delta\theta \approx \dfrac{\lambda}{Nd} = 5 \times 10^{-6}\text{rad} \sim \delta\theta$. 恰能分辨.

5.16 Let

$$s(x) = \sum_{n=-N}^{N-1} \delta(x - 6a(n + 1/2)), \quad t(x) = \begin{cases} 1, & x \le a/2, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Their Fourier transformations

$$\mathscr{F}\{s\} := \int_{-\infty}^{\infty} s(x) \mathrm{e}^{ikx\sin\theta} \mathrm{d}x = \sum_{n=-N}^{N-1} \mathrm{e}^{\mathrm{i}6ak\sin\theta(n+1/2)} = \frac{\sin 6Nak\sin\theta}{\sin 3ak\sin\theta}, \tag{2}$$

$$\mathscr{F}\{t\} := \int_{-\infty}^{\infty} t(x) \mathrm{e}^{ikx\sin\theta} \mathrm{d}x = \left.\frac{\mathrm{e}^{ikx\sin\theta}}{\mathrm{i}k\sin\theta}\right|_{-a/2}^{a/2} = a\frac{\sin\frac{1}{2}ka\sin\theta}{\frac{1}{2}ka\sin\theta}. \tag{3}$$

If we define convolution $* : \mathbb{R}^{\mathbb{R}} \times \mathbb{R}^{\mathbb{R}} \to \mathbb{R}^{\mathbb{R}}$ as follows:

$$(f * g)(x) = \int_{-\infty}^{\infty} f(t)g(x - t)\mathrm{d}t, \tag{4}$$

it is well-know that $\mathscr{F}$ is an isomorphism: $\mathscr{F}\{f * g\} = \mathscr{F}\{f\} \cdot \mathscr{F}\{g\}$.

For **(1)** even numbered slits are blocked, and **(2)** odd numbered slits are blocked, both of their aperature functions are $s * t$. Then the field distribution is

$$E(\theta) = \frac{E_0}{a}\mathscr{F}\{s * t\} = \frac{E_0}{a}\mathscr{F}\{s\} \cdot \mathscr{F}\{t\} = E_0 \left(\frac{\sin 12N\alpha}{\sin 6\alpha}\right)\left(\frac{\sin\alpha}{\alpha}\right), \tag{5}$$

where $\alpha = \frac{1}{2}ka\sin\theta$. Intensity distribution

$$I(\theta) = \frac{I_0}{E_0^2}|E(\theta)|^2 = I_0\left(\frac{\sin 12N\alpha}{\sin 6N\alpha}\right)^2\left(\frac{\sin\alpha}{\alpha}\right)^2. \tag{6}$$

(3) The aperature function is $(s(x) + s(x-2a)) * t$. By calculation:

$$\mathscr{F}\{s(x) + s(x-2a)\} = 2\mathrm{e}^{2\mathrm{i}\alpha}\cos 2\alpha \cdot \mathscr{F}\{s\}. \tag{7}$$

Therefore the field distribution is

$$E(\theta) = \frac{E_0}{a}\mathscr{F}\{(s(x) + s(x-2a)) * t\} = 2E_0\mathrm{e}^{2\mathrm{i}\alpha}\cos 2\alpha\left(\frac{\sin 12N\alpha}{\sin 6\alpha}\right)\left(\frac{\sin\alpha}{\alpha}\right), \tag{8}$$

and intensity distribution

$$I(\theta) = \frac{I_0}{E_0^2}|E(\theta)|^2 = 4I_0\cos^2 2\alpha\left(\frac{\sin 12N\alpha}{\sin 6N\alpha}\right)^2\left(\frac{\sin\alpha}{\alpha}\right)^2. \tag{9}$$

5.18 According to Fresnel-Kirchhoff formula,

$$E = -\frac{\mathrm{i}E_0}{\lambda}\int_L^{\sqrt{L^2+(D/2)^2}} 2\pi r\mathrm{d}r\frac{\mathrm{e}^{\mathrm{i}kr}}{r}\frac{1+L/r}{2} = -\mathrm{i}\pi\frac{E_0}{\lambda}\int_L^{\sqrt{L^2+(D/2)^2}}\left(1+\frac{L}{r}\right)\mathrm{e}^{\mathrm{i}kr}\mathrm{d}r, \tag{10}$$

where $L = 1.5\mathrm{m}$, $D = 2.6\mathrm{mm}$ and $\lambda = 589\mathrm{nm}$. Since $D \ll L$, we have $L/r \approx 1$, and thus

$$E \approx -\mathrm{i}kE_0\int_L^{\sqrt{L^2+(D/2)^2}}\mathrm{e}^{\mathrm{i}kr}\mathrm{d}r = 2E_0\mathrm{e}^{\mathrm{i}\delta}\sin\frac{\pi}{\lambda}\left(\sqrt{L^2+(D/2)^2} - L\right), \tag{11}$$

where $\delta = -\frac{\pi}{2} + \frac{k}{2}\left(L + \sqrt{L^2+(D/2)^2}\right)$. That is

$$I = \frac{I_0}{E_0^2}|E|^2 \approx 4I_0\sin^2\frac{\pi D^2}{8\lambda L} = 4I_0 \times 0.14. \tag{12}$$

Then center point is **dark**.

To make the center point bright, let $\frac{\pi D^2}{8\lambda L} = (2n+1)\frac{\pi}{2}$, only for small $n \in \mathbb{N}$,

$$\begin{aligned} D &= \sqrt{4\lambda L(2n+1)} = \sqrt{2n+1} \times 1.88\mathrm{mm} \\ &= 1.88\mathrm{mm},\ 3.26\mathrm{mm},\ 4.20\mathrm{mm},\ 4.97\mathrm{mm}, \ldots \end{aligned} \tag{13}$$

5.22 Using the result obtained in **5.18**,

$$\begin{aligned} E &\approx -E_0\left[\mathrm{e}^{\mathrm{i}k\sqrt{L^2+r_2^2}} - \mathrm{e}^{\mathrm{i}kL} + \frac{1}{4}\mathrm{e}^{\mathrm{i}k\sqrt{L^2+r_1^2}} - \frac{1}{4}\mathrm{e}^{\mathrm{i}k\sqrt{L^2+r_2^2}}\right] \\ &\approx -E_0\mathrm{e}^{\mathrm{i}kL}\left[\left(\mathrm{e}^{\mathrm{i}kr_2^2/(2L)} - 1\right) + \frac{1}{4}\left(\mathrm{e}^{\mathrm{i}kr_1^2/(2L)} - \mathrm{e}^{\mathrm{i}kr_2^2/(2L)}\right)\right] \\ &= -2E_0\left(1 + \frac{1}{4}\mathrm{e}^{\mathrm{i}\delta}\right)\mathrm{e}^{\mathrm{i}(\delta/2+kL)}\sin\frac{\delta}{2}, \quad \delta = \frac{kr_2^2}{2L} = \pi, \quad \text{using } r_2^2 = 2r_1^2, \\ &= -\mathrm{i}\frac{3}{2}E_0\mathrm{e}^{\mathrm{i}kL} = -\frac{3\mathrm{i}}{2}E_0, \end{aligned} \tag{14}$$

where E_0 is the amplitude on the aperature. Then the intensity is

$$I = \frac{I_0}{E_0^2}|E|^2 = \frac{9}{4}I_0. \tag{15}$$

5.23 **(1)** $\frac{2\pi}{\lambda}\left(\sqrt{L^2 + r_4^2} - L\right) = \pi + 3\times 2\pi \quad \Rightarrow \quad r_4 \approx \sqrt{7\lambda L} = 5.916\text{mm}.$

(2) $\frac{1}{10\text{m}} = \frac{(2n-1)\lambda}{r_n^2} = \frac{1}{2\text{m}} + \frac{1}{s} \quad \Rightarrow \quad s = -2.5\text{m}$, that is, S is imaged 2.5m from the Fresnel zone plate, and on the same side of S.

2 Simulation

Applying the same method in **Problem 5.16**, we have

$$I_{\text{Diffraction}}(\theta) = \left(\frac{\sin\alpha}{\alpha}\right)^2, \tag{16}$$

$$I_{\text{Interference}}(\theta) = \left(\frac{1}{N}\frac{\sin N\beta}{\sin\beta}\right)^2, \tag{17}$$

$$I(\theta) = I_0 \cdot I_{\text{Diffraction}}(\theta) \cdot I_{\text{Interference}}(\theta), \tag{18}$$

where $\alpha = \frac{1}{2}ka\sin\theta$ and $\beta = \frac{1}{2}kd\sin\theta$. $I_{\text{Diffraction}}(\theta)$ and $I_{\text{Interference}}(\theta)$ are dimensionless and normalized so that their maxima are 1.

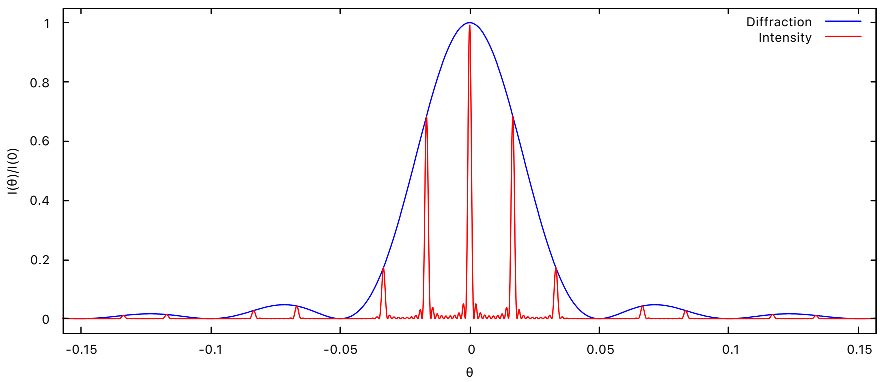

Figure 1: $\lambda = 500\text{nm}, N = 10, d = 30\mu\text{m}, a = 10\mu\text{m}$

(1) We can see in Figure 1 that the intensity $I(\theta)$ is enveloped by the diffraction factor $I_{\text{Diffraction}}(\theta)$. Thus the position of the principal maxima is determined by the interference factor $I_{\text{Interference}}(\theta)$, while their intensities are controlled by the diffraction factor.

(2) Diffraction figures of different N are ploted in Figure 2-4 ($\lambda = 500\text{nm}, d = 20\mu\text{m}, a = 7\mu\text{m}$). As N increases, the width of the principal maxima narrows.

(3) Diffraction figures of different d are ploted in Figure 5-7 ($\lambda = 500\text{nm}, N = 10, a = 7\mu\text{m}$). As d increases, the distance between two nearest principal maxima narrows and roughly proportional to $1/d$.

(4) Diffraction figures of different a are ploted in Figure 8-10 ($\lambda = 500\text{nm}, N = 10, d = 20\mu\text{m}$). As d increases, the distance between two nearest interference minima narrows and roughly proportional to $1/a$.

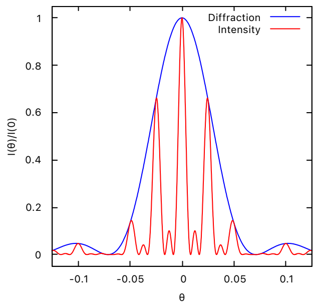

Figure 2: $N = 3$

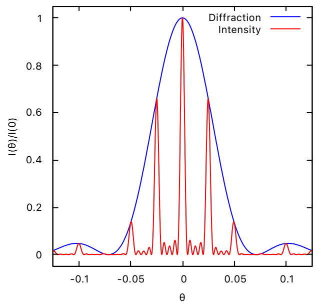

Figure 3: $N = 5$

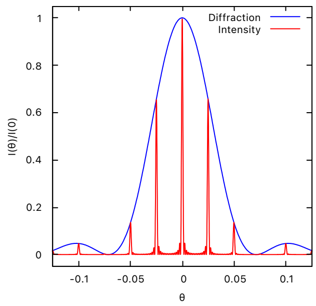

Figure 4: $N = 15$

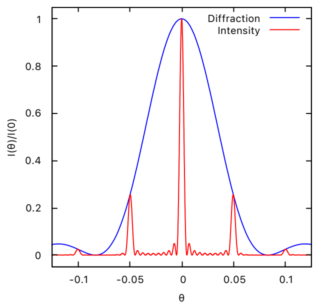

Figure 5: $d = 10\mu\text{m}$

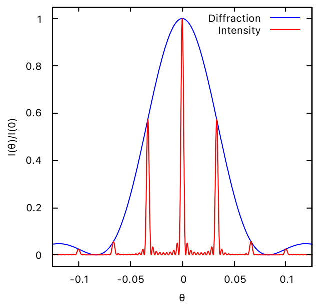

Figure 6: $d = 15\mu\text{m}$

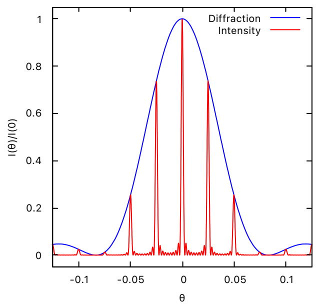

Figure 7: $d = 20\mu\text{m}$

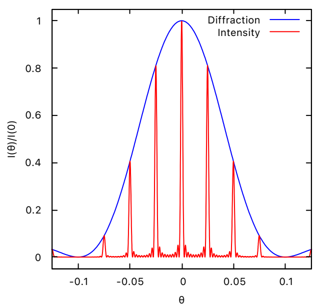

Figure 8: $a = 5\mu\text{m}$

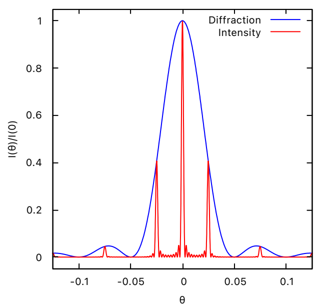

Figure 9: $a = 10\mu\text{m}$

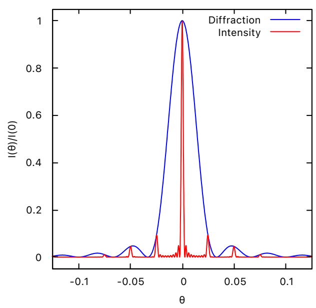

Figure 10: $a = 15\mu\text{m}$

(5) A missing order means an overlap of the maximum of interference factor and the minimum of diffraction factor. The missing orders are $\pm\left(d \cdot \text{lcm}\left(\frac{1}{a}, \frac{1}{d}\right)\right)\mathbb{N}^+$, where lcm denotes the least common multiple.

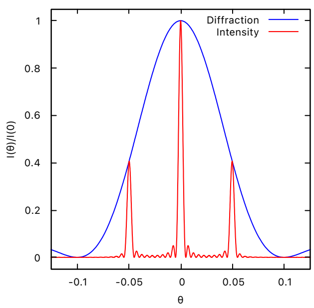

Figure 11: $a = 5\mu\text{m}, d = 10\mu\text{m}$ orders $\pm 2\mathbb{N}^+$ missed.

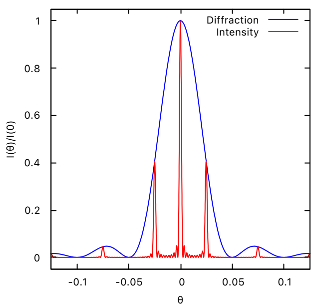

Figure 12: $a = 10\mu\text{m}, d = 20\mu\text{m}$ orders $\pm 2\mathbb{N}^+$ missed.

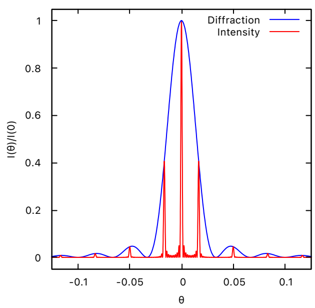

Figure 13: $a = 15\mu\text{m}, d = 30\mu\text{m}$ orders $\pm 2\mathbb{N}^+$ missed.

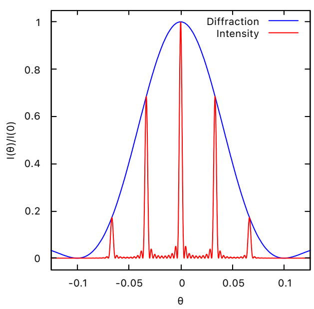

Figure 14: $a = 5\mu\text{m}, d = 15\mu\text{m}$ orders $\pm 3\mathbb{N}^+$ missed.

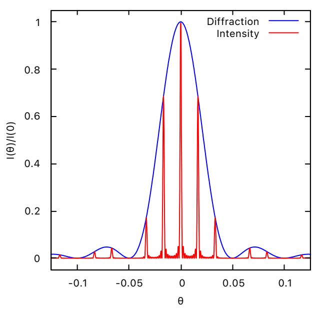

Figure 15: $a = 10\mu\text{m}, d = 30\mu\text{m}$ orders $\pm 3\mathbb{N}^+$ missed.

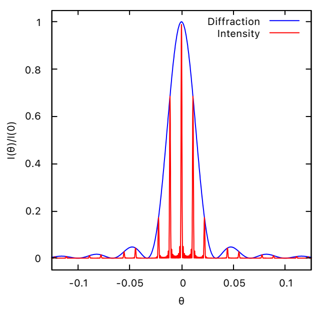

Figure 16: $a = 15\mu\text{m}, d = 45\mu\text{m}$ orders $\pm 3\mathbb{N}^+$ missed.

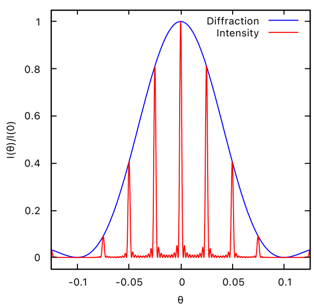

Figure 17: $a = 5\mu\text{m}, d = 20\mu\text{m}$ orders $\pm 4\mathbb{N}^+$ missed.

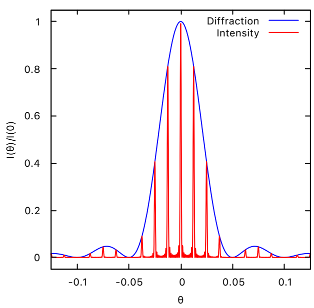

Figure 18: $a = 10\mu\text{m}, d = 40\mu\text{m}$ orders $\pm 4\mathbb{N}^+$ missed.

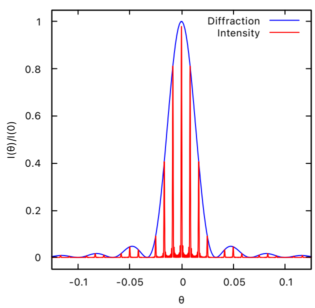

Figure 19: $a = 15\mu\text{m}, d = 60\mu\text{m}$ orders $\pm 4\mathbb{N}^+$ missed.