Regarding the second suggestion, First I will find the second moment $\hat{\mu}_2^{(3)}$ using sample size=100000 with 10 repetitions.

$$\hat{\mu}_2^{(3)} = \frac{2\hat{\sigma}^2}{3(3+k)}$$
$$\hat{\mu}_2^{(3)} = \frac{2 * (1.005)^2}{3(3+0.9994)}$$
$$\hat{\mu}_2^{(3)} = 0.1684$$

I am trying to use different way as follows:

$$\hat{\mu}_2^{(3)} = \frac{1}{N^{(3)}} \sum (x_i^{(3)})^2$$
$$\hat{\mu}_2^{(3)} = 0.1768 \approx 0.1684$$

also:

$$\hat{\mu}_2^{(3)} = \frac{2\hat{\sigma}^2}{3(3+\hat{\kappa}^{original})}$$
$$0.1768 = \frac{2 * (1.005)^2}{3(3+\hat{\kappa}^{original})}$$
$$\hat{\kappa}^{original} = 0.8085$$

$\kappa = 0.8085 \approx 0.9994$

using sample size=10000 with 10 repetitions.

$$\hat{\mu}_2^{(3)} = \frac{2\hat{\sigma}^2}{3(3+k)}$$
$$\hat{\mu}_2^{(3)} = \frac{2 * (1.0247)^2}{3(3+1.0132)}$$
$$\hat{\mu}_2^{(3)} = 0.1744$$

I am trying to use different way as follows:

$$\hat{\mu}_2^{(3)} = \frac{1}{N^{(3)}} \sum (x_i^{(3)})^2$$
$$\hat{\mu}_2^{(3)} = 0.1744 \approx 0.1765$$

also:

$$\begin{aligned}\hat{\mu}_2^{(3)} &= \frac{2\hat{\sigma}^2}{3(3+\hat{\kappa}^{original})} \\ 0.1765 &= \frac{2*(1.0247)^2}{3(3+\hat{\kappa}^{original})} \\ \hat{\kappa}^{original} &= 0.966\end{aligned}$$

$\kappa = 0.966 \approx 1.0132$

using sample size=1000 with 10 repetitions.

$$\begin{aligned}\hat{\mu}_2^{(3)} &= \frac{2\hat{\sigma}^2}{3(3+k)} \\ \hat{\mu}_2^{(3)} &= \frac{2*(1.0275)^2}{3(3+0.9716)} \\ \hat{\mu}_2^{(3)} &= 0.1772\end{aligned}$$

I am trying to use different way as follows:

$$\begin{aligned}\hat{\mu}_2^{(3)} &= \frac{1}{N^{(3)}}\sum(x_i^{(3)})^2 \\ \hat{\mu}_2^{(3)} &= 0.3020 \neq 0.1772\end{aligned}$$

When the sample size is small, the results are not good.