Figure 1 shows the pairs with a tolerance 0.5 and without repetition. The number of approximately equal pairs for all repetitions was 237 with the estimator to the parameters and you can see the fit GPD for only approximately equal pairs. The sigma from original data wher  $\hat{\sigma} = 1.02719$  and  $\hat{\kappa} = 0.972$ , then

$$\hat{\mu}_1^{(2)} = \frac{\hat{\sigma}}{2}$$

$$\hat{\mu}_1^{(2)} = \frac{1.02719}{2}$$

$$\hat{\mu}_1^{(2)} = 0.514$$

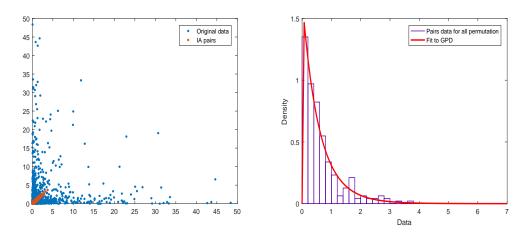


Figure 1: Tolerance= 0.5, partition samples into pairs (left), fit to GPD using pairs (right), without repetitions, sample size=1000

Figure 2 shows the pairs with a tolerance 0.1 and with repetition=10. The number of approximately equal pairs for all repetitions was 602 with the estimator to the parameters and you can see the fit GPD for only approximately equal pairs. The sigma from original data wher  $\hat{\sigma} = 1.02719$  and  $\hat{\kappa} = 0.972$ , then

$$\hat{\mu}_1^{(2)} = \frac{\hat{\sigma}}{2}$$

$$\hat{\mu}_1^{(2)} = \frac{1.02719}{2}$$

$$\hat{\mu}_1^{(2)} = 0.514$$

You can see the result of  $\hat{\mu}_1^{(2)}$  did not change in both cases!

In addition, I did not find any correlation between the values. Since I measured the correlation between the values of the vector x for ten iterations, that is, I measured the correlation between ten vectors.

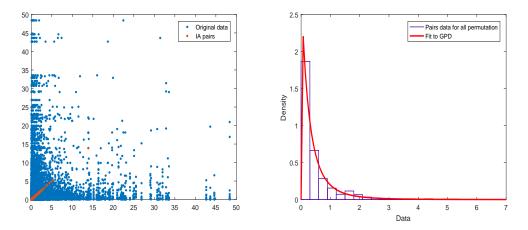


Figure 2: Tolerance= 0.1, partition samples into pairs (left), fit to GPD (right) using only pairs, with repetitions, sample size=1000

Figure 3 shows the pairs with a tolerance 0.1 and without repetition, sample size=10000. The number of approximately equal pairs 581. The sigma from original data wher  $\hat{\sigma} = 1.025$  and  $\hat{\kappa} = 1.013$ , then

$$\hat{\mu}_1^{(2)} = \frac{\hat{\sigma}}{2}$$

$$\hat{\mu}_1^{(2)} = \frac{1.025}{2}$$

$$\hat{\mu}_1^{(2)} = 0.512$$

you can see the estimator of the parameters were close to the original parameters 1. So I think increasing the sample size is better than repetition.

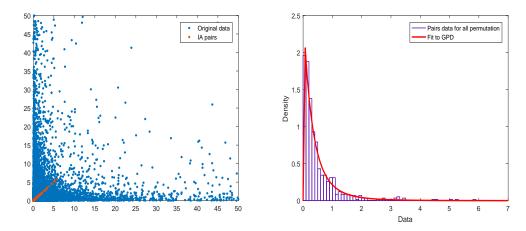


Figure 3: Tolerance= 0.1, partition samples into pairs (left), fit to GPD using pairs (right), without repetitions. Sample size=10000