Problem: estimating parameters of the Independent Approximating distribution μ, σ and κ.

We have the system:

$$\begin{aligned} \mu_1 &= \mu + \frac{\sigma}{2} \\ \mu_2 &= \mu^2 + \frac{2\mu\sigma}{3+\kappa} + \frac{2\sigma^2}{3(3+\kappa)} \\ \mu_3 &= \mu^3 + \frac{3\mu^2\sigma}{(4+2\kappa)} + \frac{12\mu\sigma^2 + 3\sigma^3}{2(4+\kappa)(4+2\kappa)} \end{aligned}$$

From first equation we ~~find~~

$$\mu = \mu_1^{(2)} - \frac{\sigma}{2}, \tag{0.1}$$

and from second equation

$$(3+\kappa)(\mu_2^{(3)} - \mu^2) = 2\mu\sigma + \frac{2\sigma^2}{3} \tag{0.2}$$

then

$$\kappa = (2\mu\sigma + \frac{2\sigma^2}{3} - 3(\mu_2^{(3)} - \mu^2))(\mu_2^{(3)} - \mu^2)^{-1}. \tag{0.3}$$

Using (2) we find

$$\kappa = (2(\mu_1^{(2)} - \frac{\sigma}{2})\sigma + \frac{2\sigma^2}{3} - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \frac{\sigma}{2})^2))(\mu_2^{(3)} - (\mu_1^{(2)} - \frac{\sigma}{2})^2)^{-1}. \tag{0.4}$$

From last equation of the system we find

$$(\mu_3^{(4)} - \mu^3)(4+2\kappa)(4+\kappa) = 3\mu^2\sigma(4+k) + 6\mu\sigma^2(4+2\kappa).$$

By (0.1) and (0.4) we find

$(\mu_3^{(4)} - (\mu_1^{(2)} - \frac{\sigma}{2})^3)(4 + 2(2(\mu_1^{(2)} - \frac{\sigma}{2})\sigma + (2\sigma^2)/3 - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2))(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2)^{-1})(4 + (2(\mu_1^{(2)} - \sigma/2)\sigma + (2\sigma^2)/3 - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2))(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2)^{-1}) = 3(\mu_1^{(2)} - \sigma/2)^2\sigma(4 + (2(\mu_1^{(2)} - \sigma/2)\sigma + (2\sigma^2)/3 - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2))(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2)^{-1}) + 6(\mu_1^{(2)} - \sigma/2)\sigma^2(4 + 2(2(\mu_1^{(2)} - \sigma/2)\sigma + (2\sigma^2)/3 - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2))(\mu_2^{(3)} - (mu_1^{(2)} - \sigma/2)^2)^{-1})$

Multiply by $(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2)^2$ we find

$$\hat{\sigma} = (\mu_3^{(4)} - (\mu_1^{(2)} - \sigma/2)^3)(4(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2) + 2(2(\mu_1^{(2)} - \sigma/2)\sigma + (2\sigma^2)/3 - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2)))(4(\mu_2^{(3)} - (\mu_1^{(2)} - \mu/2)^2) + (2(\mu_1^{(2)} - \sigma/2)\sigma + (2\sigma^2)/3 - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2))) - 3(mu_1^{(2)} - \sigma/2)^2\sigma(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2)(4(mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2) - (2(\mu_1^{(2)} - \sigma/2)\sigma + (2\sigma^2)/3 - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2))) - 6(\mu_1^{(2)} - \sigma/2)\sigma^2(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2)(4(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2) - 2(2(\mu_1^{(2)} - \sigma/2)\sigma + (2\sigma^2)/3 - 3(\mu_2^{(3)} - (\mu_1^{(2)} - \sigma/2)^2))).$$

Last equation is polynomial equation with degree 7, that mean we will get 7 root for $\hat{\sigma}$ some of them real values and some imaginary.