Light Beat

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Consider two light fields with field functions

$$E_1(x,t) = A_1 \cos\left(k_1 x - \omega_1 t\right) = A_1 \cos\left[\omega_1\left(x/c - t\right)\right], \tag{1}$$
$$E_2(x,t) = A_2 \cos\left(k_2 x - \omega_2 t\right) = A_2 \cos\left[\omega_2\left(x/c - t\right)\right]. \tag{2}$$

Total field

$$\begin{aligned} E(x,t) =& E_1(x,t) + E_2(x,t) = \frac{A_1 + A_2}{2}\Big[\cos\left(\omega_1(x/c-t)\right) + \cos\left(\omega_2(x/c-t)\right)\Big] \\ &+ \frac{A_1 - A_2}{2}\Big[\cos\left(\omega_1(x/c-t)\right) - \cos\left(\omega_2(x/c-t)\right)\Big] \\ =&(A_1 + A_2)\cos\left[\Omega(x/c-t)\right]\cos\left[\omega(x/c-t)\right] \\ &- (A_1 - A_2)\sin\left[\Omega(x/c-t)\right]\sin\left[\omega(x/c-t)\right], \end{aligned} \tag{3}$$

where $\Omega = (\omega_1 - \omega_2)/2$ and $\omega = (\omega_1 + \omega_2)/2$.

For large $\omega_1 + \omega_2 \gg |\omega_1 - \omega_2|$, we can approximately see the total field as the composition of two rapidly oscillating field with slowly varying amplitudes:

$$E(x,t) = \mathcal{A}_1(x,t)\cos\omega(x/c-t) + \mathcal{A}_2(x,t)\sin\omega(x/c-t). \tag{4}$$

Therefore, the intensity

$$\begin{aligned} I(x,t) = |E(x,t)|^2 \leq& |\mathcal{A}_1(x,t)|^2 + |\mathcal{A}_2(x,t)|^2 \\ =& A_1^2 + A_2^2 + 2A_1A_2\cos 2\Omega(x/c-t). \end{aligned} \tag{5}$$

That is, the enclosure of $I(x,t)$ is

$$I_{\text{enc}}(x,t) = A_1^2 + A_2^2 + 2A_1A_2\cos(\omega_1 - \omega_2)(x/c-t). \tag{6}$$

The wavelength of the enclosure $\lambda = \frac{2\pi}{\omega_1 - \omega_2}$ is much larger than the wavelengths of two original light fields.

An instance is demonstrated on the next page.

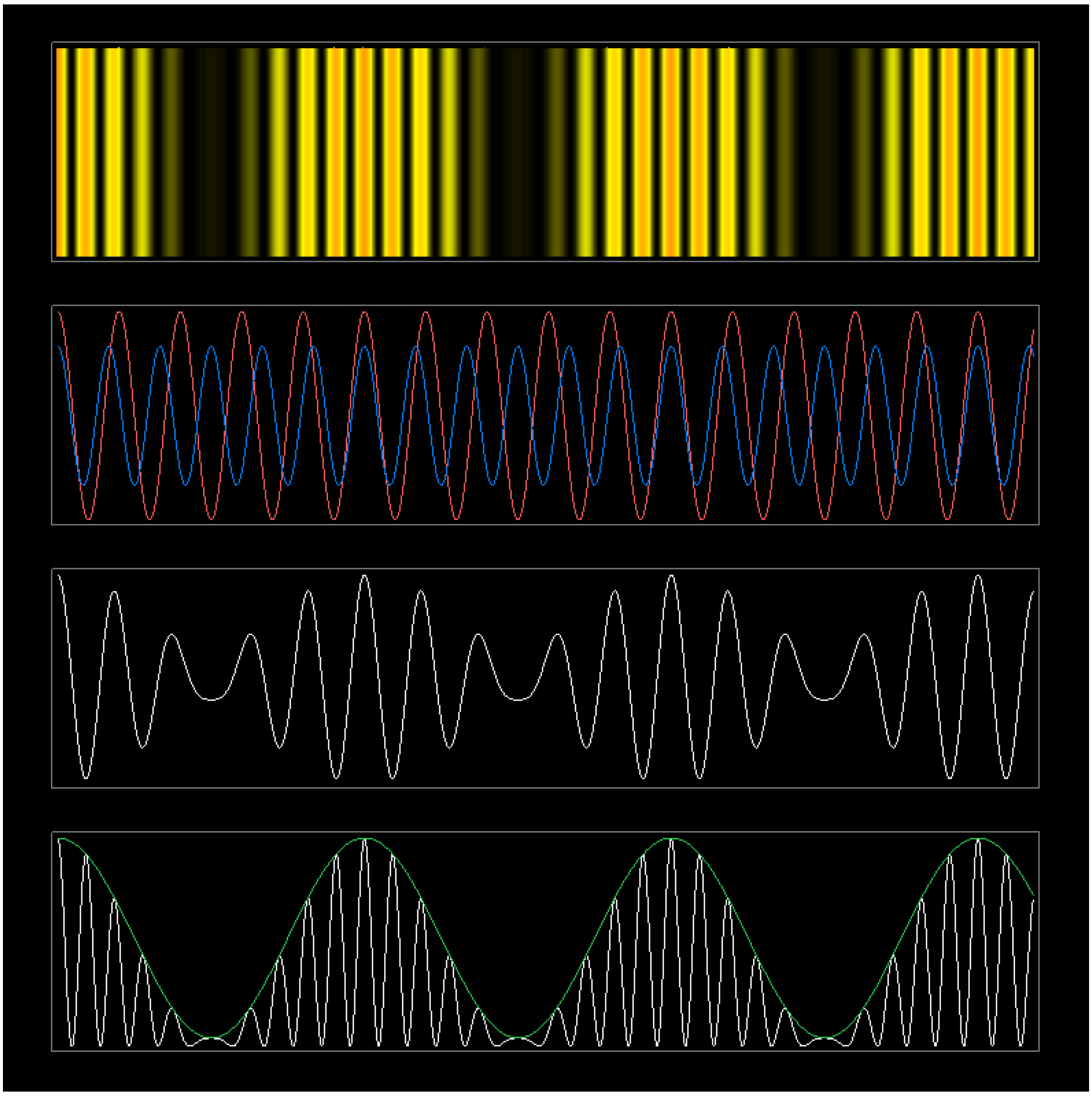

Figure 1: In this series of figures, we have taken $A_1/A_2 = 1.5$, $\omega_2/\omega_1 = 1.2$. x axis (space) is horizontal and $t = 0$.
The first figure is the simulated light intensity.
The second figure is the original two light fields.
The third figure is the total light field.
The last figure is the total light intensity with enclosure (the green curve).