

Compare Entropy Functions

See Coupled Functions file for the CoupledEntropy Function.

The Tsallis and Normalized Tsallis entropy functions are related to the non-root form of the Coupled Entropy

$$\text{TsallisNormalizedEntropy} = (1 + \kappa) \text{CoupledEntropy}$$

$$\text{TsallisEntropy} = (1 + \kappa) \text{Sum}\left(p_i^{1+\frac{\alpha\kappa}{1+\kappa}}\right) \text{CoupledEntropy}$$

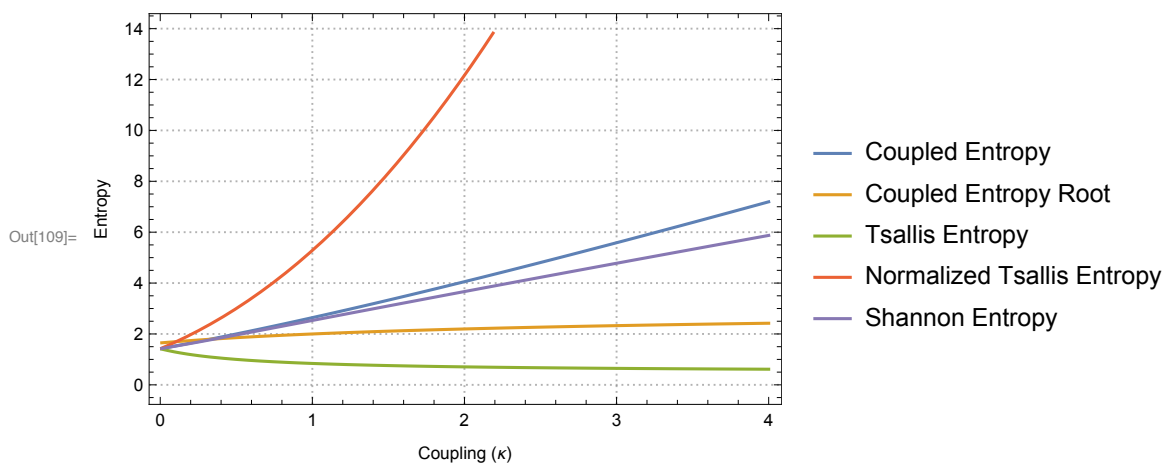
```
In[128]:= TsallisEntropy[dist_, κ_, α_:1, d_:1, limits_: {-∞, ∞}, normalize_: False] :=
  If[normalize,
    (1 + κ) CoupledEntropy[dist, κ, α, d, limits, False],
    (1 + κ) CoupledEntropy[dist, κ, α, d, limits, False]  $\int_{\text{limits}[[1]]}^{\text{limits}[[2]]} \text{PDF}[\text{dist}, x]^{1+\alpha \frac{\kappa}{1+\kappa}} dx$ 
  ];

TsallisRootEntropy[dist_, κ_, α_:1, d_:1, limits_: {-∞, ∞}, normalize_: False] :=
  If[normalize,
    (1 + κ) CoupledEntropy[dist, κ, α, d, limits, True],
    (1 + κ) CoupledEntropy[dist, κ, α, d, limits, True]  $\int_{\text{limits}[[1]]}^{\text{limits}[[2]]} \text{PDF}[\text{dist}, x]^{1+\alpha \frac{\kappa}{1+\kappa}} dx$ 
  ]
```

Plot Comparison

Compare entropies when the coupling matches the distribution

```
In[109]:= PlotCoupledEntDist[2]
```



```

In[108]:= PlotCoupledEntDist[α_] :=
  Module[{coupledDist},
    coupledDist = CoupledNormalDistribution[0., 1., κ];
    Plot[
      {
        CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
        CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
        TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
        TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
        CoupledEntropy[coupledDist, 0, 1, 1, {-∞, ∞}, False]
      }, {κ, 0.01, 4},
      PlotLegends → {"Coupled Entropy", "Coupled Entropy Root",
        "Tsallis Entropy", "Normalized Tsallis Entropy", "Shannon Entropy"},
      PlotTheme → "Detailed",
      FrameLabel → {"Coupling (κ)", "Entropy"}
    ]
  ]

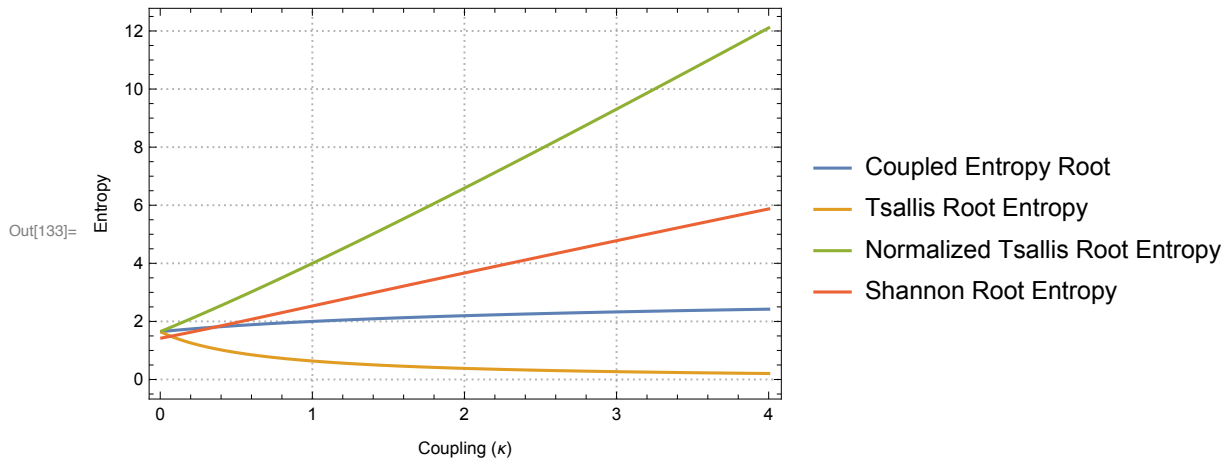
```

Plot with all entropies taking root

```

In[133]:= PlotCoupledEntDist[2]

```



```

In[132]:= PlotCoupledEntDist[α_] :=
Module[{coupledDist},
coupledDist = CoupledNormalDistribution[0., 1., κ];
Plot[
{
CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
TsallisRootEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
TsallisRootEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
CoupledEntropy[coupledDist, 0, 1, 1, {-∞, ∞}, True]
}, {κ, 0.01, 4},
PlotLegends → {"Coupled Entropy Root", "Tsallis Root Entropy",
"Normalized Tsallis Root Entropy", "Shannon Root Entropy"},
PlotTheme → "Detailed",
FrameLabel → {"Coupling (κ)", "Entropy"}
]
]

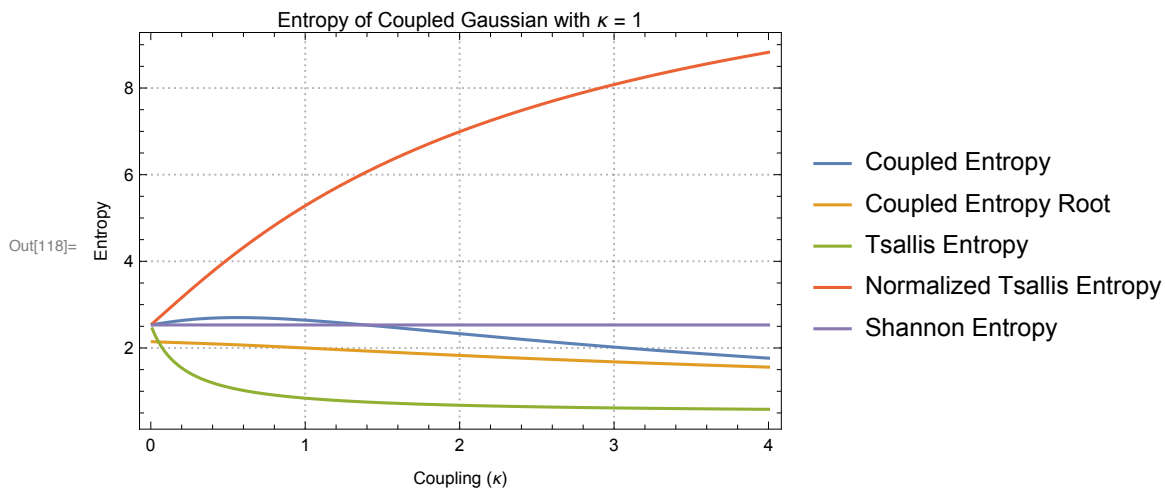
```

Compare Entropies for a Cauchy Distribution

```

In[118]:= PlotCoupledEntDist[2, 1]

```



```

In[117]:= PlotCoupledEntDist[α_, κDist_] :=
  Module[{coupledDist},
    coupledDist = CoupledNormalDistribution[0., 1., κDist];
    Plot[
      {
        CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
        CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
        TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
        TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
        CoupledEntropy[coupledDist, 0, 1, 1, {-∞, ∞}, False]
      }, {κ, 0.01, 4},
      PlotLegends → {"Coupled Entropy", "Coupled Entropy Root",
        "Tsallis Entropy", "Normalized Tsallis Entropy", "Shannon Entropy"},
      PlotTheme → "Detailed",
      FrameLabel → {"Coupling (κ)", "Entropy"},
      PlotLabel → "Entropy of Coupled Gaussian with κ = "<>ToString[κDist]
    ]
  ]

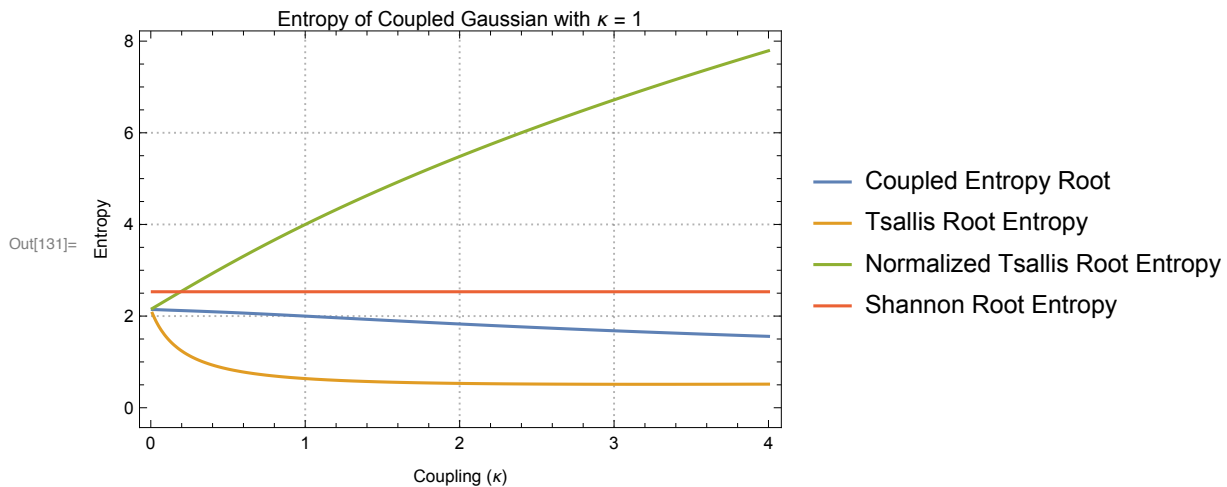
```

Comparison when other entropies also have a root term

```

In[131]:= PlotCoupledEntDist[2, 1]

```



```

In[130]:= PlotCoupledEntDist[α_, κDist_] :=
  Module[{coupledDist},
    coupledDist = CoupledNormalDistribution[0., 1., κDist];
    Plot[
      {
        CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
        TsallisRootEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
        TsallisRootEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
        CoupledEntropy[coupledDist, 0, 1, 1, {-∞, ∞}, True]
      }, {κ, 0.01, 4},
    PlotLegends → {"Coupled Entropy Root", "Tsallis Root Entropy",
      "Normalized Tsallis Root Entropy", "Shannon Root Entropy"},
    PlotTheme → "Detailed",
    FrameLabel → {"Coupling (κ)", "Entropy"},
    PlotLabel → "Entropy of Coupled Gaussian with κ = "<>ToString[κDist]
  ]
]

```

Derivative as a function of coupling for the Coupled Entropy, Coupled Root Entropy, and Tsallis Entropy

```

In[127]:= PlotDerivativeCoupledEntDist[2, 1]

```

... **General:** 0.01008151` is not a valid variable.
... **General:** 0.09151008142857144` is not a valid variable.
... **General:** 0.17293865285714288` is not a valid variable.
... **General:** Further output of General::ivar will be suppressed during this calculation.

Out[127]= \$Aborted

```

In[126]:= PlotDerivativeCoupledEntDist[α_, κDist_] :=
Module[{coupledDist},
  coupledDist = CoupledNormalDistribution[0., 1., κDist];
  Plot[
    DifferenceQuotient[
      {
        CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
        (*CoupledEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True], *)
        TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, False],
        TsallisEntropy[coupledDist, κ, α, 1, {-∞, ∞}, True],
        CoupledEntropy[coupledDist, 0, 1, 1, {-∞, ∞}, False]
      },
      {κ, 0.01}],
    {κ, 0.01, 4},
    PlotLegends → {"Coupled Entropy", "Coupled Entropy Root",
"Tsallis Entropy", "Normalized Tsallis Entropy", "Shannon Entropy"},
    PlotTheme → "Detailed",
    FrameLabel → {"Coupling (κ)", "Entropy"},
    PlotLabel →
      "Derivative of Entropy of Coupled Gaussian with κ = "<>ToString[κDist]
  ]
]

```

Limit of Coupled Root Entropy

Need symbolic definition of Coupled Root Entropy

```
In[112]:= Limit[CoupledEntropy[CoupledNormalDistribution[0., σ, κ], κ, 2, 1, {-∞, ∞}, True],
κ → ∞]
```

... **NIntegrate**: The integrand

$$\text{If}\left[\kappa == 0, \text{PDF}\left[\text{ProbabilityDistribution}\left[\frac{1}{\text{Piecewise}[\{\{2\}, \text{Times}[5]\}] \sqrt{\text{Which}[\text{Greater}[2], 6, \text{Message}[2]]}}, \text{If}\left[\kappa \geq 0, \{x\$13681176, -\text{DirectedInfinity}[1], \infty\}, \{x\$13681176, 0. + \text{Times}[2], 0. + \sqrt{1}\}\right], x\right], \text{FullSimplify}\left[\frac{\text{PDF}[\text{ProbabilityDistribution}[\text{Times}[2], \text{If}[3], x]^{1-\text{Times}[3]}}{\int_{\text{DirectedInfinity}[1]}^{\text{DirectedInfinity}[1]} \text{PDF}[2]^{Plus[2]} dy}\right]\right] \sqrt{\text{If}[1]}$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{-\infty, 0.\}$.

... **NIntegrate**: The integrand

$$\text{If}\left[\kappa == 0, \text{PDF}\left[\text{ProbabilityDistribution}\left[\frac{1}{\text{Piecewise}[\{\{2\}, \text{Times}[5]\}] \sqrt{\text{Which}[\text{Greater}[2], 6, \text{Message}[2]]}}, \text{If}\left[\kappa \geq 0, \{x\$13681176, -\text{DirectedInfinity}[1], \infty\}, \{x\$13681176, 0. + \text{Times}[2], 0. + \sqrt{1}\}\right], x\right], \text{FullSimplify}\left[\frac{\text{PDF}[\text{ProbabilityDistribution}[\text{Times}[2], \text{If}[3], x]^{1-\text{Times}[3]}}{\int_{\text{DirectedInfinity}[1]}^{\text{DirectedInfinity}[1]} \text{PDF}[2]^{Plus[2]} dy}\right]\right] \sqrt{\text{If}[1]}$$

has evaluated to non-numerical values for all sampling points in the region with boundaries $\{-\infty, 0.\}$.

... **Limit**: Warning: Assumptions that involve the limit variable are ignored.

Out[112]= \$Aborted

Test functionality of CoupledEntropy

```
In[111]:= $Assumptions = -1 < κ < ∞ && 0 < σ < ∞ && {x, κ, σ} ∈ Reals;
```

```
In[48]:= CoupledEntropy[CoupledNormalDistribution[0, 1, 1], κ, 2]
```

Out[48]= \$Aborted

```
In[51]:= CoupledEntropy[CoupledNormalDistribution[0, 1, 1], #, 2] & /@ {-0.5, 0., 0.5, 1., 1.5}
```

... **Integrate**: Integral of $3.14159 (1+y^2)^{1.}$ does not converge on $\{-\infty, \infty\}$.

$$\text{Out[51]} = \left\{ - \int_{-\infty}^{\infty} - \left(1.5708 \left((1+x^2)^2 \right)^{0.5} \text{If}\left[(1+x^2)^2 \geq 0, \right. \right. \right. \\ \left. \left. \left. \text{If}\left[-0.5 \neq 0, - \frac{\left(\pi^2 (1+x^2)^2 \right)^{-\frac{0.5}{1+(-0.5)}} - 1}{0.5}, \text{Log}\left[\pi^2 (1+x^2)^2 \right], \text{Undefined} \right] \right] \right) dy \right\} dx, 2.53102, 2.69962, 2.64159, 2.4964 \}$$

```
In[53]:= CoupledEntropy[CoupledNormalDistribution[0, 1, 0.5], #, 2] & /@ {0., 0.5, 1., 1.5}
```

Out[53]= {1.96028, 2., 1.90084, 1.76309}

```
In[54]:= CoupledEntropy[CoupledNormalDistribution[0, 1, 1.5], #, 2] & /@ {0., 0.5, 1., 1.5}
Out[54]= {3.10166, 3.45102, 3.46597, 3.32999}
```

Test functionality of CoupledEntropy with Root

```
In[68]:= CoupledEntropy[CoupledNormalDistribution[0, 1, 1], #, 2, 1, {-∞, ∞}, True] & /@
{0., 0.5, 1., 1.5}
```

```
Out[68]= {2.1464, 2.08107, 1.99952, 1.91187}
```

```
In[69]:= CoupledEntropy[CoupledNormalDistribution[0, 1, 0.5], #, 2, 1, {-∞, ∞}, True] & /@
{0., 0.5, 1., 1.5}
```

```
Out[69]= {1.90805, 1.85407, 1.77247, 1.68738}
```

```
In[70]:= CoupledEntropy[CoupledNormalDistribution[0, 1, 1.5], #, 2, 1, {-∞, ∞}, True] & /@
{0., 0.5, 1., 1.5}
```

```
Out[70]= {2.35969, 2.27791, 2.19773, 2.10942}
```

```
In[99]:= ? CoupledEntropy
```