

Your point may be correct, I think that this is because the estimated value of the parameters of the Independent Approximates distribution was not calculated as I mentioned earlier.

For example, the shape parameter in my thesis is α and it can be called κ according to your paper, I estimated it first where $\hat{\alpha} = (X'WX')^{-1}X'WY$, But according to what I understand, I have not noticed the estimated value of the parameters for the new distribution (Independent Approximates), what is the estimated value of σ', μ' and κ' ?

Let me take the following two examples, the first depends on the relationship that you found between the parameter of original distribution and Independent Approximates distribution: $\kappa' = \frac{\kappa}{1 + n + n\kappa}$, let me suppose that $n = 2$ and $\hat{\kappa}'$ is estimator of κ' then we can write $\hat{\kappa}' = \frac{\hat{\kappa}}{3 + 2\hat{\kappa}}$, If I replace $\hat{\kappa}$ with the estimate I got in my thesis (this means that we estimated the parameter of the new distribution based on the parameter estimate of the original distribution) as follows:

$$= E\left(\frac{(X'WX)^{-1}X'WY}{3 + 2((X'WX)^{-1}X'WY)}\right) \quad (0.1)$$

$$= E\left(\frac{(X'WX)^{-1}X'W(X\kappa + \epsilon)}{3 + 2((X'WX)^{-1}X'W(X\kappa + \epsilon))}\right) \quad (0.2)$$

$$= \frac{\kappa}{3 + 2\kappa} \quad (0.3)$$

$$= \kappa' \quad (0.4)$$

It can be seen that the parameter of the new estimator is unbiased.

The second example is based on a table (1), the third column, the third row, In this case, we estimate the original distribution parameter by parameter of the Independent Approximates distribution: $\hat{\kappa} = \frac{2\sigma^2 - 9\hat{\mu}_2^{(3)}}{3\hat{\mu}_2^{(3)}}$, for ease of calculation, let

me assume that the σ is known, and $=1$, then

$$= \frac{2 - 9 \frac{1}{N^{(3)}} \sum_{i=1}^{N^{(3)}} E(X_i^{(3)})}{\frac{3}{N^{(3)}} \sum_{i=1}^{N^{(3)}} E(X_i^{(3)})} \quad (0.5)$$

$$= \frac{2 - 9\mu_2^{(3)}}{3\mu_2^{(3)}} \quad (0.6)$$

$$= \kappa \quad (0.7)$$

Also, these two examples lead me to the following question: Is the goal to estimate and know the estimation properties for the parameters of the original distribution or for the Independent Approximates distribution?