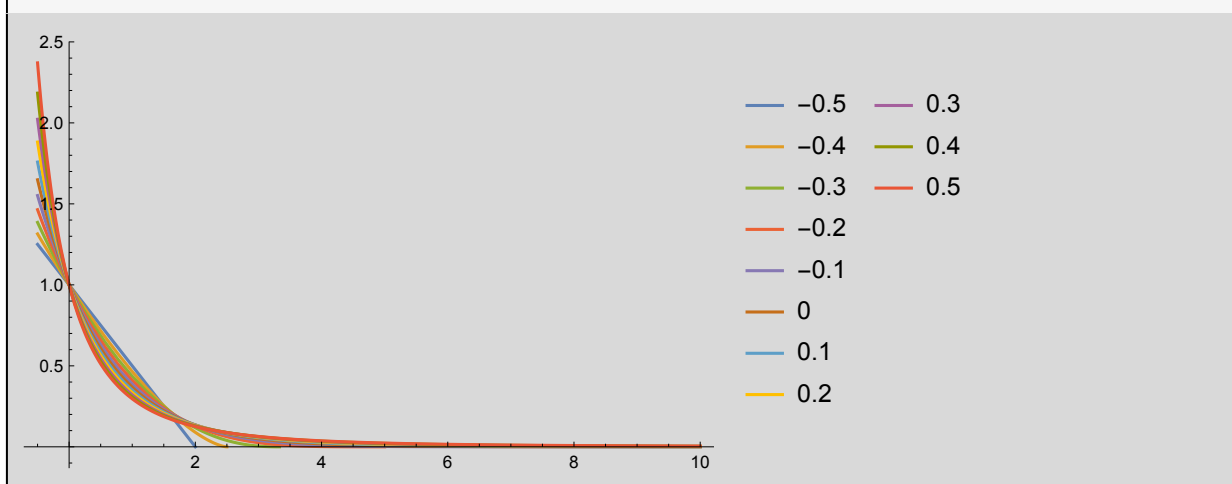# Coupled Exponentials & Logarithms

Graphic of Coupled Exponential

Graph shows curves from linear (κ = -0.5) to exponential (κ = 0)

```
In[●]:=    CouplingValues = {-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5};
           Plot[CoupledExponential[x, #]⁻¹ & /@ CouplingValues // Evaluate,
             {x, -0.5, 10}, PlotLegends → CouplingValues, PlotRange → Full]
```
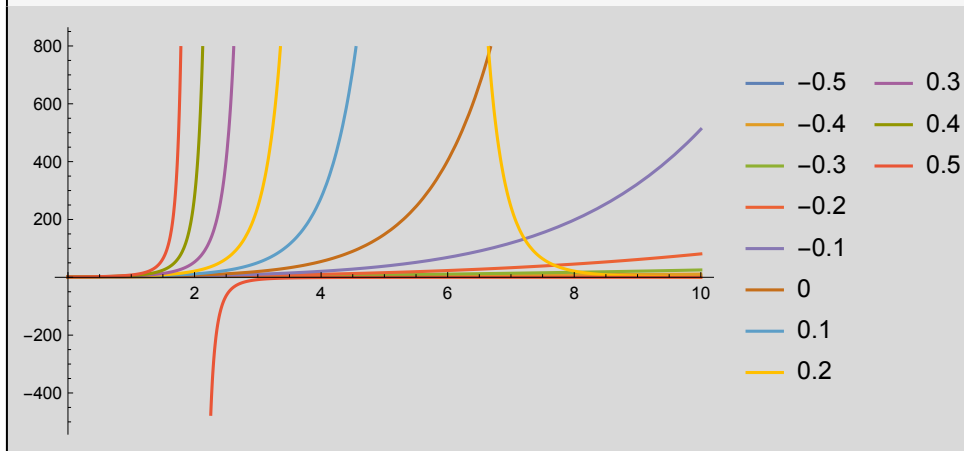

The curves are produced by the Coupled Exponential Function

$$(1 + \kappa \, x)^{-\frac{1+\kappa}{\kappa}}$$

```
In[●]:=   CouplingValues = {-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5};
          Plot[CoupledExponential[-x, #]^-1 & /@ CouplingValues // Evaluate,
           {x, 0, 10}, PlotLegends → CouplingValues]
```

Out[●]=

The curves are produced by the Coupled Exponential Function
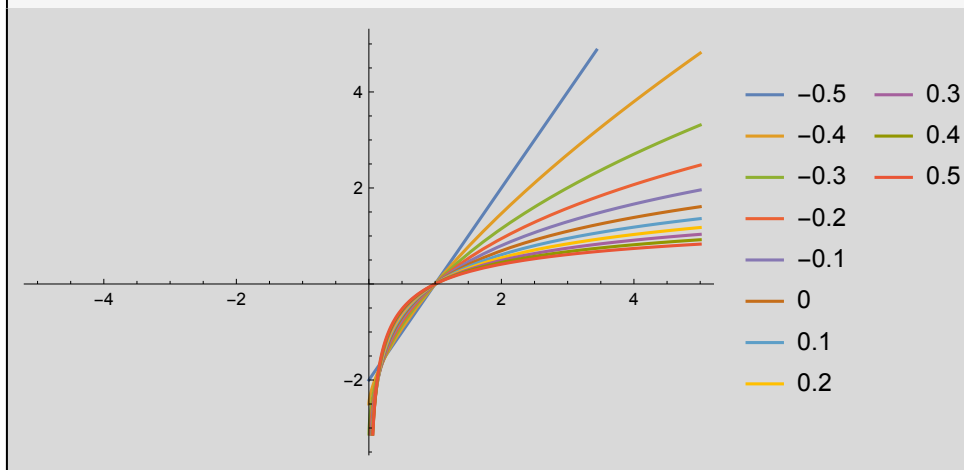
$$(1 - \kappa\, x)^{\frac{1 + \kappa}{-\kappa}}$$

Graphic of Coupled Logarithm

Graph shows curves from linear to logarithmic

```
In[●]:=   CouplingValues = {-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5};
          Quiet[Plot[
             -CoupledLogarithm[x^-1, #] & /@ CouplingValues // Evaluate, {x, -5, 5},
             PlotLegends → CouplingValues],
            {Power::infy}]
```

Out[●]=

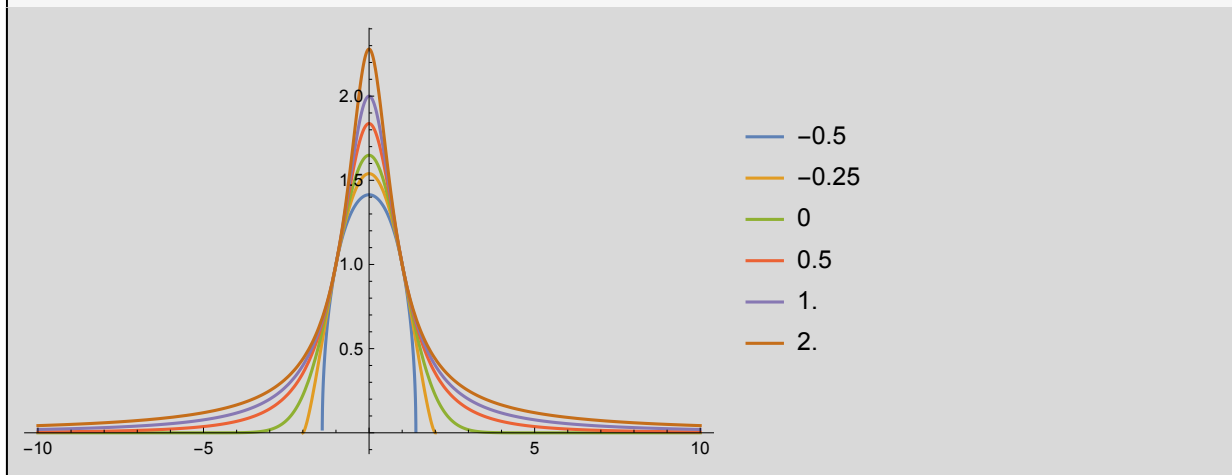The curves are produced by the Coupled Logarithmic Function

$$\frac{1}{-\kappa}\left(x^{\frac{-\kappa}{1 + \kappa}} - 1\right)$$

Coupled Normal Distribution

In[●]:=
```
CouplingValues = {-0.5, -0.25, 0, 0.5, 1.0, 2.0};
Quiet[Plot[
  PDF[CoupledNormalDistribution[0, 1, #], {x}] /
     PDF[CoupledNormalDistribution[0, 1, #], {1}] & /@
    CouplingValues // Evaluate, {x, -10, 10},
  PlotLegends → CouplingValues,
  PlotRange → Full],
  {Power::infy}]
```

Out[●]=

Coupled Gaussian is Scale-Free as $\sigma \to 0$

```
In[ ]:=   Parameters = {{1, 1, 0.5, 0.05, 0.005}, {0, 1, 1, 1, 1}};
          Quiet[LogLogPlot[MapThread[
               PDF[CoupledNormalDistribution[0, #1, #2], {x}] &, Parameters] // Evaluate,
            {x, 0.1, 100},
            PlotLegends → {"Normal κ = 0, σ = 1",
               "Cauchy κ = 1, σ = 1", "Cauchy σ = 0.5",
               "Cauchy σ = 0.05", "Cauchy σ = 0.005"},
            LabelStyle → Directive[Gray, Smaller],
            PlotRange → {{0.1, 100}, {10⁻⁴, 1}},
            PlotTheme → {"Detailed"},
            FrameLabel → {"x", "Density"},
            PlotLabel → "Coupled Gaussian Distributions"],
          {Power::infy}]
```
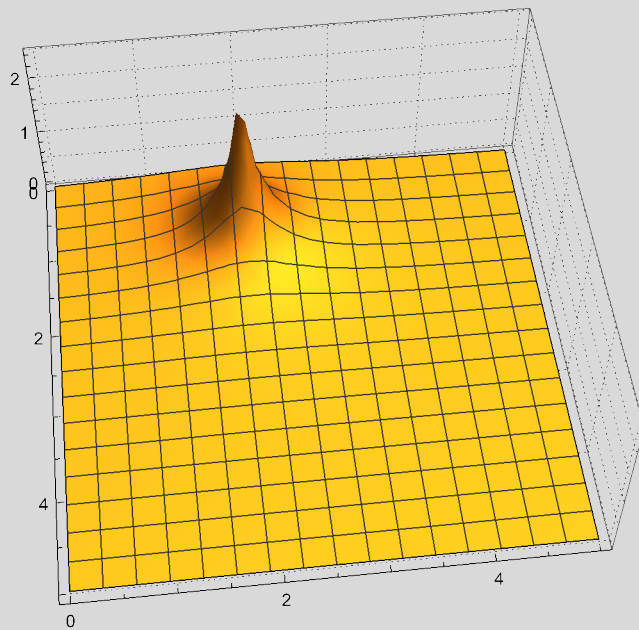
Out[ ]=
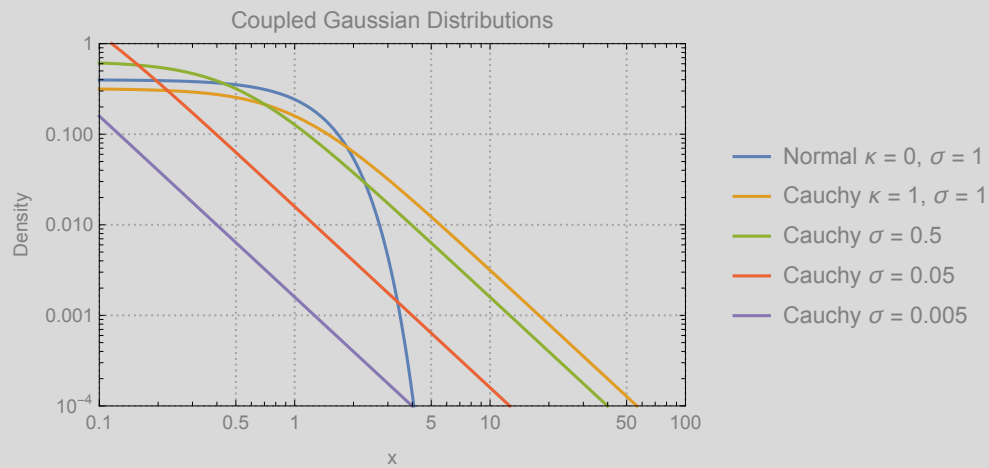
Multivariate Coupled Distribution

Multivariate Coupled Exponential

In[228]:=
```
Plot3D[PDF[MultivariateCoupledDistribution[{1, 2}, {{1, 0}, {0, 1}}, 2, 1],
  {x, y}],
 {x, 0, 5}, {y, 0, 5},
 PlotLegends → None,
 PlotTheme → "Detailed",
 PlotRange → Full
]
```

Out[228]=

Multivariate Coupled Gaussian

In[230]:=
```
Plot3D[
 PDF[MultivariateCoupledDistribution[{1, 2}, {{1, -0.01}, {0.01, 1}}, 0.01, 2],
  {x, y}],
 {x, -5, 5}, {y, -5, 5},
 PlotLegends → None,
 PlotTheme → "Detailed",
 PlotRange → Full
]
```

Out[230]=