A Cylindrical Lens

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Mark every parallel ray with its position x. For every ray, their optical path length should be equal. That is,

$$C = \sqrt{f^2 + \left(\frac{L}{2}\right)^2} = nh + \sqrt{(f - h(x))^2 + x^2}, \tag{1}$$

where $h(x)$ is the thickness of the lens at point x. We can rewrite equation (1) as

$$h(x) = \frac{nC - f}{n^2 - 1} - \sqrt{\left(\frac{nC - f}{n^2 - 1}\right)^2 - \frac{\left(\frac{L}{2}\right)^2 - x^2}{n^2 - 1}}. \tag{2}$$

We took a reasonable solution where $h(\pm L/2) = 0$. Then $h(0)$ is easily obtained.

In fact, $h(x)$ is a branch of the hyperbola

$$\frac{\left(h - \frac{nC-f}{n^2-1}\right)^2}{\left(\frac{nC-f}{n^2-1}\right)^2 - \frac{(L/2)^2}{n^2-1}} - \frac{x^2}{\frac{(nC-f)^2}{n^2-1} - \left(\frac{L}{2}\right)^2} = 1. \tag{3}$$

Therefore the radius at $x = 0$ is easily calculated with the result of hyperbolae,

$$R = \sqrt{(nC - f)^2 - (n^2 - 1)(L/2)^2}. \tag{4}$$

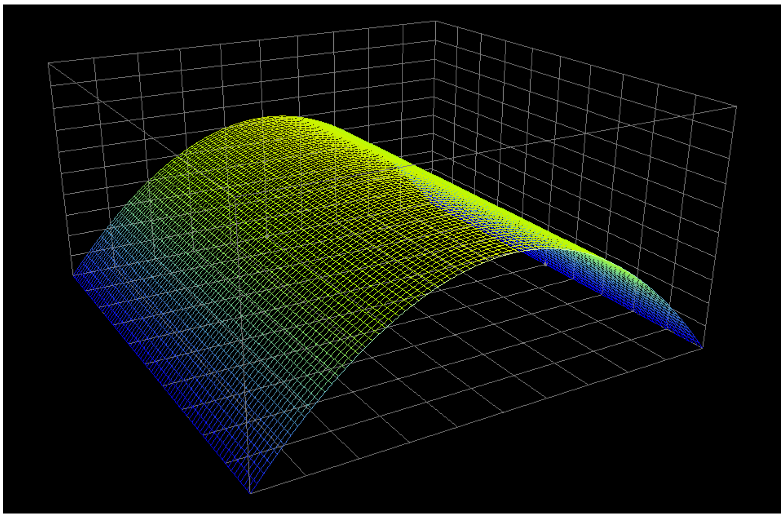

Figure 1: Surface of the lens. $L = 100$, $f = 400$. Height $h(0) = 6.225775$, radius at $x = 0$ is 196.8871.