Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 1
Basic Concepts

Basic Concepts - Chapter 1

1.1 Systems of Units.
1.2 Electric Charge.
1.3 Current.
1.4 Voltage.
1.5 Power and Energy.
1.6 Circuit Elements.

What is an Electric Circuit?

- In electrical engineering, we are usually interested in transferring energy or communicating signals from one point to another.

  To do this, we often require an interconnection of electrical components.

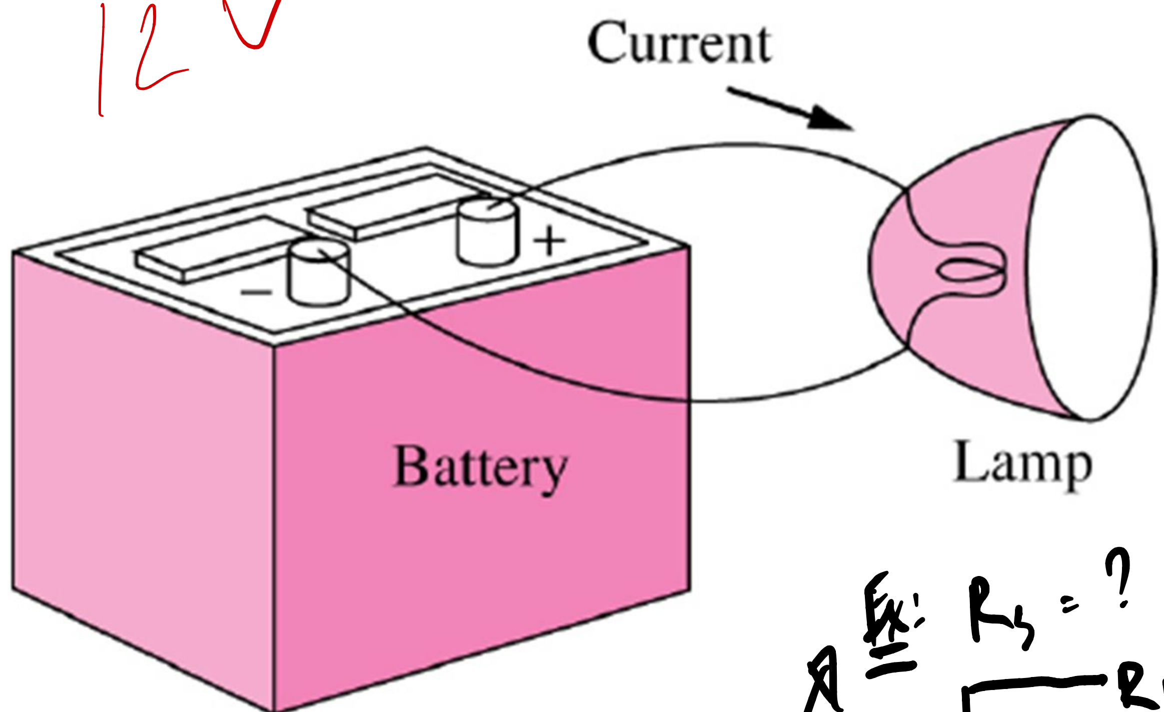
  "An electric circuit is an interconnection of electrical components."

- Typical circuit or electrical components that we will see in this year:

  batteries or voltage sources, current sources, resistors, switches, capacitors, inductors, diodes, transistors, operational amplifiers, ...

A Simple Circuit

12 V



Ex: R_3 = ? voltage

R_1 47

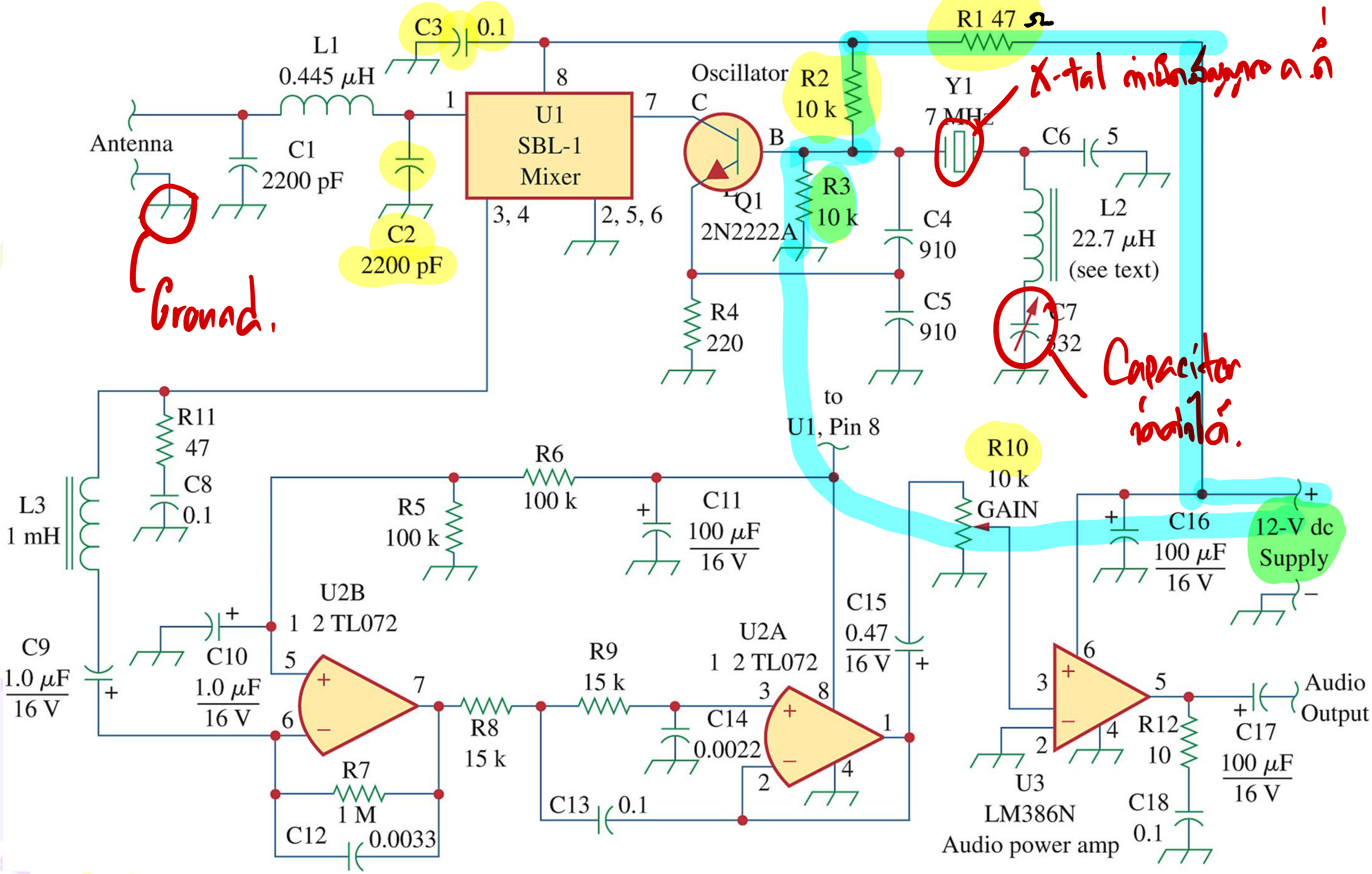
R_2 10k

R_3 10k

12-V dc Supply

$R_1 \to 0\,\Omega \quad \therefore V_{R_2} = \frac{12}{2} = 6$

1



C3 0.1
R1 47 Ω
L1 0.445 μH
8
Oscillator
R2 10 k
Y1 7 MHz
X-tal
Antenna
1
U1 SBL-1 Mixer
7
C
B
C6 5
C1 2200 pF
Q1 2N2222A
R3 10 k
C4 910
L2 22.7 μH (see text)
C2 2200 pF
3, 4
2, 5, 6
Ground.
R4 220
C5 910
C7 532
Capacitor
to U1, Pin 8
R11 47
R6 100 k
R10 10 k GAIN
L3 1 mH
C8 0.1
R5 100 k
C11 100 μF 16 V
C16 100 μF 16 V
12-V dc Supply
U2B 1 2 TL072
C15 0.47 16 V
U2A 1 2 TL072
C9 1.0 μF 16 V
C10 1.0 μF 16 V
R9 15 k
R8 15 k
C14 0.0022
R7 1 M
C12 0.0033
C13 0.1
U3 LM386N Audio power amp
R12 10
C17 100 μF 16 V
C18 0.1
Audio Output

1.1 System of Units (1)

Six basic units

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	Kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

1.1 System of Units (2)

The derived units commonly used in electric circuit theory

Quantity	Unit	Symbol
electric charge	coulomb	C
electric potential	volt	V
resistance	ohm	Ω
conductance	siemens	S
inductance	henry	H
capacitance	farad	F
frequency	hertz	Hz
force	newton	N
energy, work	joule	J
power	watt	W
magnetic flux	weber	Wb
magnetic flux density	tesla	T

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

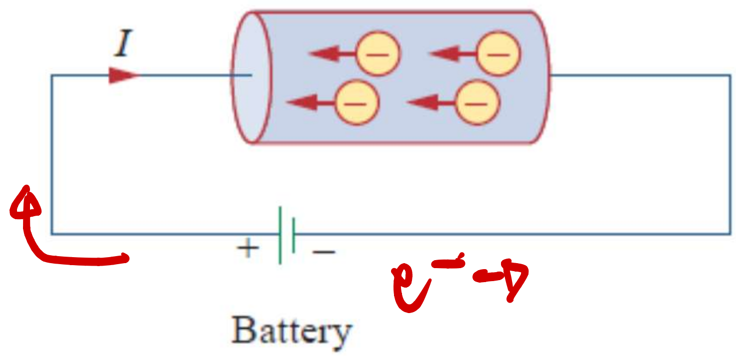
Decimal multiples and submultiples of SI units

1.2 Electric Charges

- Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

- The charge e on one electron is negative and equal in magnitude to 1.602×10^{-19} C which is called as electronic charge. The charges that occur in nature are integral multiples of the electronic charge.

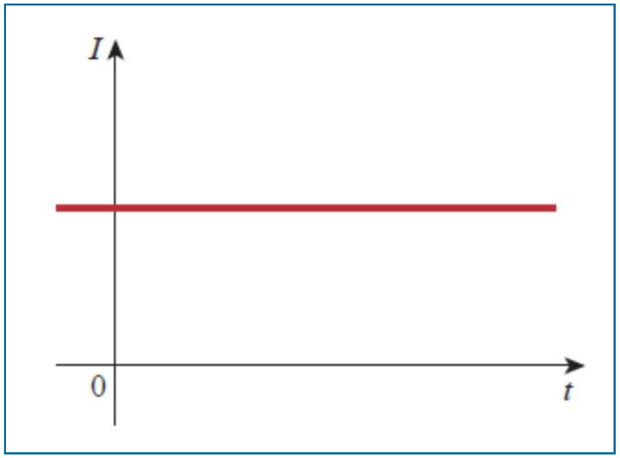
1.3 Current



- Electric current $\boxed{i = dq/dt}$.
- The unit of ampere can be derived as

  1 Ampere = 1 Coulomb/second.

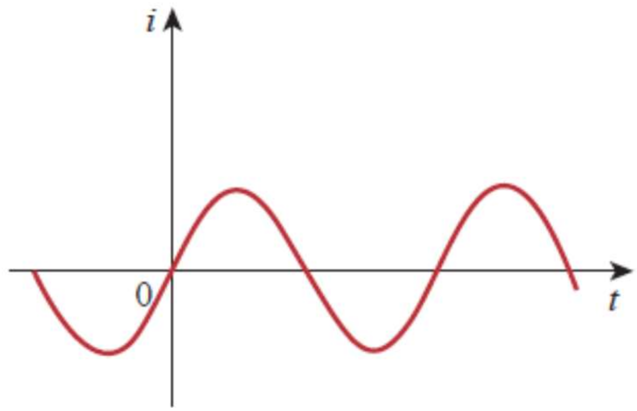
1.3 Direct Current



- A direct current (dc) is a current that remains constant with time.

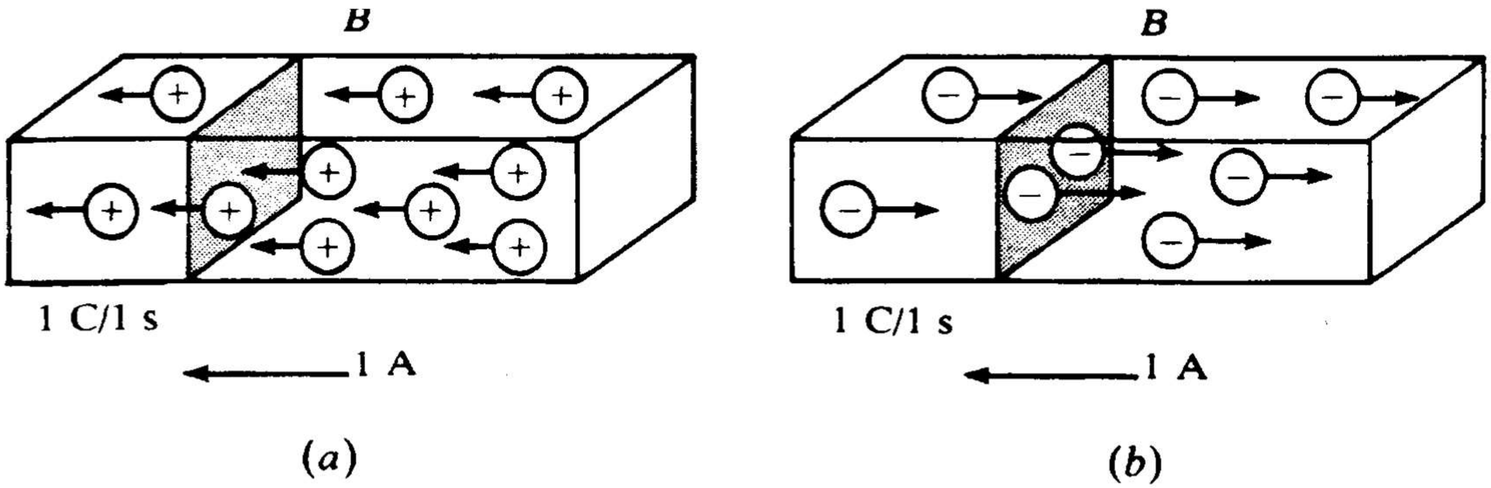
1.3 Alternating Current

$\boxed{i(t) = i_0 \sin t}$



- An alternating current (ac) is a current that varies sinusoidally with time. (reverse direction)

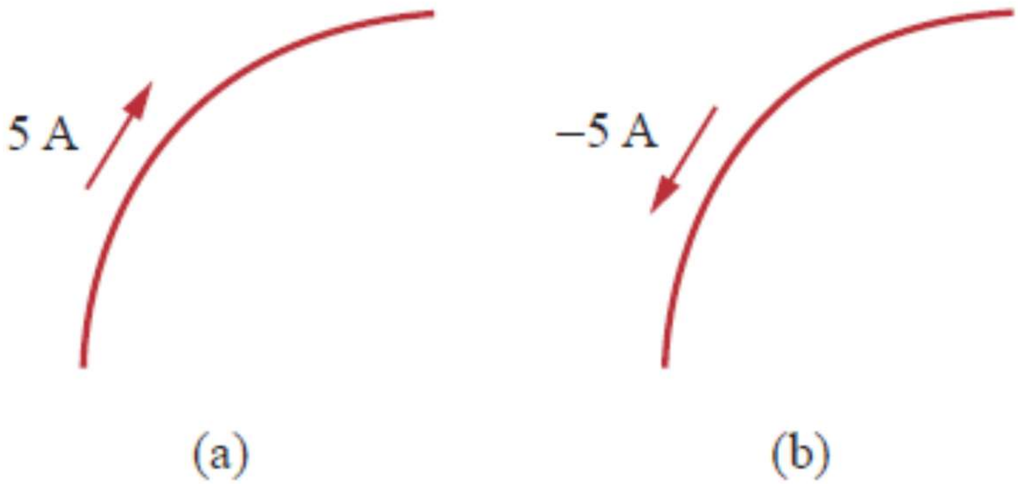
1.3 Current Flow



Positive ions

Negative ions

1.3 Current Flow



1.3 Current

Example 1

Q

How much charge represented by 4,600 electrons?

$4{,}600 \times (-1.602 \times 10^{-19})$ #

1.3 Current

Example 2

Q

Calculate the amount of charge represented by four million protons.

$(4 \times 10^{6}) \times (1.602 \times 10^{-19})$

1.3 Current

Example 3

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC.

Calculate the current at t=0.5 s.

$I = dq/dt$

$\therefore d(5t \sin 4\pi t)$

$\rightarrow 5t(\cos 4\pi t)(4\pi) + 5(\sin 4\pi t)$

$\rightarrow \frac{5}{2} \cdot (1)(4\pi) + 0$

$\rightarrow 10\pi$ mA

C.4 Derivatives

If $U = U(x)$, $V = V(x)$, and a = constant,

$$\frac{d}{dx}(aU) = a\frac{dU}{dx}$$

$$\frac{d}{dx}(UV) = U\frac{dV}{dx} + V\frac{dU}{dx}$$

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

1.3 Current

$$\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$$

$$\frac{d}{dx}(aU^n) = naU^{n-1}$$

$$\frac{d}{dx}(a^U) = a^U \ln a\frac{dU}{dx}$$

$$\frac{d}{dx}(e^U) = e^U\frac{dU}{dx}$$

$$\frac{d}{dx}(\sin U) = \cos U\frac{dU}{dx}$$

$$\frac{d}{dx}(\cos U) = -\sin U\frac{dU}{dx}$$

<u>Example 4</u>

The total charge entering a terminal is given by

$$q = (10 - 10e^{-2t})\ \text{mC}$$

Calculate the current at t=0.5 s.

$I = dq/dt.$

$\therefore 0 - 10e^{-2t}(-2) = 20e^{-2t}$

$I|_{t=0.5} = \frac{20}{2.7}$ ##

1.3 Current

Example 5

Determine the total charge entering a terminal between t=1 s and t = 2 s if the current passing the terminal is $i = (3t^2 - t)$ A.

$$Q = \int_{t=1}^{2} i\,dt.$$

$$t^3 - \frac{t^2}{2} \Big|_1^2 = (8-2) - (1 - \tfrac{1}{2})$$

$$= 6 - \frac{1}{2} = \frac{11}{2}$$

$$= 5.5 \text{ C}$$

จาก $I = dq/dt$

$\therefore dq = I\,dt$

$\therefore q = \int i\,dt.$

1.3 Current

Example 6

A conductor has a constant current of I 5 A. How many electrons e^- pass a fixed point on the conductor in one minute? 60 s

$$\boxed{I = Q/t}$$

$$Q = It = 5 \times 60 = 300 \text{ C/min.}$$

$$\therefore \quad \frac{300 \text{ C/min}}{1.602 \times 10^{-19} \text{ C}/e^-} = 1.87 \times 10^{21} e^-/\text{min.}$$

1.3 Current

<u>Example 6</u>

A conductor has a constant current of 5 A. How many electrons pass a fixed point on the conductor in one minute?

$$\frac{300 \text{ C/min}}{1.602x10^{-19} \text{ C/electron}} = 1.87x10^{21} \text{ electrons/min}$$

1.3 Current

Example 7

Q $t = 60 \times 1.5 = 90s$

ถ้าประจุไฟฟ้าเคลื่อนที่ผ่านตัวนำ 900 Coulombs ในเวลา 1.5 นาที จะมีกระแสไฟฟ้าไหลผ่าน กี่แอมแปร์

$$Q = It \quad \therefore \quad I = Q/t = \frac{900}{90} = 10\ A$$

\#

1.3 Current

Example 8

e^-

อิเลกตรอนอิสระเคลื่อนที่ในโลหะตัวนำ โดยใน 2 วินาทีจะเคลื่อนผ่านพื้นที่หน้าตัดแห่งหนึ่ง 4×10^{10} ตัว จงหากระแสไฟฟ้าที่ผ่านตัวนำนี้

$$I = \frac{Q}{t} = \frac{(4\times10^{10})(-1.602)\times10^{-19}}{2}$$

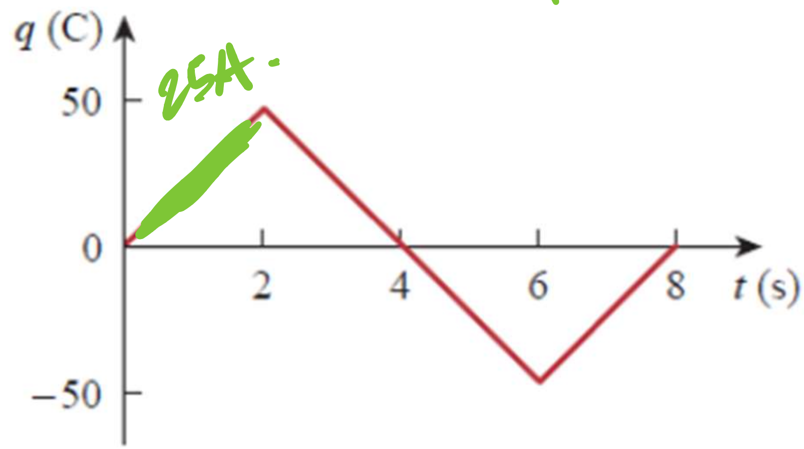
#

1.3 Current

$m = \text{slope} = I$

Example 9

จงเขียนกราฟ ของกระแสและเวลา



$m = \text{slope}$

$= \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \frac{dq}{dt} = I$

$\therefore m = \text{current}$ #

1.4 Voltage

- Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V).

$$v_{ab} \triangleq \frac{dw}{dq}$$

$\frac{\text{Joule}}{\text{Coulomb}}$. or $\frac{\text{N}\cdot\text{m}}{\text{Coulomb}}$. (W = FS)

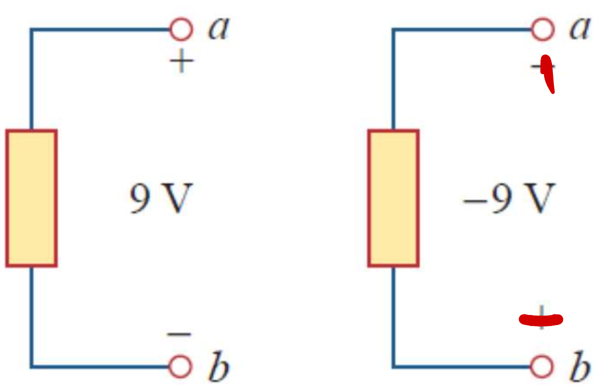
– W is energy in joules (J) and q is charge in coulomb (C).

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton-meter/coulomb}$$

จุด a,b → จุดอ้างอิง.

1.4 Voltage

- Electric voltage, v_{ab}, is always across the circuit element or between two points in a circuit.
  - $v_{ab} > 0$ means the potential of a is higher than potential of b.
  - $v_{ab} < 0$ means the potential of a is lower than potential of b.



$$v_{ab} = -v_{ba}$$

Note! if voltage "-" change - to + / + to -

1.4 Voltage

Example 1

ถ้าความต่างศักย์ระหว่าง 2 จุดมีค่าเท่ากับ 42 V จงหาค่าของงานที่ต้องการในการนำประจุขนาด 6 C จากจุดหนึ่งไปยังอีกจุดหนึ่ง

$\rightarrow W = VQ = 42 \times 6 = 252$ J. ✱

1.4 Voltage

Example 2

แรงดันของแบตเตอรี่ควรจะมีค่าเท่าใดโดยที่แบตเตอรี่ซึ่งใช้พลังงาน 800 J ในการเคลื่อนย้ายประจุจำนวน 40 C ผ่านความต้านทาน

$$V = \frac{800}{40} = 20V.$$

1.5 Power and Energy

- Power is the time rate of expending or absorbing energy, measured in watts (W).

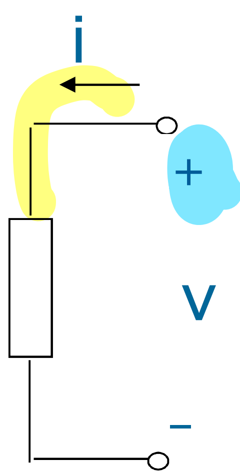
$$\boxed{p \triangleq \frac{dw}{dt}} \sim \frac{\text{Joule}}{\text{sec}}$$

- Mathematical expression：

$$p = \frac{dw}{dt} = \overbrace{\frac{dw}{dq}}^{v} \cdot \underbrace{\frac{dq}{dt}}_{i} = vi$$

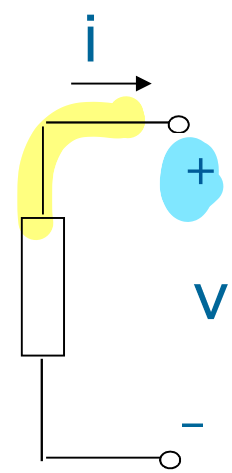
$$\boxed{p = vi}$$

1.5 Power and Energy



$P = +vi$

absorbing power

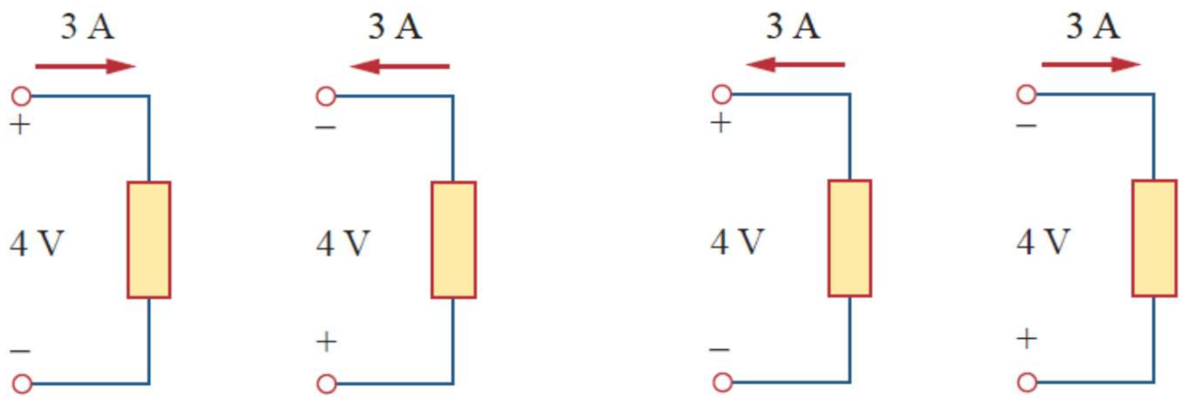


$P = -vi$

supplying power

1.5 Power and Energy

Absorb Absorb. Supply Supply



A B C D

1.5 Power and Energy (2)

- The law of conservation of energy

$$\sum p = 0$$

- Energy is the capacity to do work, measured in joules (J).
- Mathematical expression $w = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} vi\,dt$

1.5 Power and Energy

Example 1

An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

$I \quad t$

$$V = \frac{W}{Q} = \frac{2.3 \times 10^{3}}{20} = \frac{230\cancel{0}}{2\cancel{0}} = 115\ \text{V}$$ #

1.5 Power and Energy

Example 2

To move charge q from point a to point b requires -30 J. Find the voltage drop v_{ab} if: (a) $q = 2$ C, (b) $q = -6$ C.

$$\underset{2C}{a} \xrightarrow{30J.} \underset{-6C}{b.}$$

$$\therefore V_{ab} = \frac{0-(-30)}{2-(-6)} = \frac{30}{8}$$

$$= 3.75 \text{ V} \#$$

1.5 Power and Energy

Example 3

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5\cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3\,di/dt$.

(a) $p = iv$

$\therefore p = 5\cos(60\pi 3)$

$= 5 \cdot (1) = 5$

(b) $v = 3di/dt$

$= 5(-\sin(60\pi t))(60\pi)$

1.5 Power and Energy

Example 4

Find the power delivered to the element in Example 3 at $t = 5$ ms if the current remains the same but the voltage is: (a) $v = 2i$ V,

1.5 Power and Energy

Example 5

How much energy does a 100-W electric bulb consume in two hours?

$P = \frac{W}{t}$

$\therefore$ Work $= Pt$

$= 100 \times 60 \times 60 \times 2$

$= 720{,}000$ J #

1.5 Power and Energy

Example 6

I V

A stove element draws 15 A when connected to a 240-V line. How long does it take to consume 60 kJ?

W

1.5 Power and Energy

Example 7

P t

จงหาพลังงานที่ใช้ เมื่อหลอดไฟ 60 W ใช้ไปในเวลา 2 ชั่วโมง

$$W = \frac{P}{t} = \frac{60}{60 \times 60 \times 2} = \frac{1}{120}$$

1.5 Power and Energy

Example 8

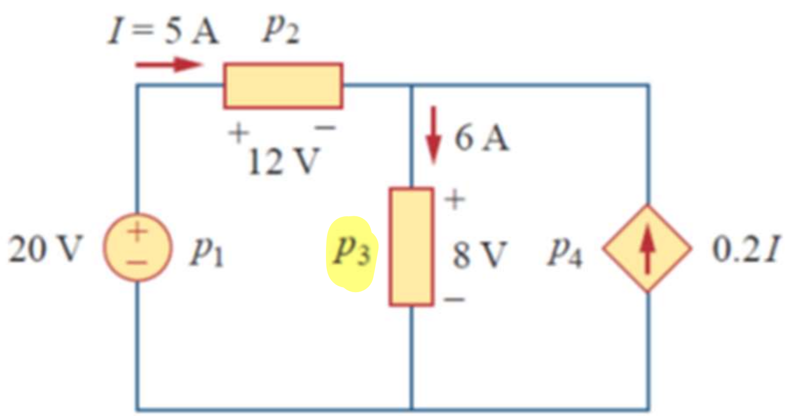
จงหาจำนวนของอิเล็กตรอนต่อวินาทีที่ไหลผ่านหลอดไฟเมื่อหลอดไฟนั้นต่ออยู่กับแหล่งจ่ายแรงดันขนาด 120 V โดยที่หลอดไฟดังกล่าวมีขนาด 75 W

1.5 Power and Energy

Example 9

$p = VI$.

กำลังงานดูดกลืน (absorbed power) ที่ต้องการโดย อุปกรณ์ P3 คือเท่าใด

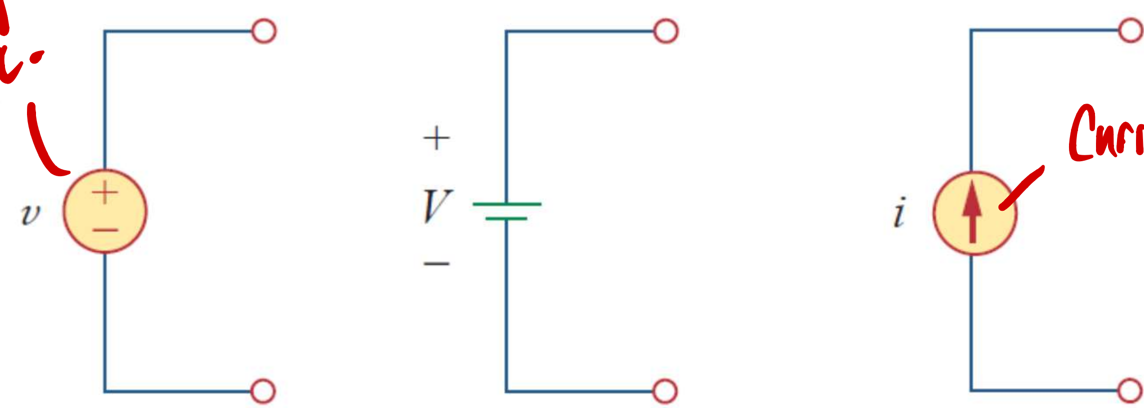


$p = VI = 8 \times 6 = 48$ Watt.

1.6 Circuit Elements

An **ideal independent source** is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

Voltage source. → ค่าง v คงที่.



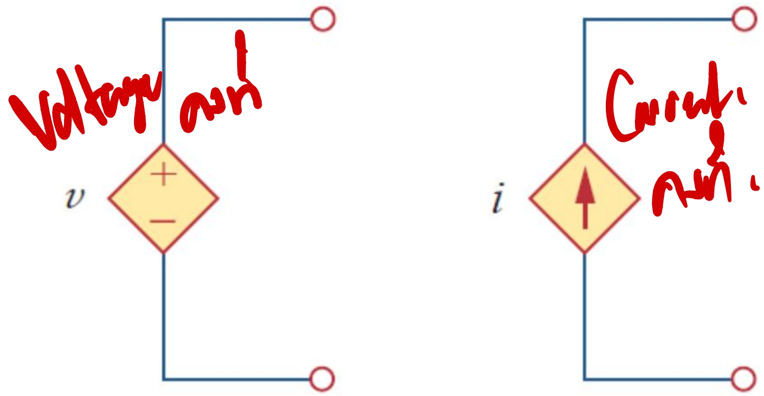
Current source

voltage source   current source

Independent = อิสระ

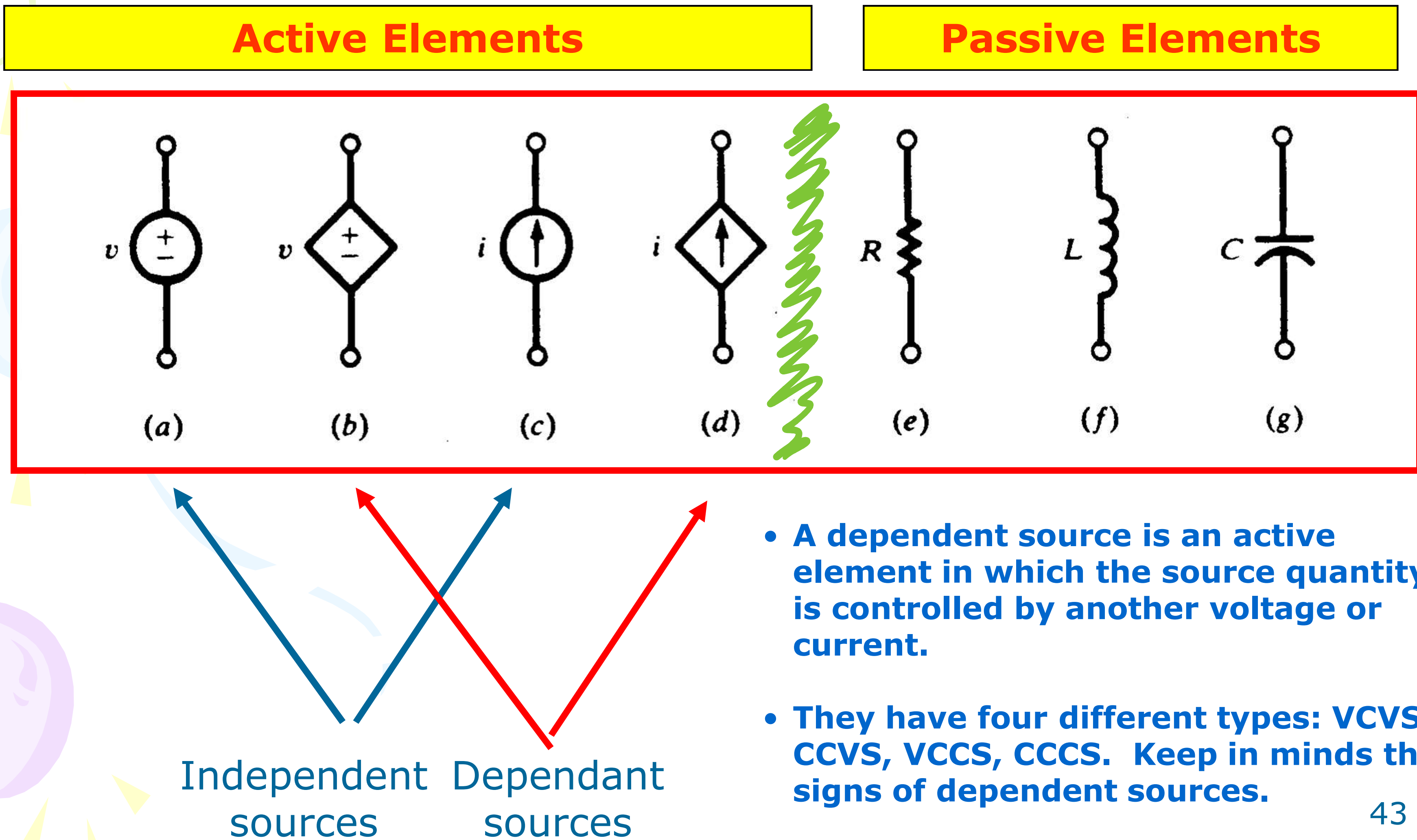
1.6 Circuit Elements

An **ideal dependent** (or **controlled**) **source** is an active element in which the source quantity is controlled by another voltage or current.



1. A voltage-controlled voltage source (VCVS). V คุม V
2. A current-controlled voltage source (CCVS). I คุม V
3. A voltage-controlled current source (VCCS). V คุม I
4. A current-controlled current source (CCCS). I คุม I

1.6 Circuit Elements



- **A dependent source is an active element in which the source quantity is controlled by another voltage or current.**
- **They have four different types: VCVS, CCVS, VCCS, CCCS. Keep in minds the signs of dependent sources.**

Fundamentals of Electric Circuits

Chapter 2
Basic Laws

Basic Laws - Chapter 2

2.1 Ohm's Law.

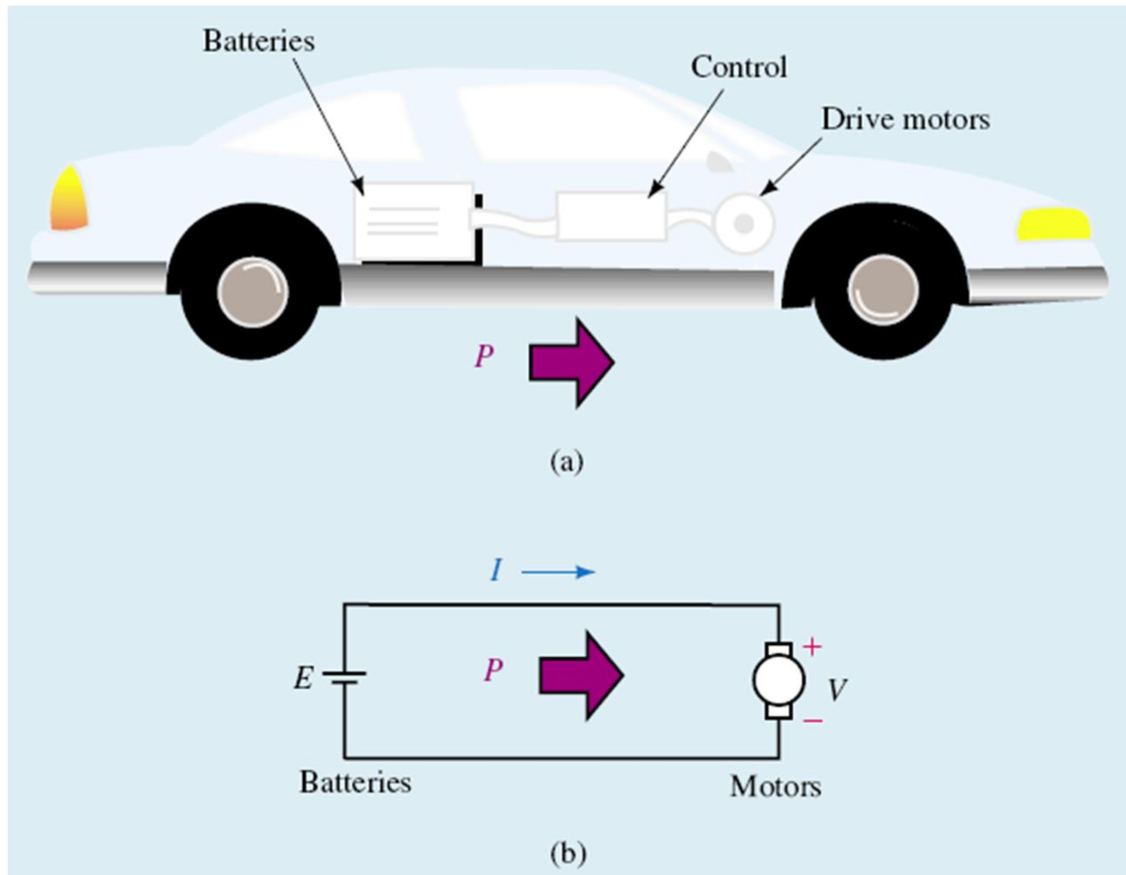
2.2 Nodes, Branches, and Loops.

2.3 Kirchhoff's Laws.

2.4 Series Resistors and Voltage Division.

2.5 Parallel Resistors and Current Division.

2.6 Wye-Delta Transformations.



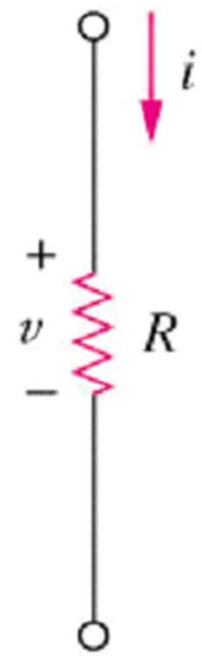
2.1 Ohms Law (1)

- Ohm’s law states that the voltage across a resistor is directly proportional to the current I flowing through the resistor.

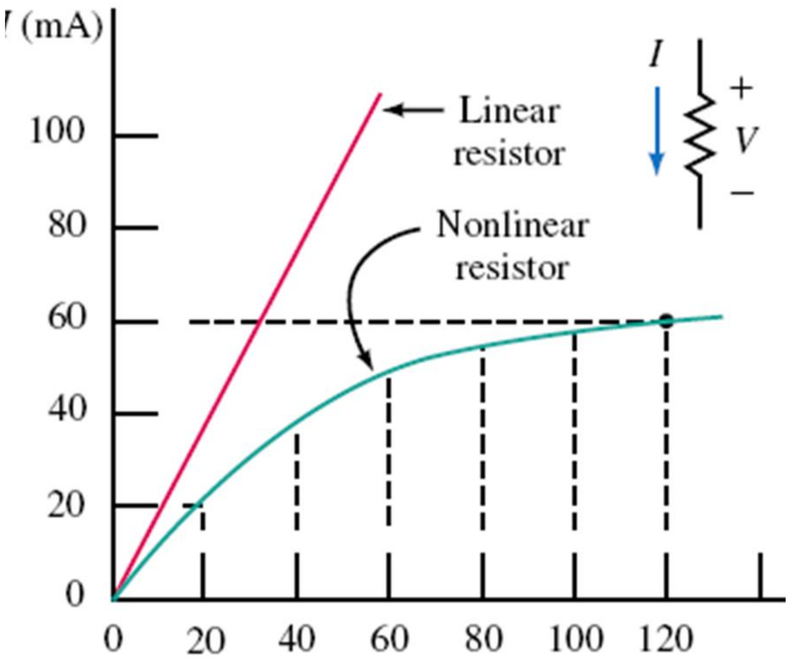
- Mathematical expression for Ohm’s Law is as follows:

$$v = iR$$

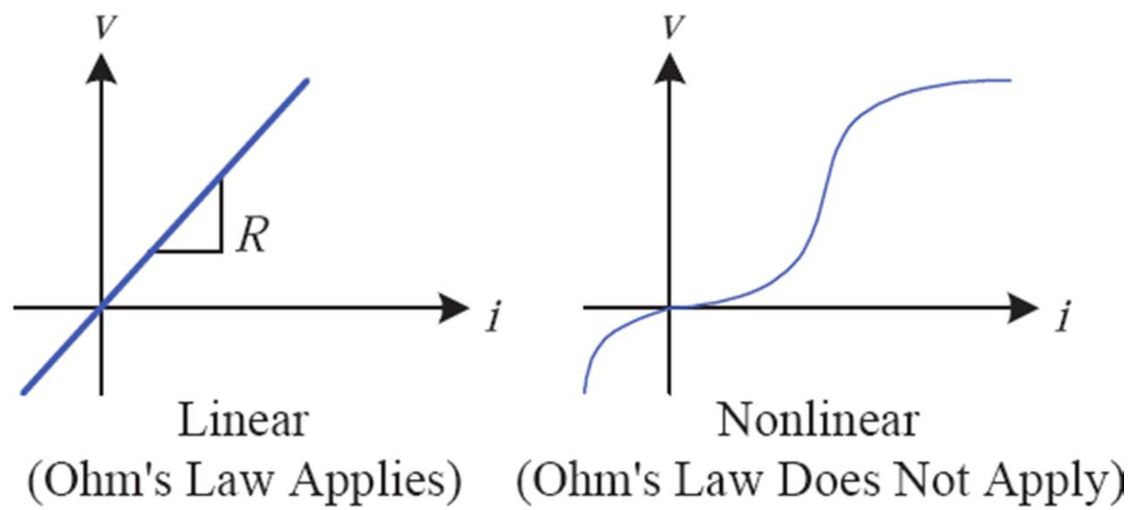
- Two extreme possible values of R: **0 (zero) and ∞ (infinite)** are related with two basic circuit concepts: short circuit and open circuit.



Linear and nonlinear resistance

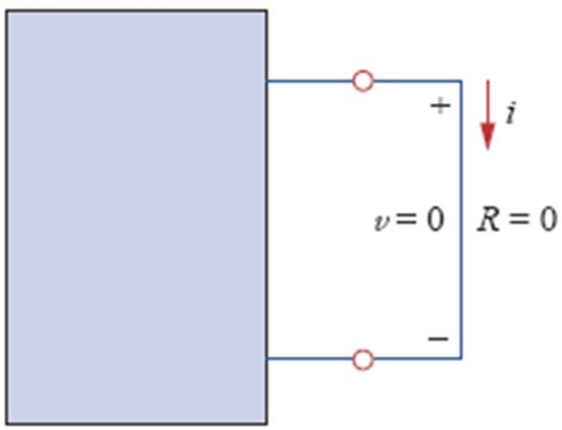


Linear and nonlinear resistance characteristics.

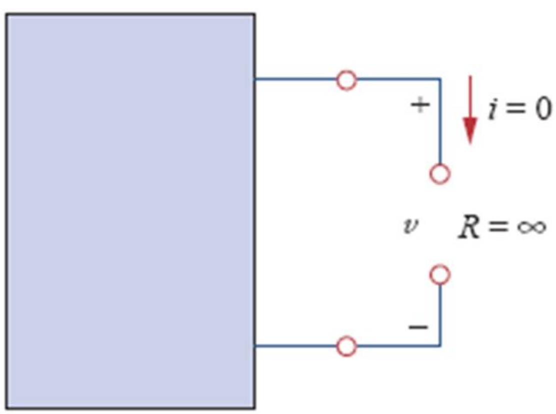


Short and open circuit

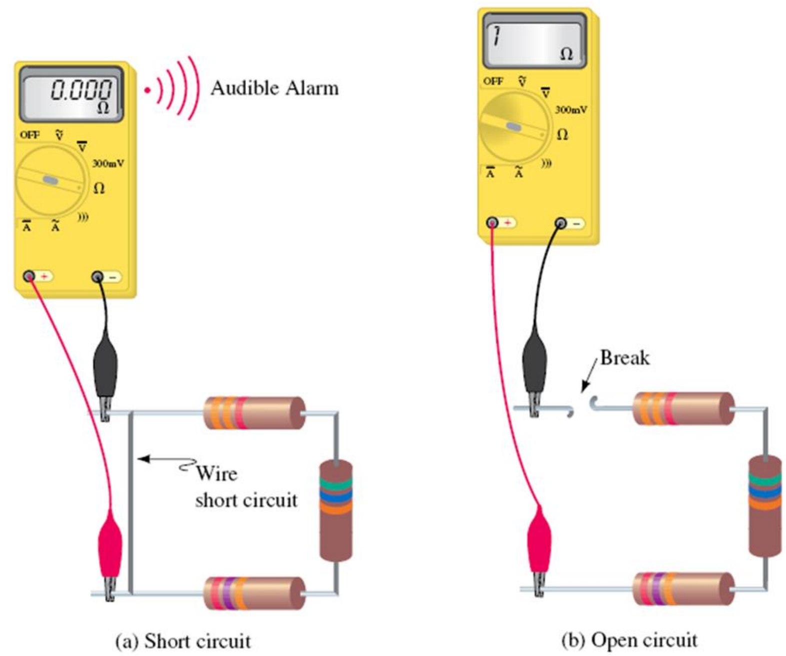
A short circuit is a circuit element with resistance approaching zero.



An open circuit is a circuit element with resistance approaching infinity.



(b)



2.1 Ohms Law (2)

- *Conductance* is the ability of an element to conduct electric current; it is the reciprocal of resistance R and is measured in mhos or siemens.

$$G = \frac{1}{R} = \frac{i}{v}$$

ความเหนี่ยวนำไฟฟ้า

- The power dissipated by a resistor:

$$p = vi = i^2 R = \frac{v^2}{R}$$

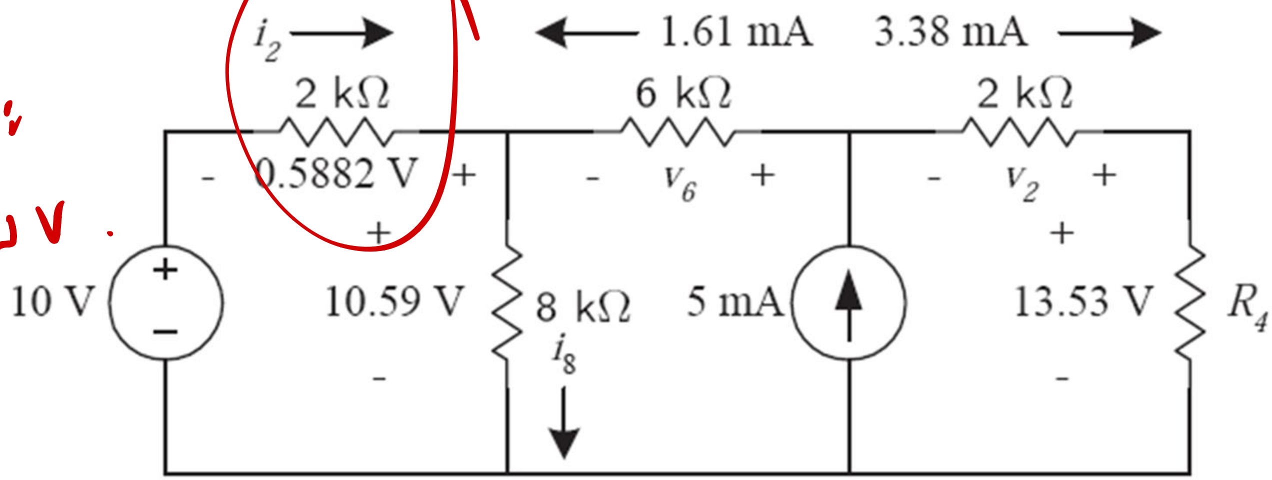
$V = IR$ / $I = \frac{V}{R}$ / $R = \frac{V}{I}$

Example

$i_2 \rightarrow$ + − ; − v +

∴ i_2 ติดลบ.

i_2 ติดลบเพราะ ไหลไปตามทิศลบ v.



$$i_2 = -0.5882/2000 = 0.294\ \text{mA}$$

$$v_6 = 1.61\ \text{mA} \times 6\text{k} = 9.66\ \text{V}$$

$$R_4 = 13.53/3.38\ \text{mA} = 4\ \text{k}\Omega$$

$$v_2 = 3.38\ \text{mA} \times 2\ \text{k}\Omega = 6.76\ \text{V}$$

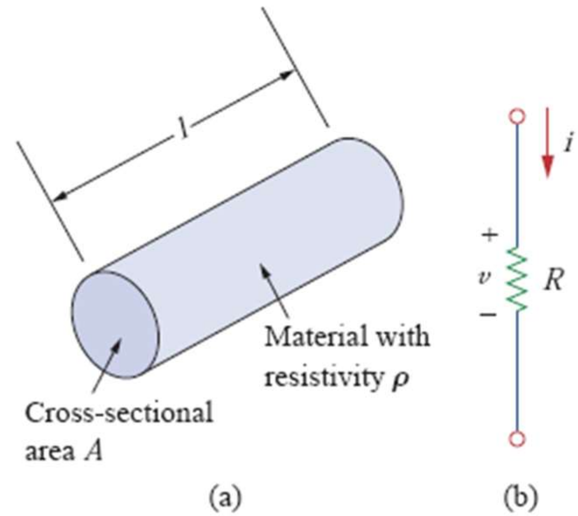
$$i_8 = 10.59/8\text{k} = 1.32\ \text{mA}$$

Conductance and resistivites

TABLE 2.1

Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator



ค.ยาวลวด.

$$\boxed{R = \rho\frac{L}{A} \quad \text{or} \quad R = \frac{L}{\sigma A}.}$$

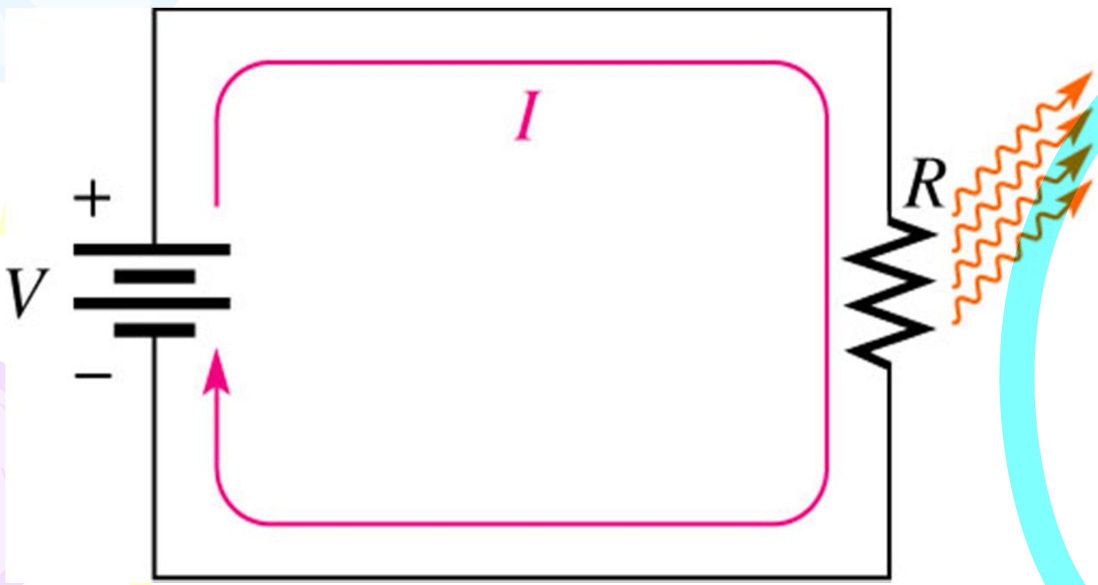
ความต้านทานไฟฟ้า.

พท.หน้าตัด.

ค.นำไฟฟ้า.

Heat produced by Current

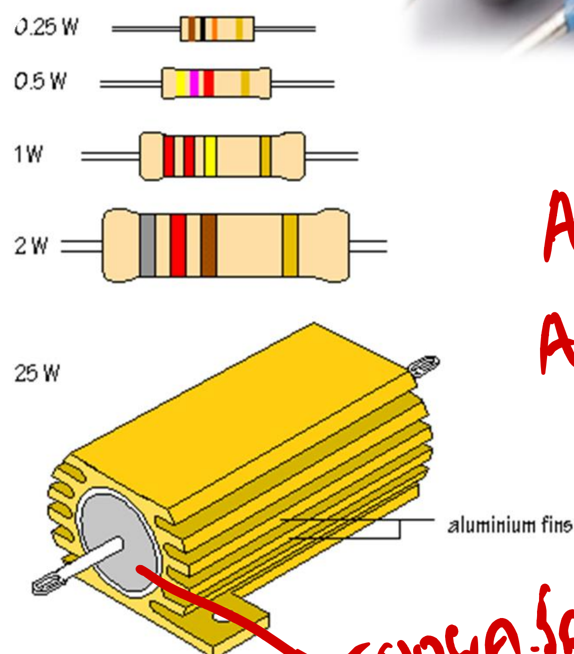
- When there is current through resistance, the collisions of the electrons produce heat, as a result of the conversion of electrical energy.



Heat produced by current through resistance is a result of energy conversion.

Resistor Power Rating

Resistor power rating is not related to ohmic value (resistance) but rather is determined by the physical composition, size and shape of the resistor.



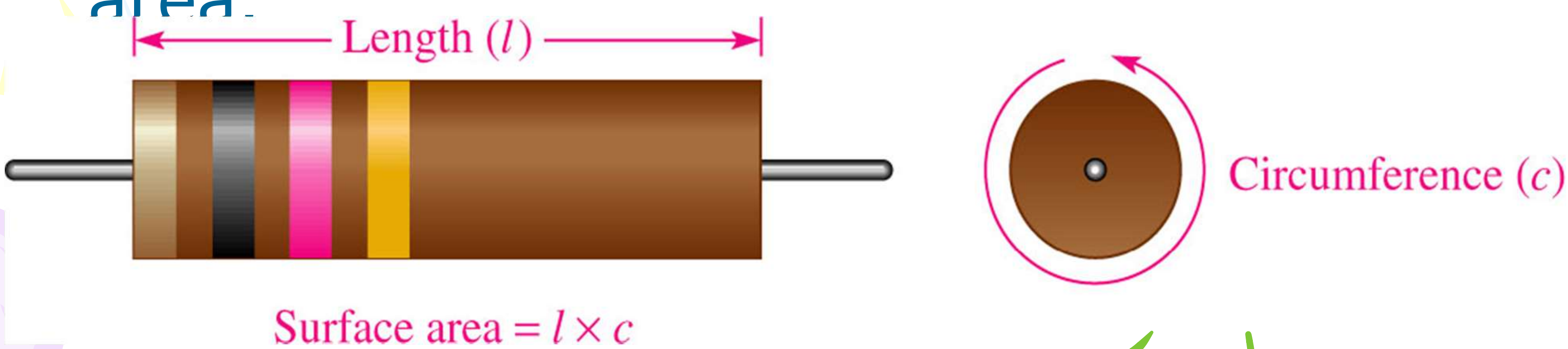
A เยอะ = ระบายดี

A น้อย = ระบายยาก,

ระบาย ค.ร้อน.

Resistor Power Rating

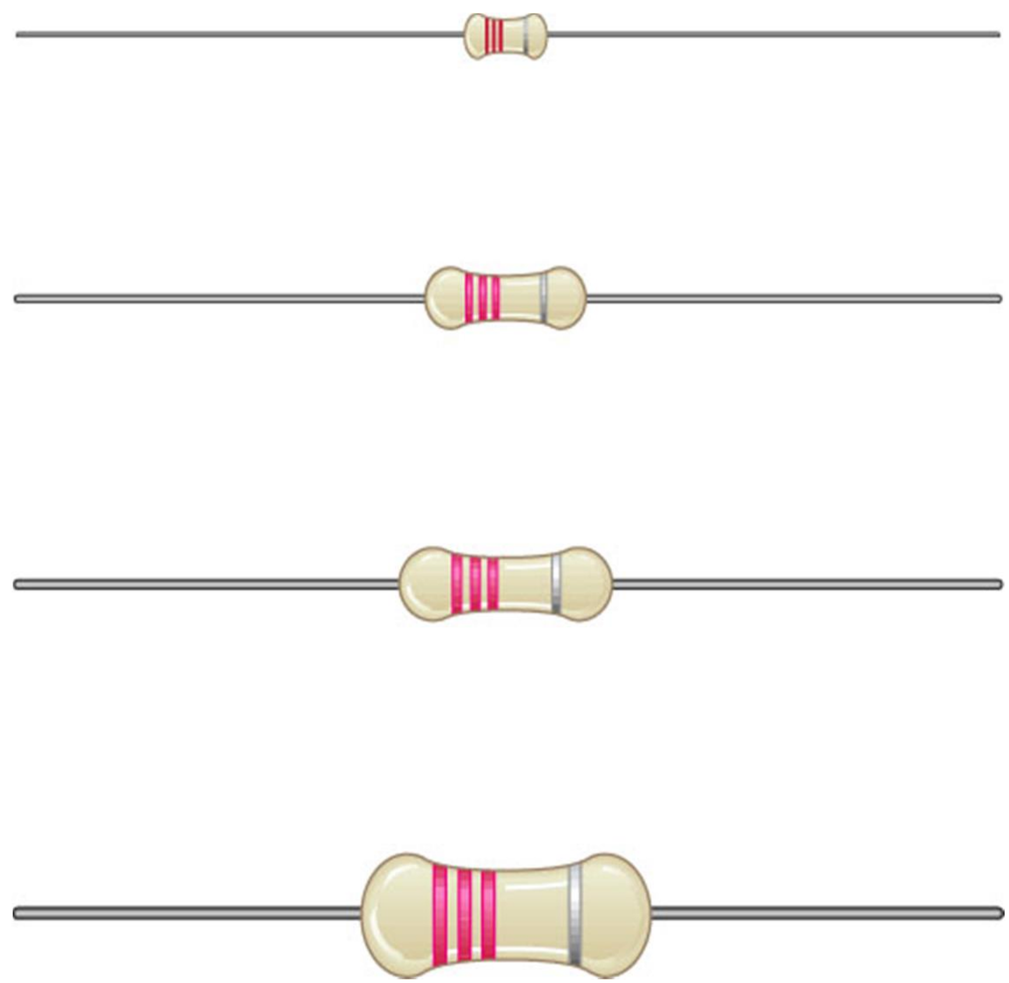
- Power rating of a resistor is the maximum amount of power that a resistor can dissipate without being damaged by excessive heat buildup.
- Power rating is directly related to surface area.



$2\pi r \times L$

Metal-film Resistors

Metal-film resistors have standard power ratings of 1/8 W, 1/4 W, 1/2 W, and 1 W.



Selecting the Proper Power Rating

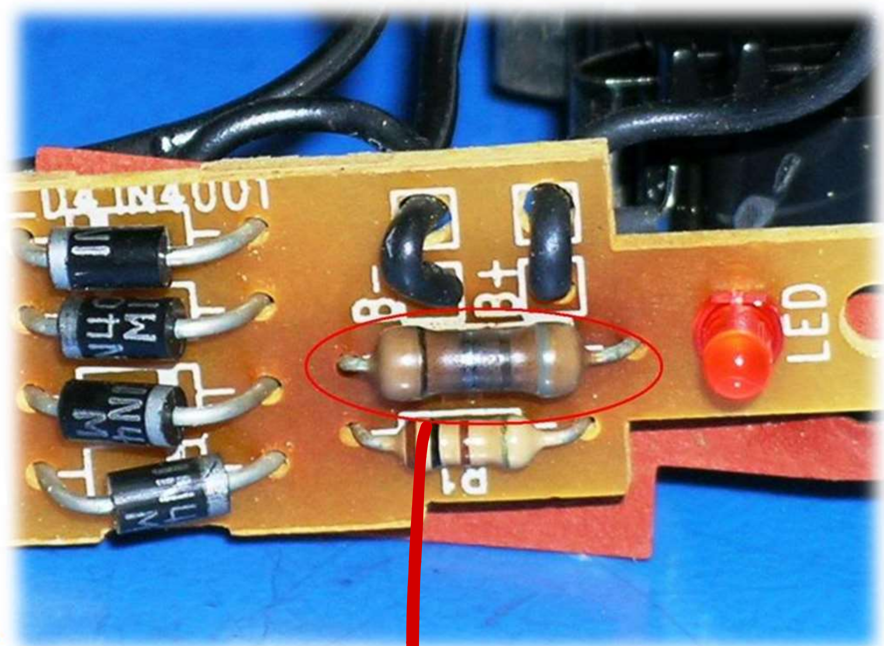
- A resistor used in a circuit must have a power rating in excess of what it will have to handle.

Rx2 ומ'ר.

- Ideally, a rating that is approximately twice the actual power should be used when possible.

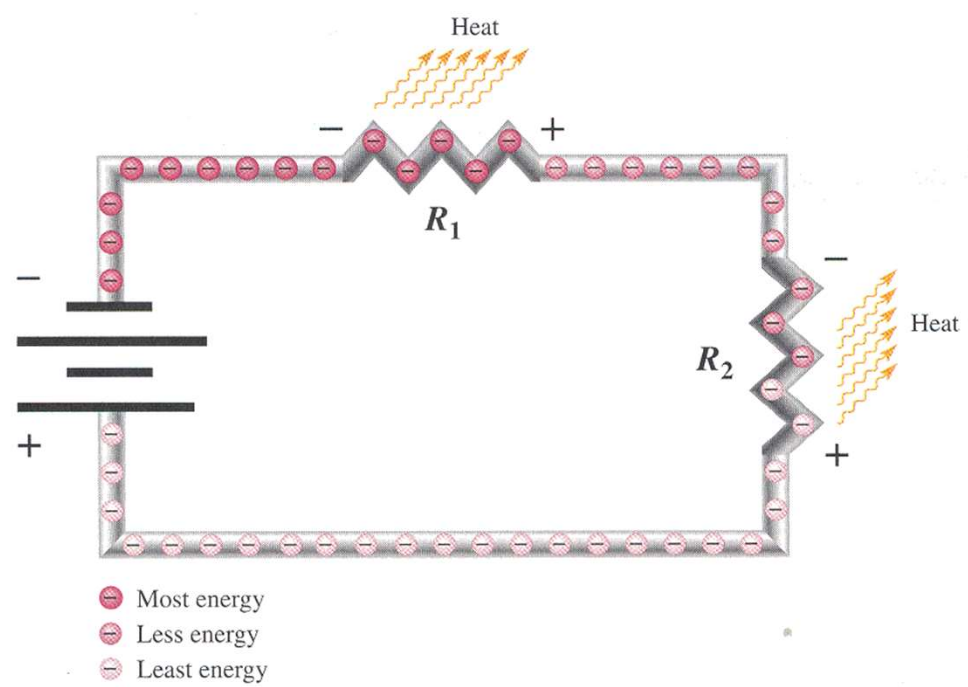
Resistor Failures

- When excessive power is applied to a resistor, the resistor will overheat.
- The resistor will burn open, or its resistance value will be greatly altered.
- Overheated resistors may be charred, or the surface color may change.
- Resistors suspected of being damaged should be removed from the circuit and checked with an ohmmeter.



Energy Conversion and Voltage Drop in Resistance

- As electrons flow through resistors, some of their energy is given up as heat.
- The same number of electrons entering a resistor will exit it, only their energy will be less, so the voltage exiting a resistor is less than the voltage entering the resistor.



2.2 Nodes, Branches and Loops (1)

- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.
- Network = interconnection of elements or devices
- Circuit = a network with closed paths
- A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

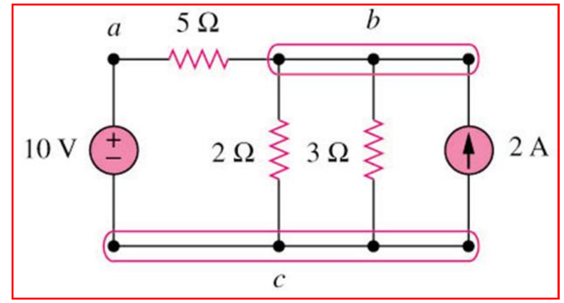
$$b = l + n - 1$$

2.2 Nodes, Branches and Loops (2)

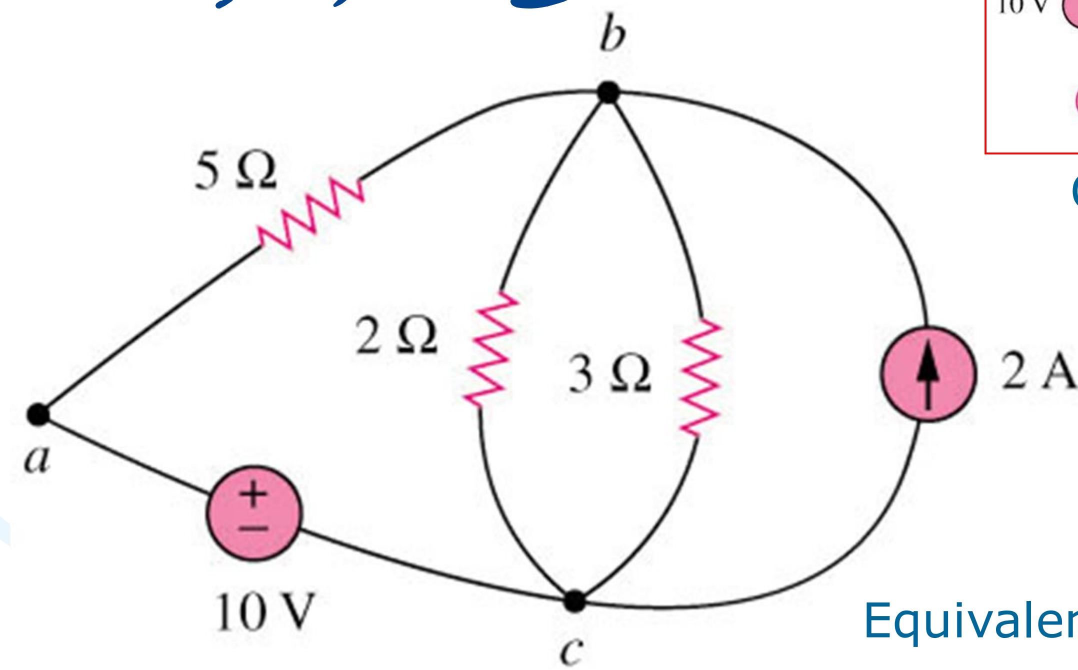
Example 1

$b = l + n - 1$

$5 = 3 + 3 - 1$



Original circuit

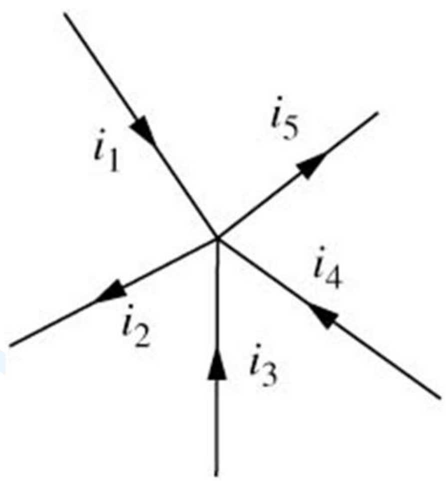


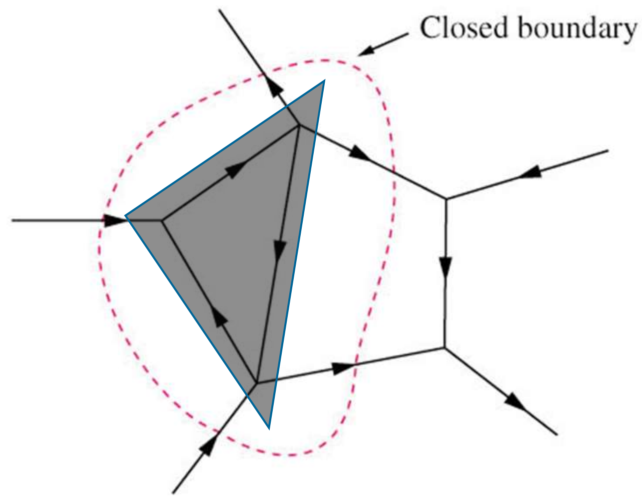
Equivalent circuit

How many branches, nodes and loops are there?

2.3 Kirchhoff's Laws (1)

- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.





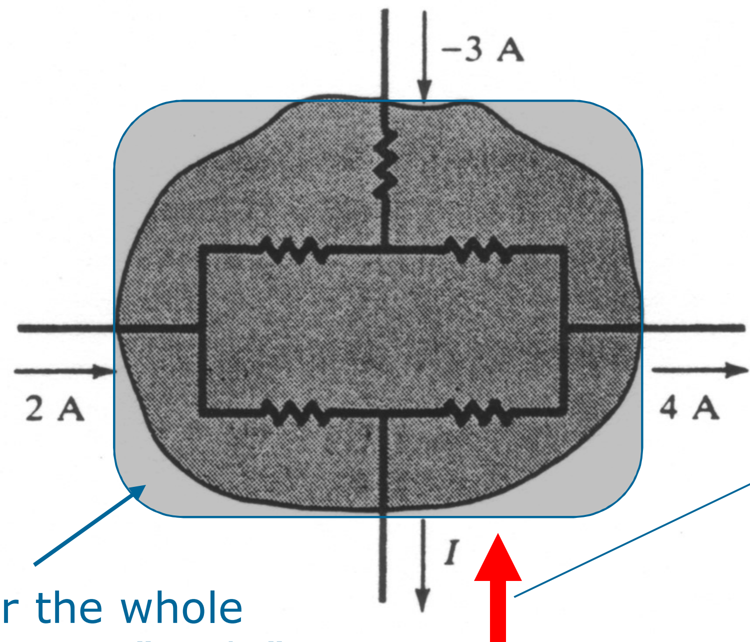
Mathematically, $$\sum_{n=1}^{N} i_n = 0$$

$I_{in} = I_{out}$ #

2.3 Kirchhoff’s Laws (2)

Example 4

- Determine the current I for the circuit shown in the figure below.



$I + 4-(-3)-2 = 0$

$\Rightarrow I = -5A$

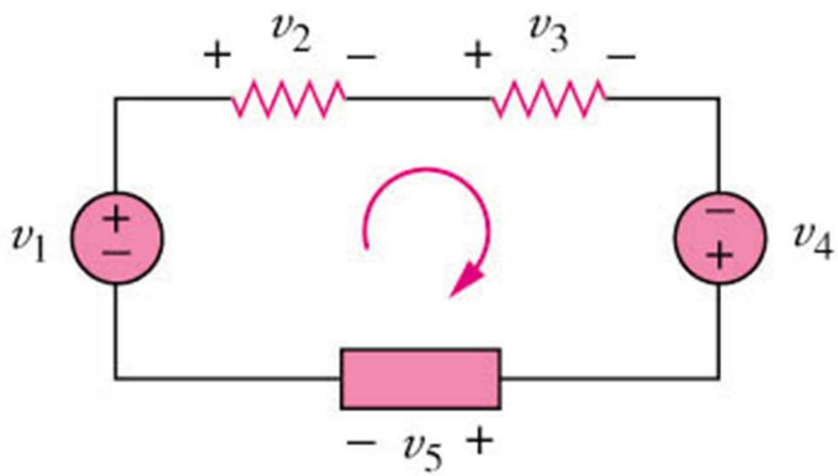
This indicates that the actual current for I is flowing in the opposite direction.

We can consider the whole enclosed area as one “node”.

2.3 Kirchhoff's Laws (3)

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

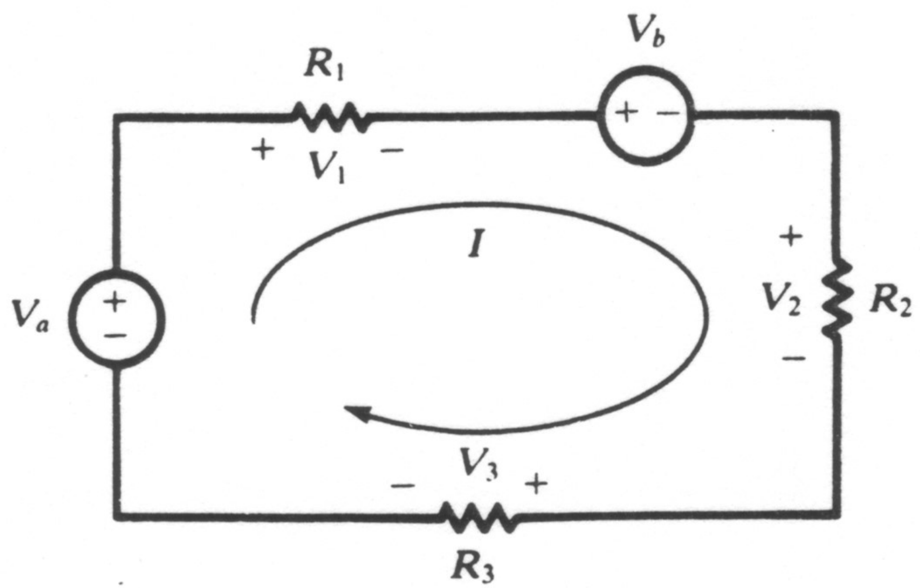
$\Sigma V = v_1 - v_2 - v_3 + v_4 - v_5 = 0$

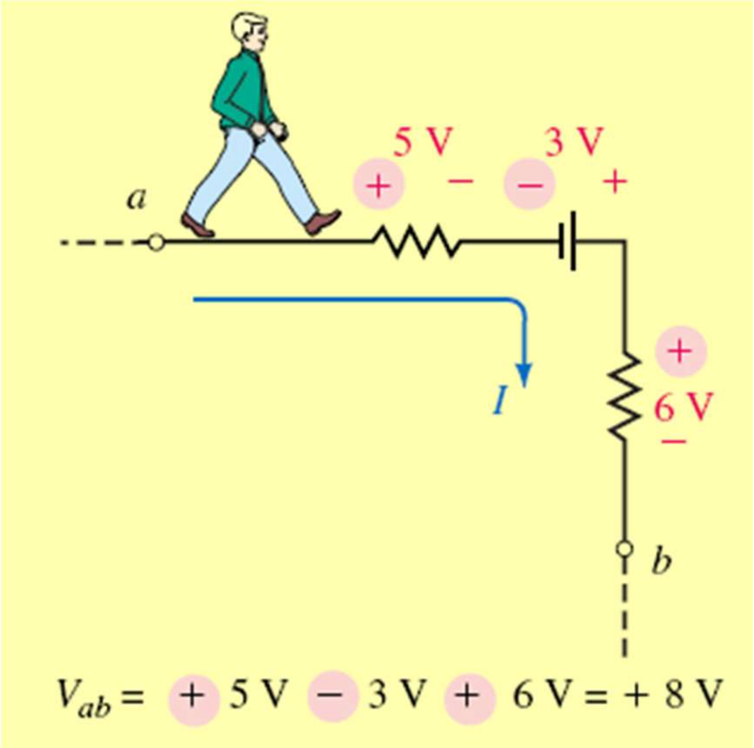


Mathematically, $$\sum_{m=1}^{M} v_n = 0$$

2.3 Kirchhoff's Laws (4)

Example 5 Applying the KVL equation for the circuit of the figure below.





$v_a - v_1 - v_b - v_2 - v_3 = 0$

$V_1 = IR_1 \quad v_2 = IR_2 \quad v_3 = IR_3$

$\Rightarrow v_a - v_b = I(R_1 + R_2 + R_3)$

$$I = \frac{v_a - v_b}{R_1 + R_2 + R_3}$$

2.4 Series Resistors and Voltage Division (1)

- Series: Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current.

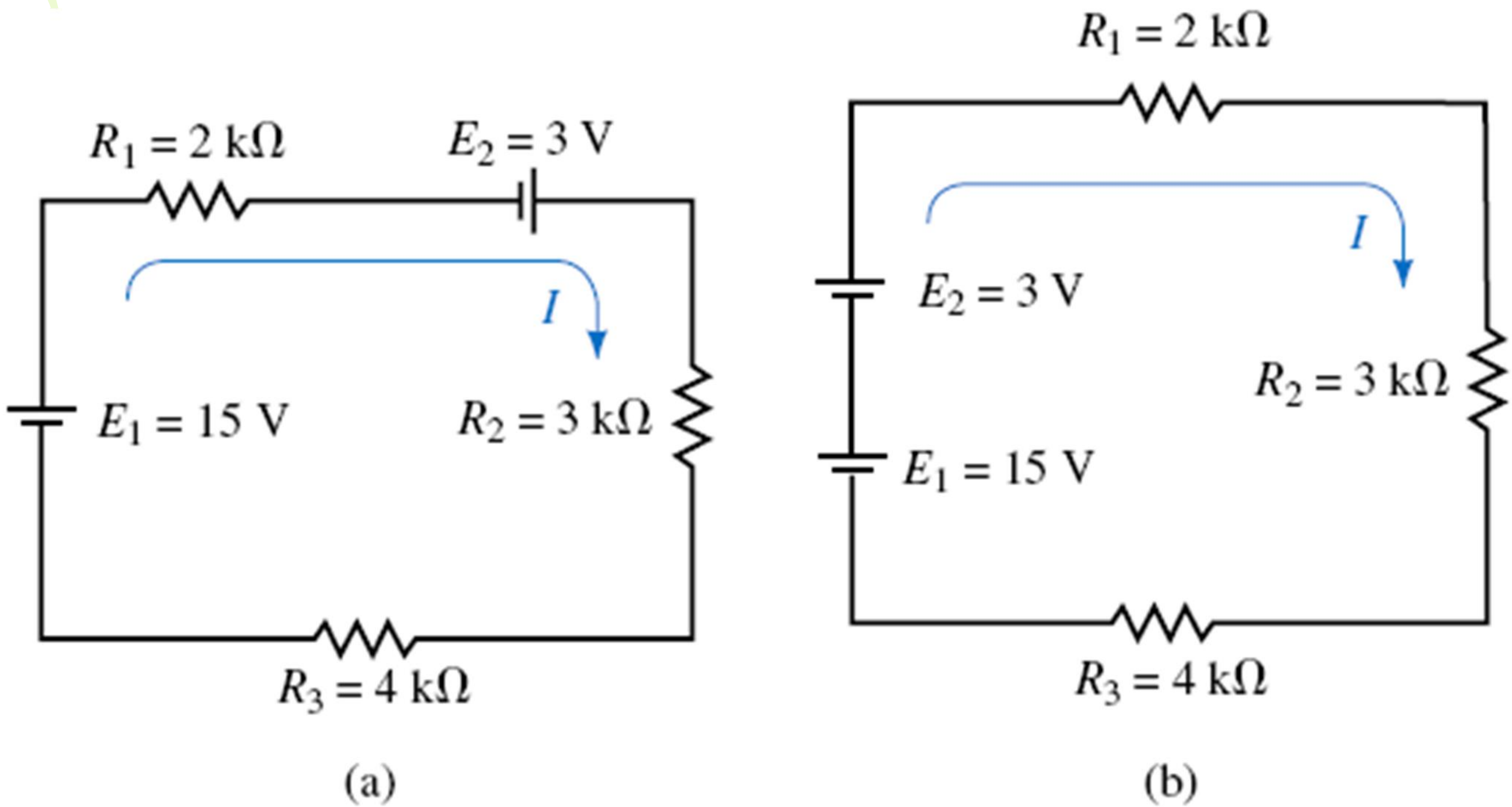
- The equivalent resistance of any number of resistors connected in a series is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

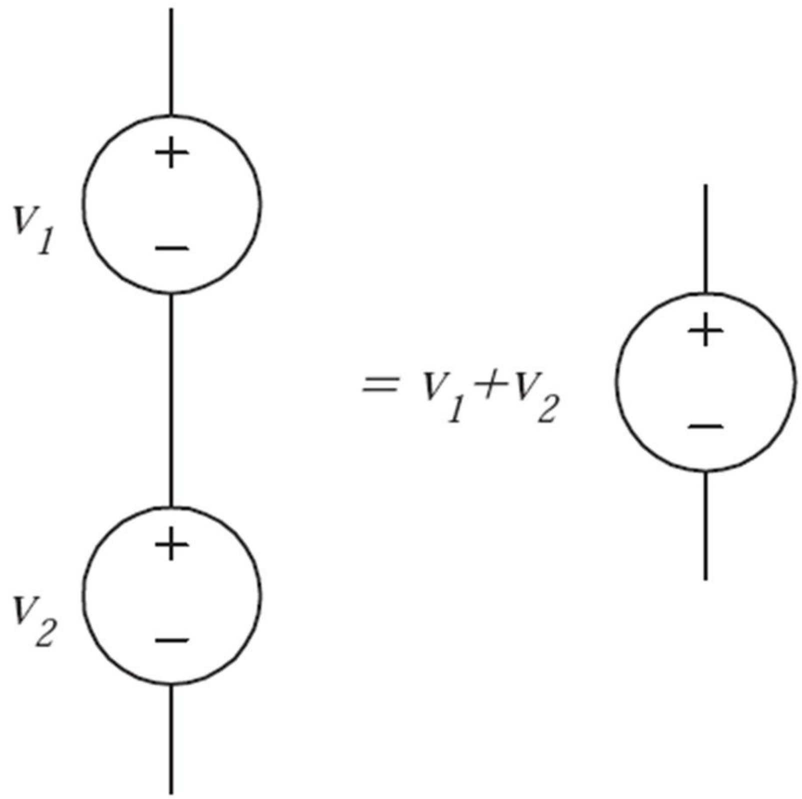
- The voltage divider can be expressed as

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v$$

Interchanging Series Components

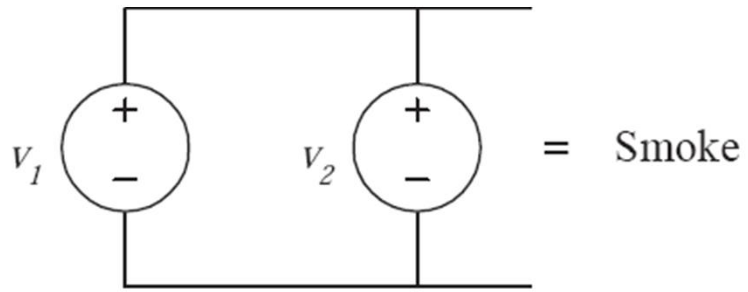


Voltage source in series



- Ideal voltage sources connected in series add

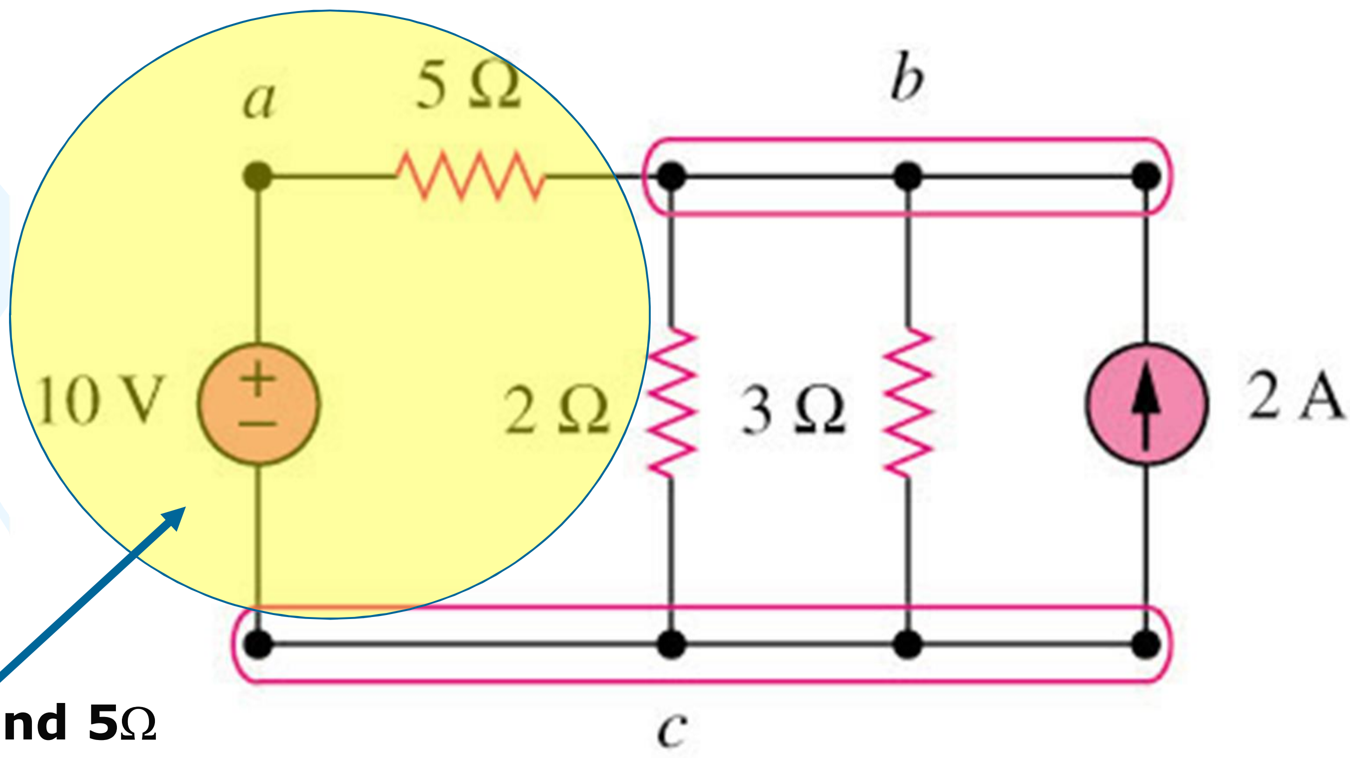
Voltage source in parallel



- Ideal voltage sources *cannot* be connected in parallel
- Recall: ideal voltage sources guarantee the voltage between two terminals is at the specified potential (voltage)
- Immovable object meets unstoppable force
- In practice, the stronger source would win
- Could easily cause component failure (smoke)
- Ideal sources do not exist
- Technically allowed if $V_1 = V_2$, but is a bad idea

2.4 Series Resistors and Voltage Division (1)

Example 3



10V and 5Ω are in series

2.5 Parallel Resistors and Current Division (1)

- Parallel: Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

- The equivalent resistance of a circuit with N resistors in parallel is:

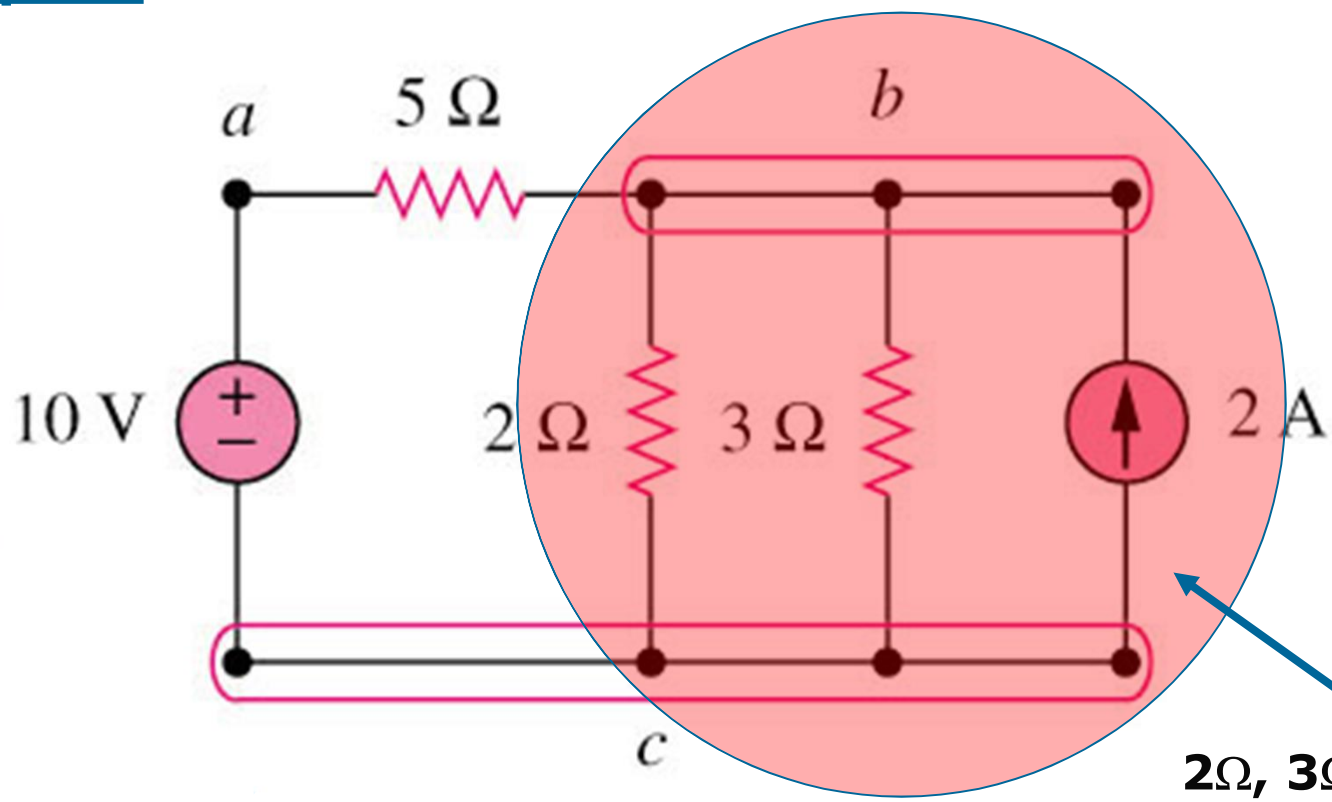
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- The total current i is shared by the resistors in inverse proportion to their resistances. The current divider can be expressed as:

$$i_n = \frac{v}{R_n} = \frac{iR_{eq}}{R_n}$$

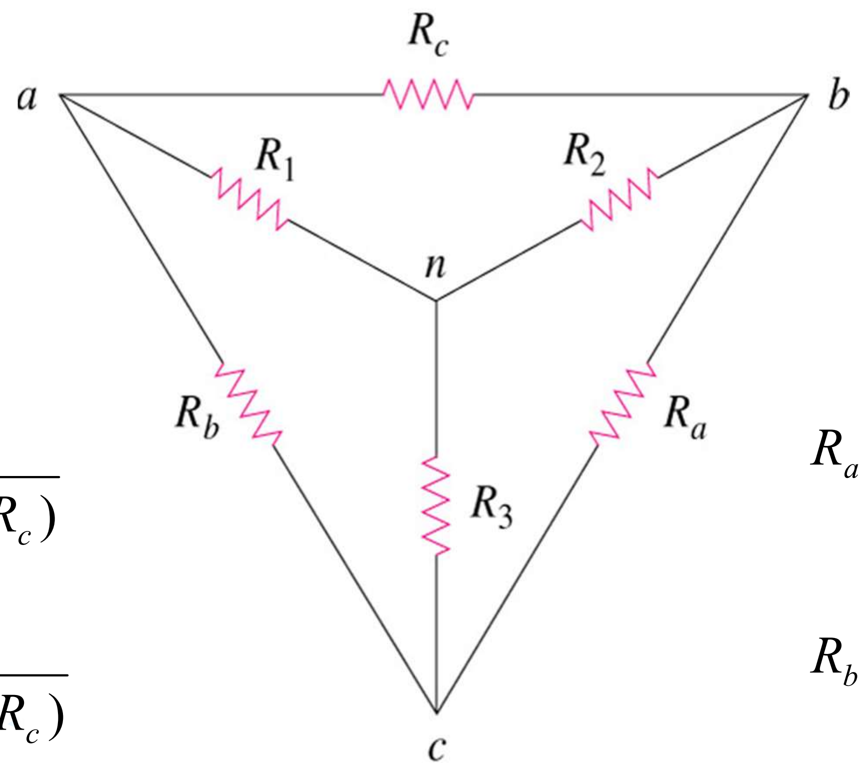
2.5 Parallel Resistors and Current Division (1)

Example 4



2Ω, 3Ω and 2A are in parallel

2.6 Wye-Delta Transformations



Delta -> Star

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

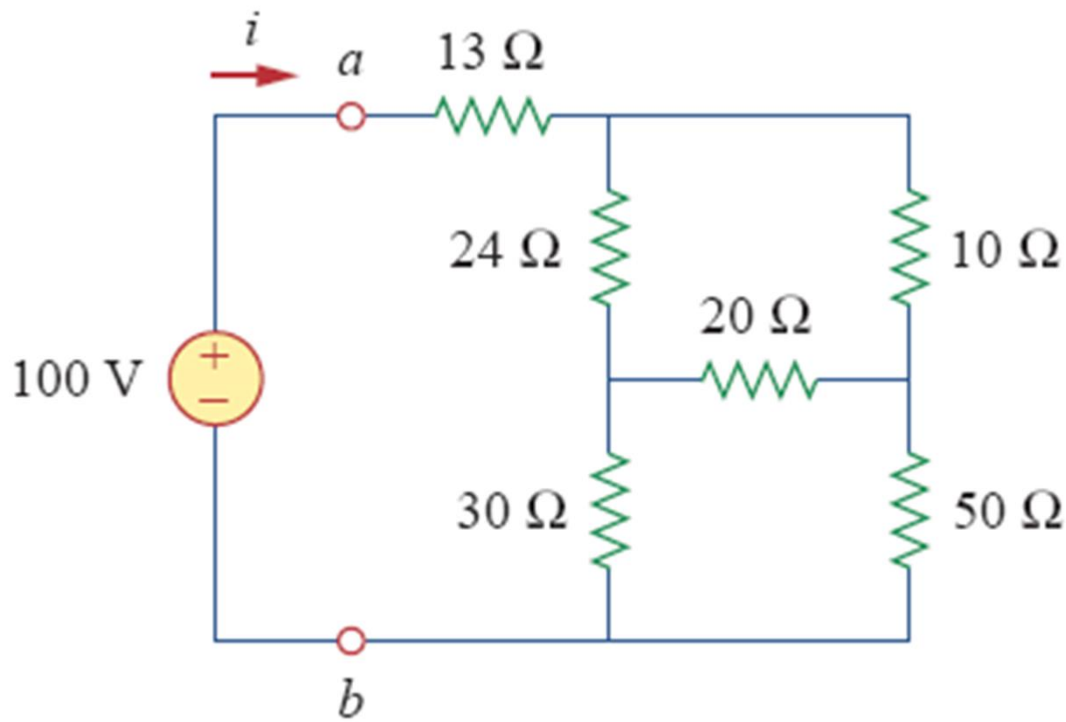
$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

Star -> Delta

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

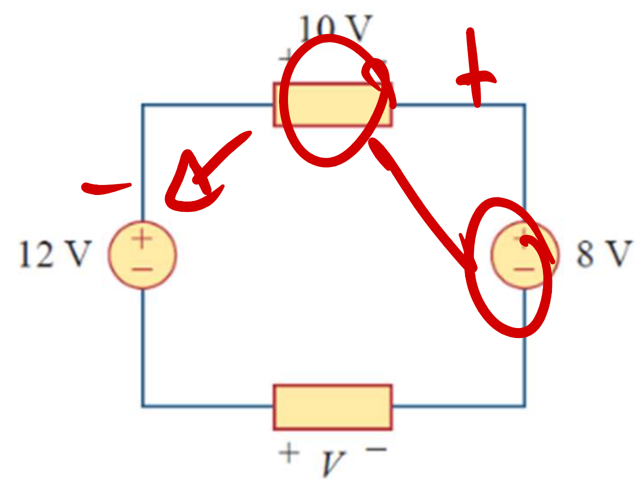

i
a
13 Ω
24 Ω
10 Ω
20 Ω
100 V
+
−
30 Ω
50 Ω
b

1.6 Circuit Elements

Example 1

จากรูป V มีค่าเท่าใด

$= 8+10-12 = 6 \, V.$



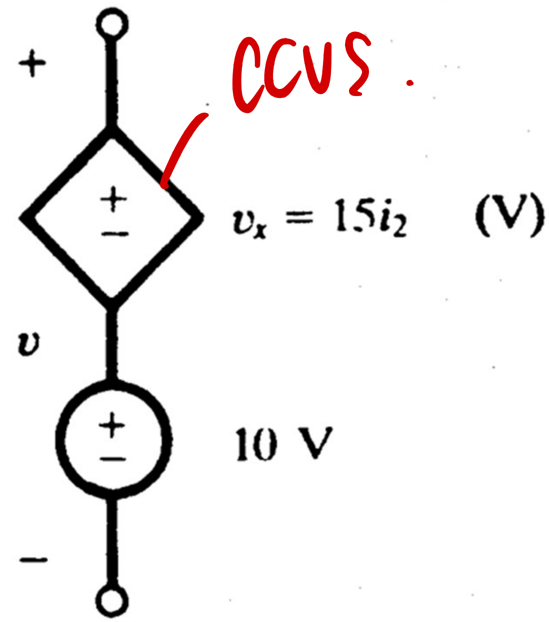
1.6 Circuit Elements

<u>Example 2</u>

$i_2 = 1A$

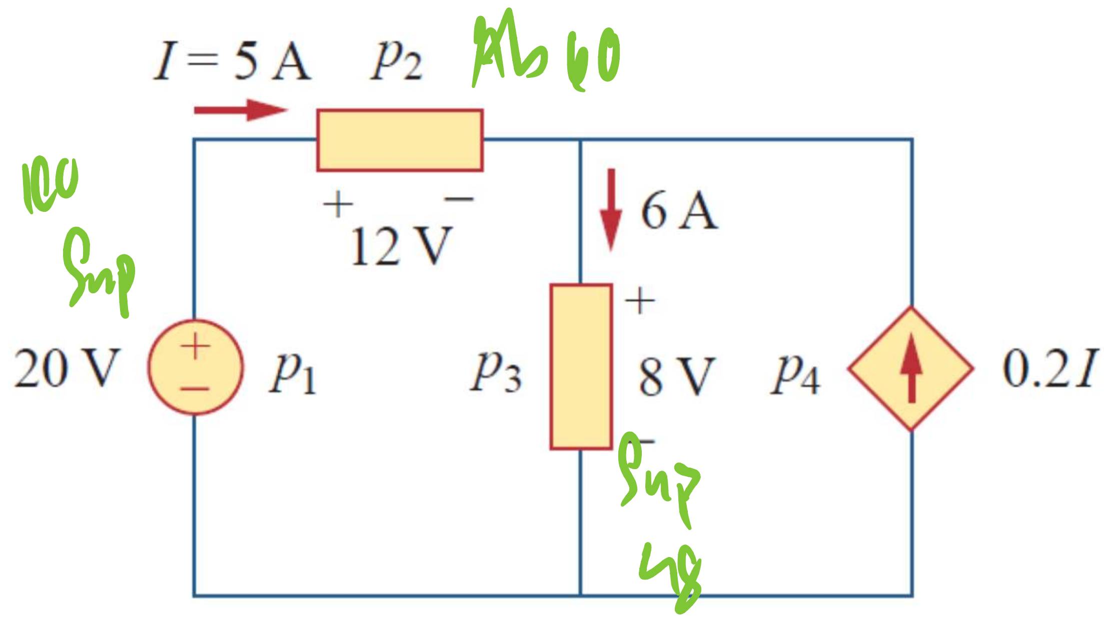
V มีค่าเท่าใด

$15(1) + 10 = 25$ V #



1.6 Circuit Elements

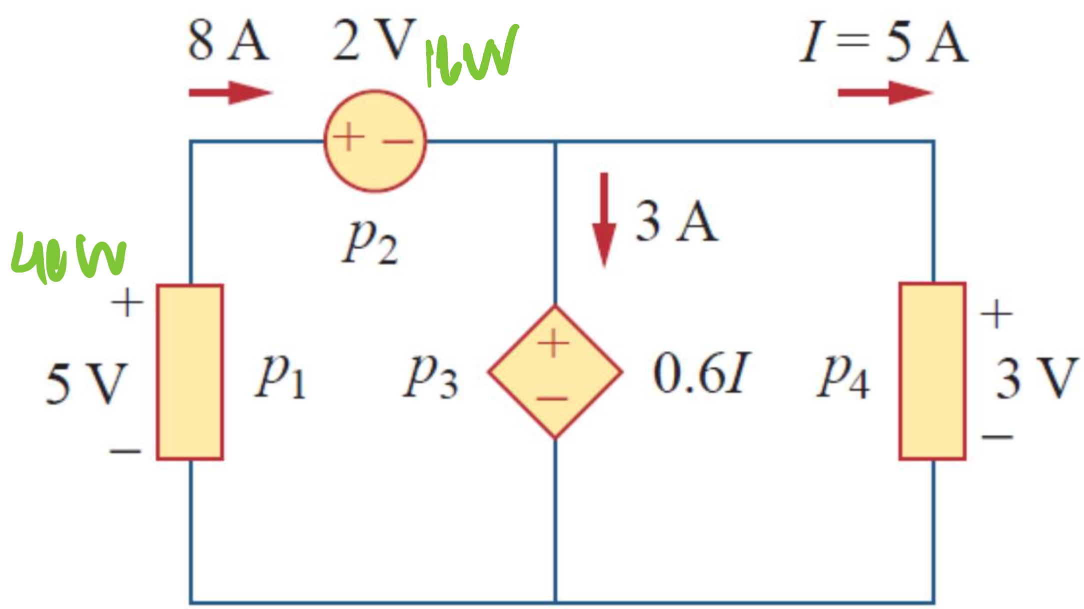
Example 3



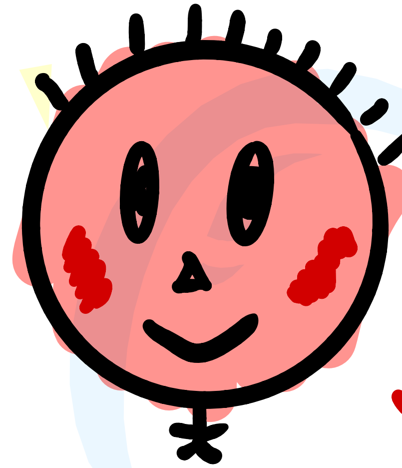
Calculate the power supplied or absorbed by each element

1.6 Circuit Elements

Example 4



Compute the power absorbed or supplied by each component



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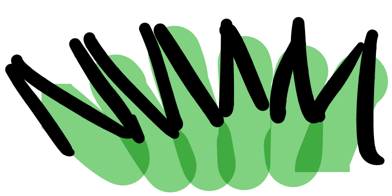
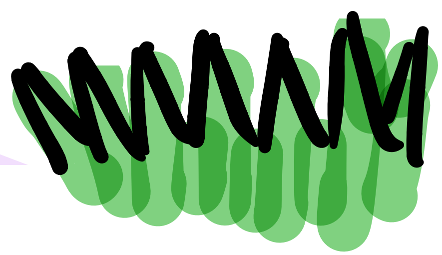
Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 3
Methods of Analysis

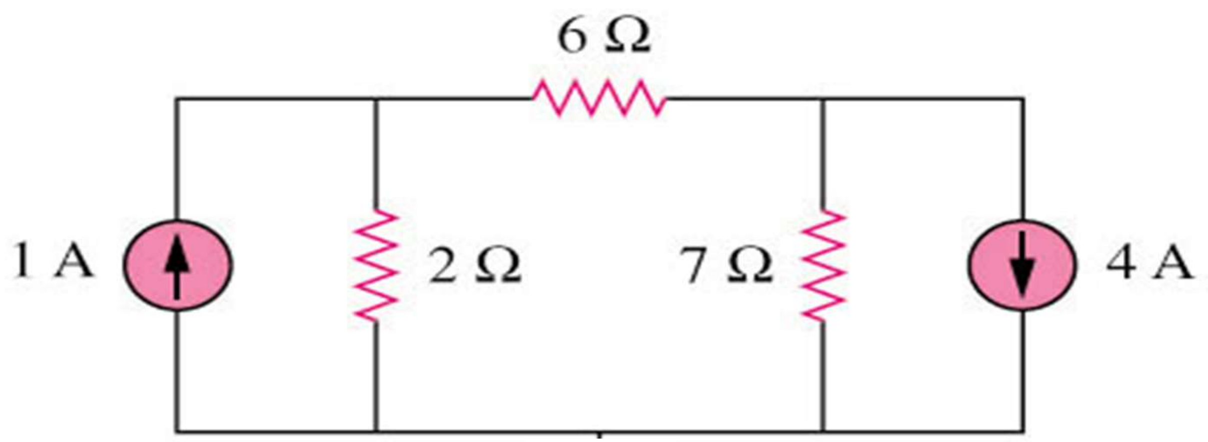




Methods of Analysis - Chapter 3

3.1 Motivation (1)

If you are given the following circuit, how can we determine (1) the voltage across each resistor, (2) current through each resistor. (3) power generated by each current source, etc.



What are the things which we need to know in order to determine the answers?

3.1 Motivation (2)

Things we need to know in solving any resistive circuit with current and voltage sources only:

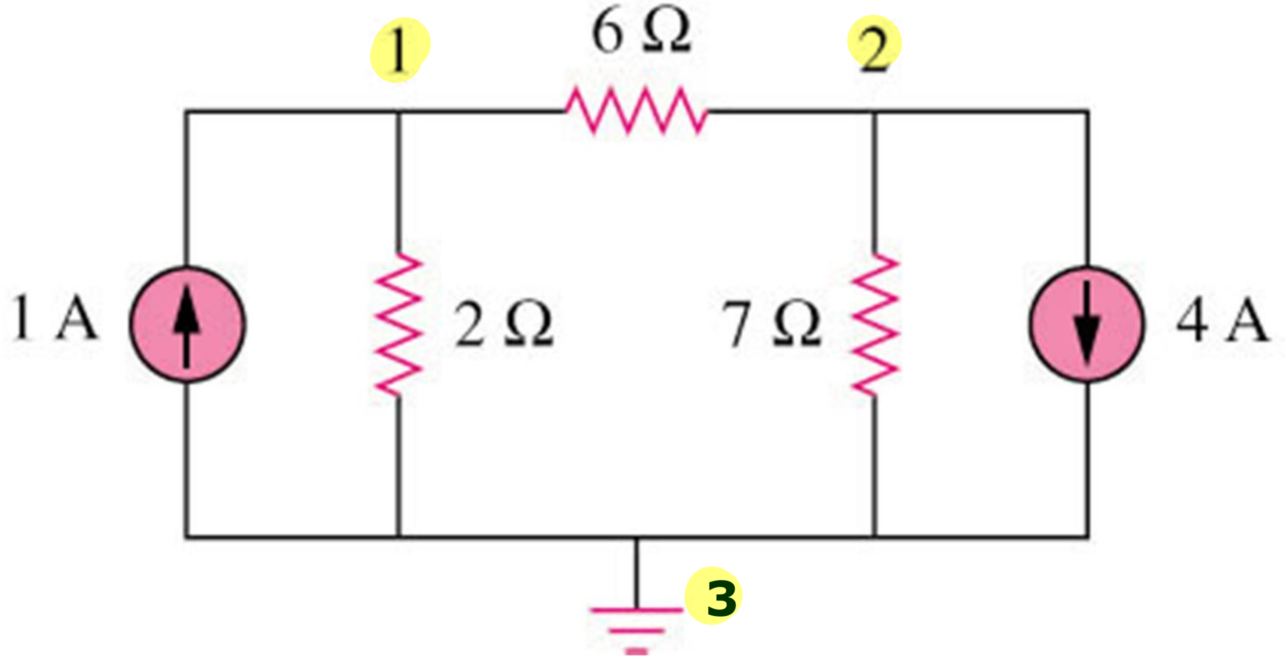
- Kirchhoff's Current Laws (KCL)
- Kirchhoff's Voltage Laws (KVL)
- Ohm's Law

How should we apply these laws to determine the answers?

3.2 Nodal Analysis (1)

It provides a general procedure for analyzing circuits using node voltages as the circuit variables.

Example 1



3.2 Nodal Analysis (2)

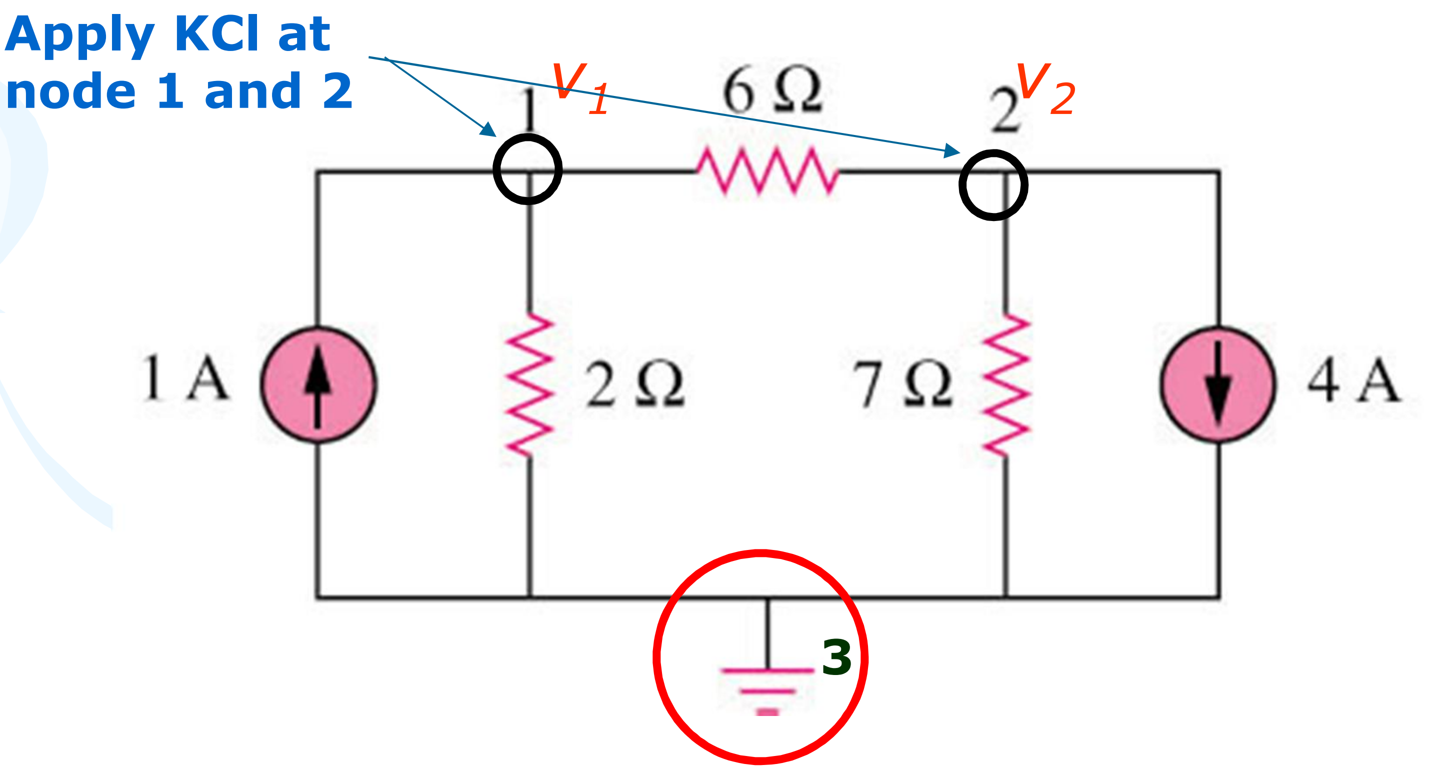
Steps to determine the node voltages:

into a volt

1. Select a node as the reference node.
2. Assign voltages v1,v2,…,vn-1 to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

3.2 Nodal Analysis (3)

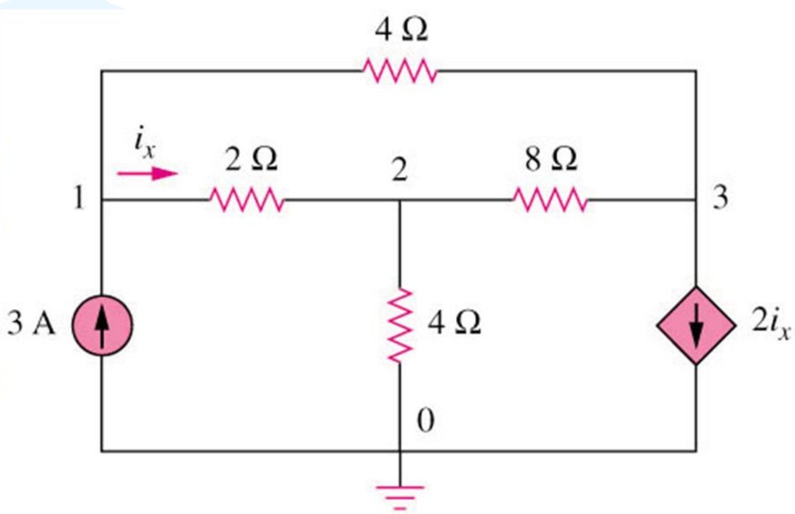
<u>Example 2</u> – circuit independent current source only



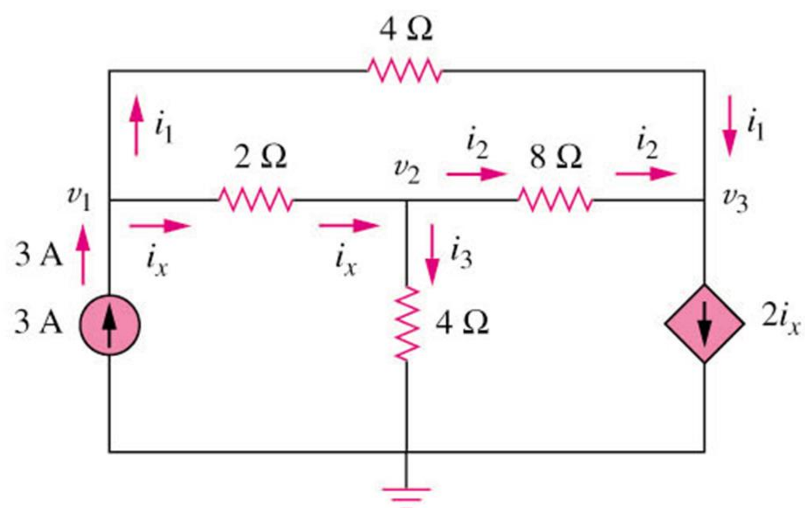
*Refer to in-class illustration, textbook, answer v_1 = -2V, v_2 = -14V

3.2 Nodal Analysis (4)

<u>Example 3</u> – current with dependant current source



(a)



(b)

*Refer to in-class illustration, textbook, answer v_1= 4.8V, v_2 = 2.4V, v_3 = -2.4V

$3 = i_x + i_1$ — (1) ; $3 = \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4}$ — (1)'

$i_x = i_2 + i_3$ — (2) ; $\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2}{4}$ — (2)'

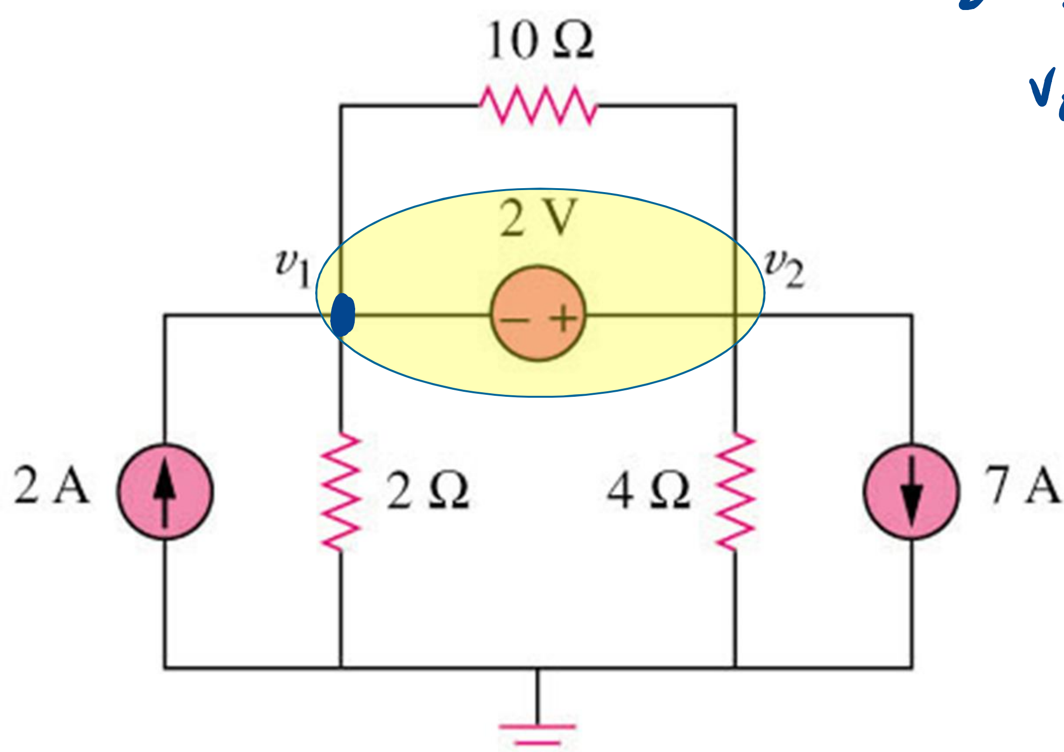
$i_1 + i_2 = 2i_x$ — (3) ; $\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = v_1 - v_2$ — (3)'

3.3 Nodal Analysis with Voltage Source (1)

<u>Example 4</u> –circuit with independent voltage source

$2 = \frac{v_1}{2} + \frac{v_2}{4} + 7$ — ①

$v_2 - v_1 = 2$ — ②



How to handle the 2V voltage source?

3.3 Nodal Analysis with Voltage Source (2)

A super-node is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

*Note: We analyze a circuit with super-nodes using the same three steps mentioned above except that the super-nodes are treated differently.

3.3 Nodal Analysis with Voltage Source (3)

Basic steps:

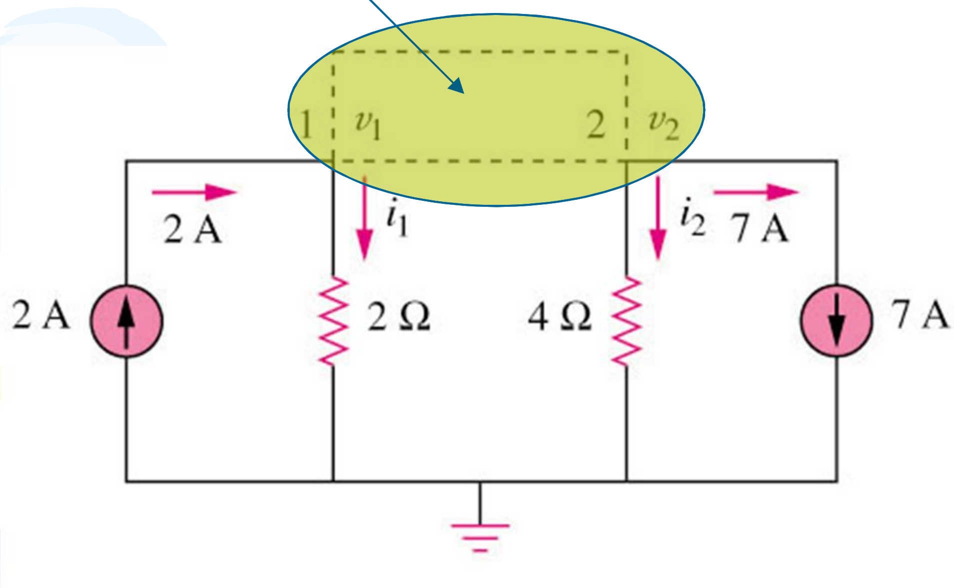
1. Take off all voltage sources in super-nodes and apply KCL to super-nodes.

2. Put voltage sources back to the nodes and apply KVL to relative loops.

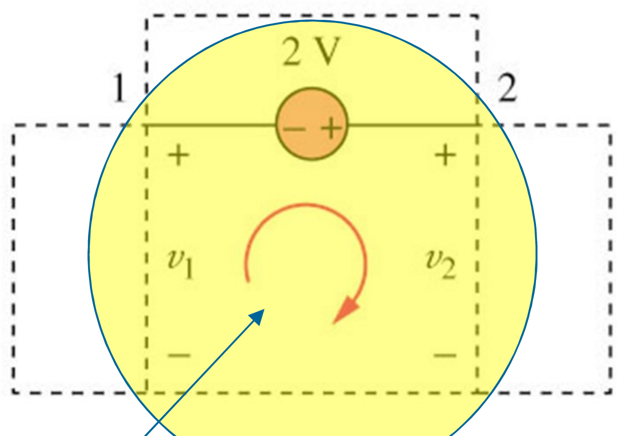
3.3 Nodal Analysis with Voltage Source (4)

Example 5 – **circuit with independent voltage source**

Super-node => $2 - i_1 - i_2 - 7 = 0$



(a)

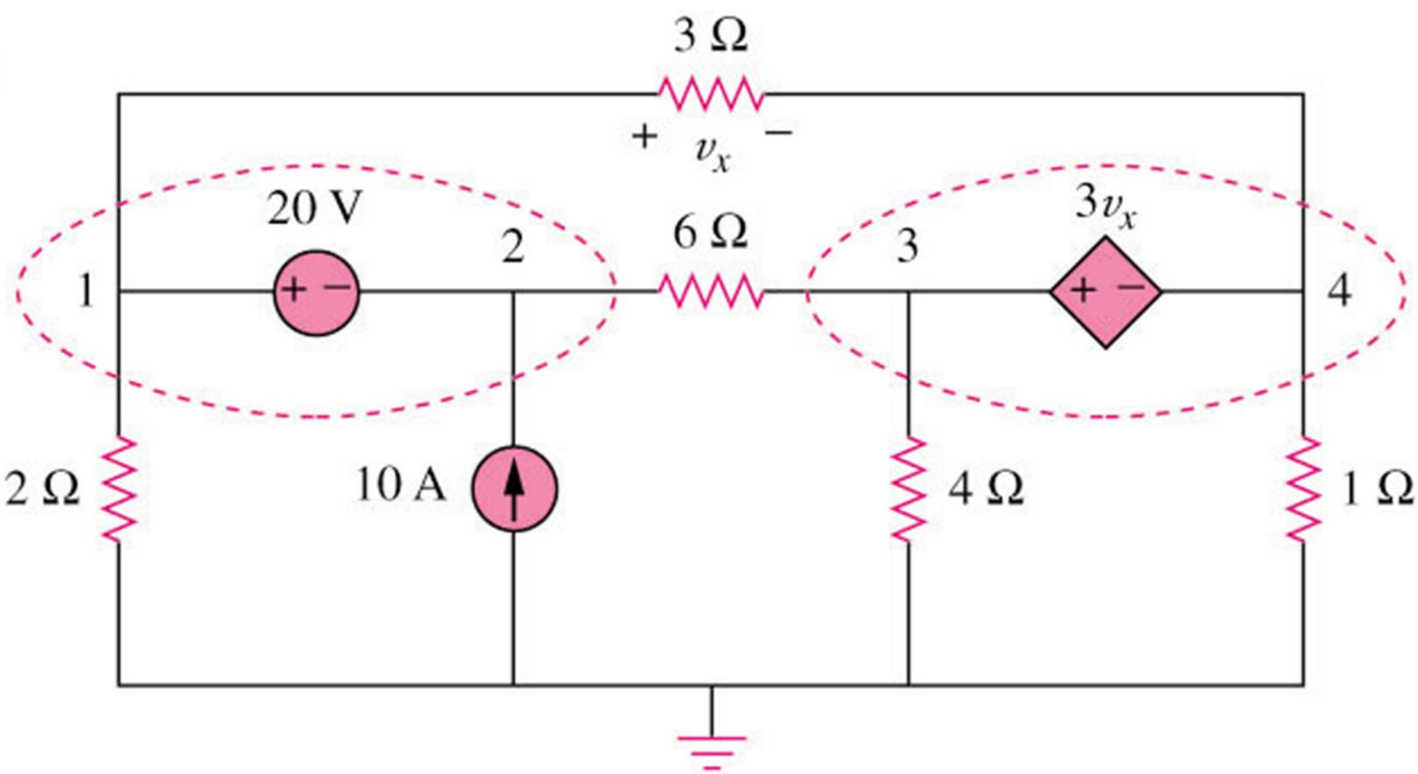


(b)

Apply KVL => $v_1 + 2 - v_2 = 0$

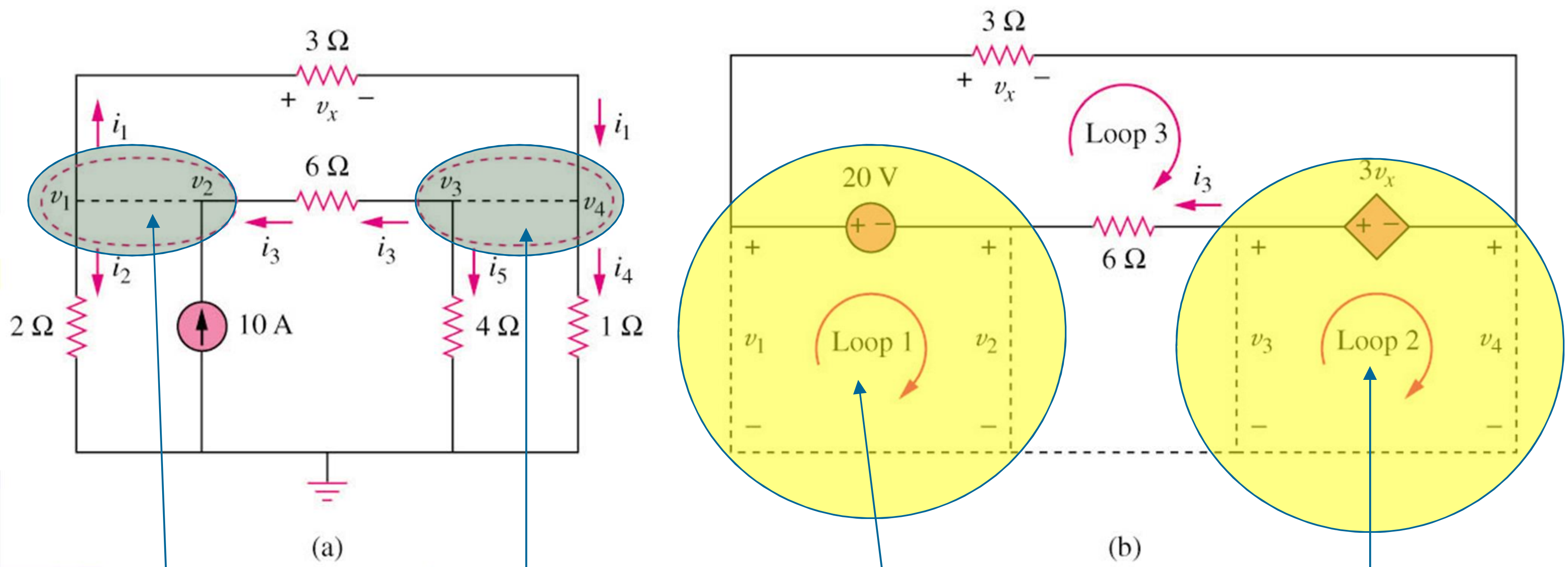
3.3 Nodal Analysis with Voltage Source (5)

Example 6 – circuit with two independent voltage sources



3.3 Nodal Analysis with Voltage Source (6)

Example 7 – circuit with two independent voltage sources



$10 + i_3 = i_1 + i_2$ $\quad i_1 = i_3 + i_5 + i_4$ $\quad v_1 - 20 - v_2 = 0$ $\quad v_3 - 3v_x - v_4 = 0$

3.4 Mesh Analysis (1)

1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.

2. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.

3. A mesh is a loop which does not contain any other loops within it.

3.4 Mesh Analysis (2)

Steps to determine the mesh currents:

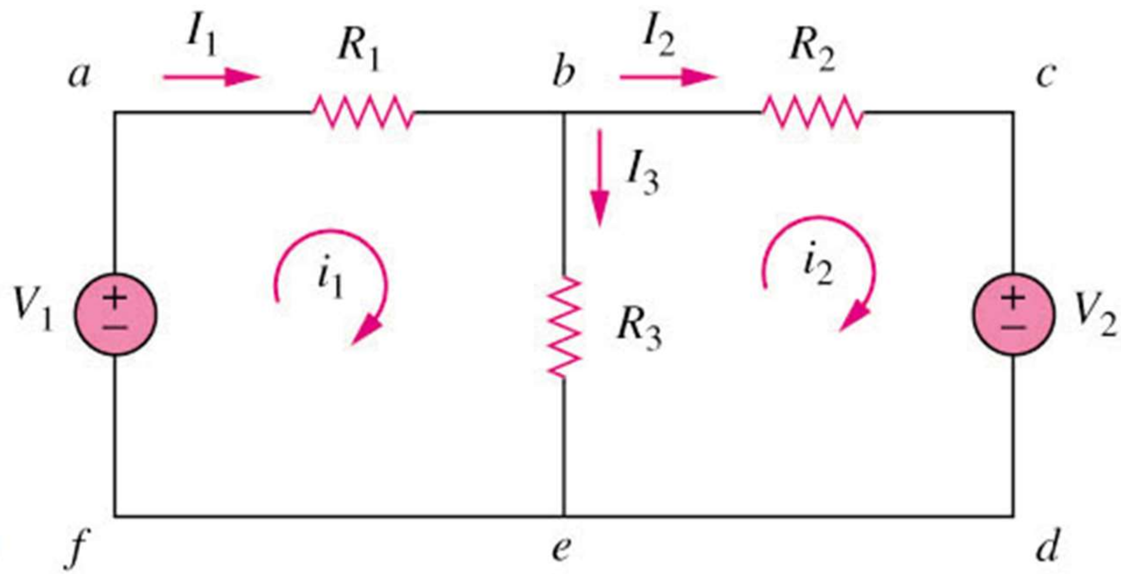
1. Assign mesh currents i_1, i_2, …, in to the n meshes.

2. Apply KCL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting n simultaneous equations to get the mesh currents.

3.4 Mesh Analysis (3)

<u>Example 8</u> – circuit with independent voltage sources



<u>Note</u>:

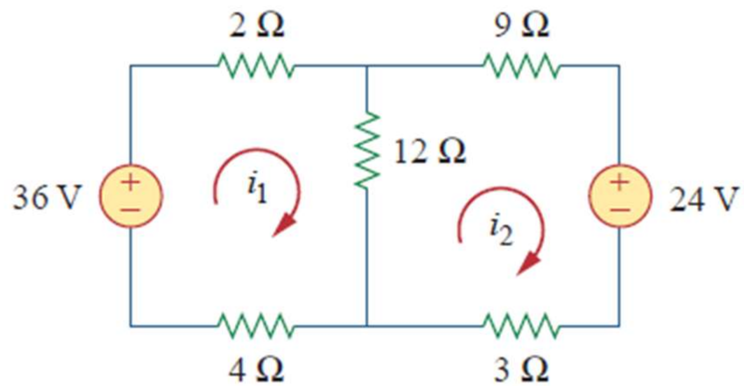
i_1 and i_2 are mesh current (imaginative, not measurable directly)

I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

*Refer to in-class illustration, textbook

3.4 Mesh Analysis

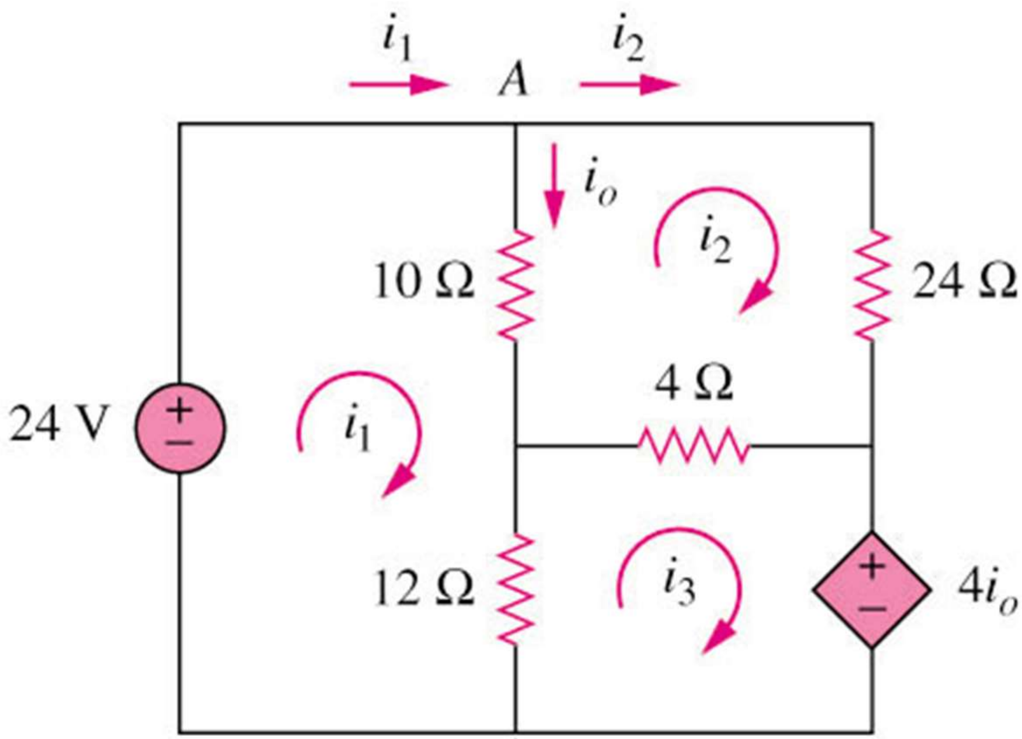


$36 = 2i_1 + 12(i_1 - i_2) + 4i_1$ — 1

$-24 =$

3.4 Mesh Analysis (4)

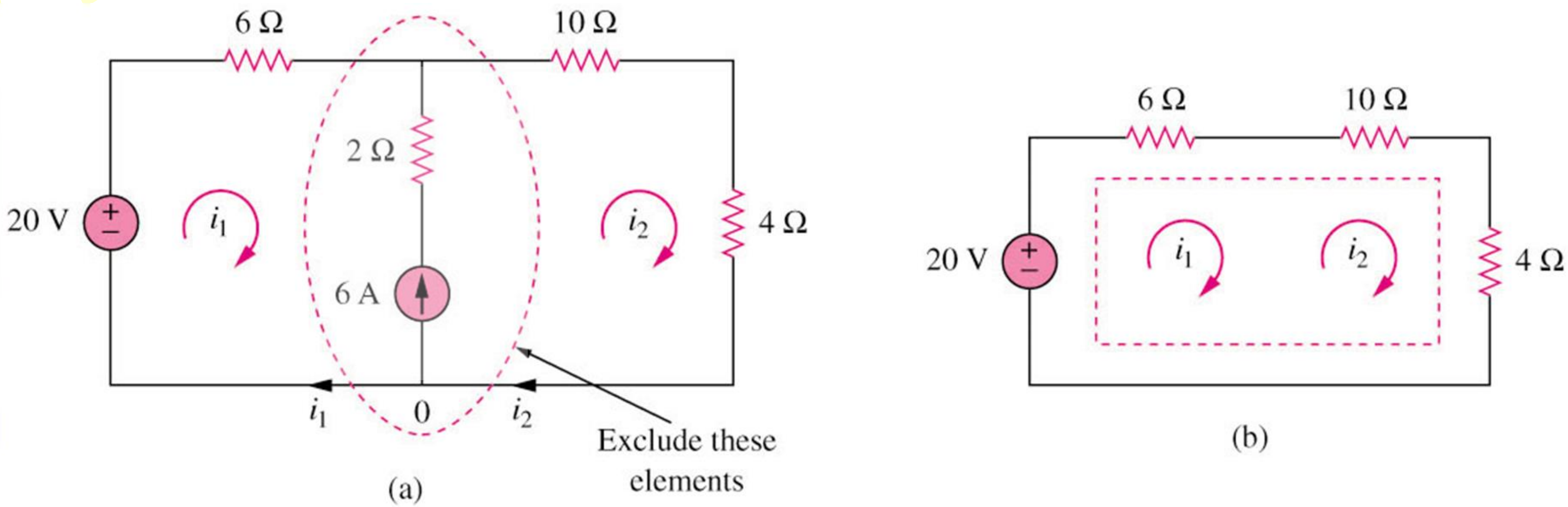
circuit with dependent voltage source



*Refer to in-class illustration, textbook, answer I_o = 1.5A

3.5 Mesh Analysis with Current Source (1)

Circuit with current source



A **super-mesh** results when two meshes have a (dependent or independent) current source in common as shown in (a). We create a super-mesh by excluding the current source and any elements connected in series with it as shown in (b).

*Refer to in-class illustration, textbook

3.5 Mesh Analysis with Current Source (2)

The properties of a super-mesh:

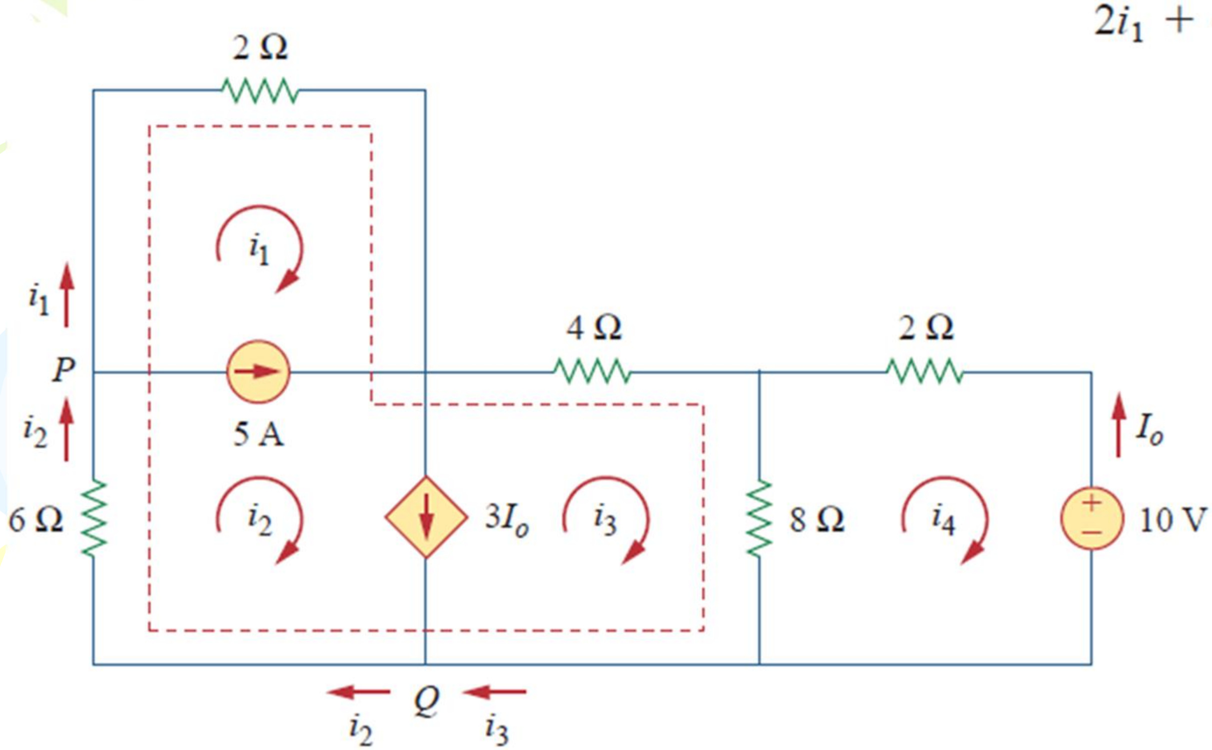
1. The current source in the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

2. A super-mesh has no current of its own.

3. A super-mesh requires the application of both KVL and KCL.

3.5 Mesh Analysis with Current Source

find i_1 to i_4



$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$

$$i_2 = i_1 + 5$$

$$i_2 = i_3 + 3I_o$$

$$I_o = -i_4,$$

$$i_2 = i_3 - 3i_4$$

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

$$5i_4 - 4i_3 = -5$$

$$i_1 = -7.5 \text{ A}, \qquad i_2 = -2.5 \text{ A}, \qquad i_3 = 3.93 \text{ A}, \qquad i_4 = 2.143 \text{ A}$$

$i_1 - i_2 = 3 \quad \therefore \; i_1 = i_2 + 3$

$\therefore I_1 = \frac{36}{76} + \frac{228}{76} = \boxed{\frac{264}{76}}$

3.5 Mesh Analysis with Current Source

Loop 1+2

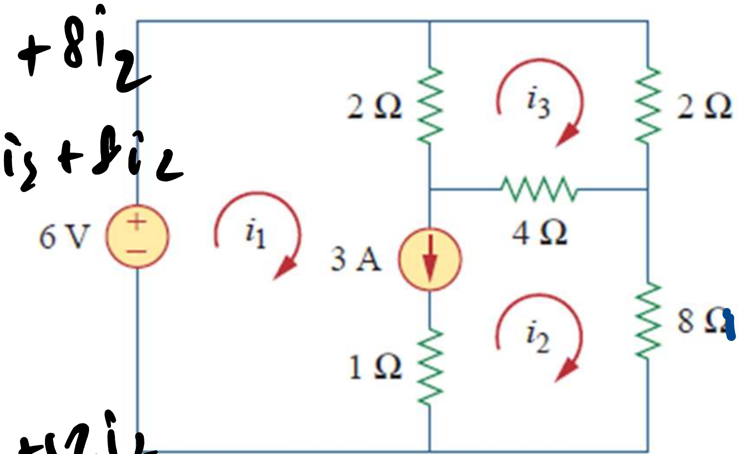
$-6 = 2(i_1 - i_3) + 4(i_2 - i_3) + 8i_2$

$; \; 6 = 2i_1 - 2i_3 + 4i_2 - 4i_3 + 8i_2$

$; \; 6 = 2i_1 - 6i_3 + 12i_2$

$; \; 6 = 2i_2 + 6 - 6i_3 + 12i_2$

$\therefore 0 = 14i_2 - 6i_3 \; — \; (1)$



$8i_3 = 6 + 6i_2$

$8i_3 = \frac{456 + 216}{76}$

$\therefore 8i_3 = \frac{672}{76} \quad \therefore i_3 = \boxed{1.105 \, A}$

$(1) \times 8; \; 0 = 112i_2 - 48i_3 \; — \; (3)$

$(2) \times 6; \; 36 = 48i_3 - 36i_2 \; — \; (4)$

$(3) + (4); \; 36 = 76i_2$

$\therefore i_2 = \boxed{\frac{36}{76}}$

Loop 3

$2i_3 + 4i_3 - 4i_2 + 2i_3 - 2i_1 = 0$

$8i_3 - 4i_2 - 2i_2 - 6 = 0$

$\therefore 6 = 8i_3 - 6i_2 \; — \; (2)$

3.6 Nodal and Mesh Analysis with Inspection (1)

The properties of a super-mesh:

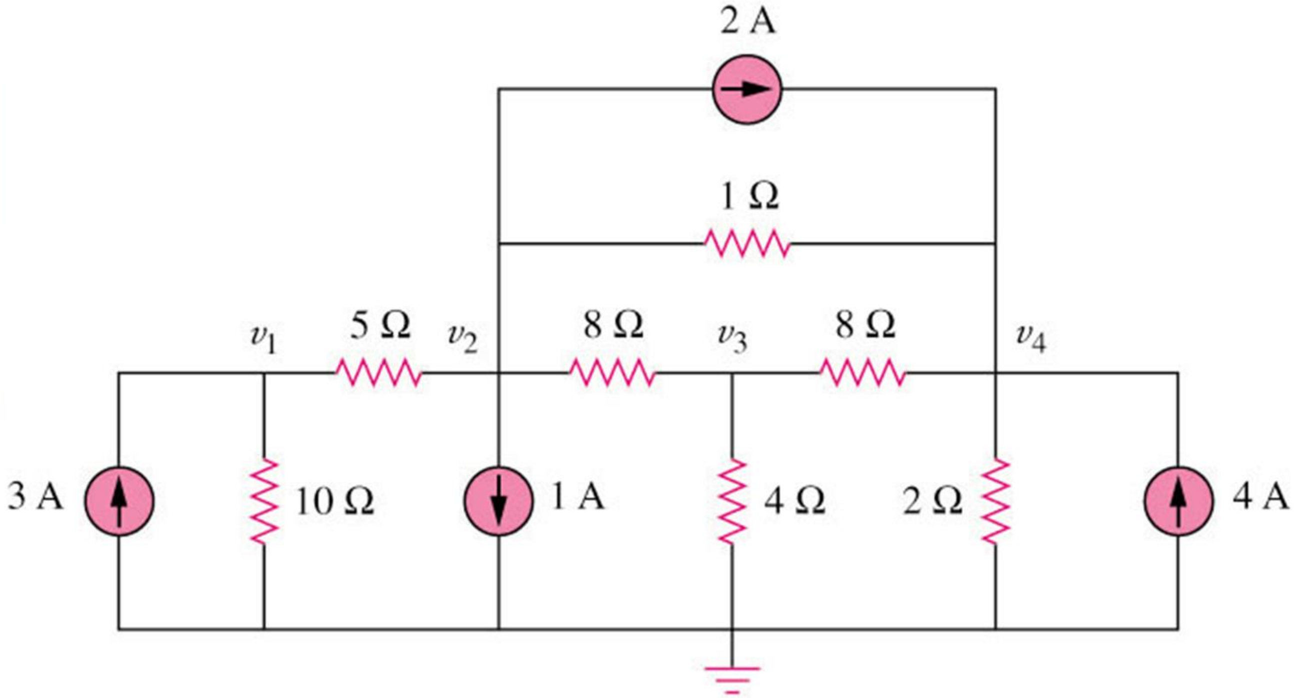
1. The current source in the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

2. A super-mesh has no current of its own.

3. A super-mesh requires the application of both KVL and KCL.

3.6 Nodal and Mesh Analysis with Inspection (2)

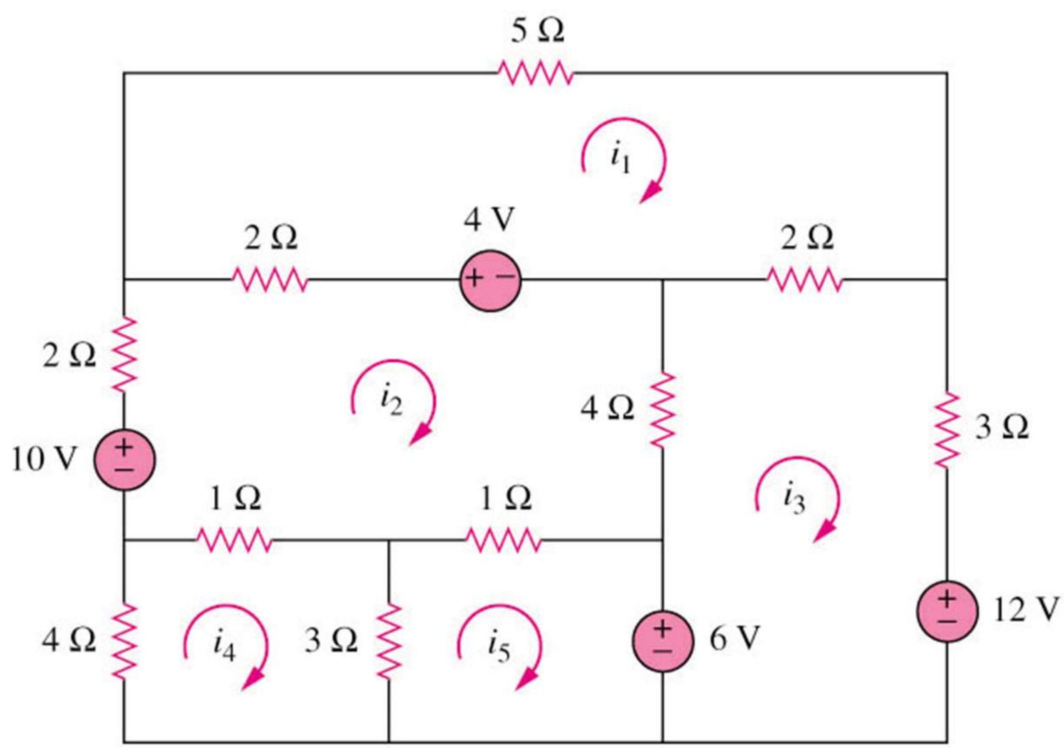
<u>Example 10</u> – By inspection, write the nodal voltage equations for the circuit



*Refer to in-class illustration, textbook

3.6 Nodal and Mesh Analysis with Inspection (3)

Example 11 – By inspection, write the mesh-current equations for the circuit



*Refer to in-class illustration, textbook

3.7 Nodal versus Mesh Analysis (1)

To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.
   *Choose mesh analysis for circuit with fewer meshes than nodes.
   *Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.
   *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.
2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

Alexander-Sadiku
Fundamentals of Electric Circuits

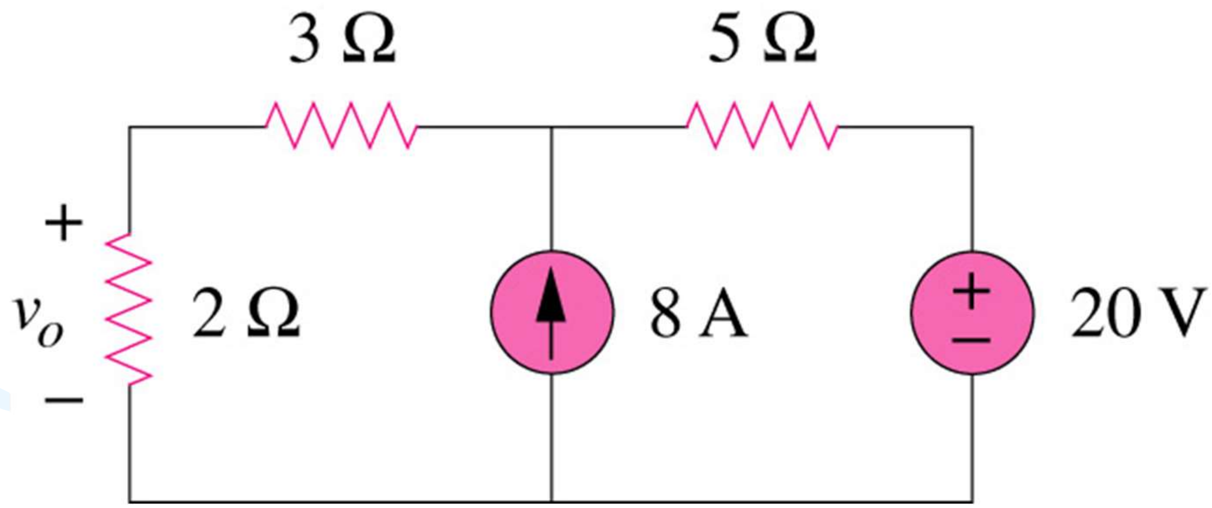
Chapter 4
Circuit Theorems

Circuit Theorems - Chapter 4

4.1 Motivation
4.2 Linearity Property
4.3 Superposition
4.4 Source Transformation
4.5 Thevenin's Theorem
4.6 Norton's Theorem
4.7 Maximum Power Transfer

4.1 Motivation (1)

If you are given the following circuit, are there any other alternative(s) to determine the voltage across 2Ω resistor?



What are they? And how?

Can you work it out by inspection?

4.2 Linearity Property (1)

It is the property of an element describing a linear relationship between cause and effect.

A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Homogeneity (scaling) property

$$v = i\,R \quad \longrightarrow \quad k\,v = k\,i\,R$$

Additive property

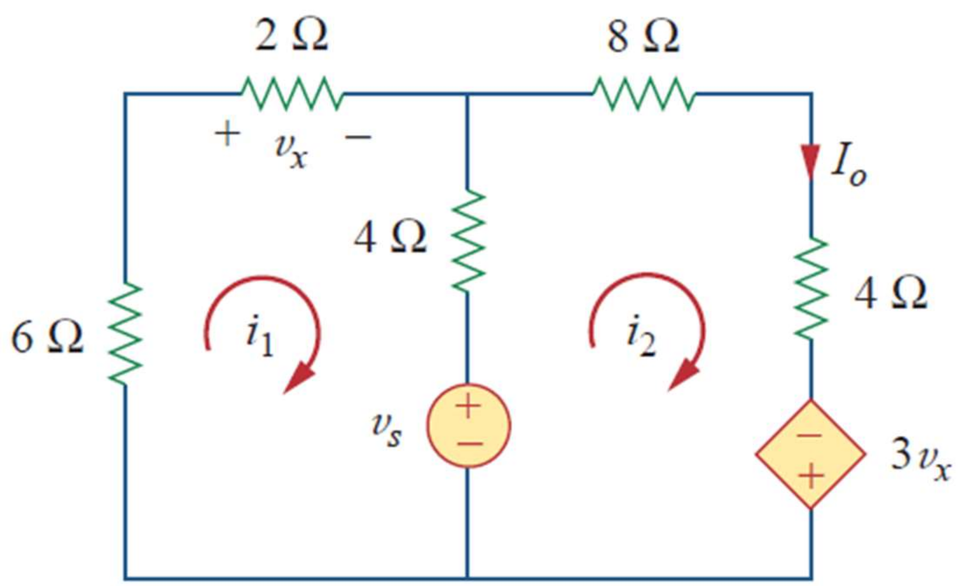
$$v_1 = i_1\,R \text{ and } v_2 = i_2\,R$$

$$\longrightarrow \; v = (i_1 + i_2)\,R = v_1 + v_2$$

4.2 Linearity Property (2)

Example 1

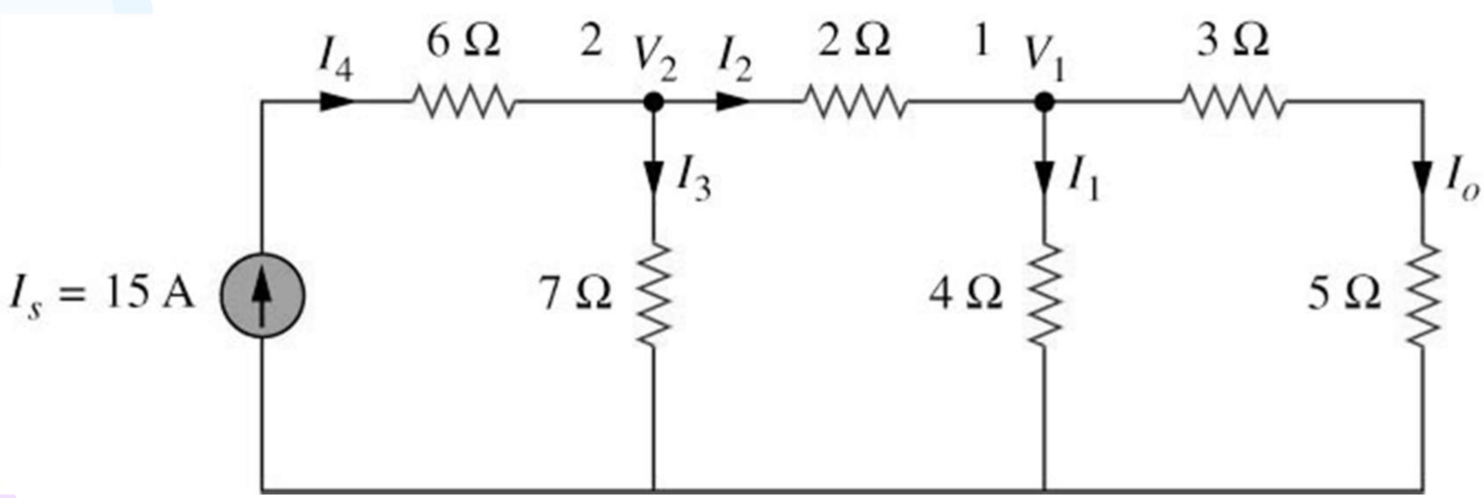
Find I_o when Vs = 12 V and Vs = 24 V



4.2 Linearity Property (3)

Example 2

By assume I_o = 1 A, use linearity to find the actual value of Io in the circuit shown below.



*Refer to in-class illustration, text book, answer $I_o = 3\text{A}$

4.2 Linearity Property (4)

Assume that $V_o = 1$ V and use linearity to calculate the actual value of V_o in the circuit of Fig. 4.5.

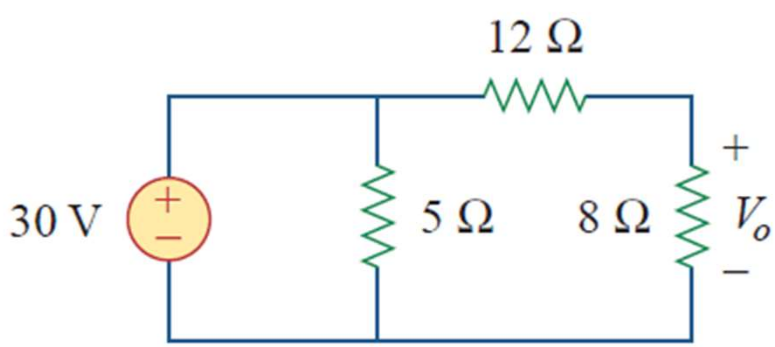


Figure 4.5

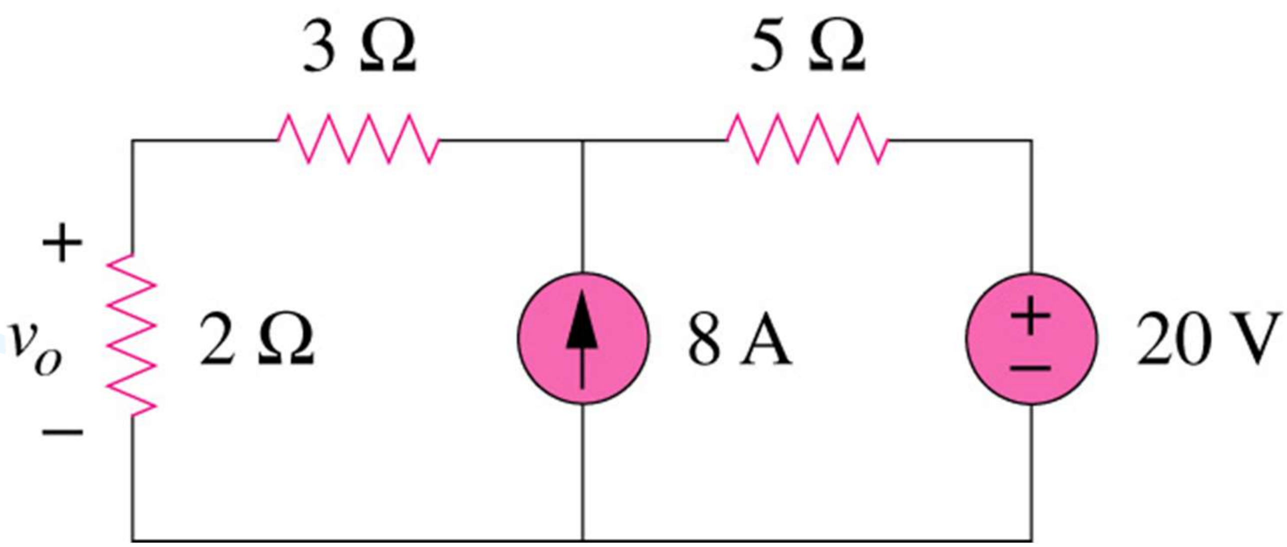
4.3 Superposition Theorem (1)

It states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltage across (or currents through) that element due to EACH independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

4.3 Superposition Theorem (2)

We consider the effects of 8A and 20V one by one, then add the two effects together for final v_o.



4.3 Superposition Theorem (3)

Steps to apply superposition principle

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

4.3 Superposition Theorem (4)

Two things have to be keep in mind:

1. When we say turn off all other independent sources:
   - Independent voltage sources are replaced by 0 V (short circuit) and
   - Independent current sources are replaced by 0 A (open circuit).

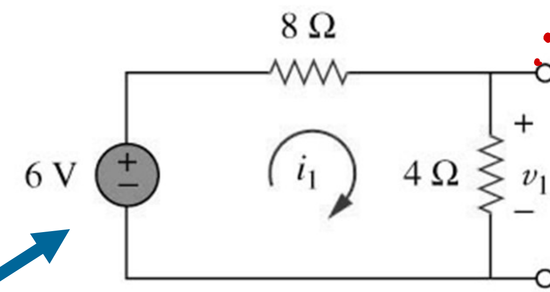
2. Dependent sources are left intact because they are controlled by circuit variables.

4.3 Superposition Theorem (5)

$6 = 12I \therefore I = \frac{1}{2}$

Example 3

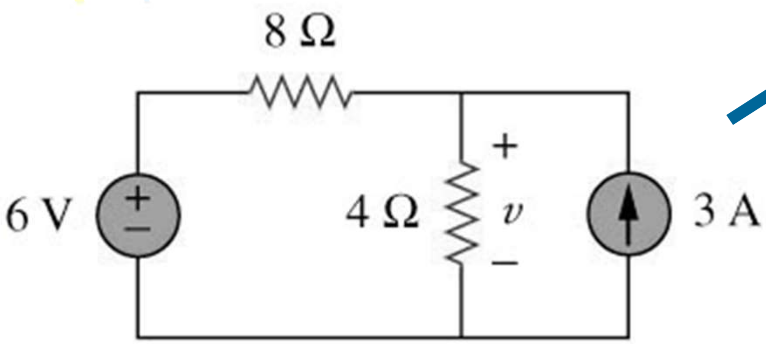
Use the superposition theorem to find v in the circuit shown below.

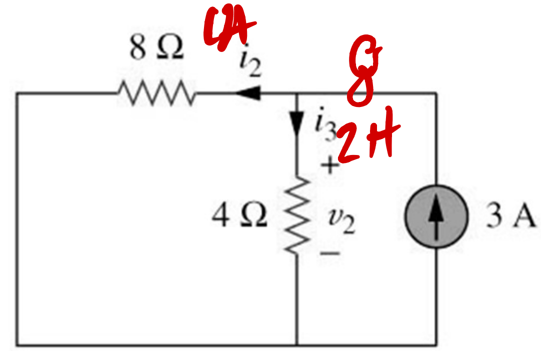


$\therefore v_1 = 2V$

(a)

3A is discarded by open-circuit





4A

8

2A

(b)

6V is discarded by short-circuit

*Refer to in-class illustration, text book, answer *v* = 10V

4.3 Superposition Theorem (6)

Using the superposition theorem, find v_o in the circuit of Fig. 4.8.

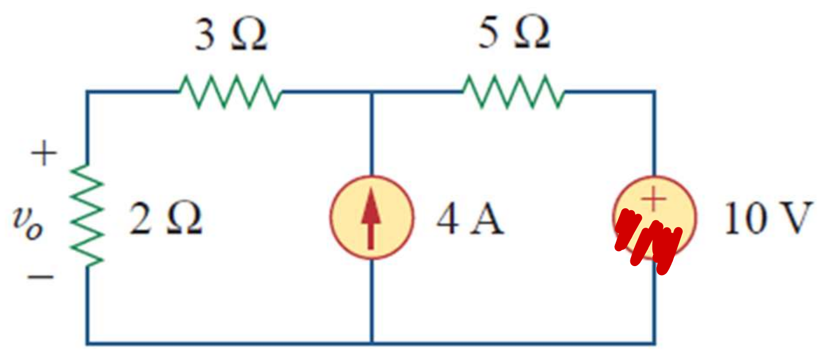


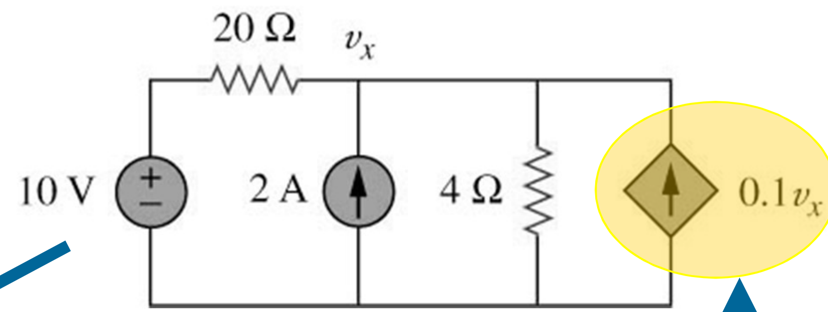
Figure 4.8

5A

4.3 Superposition Theorem (7)

Example 4

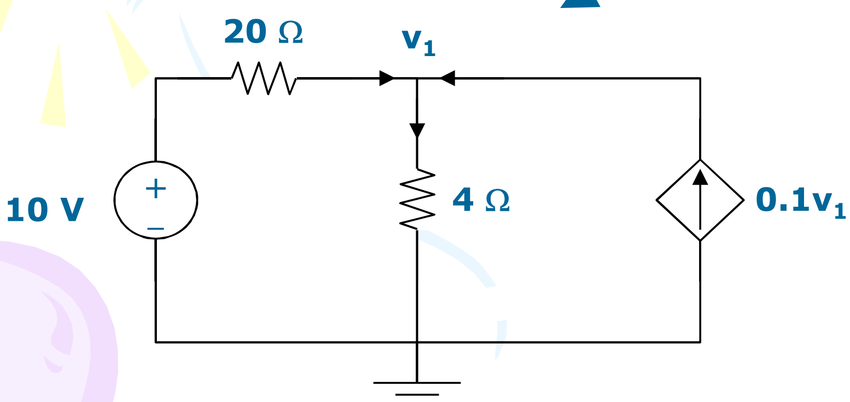
Use superposition to find v_x in the circuit below.



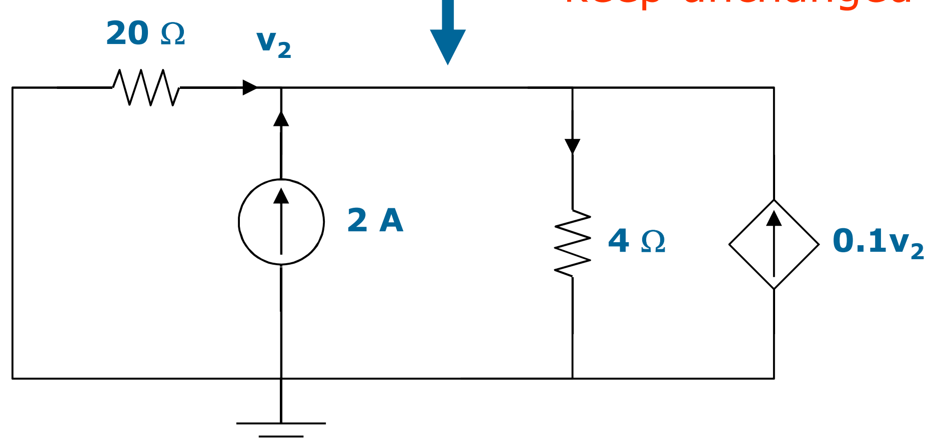
2A is discarded by open-circuit

10V is discarded by open-circuit

Dependant source keep unchanged



(a)

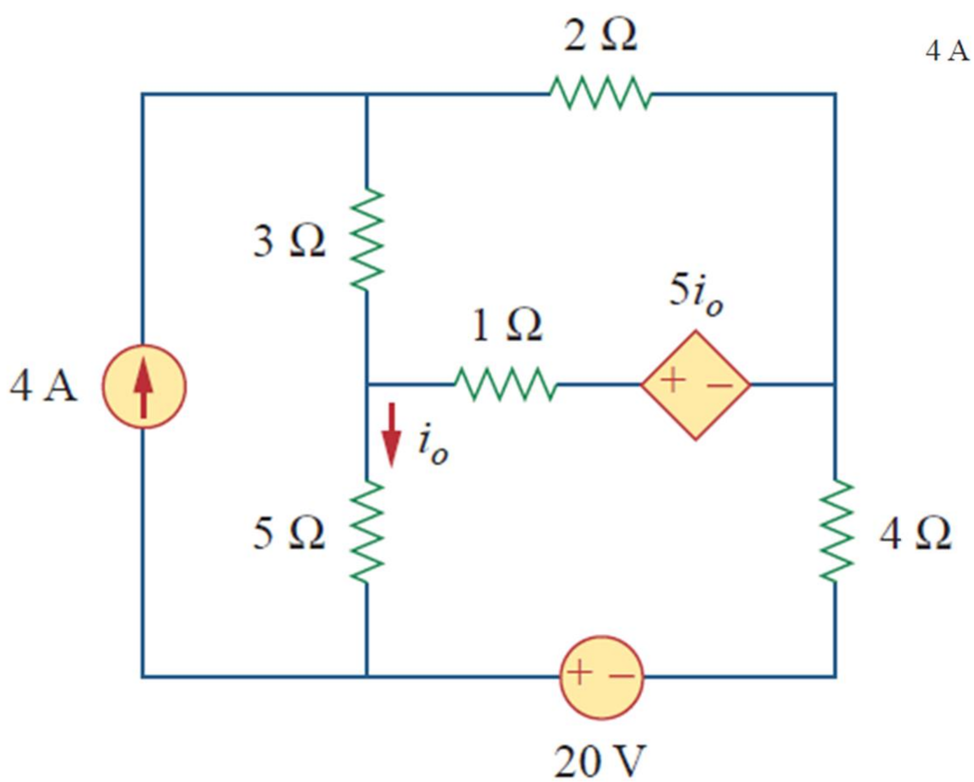


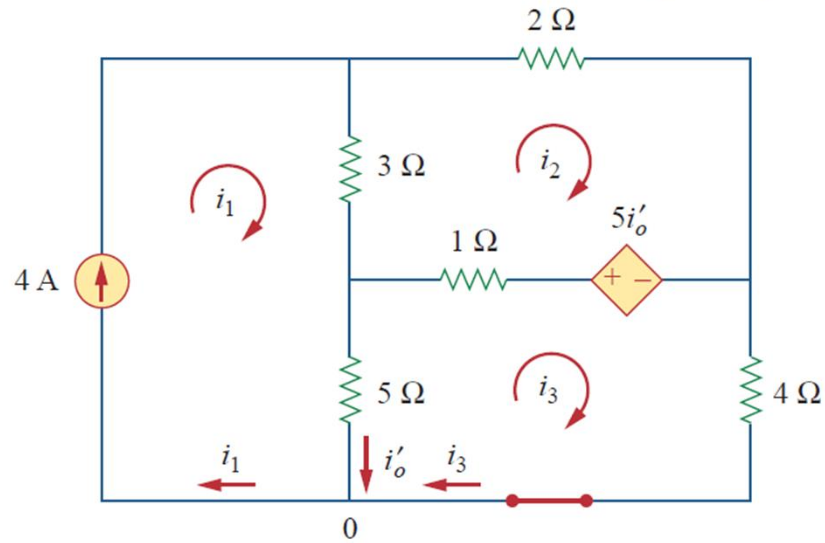
(b)

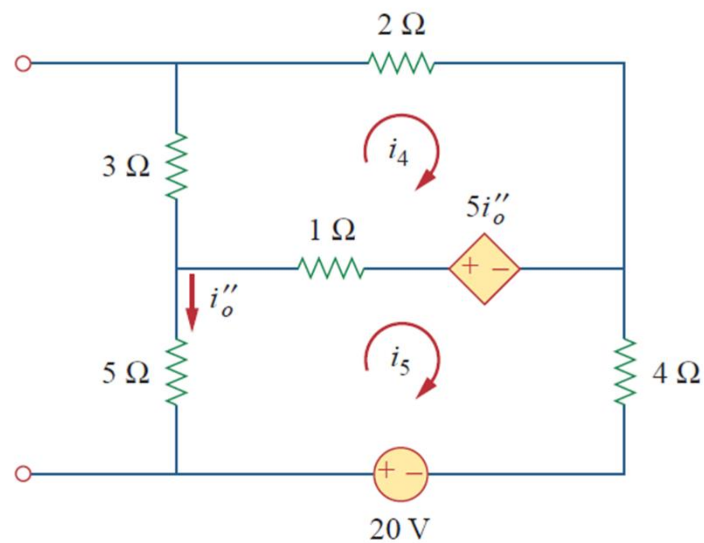
*Refer to in-class illustration, text book, answer $V_x = 12.5V$

4.3 Superposition Theorem (8)

Example 5 Find Io

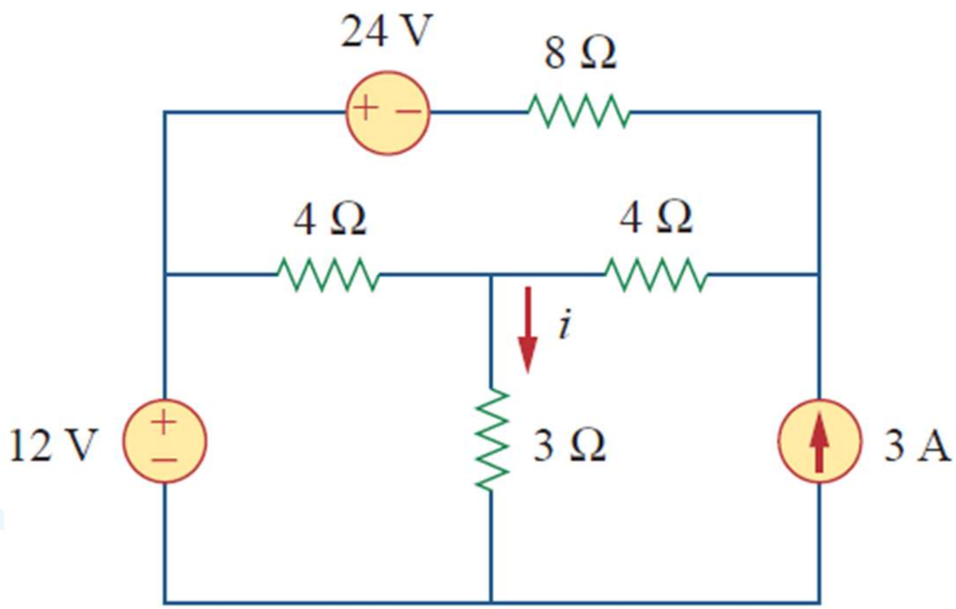






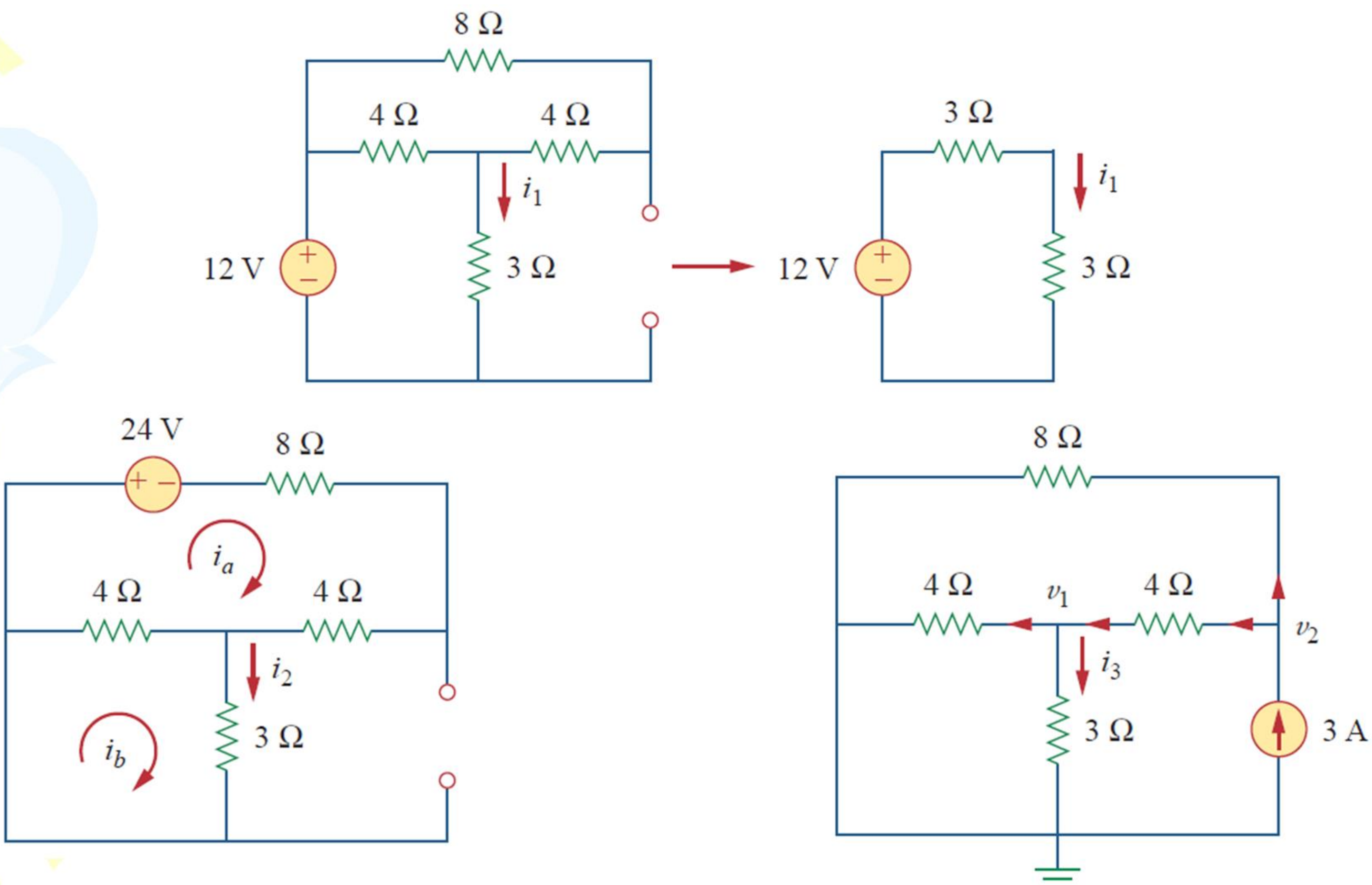
4.3 Superposition Theorem (9-1)

Example 6 Find I by superposition



4.3 Superposition Theorem (9-2)

Example 6   Find i   by superposition

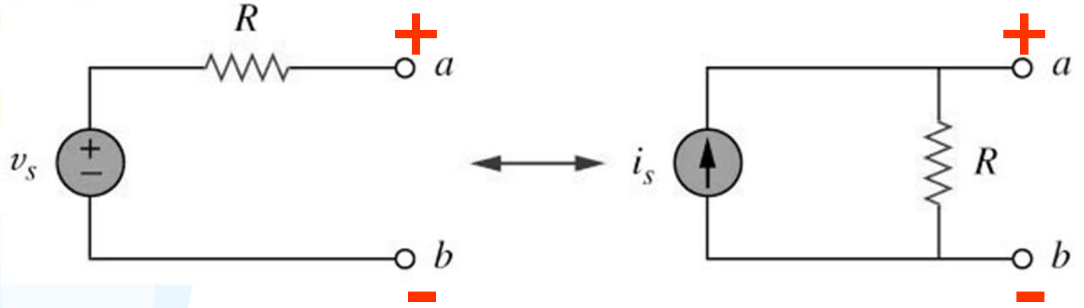


4.4 Source Transformation (1)

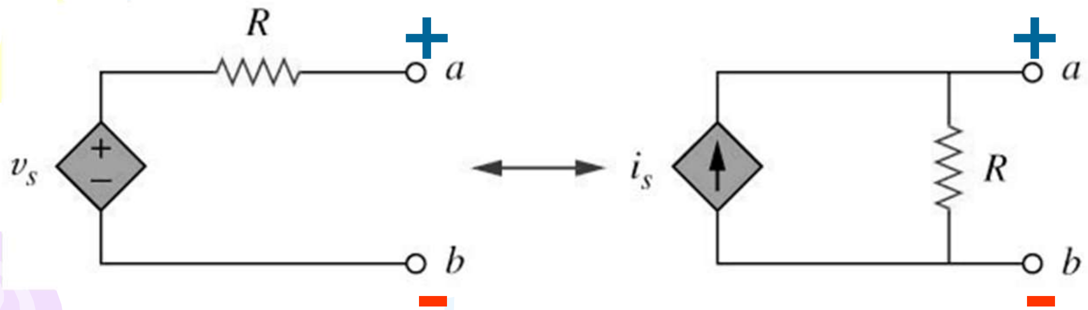
- An equivalent circuit is one whose *v-i* characteristics are identical with the original circuit.

- It is the process of replacing a voltage source v_S in series with a resistor R by a current source i_S in parallel with a resistor R, or vice versa.

4.4 Source Transformation (2)



(a) Independent source transform



(b) Dependent source transform

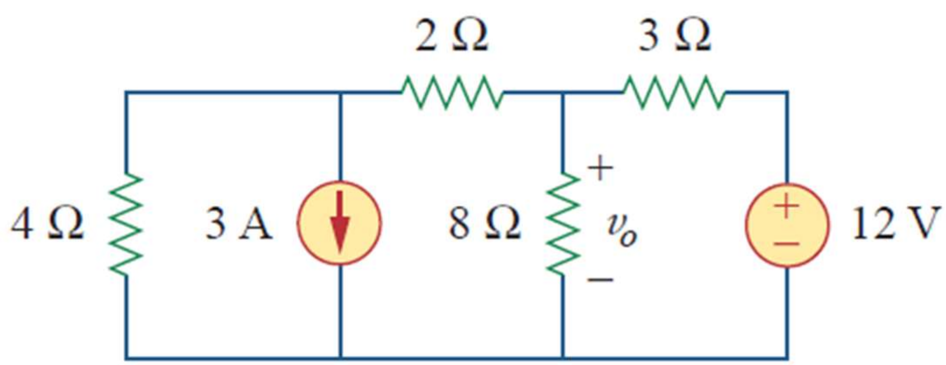
- The arrow of the current source is directed toward the positive terminal of the voltage source.

- The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.

4.4 Source Transformation (3)

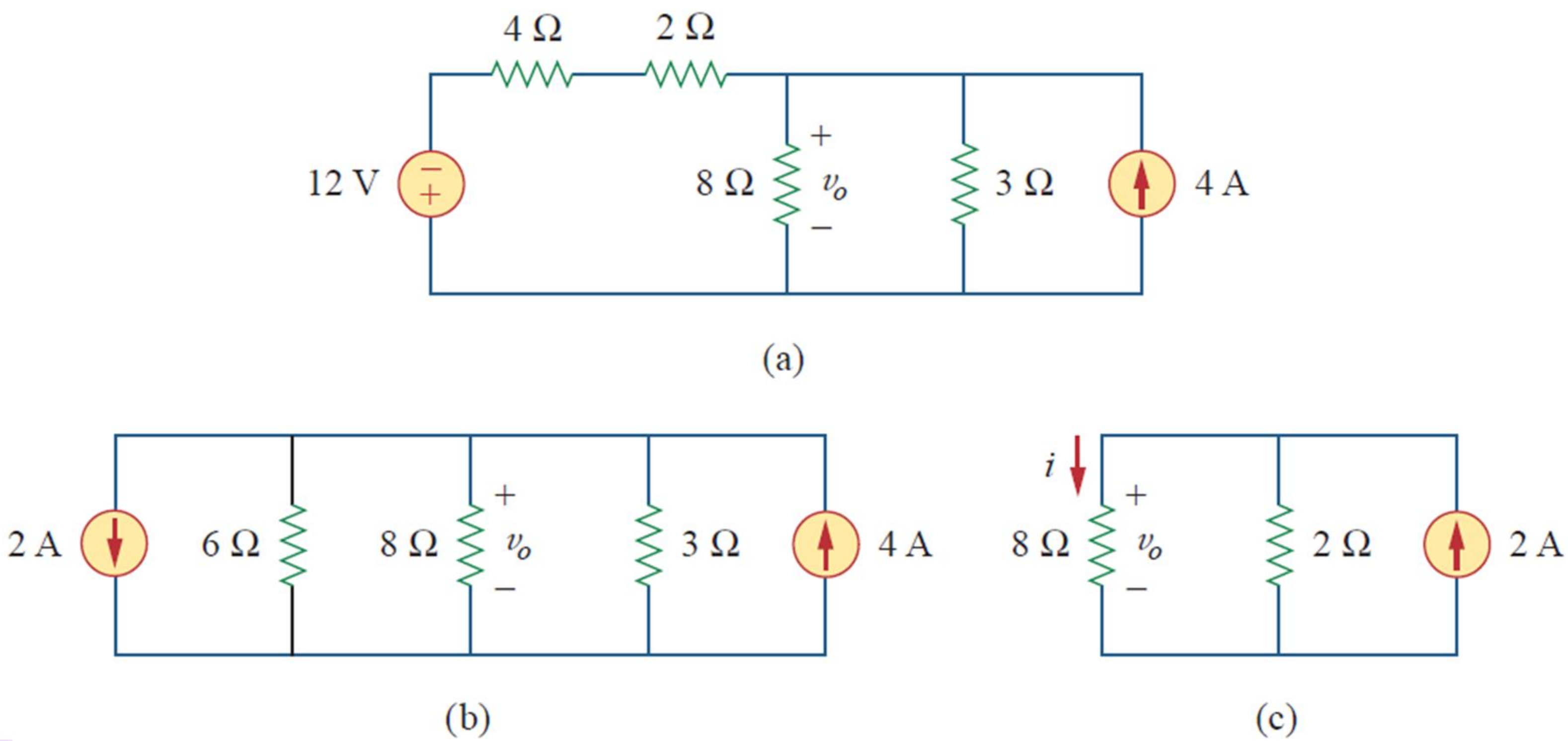
Example 7

Find v_o in the circuit shown below using source transformation.



4.4 Source Transformation (4)

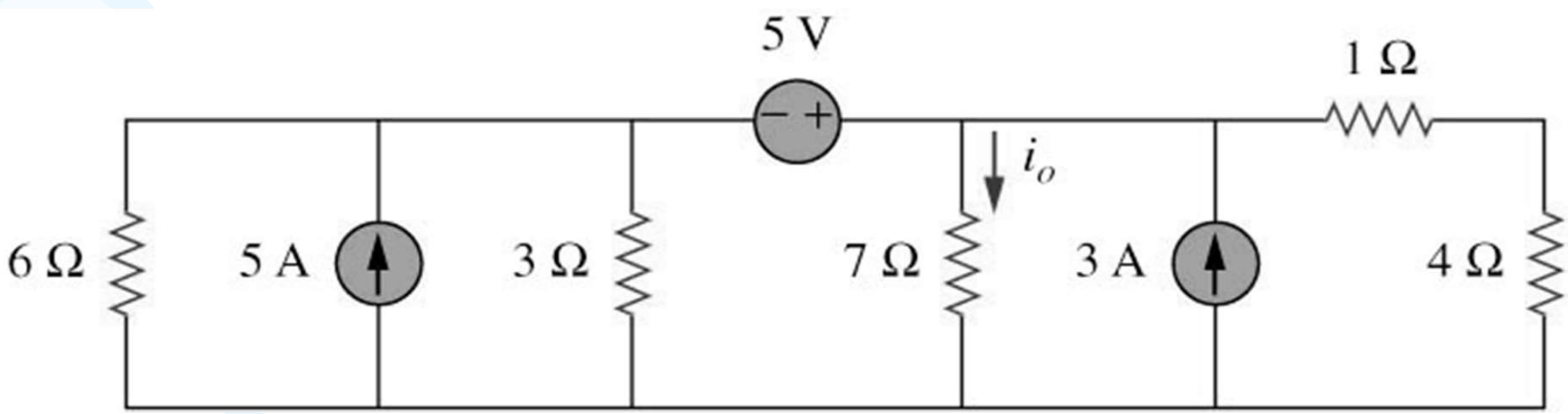
Example 7



4.4 Source Transformation (5)

Example 8

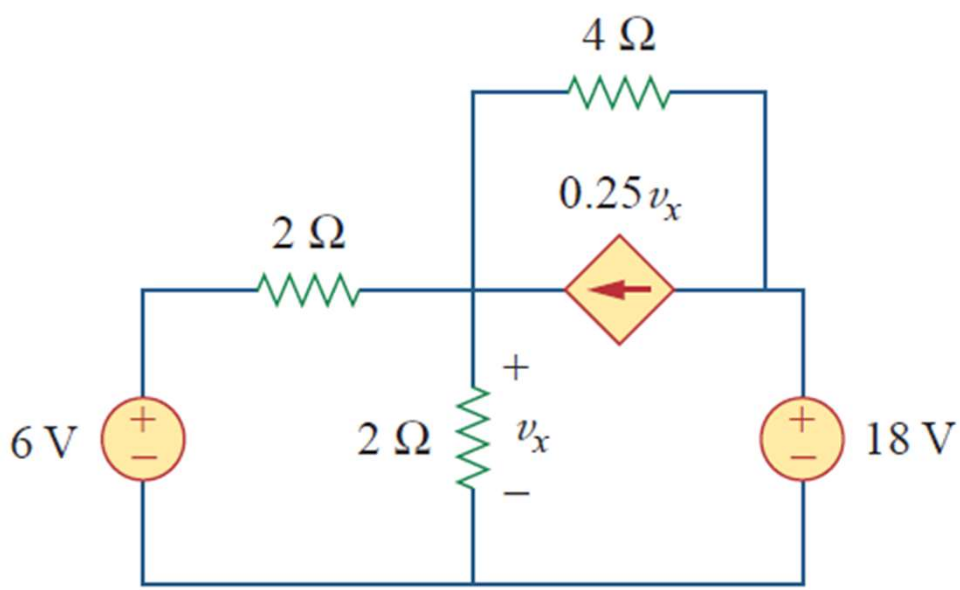
Find i_o in the circuit shown below using source transformation.



*Refer to in-class illustration, textbook, answer $i_o = 1.78A$

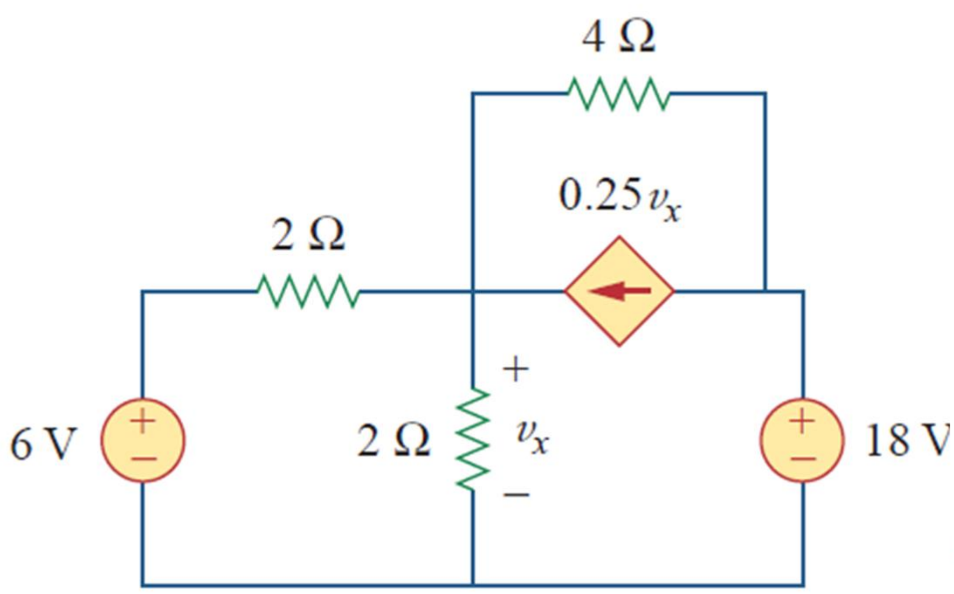
4.4 Source Transformation (6)

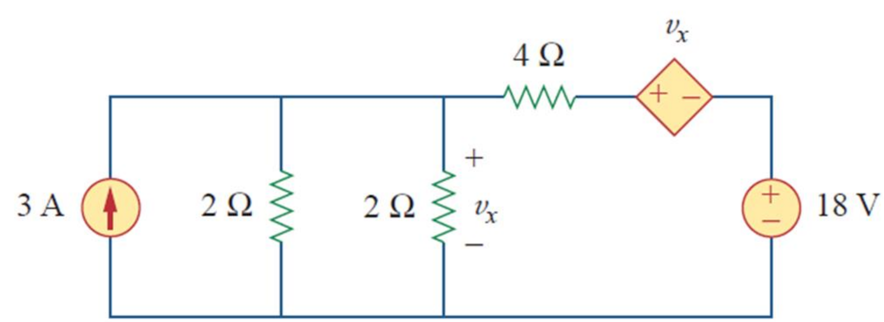
Example 9

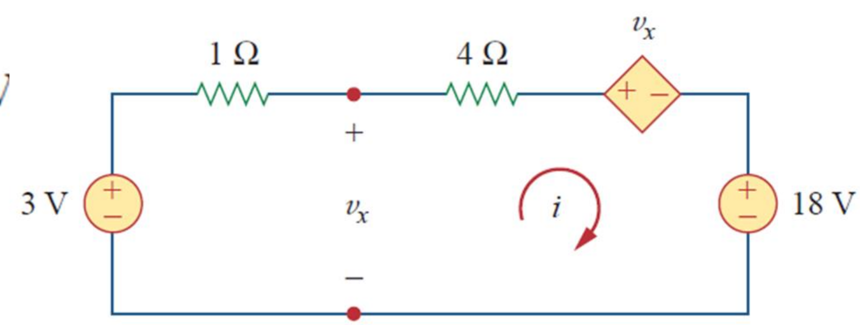


4.4 Source Transformation (7)

Example 9 Find Vx using source transformation.







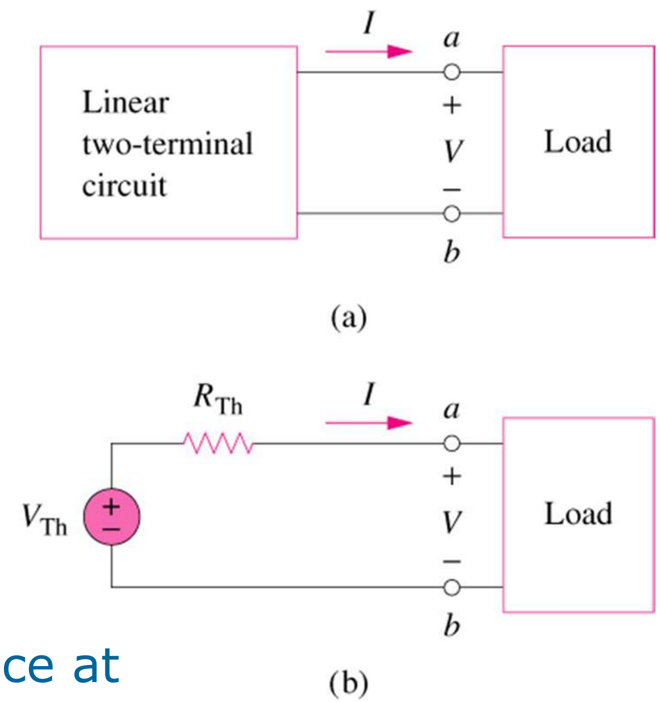
4.5 Thevenin's Theorem (1)

It states that a linear two-terminal circuit (Fig. a) can be replaced by an equivalent circuit (Fig. b) consisting of a voltage source V_{TH} in series with a resistor R_{TH},

where

- *VTH* is the open-circuit voltage at the terminals.

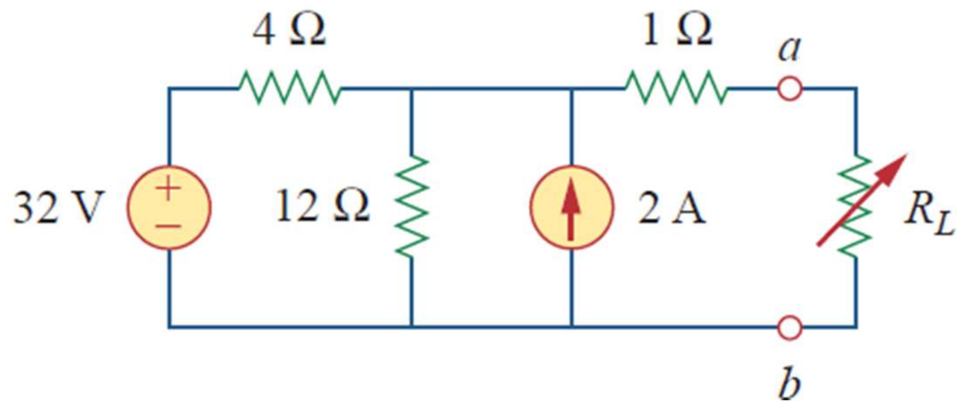
- *RTH* is the input or equivalent resistance at the terminals when the independent sources are turned off.

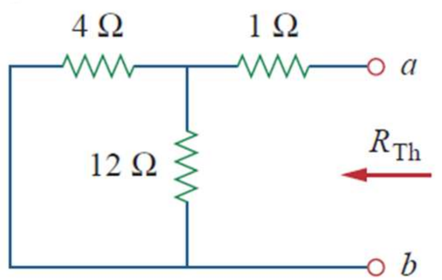


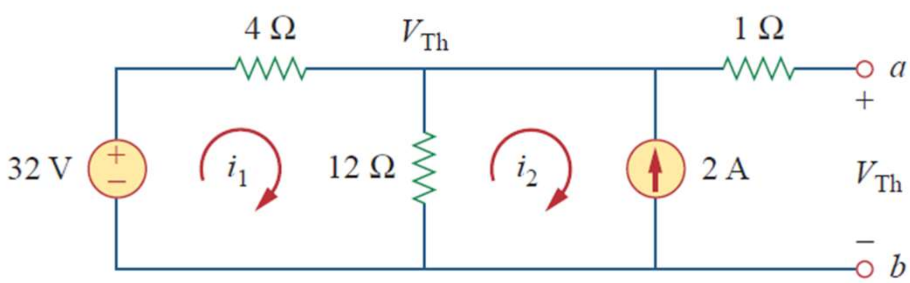
4.5 Thevenin's Theorem (2)

Example 10

Using Thevenin's theorem, find the current through R_L=6,16 and 36 Ohms







4.5 Thevenin's Theorem (2)

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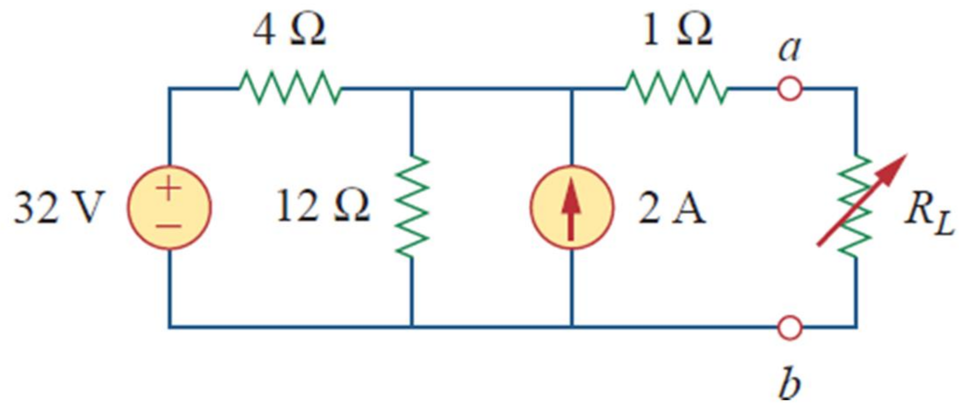
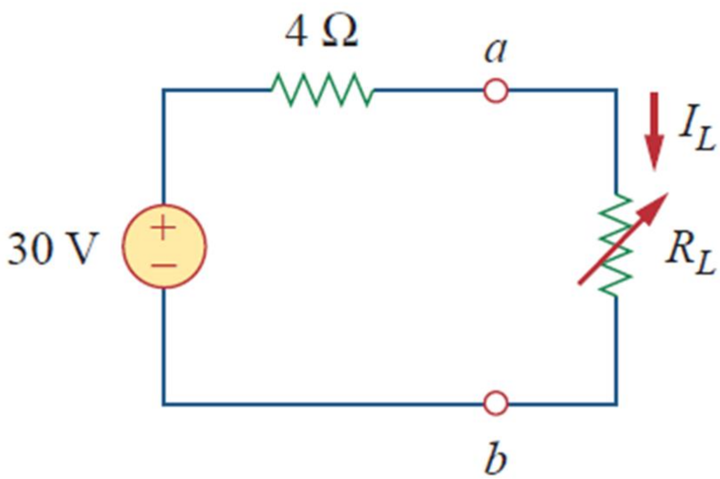
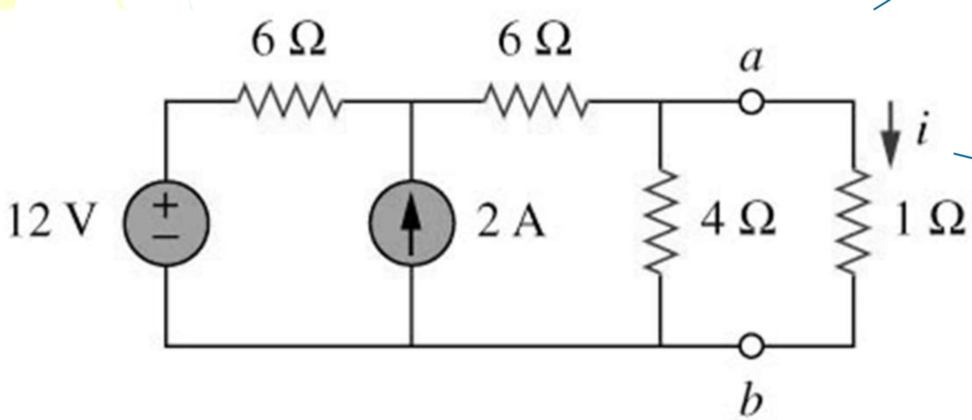


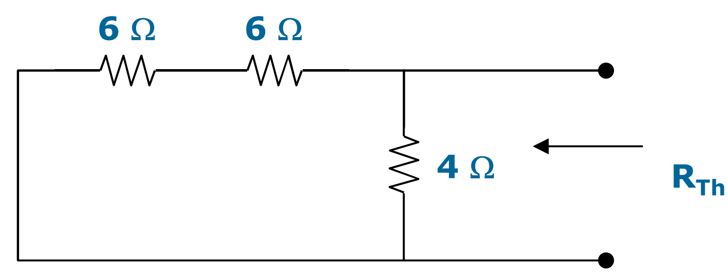


4.5 Thevenin's Theorem (3)

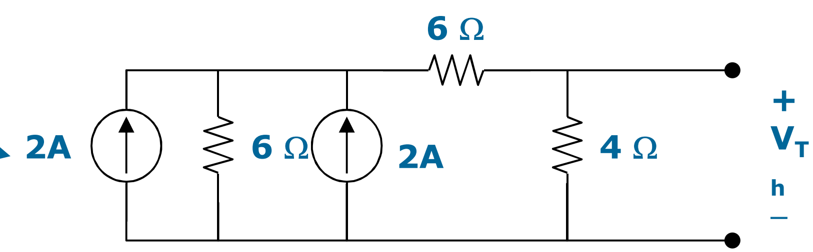
Example 11

Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit shown below. Hence find i.





(a)

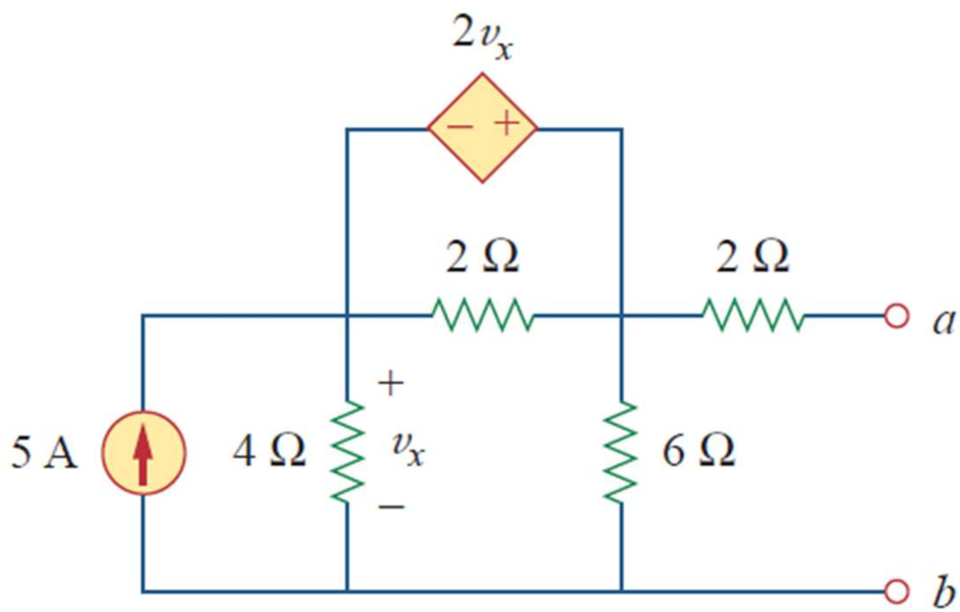


(b)

*Refer to in-class illustration, textbook, answer V_{TH} = 6V, R_{TH} = 3Ω, i = 1.5A

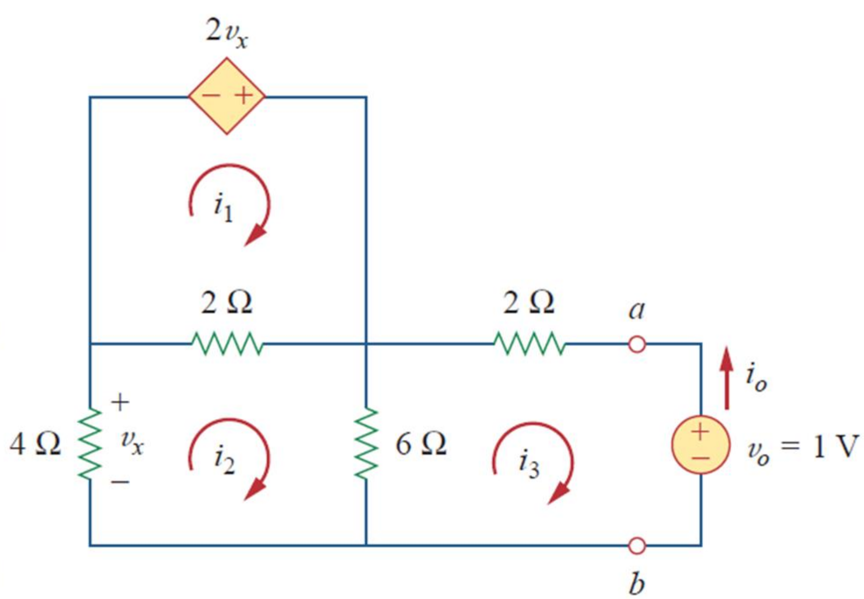
4.5 Thevenin's Theorem (4)

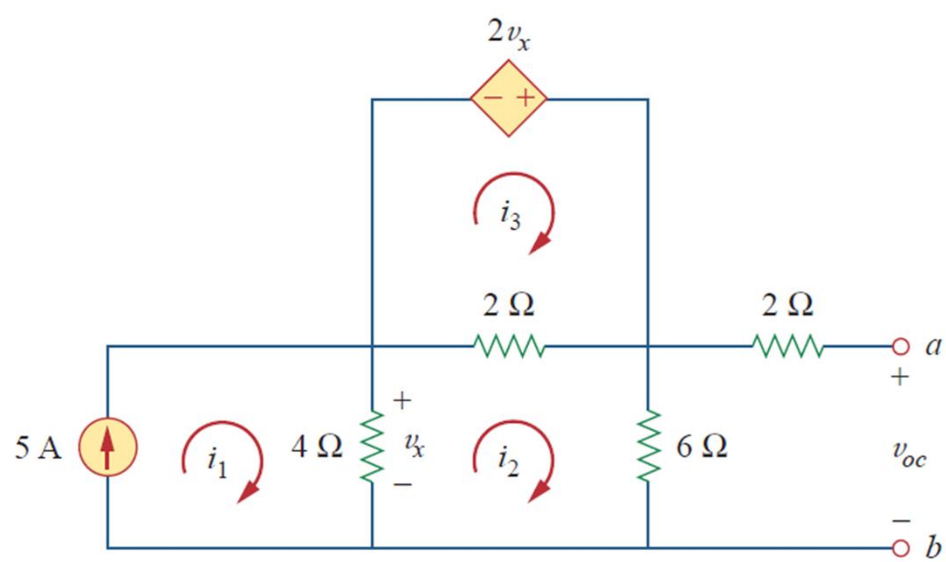
Find the Thevenin equivalent of the circuit in Fig. 4.31 at terminals *a*-*b*.



4.5 Thevenin’s Theorem (4)

Find the Thevenin equivalent of the circuit in Fig. 4.31 at terminals *a*-*b*.



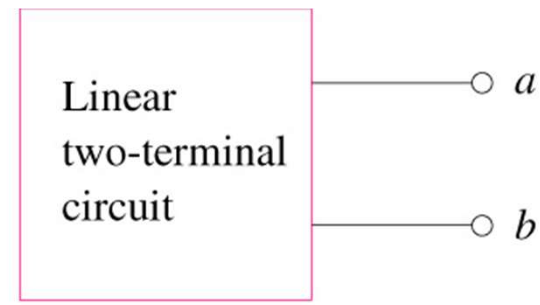


4.6 Norton's Theorem (1)

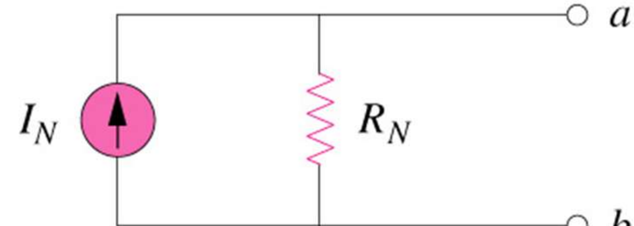
It states that a linear two-terminal circuit can be replaced by an equivalent circuit of a current source I_N in parallel with a resistor R_N,

Where

- I_N is the short circuit current through the terminals.
- R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



(a)



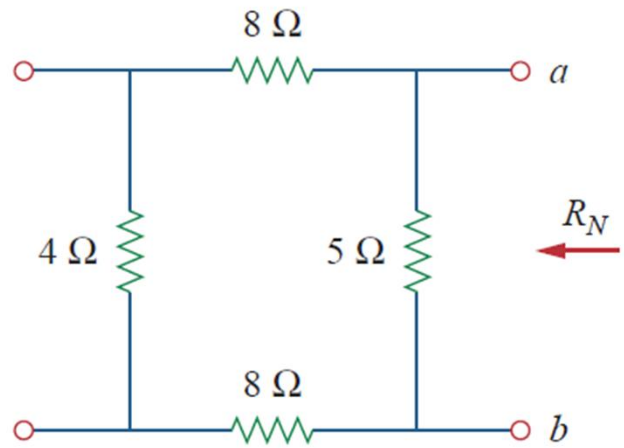
(b)

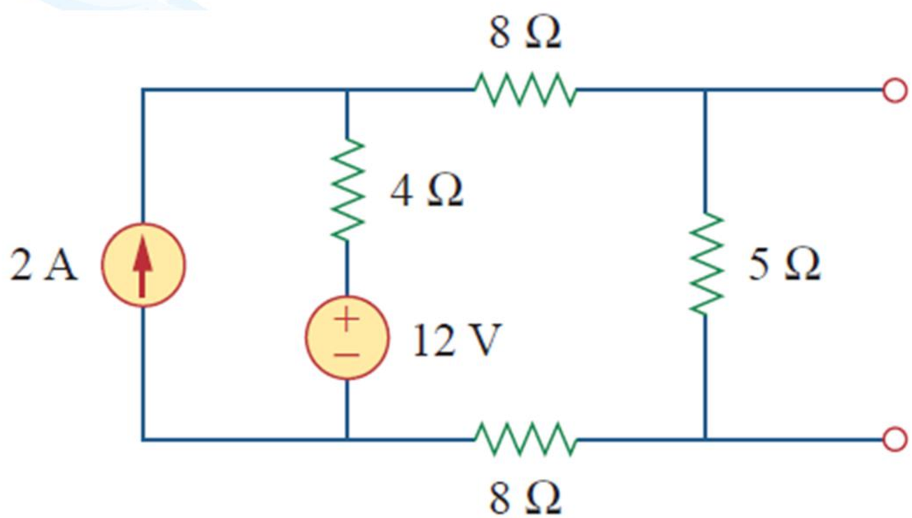
The Thevenin's and Norton equivalent circuits are related by a source transformation.

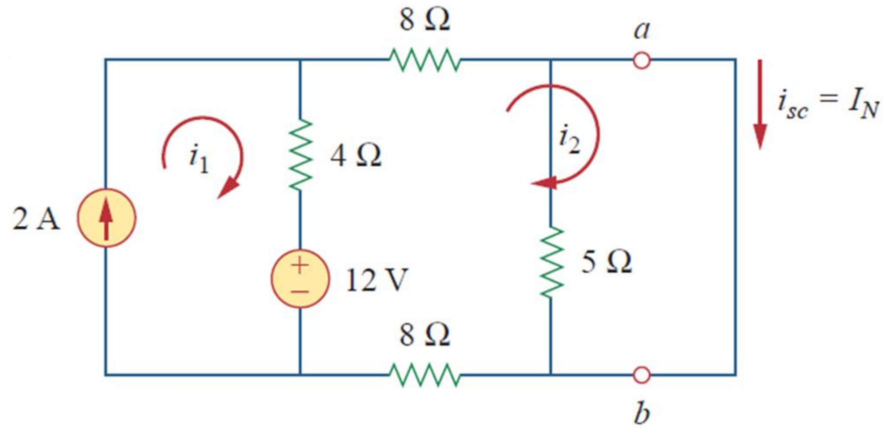
4.6 Norton's Theorem (2)

Example 13

Find the Norton equivalent circuit of the circuit shown below.



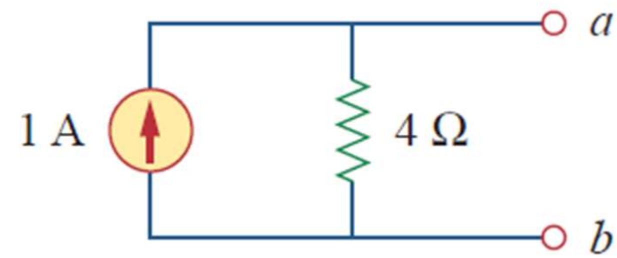


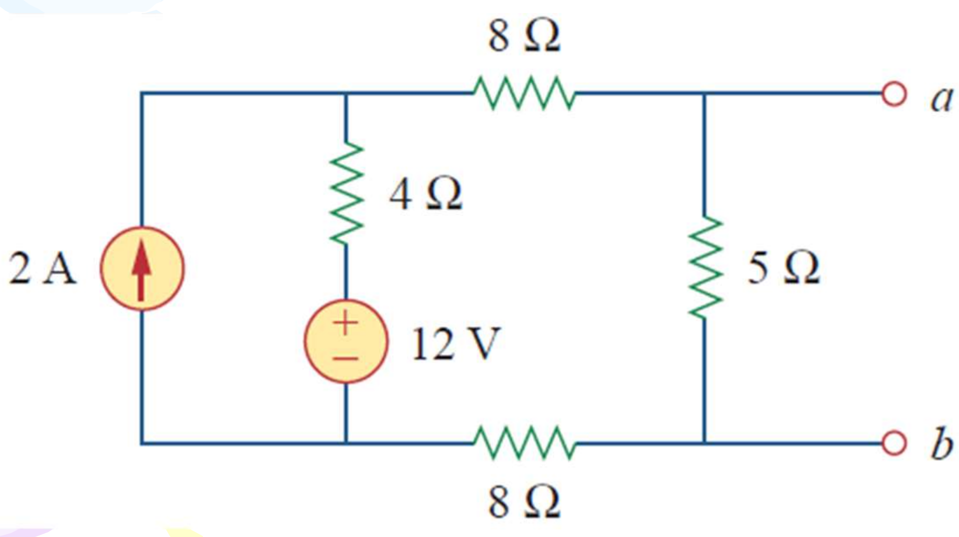


4.6 Norton's Theorem (2)

Example 13

Find the Norton equivalent circuit of the circuit shown below.

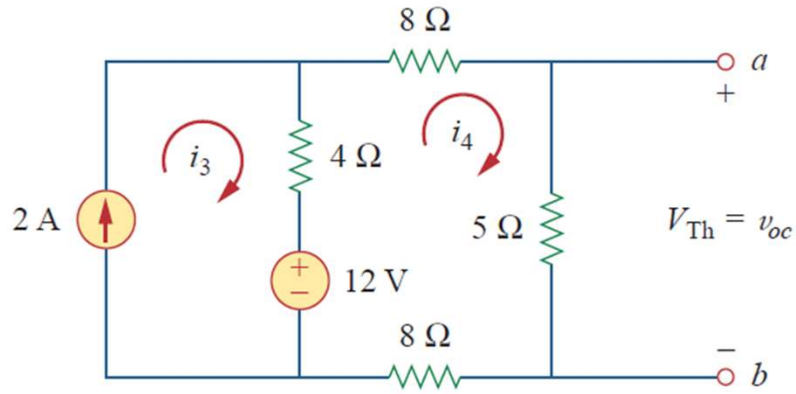


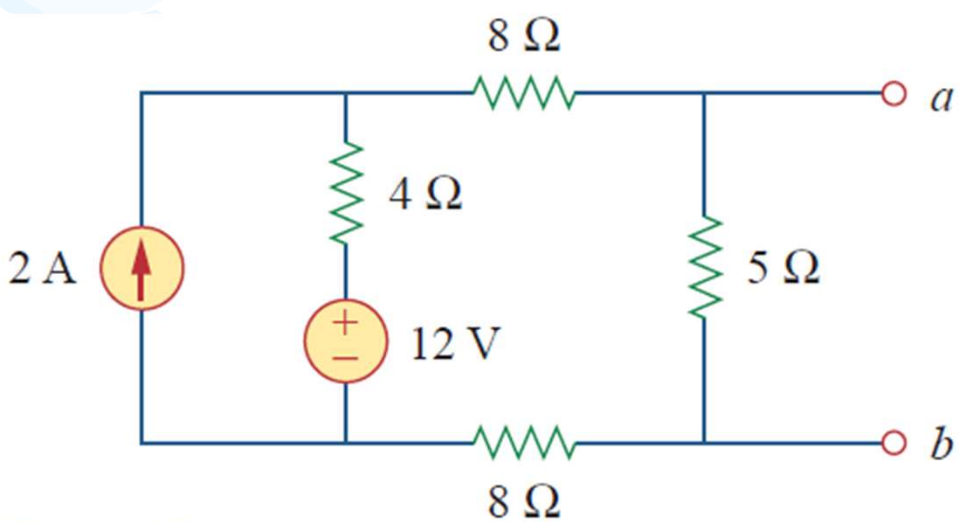


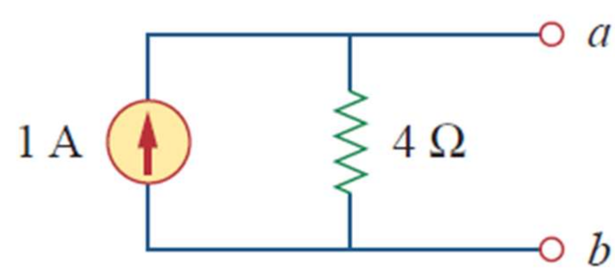
4.6 Norton's Theorem (2)

Example 13

Find the Norton equivalent circuit of the circuit shown below.

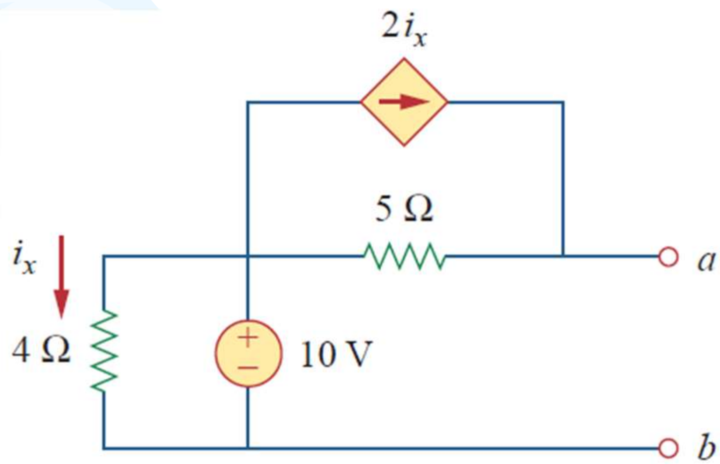


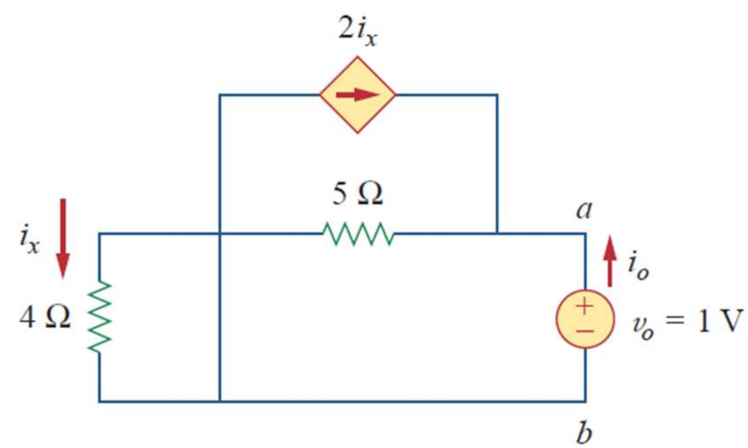


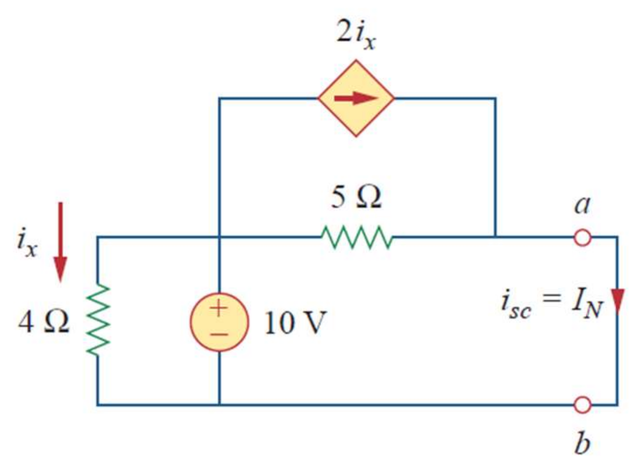


4.6 Norton's Theorem (3)

Example 14 Find I_N and R_N



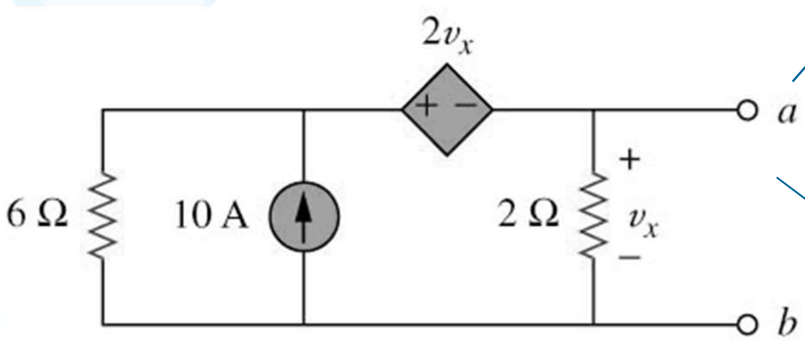


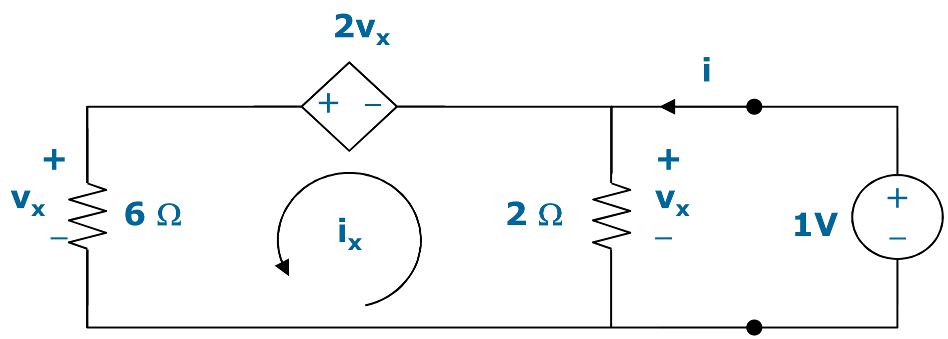


4.6 Norton's Theorem (4)

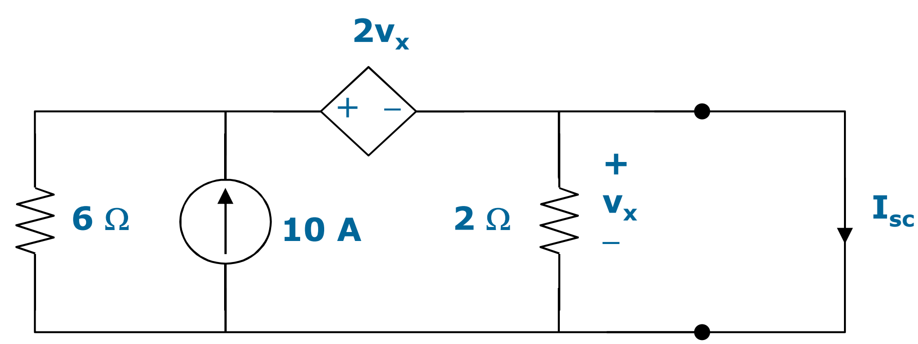
Example 15

Find the Norton equivalent circuit of the circuit shown below.





(a)



(b)

*Refer to in-class illustration, textbook, $R_N = 1\Omega$, $I_N = 10\text{A}$.

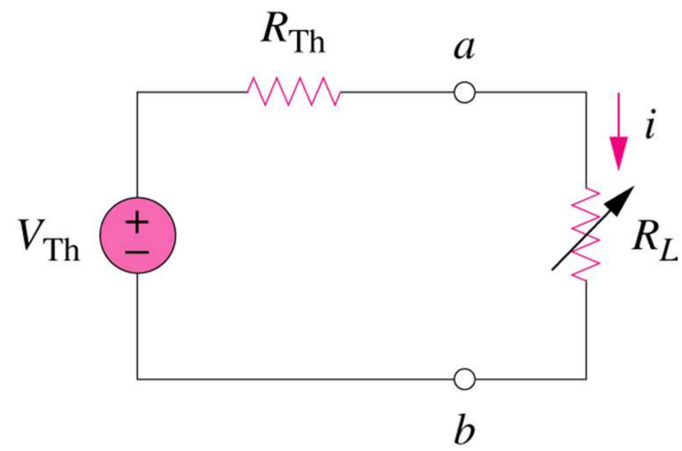
4.7 Maximum Power Transfer (1)

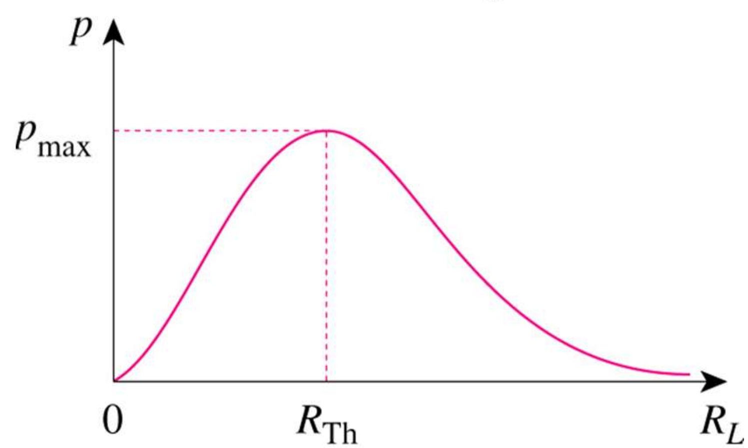
If the entire circuit is replaced by its Thevenin equivalent except for the load, the power delivered to the load is:

$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

For maximum power dissipated in R_L, P_{max}, for a given R_{TH}, and V_{TH},

$$R_L = R_{TH} \quad \Rightarrow \quad P_{max} = \frac{V_{Th}^2}{4R_L}$$



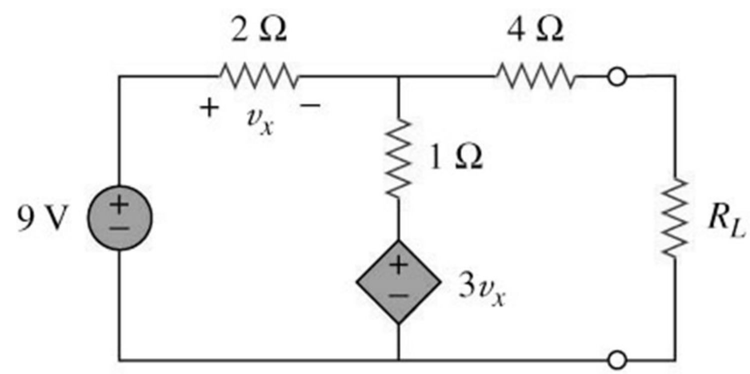


The power transfer profile with different R_L

4.7 Maximum Power Transfer (2)

Example 8

Determine the value of R_L that will draw the maximum power from the rest of the circuit shown below. Calculate the maximum power.



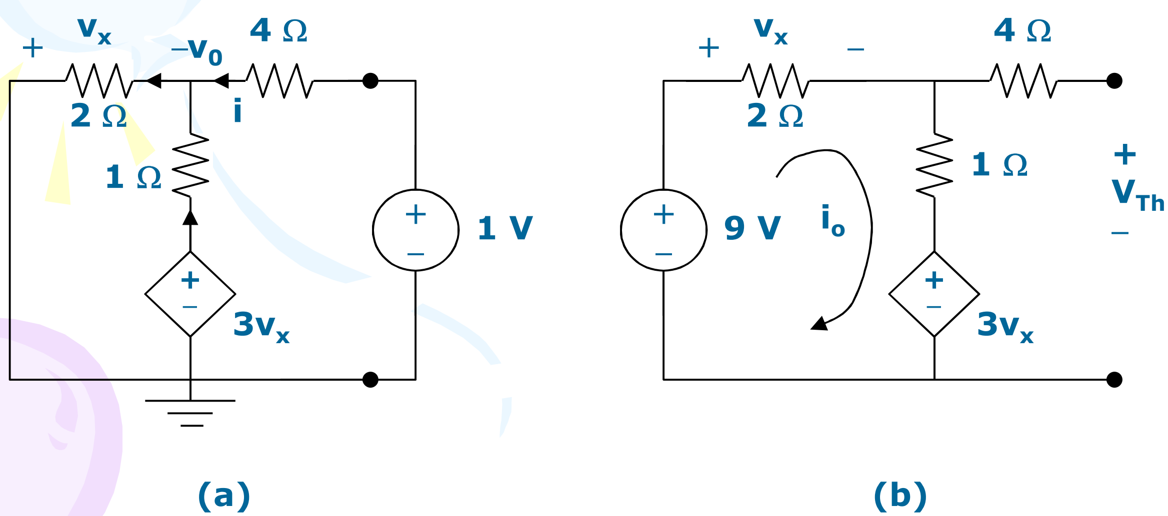


Fig. a

=> To determine R_{TH}

Fig. b

=> To determine V_{TH}

*Refer to in-class illustration, textbook, R_L = 4.22Ω, P_m = 2.901W

Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 6
Capacitors and Inductors

Capacitors and Inductors
Chapter 6

6.1 Capacitors

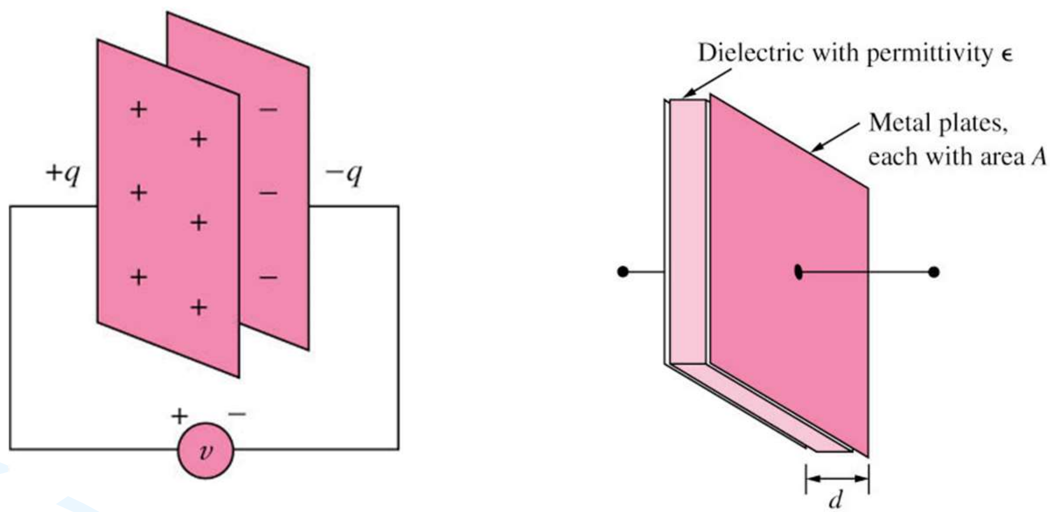
6.2 Series and Parallel Capacitors

6.3 Inductors

6.4 Series and Parallel Inductors

6.1 Capacitors (1)

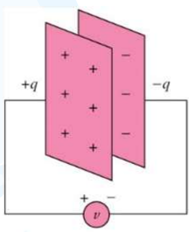
- A capacitor is a passive element designed to **store energy** in its **electric field.**



- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.1 Capacitors (2)

- **Capacitance** *C* is the ratio of the charge *q* on one plate of a capacitor to the voltage difference *v* between the two plates, measured in farads (F).

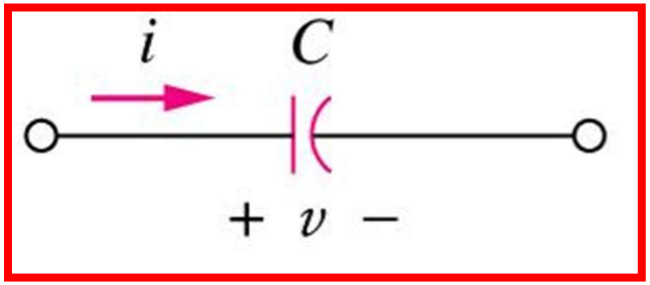


$$q = C\,v \quad \text{and} \quad C = \frac{\varepsilon\, A}{d}$$

- Where ε is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and **μF (10^{-6})**

6.1 Capacitors (3)

- If ***i*** is flowing into the +ve terminal of C
  - Charging => ***i*** is +ve
  - Discharging => ***i*** is –ve



- The current-voltage relationship of capacitor according to above convention is

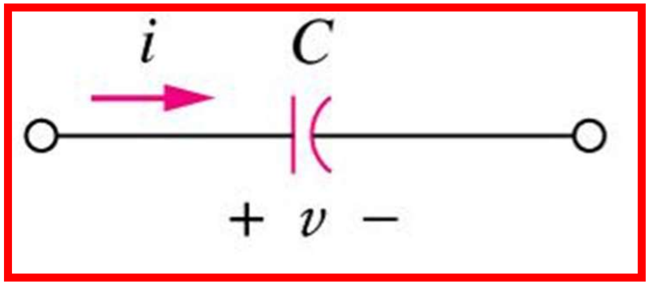
$$\boxed{i = C\frac{dv}{dt}}$$

and

$$\boxed{v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$

6.1 Capacitors (4)

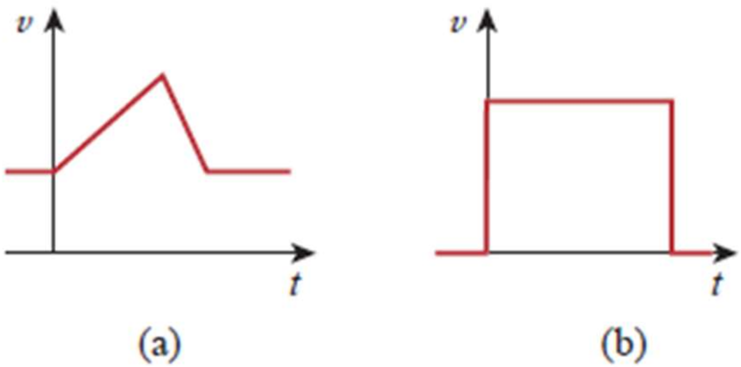
- The energy, ***w***, stored in the capacitor is

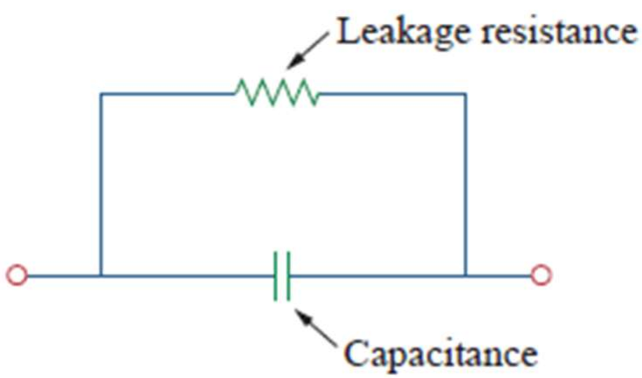


$$\boxed{w = \frac{1}{2} C v^2}$$

- A capacitor is
  - an **open circuit** to dc (dv/dt = 0).
  - its voltage **cannot change abruptly**.

6.1 Capacitors (5)





6.1 Capacitors (6)

(a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

(a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

(b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

6.1 Capacitors (7)

The voltage across a 5-μF capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:

By definition, the current is

$$i(t) = C\frac{dv}{dt} = 5 \times 10^{-6}\frac{d}{dt}(10 \cos 6000t)$$
$$= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A}$$

6.1 Capacitors (8)

Determine the voltage across a 2-μF capacitor if the current through it is

$$i(t) = 6e^{-3000t} \text{ mA}$$

Assume that the initial capacitor voltage is zero.

Solution:

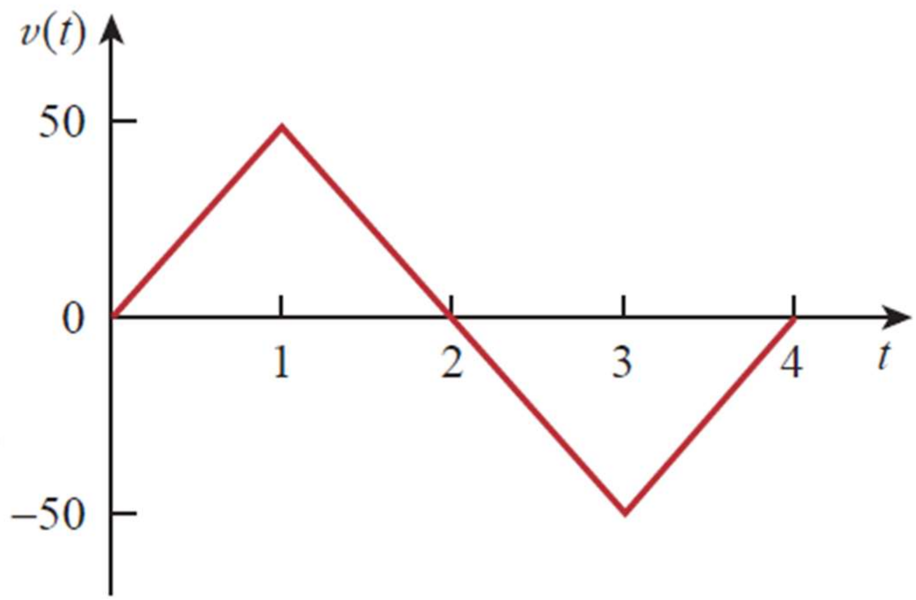
Since $v = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + v(0)$ and $v(0) = 0$,

$$v = \frac{1}{2 \times 10^{-6}} \int_0^t 6e^{-3000t}\,dt \cdot 10^{-3}$$

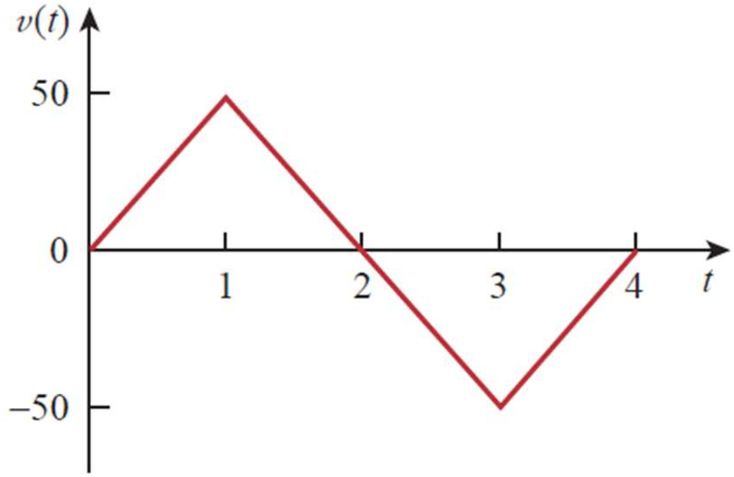
$$= \frac{3 \times 10^3}{-3000} e^{-3000t} \Big|_0^t = (1 - e^{-3000t}) \text{ V}$$

6.1 Capacitors (9)

Determine the current through a 200-μF capacitor whose voltage is shown in Fig. 6.9.



6.1 Capacitors (10)



The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50t \text{ V} & 0 < t < 1 \\ 100 - 50t \text{ V} & 1 < t < 3 \\ -200 + 50t \text{ V} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

6.1 Capacitors (11)

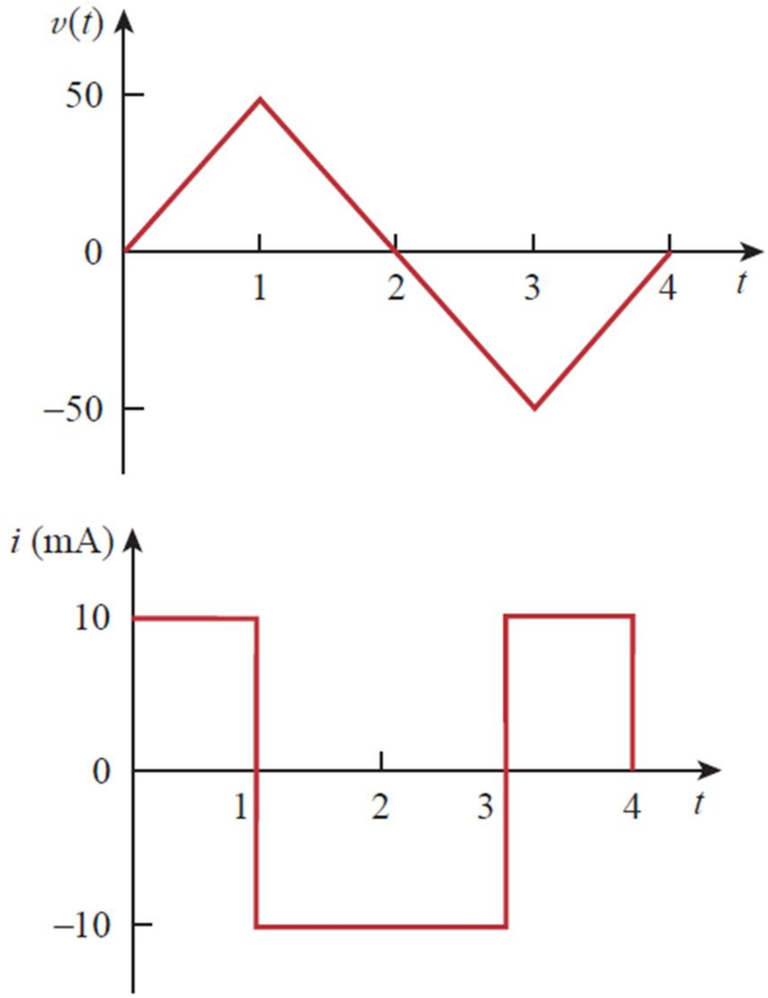
Since $i = C\,dv/dt$ and $C = 200\ \mu\text{F}$, we take the derivative of v to obtain

$$i(t) = 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 10\ \text{mA} & 0 < t < 1 \\ -10\ \text{mA} & 1 < t < 3 \\ 10\ \text{mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus the current waveform is as shown in Fig. 6.10.

6.1 Capacitors (12)

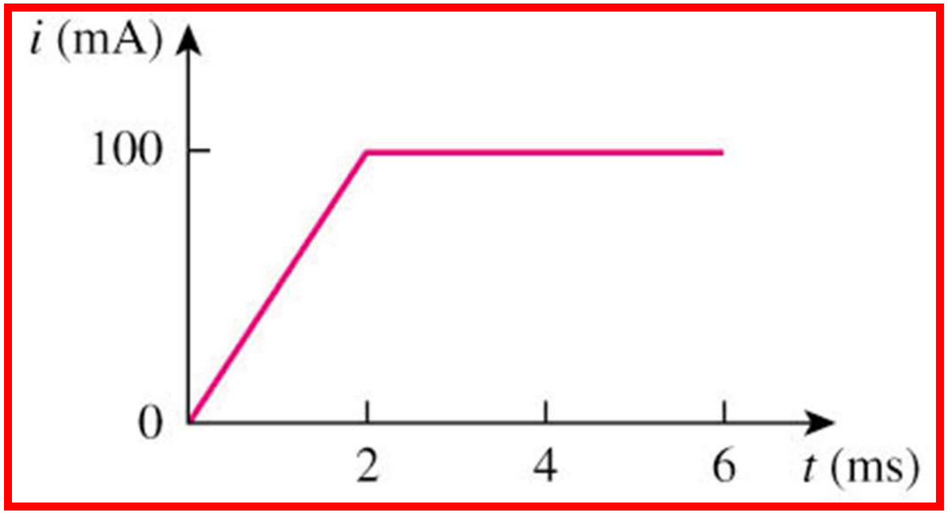


6.1 Capacitors (13)

Example

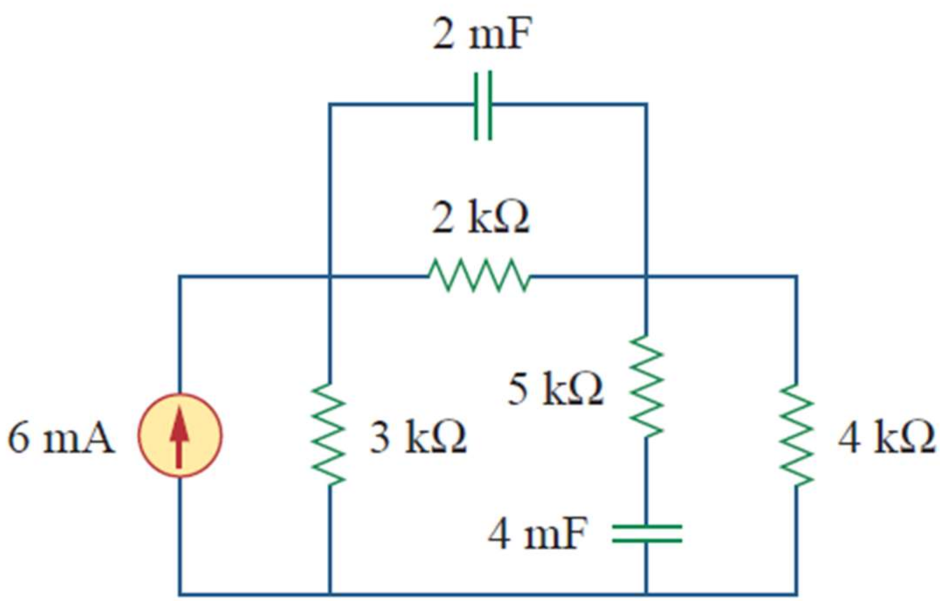
An initially uncharged 1-mF capacitor has the current shown below across it.

Calculate the voltage across it at t = 2 ms and t = 5 ms.

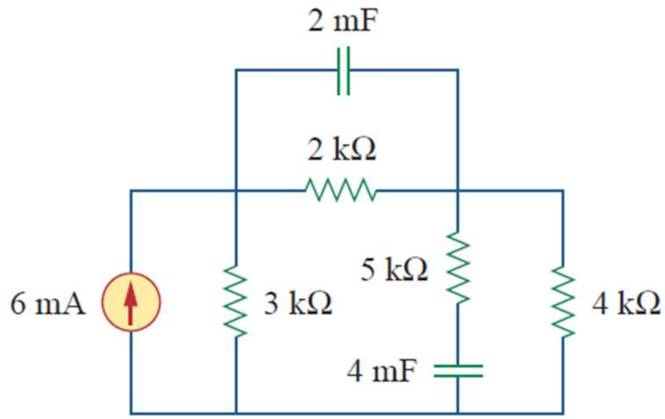


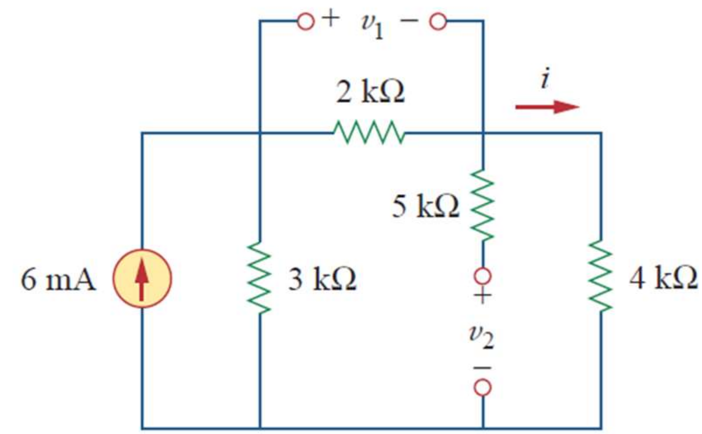
6.1 Capacitors (14)

Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.



6.1 Capacitors (15)

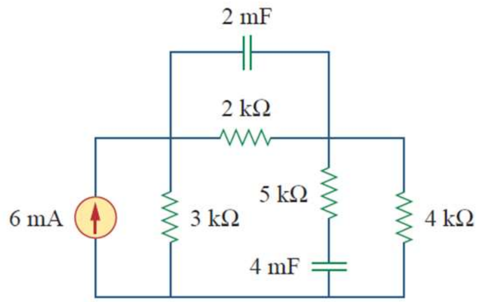


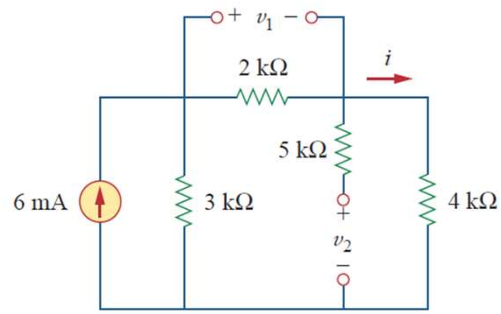


Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3+2+4}(6\text{ mA}) = 2\text{ mA}$$

6.1 Capacitors (16)





Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \qquad v_2 = 4000i = 8 \text{ V}$$

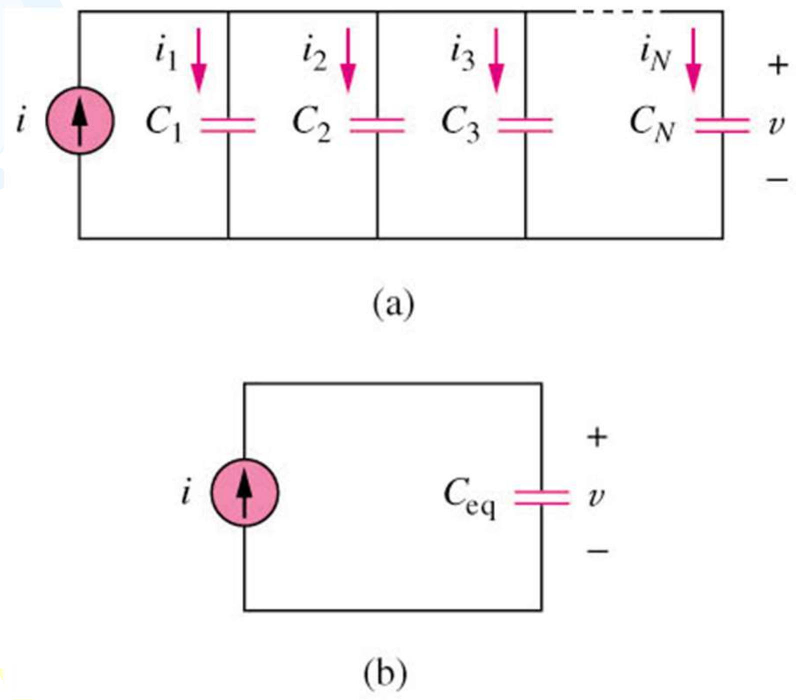
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

6.2 Series and Parallel Capacitors (1)

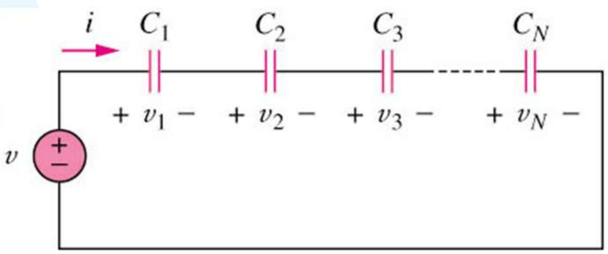
- The equivalent capacitance of *N* **parallel-connected** capacitors is the sum of the individual capacitances.



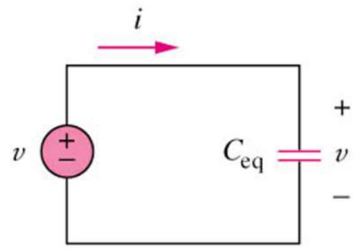
$$C_{eq} = C_1 + C_2 + \ldots + C_N$$

6.2 Series and Parallel Capacitors (2)

- The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)

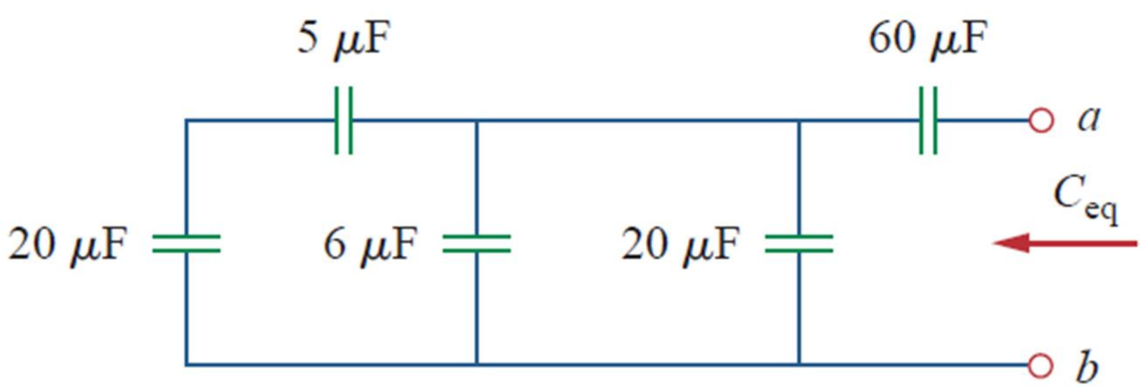


(b)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

6.2 Series and Parallel Capacitors (4)

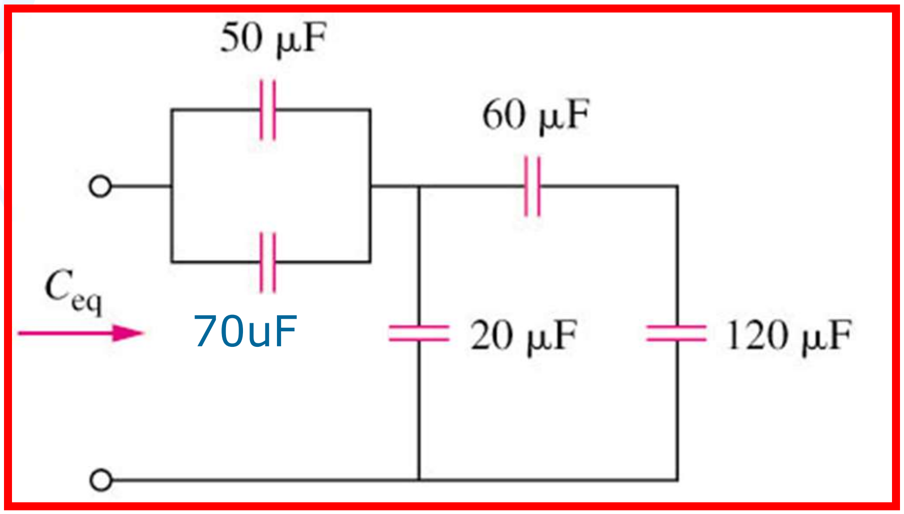
Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.



6.2 Series and Parallel Capacitors (5)

Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:

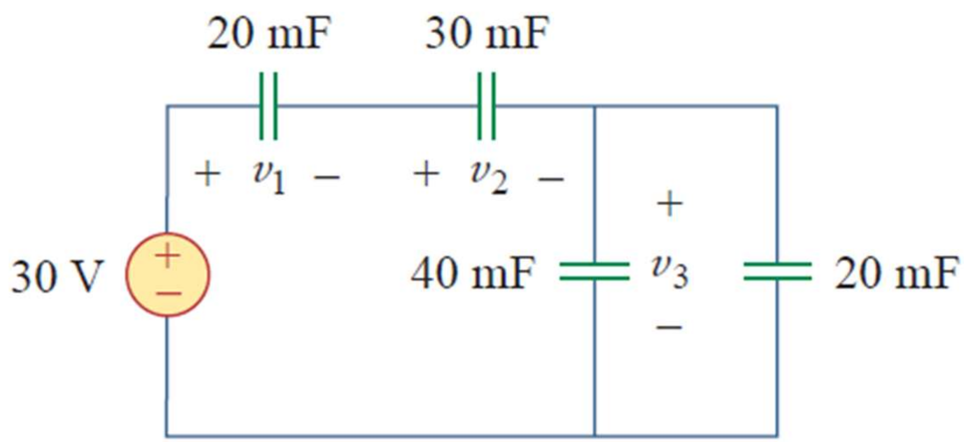


Answer:

C_{eq} = 40μF

6.2 Series and Parallel Capacitors (6)

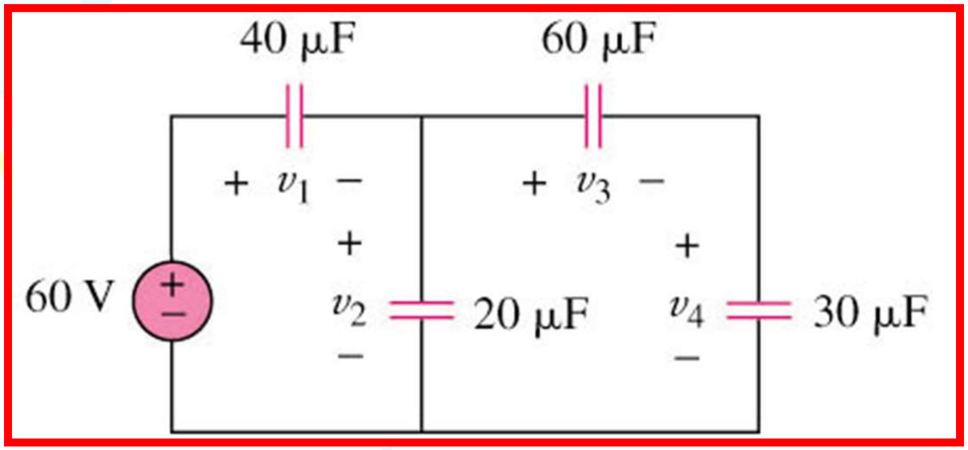
For the circuit in Fig. 6.18, find the voltage across each capacitor.



6.2 Series and Parallel Capacitors (7)

Example 4

Find the voltage across each of the capacitors in the circuit shown below:



Answer:

v_1 = 30V

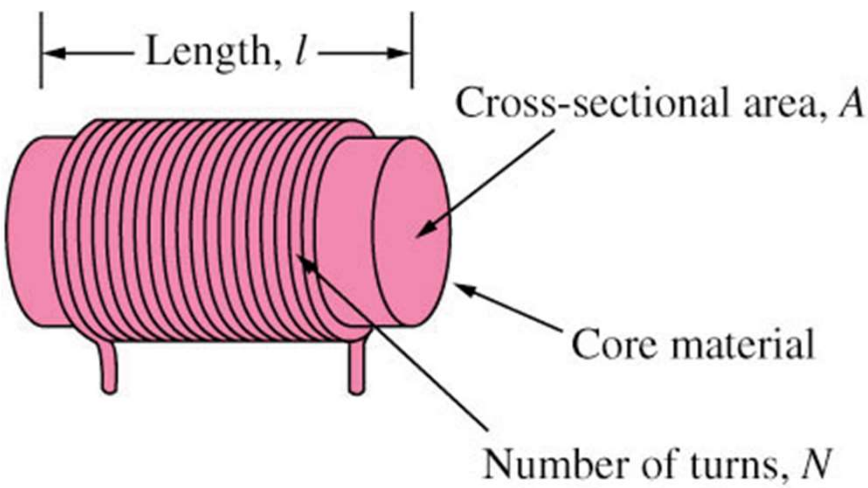
v_2 = 30V

v_3 = 10V

v_4 = 20V

6.3 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.



- An inductor consists of a coil of conducting wire.

6.3 Inductors (2)

- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

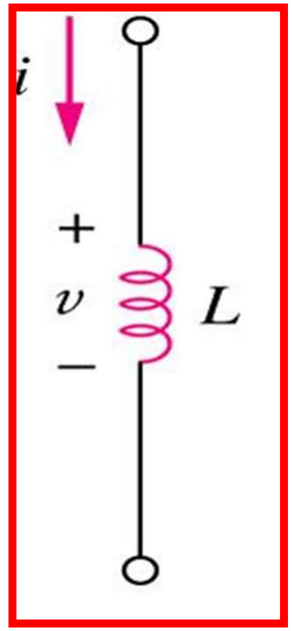
$$v = L \frac{d\,i}{d\,t} \qquad \text{and} \qquad L = \frac{N^2 \mu A}{l}$$

- The unit of inductors is Henry (H), mH (10^{-3}) and μH (10^{-6}).

6.3 Inductors (3)

- The current-voltage relationship of an inductor:

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

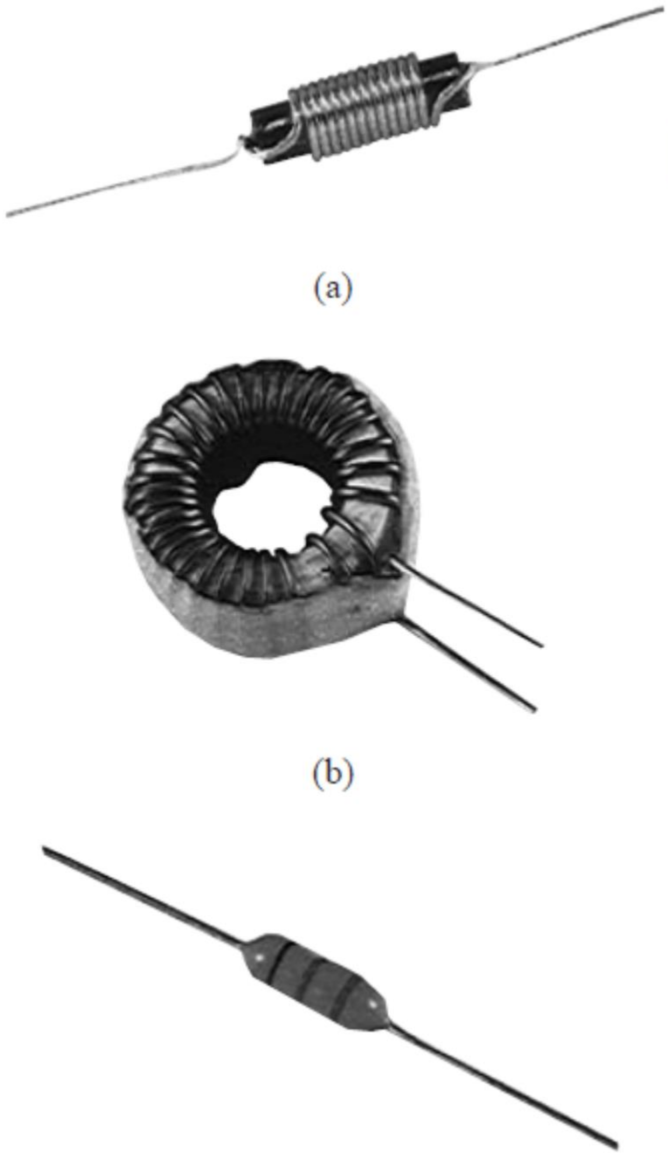


- The power stored by an inductor:

$$w = \frac{1}{2} L\, i^2$$

- An inductor acts like a short circuit to dc (di/dt = 0) and its current **cannot change abruptly.**

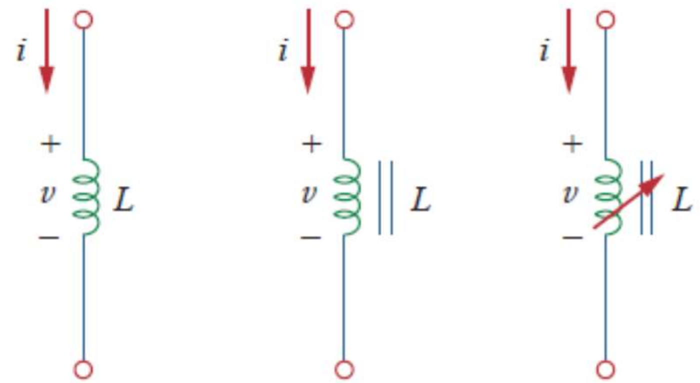
ıductors (3)



(a)

(b)

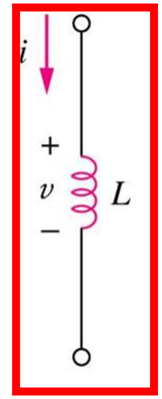
(c)



(a) (b) (c)

6.3 Inductors (4)

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.



Solution:

Since $v = L\,di/dt$ and $L = 0.1$ H,

$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t)\text{ V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t}\text{ J}$$

6.3 Inductors (5)

Find the current through a 5-H inductor if the voltage across it is

$$v(t) = \begin{cases} 30t^2, & t > 0 \\ 0, & t < 0 \end{cases}$$

Also, find the energy stored at $t = 5$ s. Assume $i(v) > 0$.

Solution:

Since $i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$ and $L = 5$ H,

$$i = \frac{1}{5}\int_0^t 30t^2\,dt + 0 = 6 \times \frac{t^3}{3} = 2t^3 \text{ A}$$

6.3 Inductors (6)

The power $p = vi = 60t^5$, and the energy stored is then

$$w = \int p\, dt = \int_0^5 60t^5\, dt = 60\frac{t^6}{6}\bigg|_0^5 = 156.25 \text{ kJ}$$

Alternatively, we can obtain the energy stored using Eq. (6.24), by writing

$$w\big|_0^5 = \frac{1}{2}Li^2(5) - \frac{1}{2}Li(0) = \frac{1}{2}(5)(2 \times 5^3)^2 - 0 = 156.25 \text{ kJ}$$

as obtained before.

6.3 Inductors (7)

Example 5

The terminal voltage of a 2-H inductor is

$$v = 10(1-t) \text{ V}$$

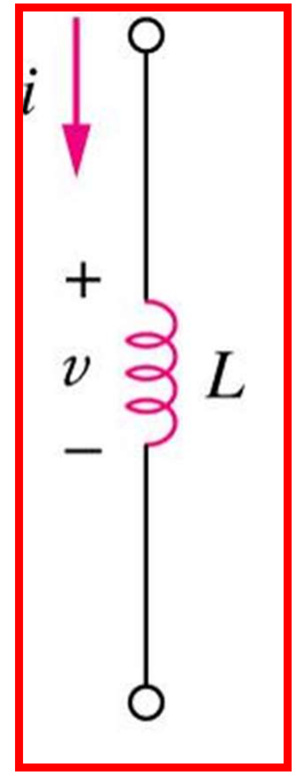
Find the current flowing through it at $t = 4$ s and the energy stored in it within $0 < t < 4$ s.

Assume $i(0) = 2$ A.

Answer:

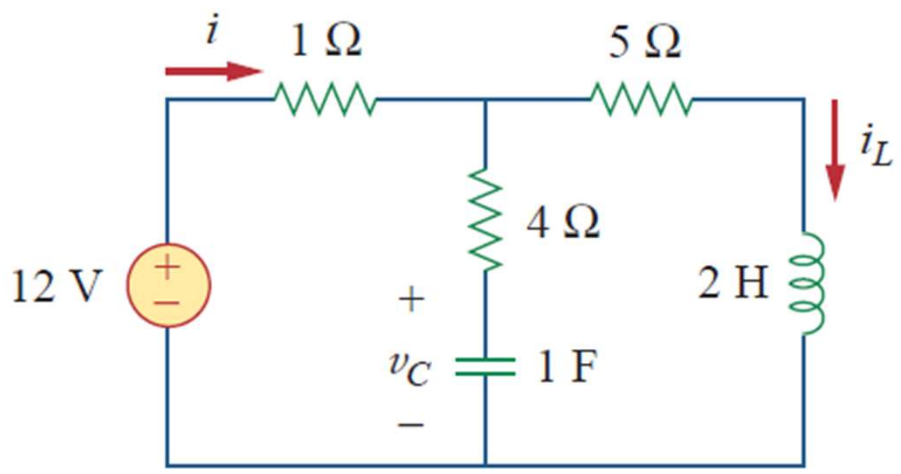
$i(4s) = -18A$

$w(4s) = 320J$



6.3 Inductors (8)

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.

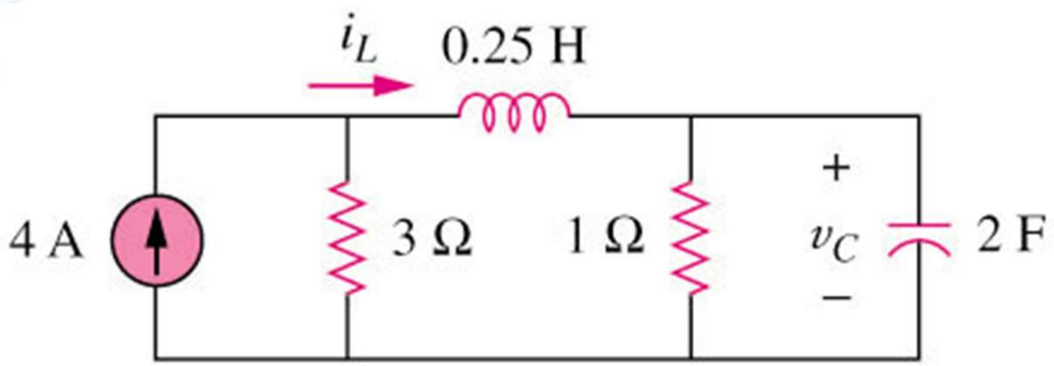


(a)

6.3 Inductors (9)

Example 6

Determine v_c, i_L, and the energy stored in the capacitor and inductor in the circuit of circuit shown below under dc conditions.



Answer:

i_L = 3A

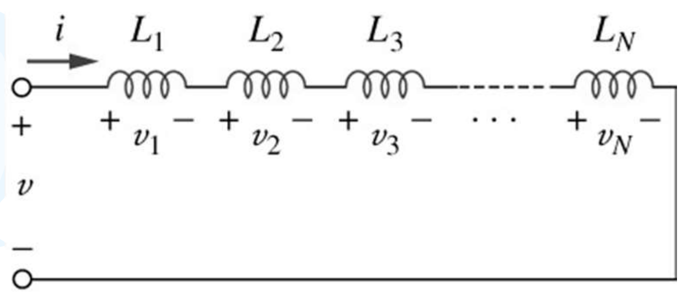
v_C = 3V

w_L = 1.125J

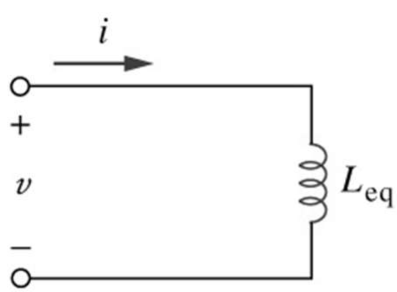
w_C = 9J

6.4 Series and Parallel Inductors (1)

- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



(a)

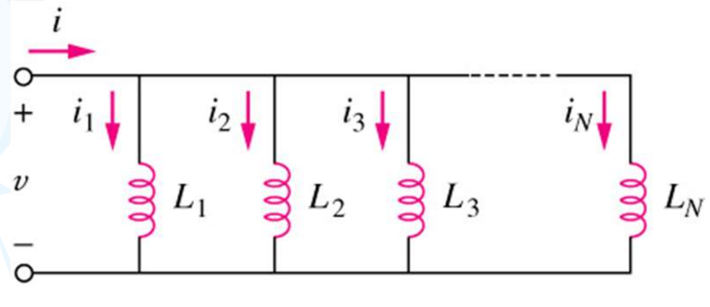


(b)

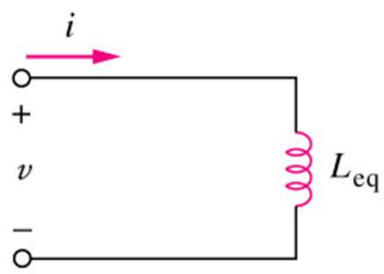
$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

6.4 Series and Parallel Inductors (2)

- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



(a)

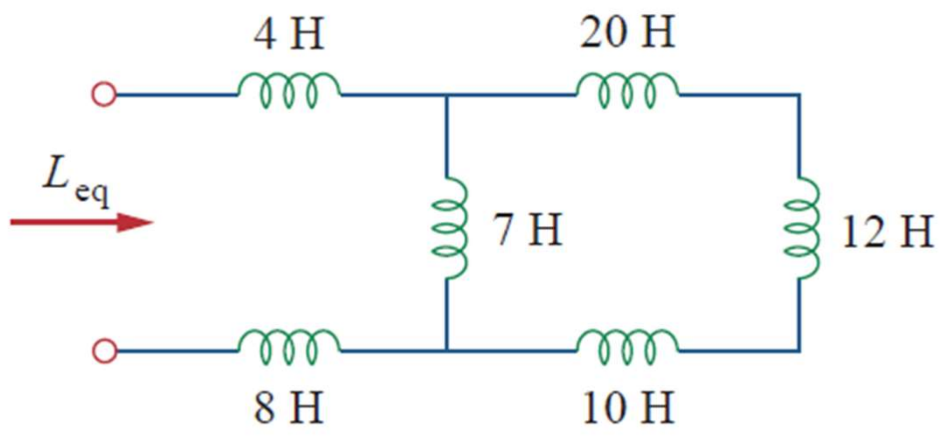


(b)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

6.4 Series and Parallel Capacitors (3)

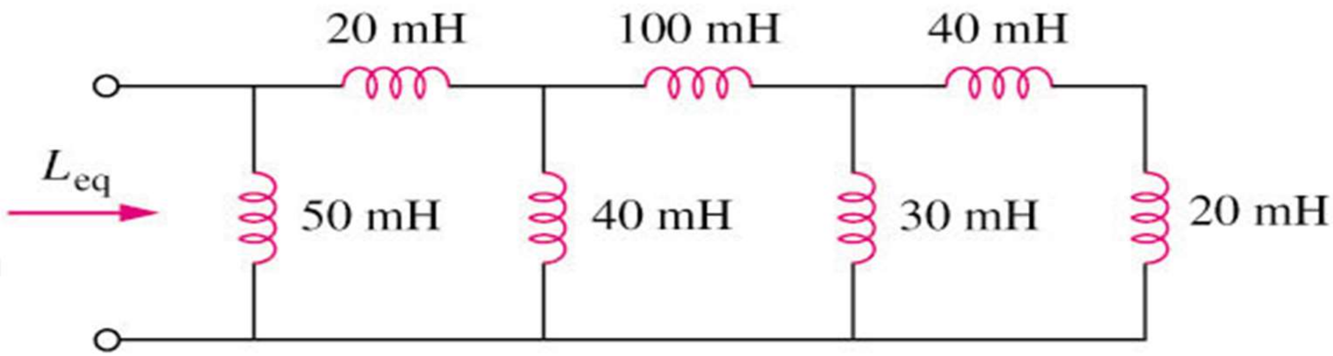
Find the equivalent inductance of the circuit



6.4 Series and Parallel Capacitors (3)

Example 7

Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:

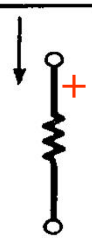


Answer:

L_{eq} = 25mH

6.4 Series and Parallel Capacitors (4)

- Current and voltage relationship for R, L, C

Circuit element	Units	Voltage	Current	Power
 Resistance	ohms (Ω)	$v = Ri$ (Ohm's law)	$i = \frac{v}{R}$	$p = vi = i^2R$
 Inductance	henries (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + k_1$	$p = vi = Li\frac{di}{dt}$
 Capacitance	farads (F)	$v = \frac{1}{C}\int i\,dt + k_2$	$i = C\frac{dv}{dt}$	$p = vi = Cv\frac{dv}{dt}$