Let $X^{(2)}$ drawn from a 2-power Generalized Pareto distribution. The estimate of the location $\hat{\mu} = E[\mu_1^{(2)} - \frac{\sigma}{2}]$, which is an unbiased estimate in both cases (scale is known or unknown) as follows,

1. Estimate location parameter

   - In cases σ is known

     $$\begin{aligned}&=\frac{1}{N^{(2)}}\sum_{i=1}^{N^{(2)}} E(X_i^{(2)}) - \frac{\sigma}{2}\\&=\mu + \frac{\sigma}{2} - \frac{\sigma}{2}\\&=\mu\end{aligned}$$

     Estimate location parameter when σ in known is unbiased.

   - In cases σ is unknown is unknown instead I will use an estimate of σ where $\hat{\sigma} = 2E(\mu_1^{(2)} - \mu)$,(Table 1 column 2)

     $$\begin{aligned}&\hat{\mu} = E[\mu_1^{(2)} - \frac{\sigma}{2}]\\&=E(\mu_1^{(2)} - \frac{1}{2}2(\mu_1^{2} - \mu))\\&=E(\mu_1^{(2)} - \mu_1^{(2)} + \mu)\\&=\mu\end{aligned}$$

     Also,estimate location parameter when σ in known is unbiased

   - In case I use estimate of σ as $\hat{\sigma} = 2E(\mu_1^{(2)})$ then

     $$\begin{aligned}&=\frac{1}{N^{(2)}}\sum_{i=1}^{N^{(2)}} E(X_i^{(2)}) - 2\frac{1}{2N^{(2)}}\sum_{i=1}^{N^{(2)}} E(X_i^{(2)})\\&=0 \neq \mu\end{aligned}$$

The estimate of the scale $\hat{\sigma} = 2E[\mu_1^{(2)}$, which is an unbiased estimate in both cases (location is known or unknown) as follows:

2. Estimate scale parameter

   - Whene (μ is known=0) Table 1 column 1

$$
\begin{aligned}
&= \frac{2}{N^{(2)}} \sum_{i=1}^{N^{(2)}} E[X_i^{(2)}] \\
&= 2.\frac{\sigma}{2} \\
&= \sigma
\end{aligned}
$$

   - Whene (μ is unknown) I used the Table 1 column 2

$$
\begin{aligned}
&= 2\mu + \frac{2\sigma}{2} - 2\mu \\
&= \sigma
\end{aligned}
$$