Problem: estimating parameters of the Independent Approximating distribution (σ, κ), we suppose that the location parameter $\mu = 0$

1 Traditional methods for estimating the parameters of the GPD

1.1 Method of moments (MOM)

This method for estimating the two parameters of GP distribution using the first two moments of the sample. The expressions of the mean, variance are given by (Table 1) as follows:

$$Mean = \frac{\sigma}{2} \tag{1.1}$$

$$Variance = \frac{2\sigma^2}{3(3+\kappa)} \tag{1.2}$$

Because this method sometimes yields parameter estimates that are not practical, we estimate the only location and scale parameter. The MOM estimates of parameters k and sigma are now obtained using the expressions for the mean and the variance in 1.1, 1.2. calculations show that the MOM estimates of k and sigma are:

$$\hat{\sigma} = 2\hat{\mu}_1^{(2)}$$

$$\hat{\kappa} = \frac{8\bar{X}^2 - 9S^2}{3S^2}$$

where $\hat{\mu}_1^{(2)} = \bar{X} = \frac{1}{N^{(2)}} \sum_{i=1}^{N^{(2)}} x_i^{(2)}$ and $S^2 = \frac{1}{N^{(3)}} \sum_{i=1}^{N^{(3)}} x_i^{(3)}$ the sample mean and the sample variance, respectively.

2 Bias

The bias of the scale and shape has the following properties.
The estimate of the shape has the following Bias:

$$E(\hat{\kappa}) = E(\frac{8\bar{X}^2}{3S^2} - 3) \tag{2.1}$$

$$= \frac{8E(\bar{X}^2)}{3E(S^2)} - 3 \tag{2.2}$$

We know that

$$E(\bar{X^2}) = var(\bar{X}) + (E(\bar{X}))^2$$

$$\begin{aligned}
(E(\bar{X}))^2 =& (\frac{1}{N^{(2)}} \sum_{i=1}^{N^{(2)}} E(x_i^{(2)}))^2 && (2.3)\\
=& (\frac{1}{N^{(2)}} \sum_{i=1}^{N^{(2)}} (\frac{\sigma}{2}))^2 && (2.4)\\
=& \frac{\sigma^2}{4} && (2.5)
\end{aligned}$$

and

$$\begin{aligned}
E(S^2) =& \frac{1}{N^{(3)}} \sum_{i=1}^{N^{(3)}} E(x_i)^2 && (2.6)\\
=& \frac{1}{N^{(3)}} \sum_{i=1}^{N^{(3)}} (\frac{2\sigma^2}{3(3+\kappa)}) && (2.7)\\
=& \frac{2\sigma^2}{3(3+\kappa)} && (2.8)
\end{aligned}$$

and

$$var = \frac{\sigma^2}{N^{(2)}}$$

we replace (2.5), (2.8) and (2.10) in (2.2)

$$\begin{aligned}
E(\hat{\kappa}) =& \frac{8\frac{\sigma^2}{N^{(2)}} + 8\frac{\sigma^2}{4}}{3\frac{2\sigma^2}{3(3+\kappa)}} - 3\\
=& \frac{(3+\kappa)(8+N^{(2)})}{2N^{(2)}} - 3
\end{aligned}$$

then $E(\hat{\kappa})$ has a bias $\frac{(3+\kappa)(8+N^{(2)})}{2N^{(2)}} - 3$