5 Key Concepts in Linear Algebra

1. Minor

The minor of an element a_{ij} in a matrix is the determinant of the submatrix formed by deleting the i-th row and j-th column that contains the element.

Example:

Given matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

The minor of a_{11} is the determinant of:

$$\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} = 5 \cdot 9 - 6 \cdot 8 = -3$$

2. Cofactor

The cofactor of an element a_{ij} is its minor multiplied by $(-1)^{i+j}$.

Formula:

$$\text{Cofactor}(a_{ij}) = (-1)^{i+j} \cdot \text{Minor}(a_{ij})$$

Example: The cofactor of a_{11} is $(+1) \cdot (-3) = -3$

3. Determinant

The determinant is a scalar value that can be computed from a square matrix. It reflects many linear properties, including invertibility.

Example (2x2 matrix):

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

4. Adjugate

The adjugate (or classical adjoint) of a matrix A is the transpose of its cofactor matrix.

What is transpose?

The transpose of a matrix is obtained by swapping its rows and columns.

Example:

$$\text{transpose}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

General formula:

$$\text{adj}(A) = \text{transpose of the cofactor matrix of } A$$

5. Inverse Matrix

The inverse of a matrix A^{-1} satisfies:

$$A \cdot A^{-1} = A^{-1} \cdot A = I$$

What is I?

I is the identity matrix — a square matrix with 1s on the main diagonal and 0s elsewhere.

Example:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Formula to compute the inverse:

$$A^{-1} = \frac{1}{\det(A)} \cdot \text{adj}(A)$$