Let $X^{(2)}$ drawn from a 2-power Generalized Pareto distribution, the estimate of the location is biased

$$\begin{aligned}
2E(\mu_1^{(2)}) &= \frac{1}{N^{(2)}}\sum_{i=1}^{N^{(2)}} E[X_i^{(2)}] \\
&= \frac{1}{N^{(2)}}\sum_{i=1}^{N^{(2)}} (\mu + \frac{\sigma}{2}) \\
&= \mu + \frac{\sigma}{2}
\end{aligned}$$

then the estimate of the location is unbiased.

Let $X^{(3)}$ drawn from a 3-power Generalized Pareto distribution, the estimate of the scale is unbiased:

$$\begin{aligned}
\mu_2^{(3)} &= \frac{2\sigma^2}{3(3+k)} \\
3(3+k)\mu_2^{(3)} &= 2.\sigma^2 \\
\sigma^2 &= \frac{3(3+\kappa)\mu_2^{(3)}}{2} \\
&= \frac{3(3+\kappa)E(\mu_2^{(3)})}{2} \\
&= \frac{3(3+\kappa)}{2N^{(3)}}\sum_{i=1}^{N^{(3)}} E[X_i^{(3)}] \\
&= \frac{3(3+\kappa)}{2N^{(2)}}\sum_{i=1}^{N^{(3)}} (\frac{2\sigma^2}{3(3+k)} \\
&= \sigma^2
\end{aligned}$$

the estimate of the scale is unbiased.
and

$$\begin{aligned}
2E(\mu_1^{(2)}) &= \frac{2}{N^{(2)}}\sum_{i=1}^{N^{(2)}} E[X_i^{(2)}] \\
&= 2.\frac{\sigma}{2} \\
&= \sigma
\end{aligned}$$

To prove consistency we will use Chebyshev inequality:

$$\begin{aligned}
\lim P\left\{|\mu_1^{(2)} - \sigma^2| > \epsilon\right\} &\leq Var(\mu_1^{(2)}) \\
&\leq Var(2\mu_1^{(2)}) \\
&\leq 4Var(\frac{1}{N^{(2)}}\sum[X_i^{(2)}]) \\
&\leq 4\frac{1}{(N^{(2)})^2}\sum.Var(X_i^{(2)}) \\
&\leq \frac{4}{(N^{(2)})^2}\sum_{i=1}^{N_{(3)}} Var(X_i^{(2)})^2 \\
&\leq \frac{4}{N^{(2)}}\sigma^2 \\
&\leq 0
\end{aligned}$$

Similarly, we prove consistency for scale parameter:

$$\begin{aligned}
\lim P\left\{|\mu_2^{(3)} - \sigma^2| > \epsilon\right\} &\leq Var(\mu_2^{(3)}) \\
&\leq Var(\frac{3(3+\kappa)\mu_2^{(3)}}{\epsilon^2}) \\
&\leq \frac{9(3+\kappa)}{4\epsilon^2} \\
&\leq C.Var(\frac{1}{N^{(3)}}\sum_{i=1}^{N^{(3)}} X_i^{(3)}) \\
&\leq \frac{C}{(N^{(3)})^2}\sum_{i=1}^{N_{(3)}} Var(X_i^{(3)}) \\
&\leq \frac{C}{N^{(3)}}\sigma^2 \\
&\leq 0
\end{aligned}$$

σ is a consistent estimator of the population σ, where $C = \dfrac{9(3+\kappa)^2}{4\epsilon^2}$.

In both cases, I used pairs because I got good previous estimation results, but so far I'm looking for good approximation results for triples because according to my previous report, I told you the approximation was good, but not to the required level.

Regarding the reference (Maximum likelihood estimation for q-exponential (Tsallis) distributions." arXiv preprint math/0701854 (2007).) I am trying to apply the formula $\hat{\sigma} = \frac{\theta + 1}{n} \sum \frac{x_i}{1 + \frac{x_i}{\hat{\sigma}}}$ using the following code:

$$NSolve[\hat{\sigma} == 2/n * Sum[x[[i]]/(1 + x[[i]]/\hat{\sigma}), i, 1, n], u, Reals],$$

where $n = 95$, but I got a vector of results?