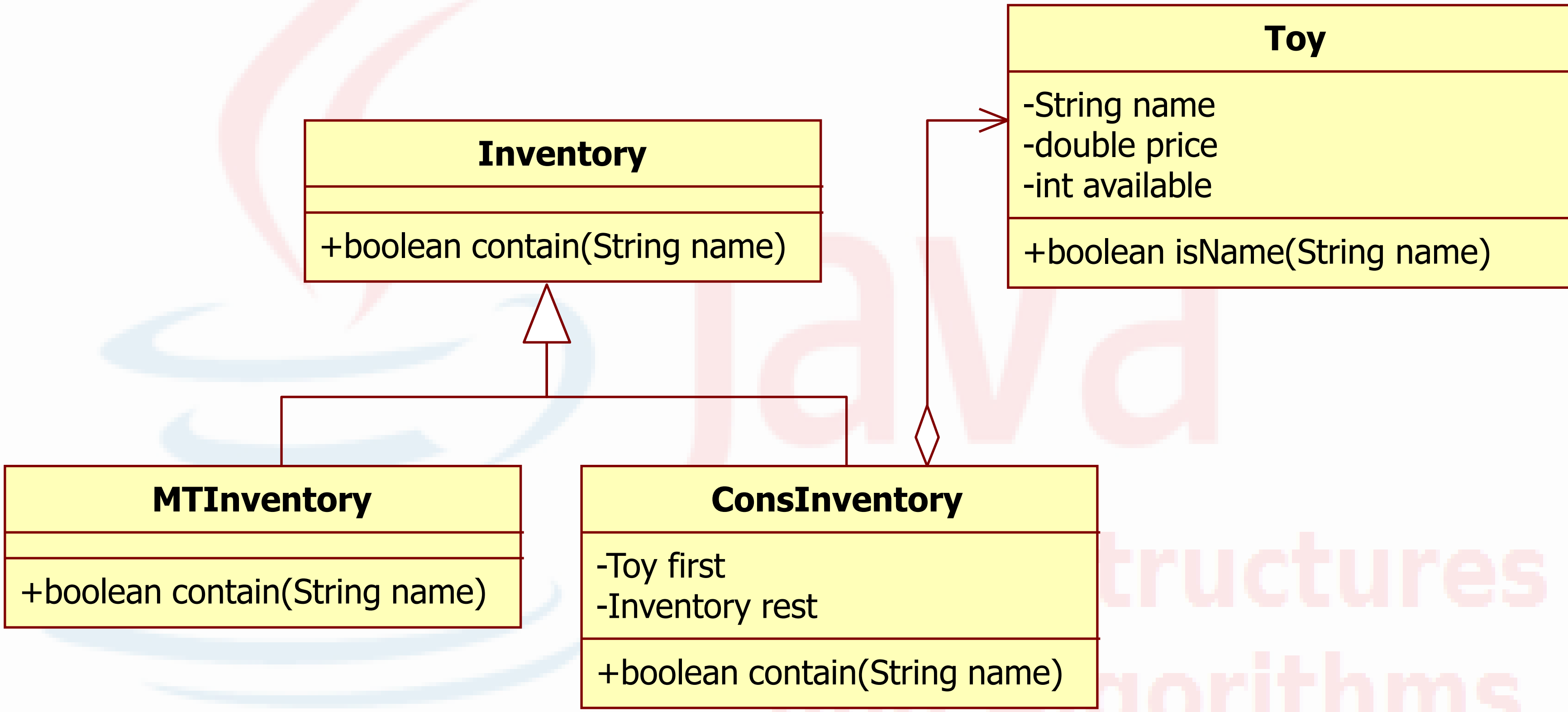
FACULTY OF INFORMATION TECHNOLOGY

DATA STRUCTURES (CTDL)

Semester 1, 2024/2025

Example 1. (BP Review)



contains() for MTInventory and ConsInventory

```
//in class MTInventory
public boolean contains(String toyName) {
   return false;
}
```

```
// in class ConsInventory
public boolean contains(String toyName) {
   return this.first.isName(toyName)
        || this.rest.contains(toyName);
}
```

```
//in class Toy
 public boolean isName(String toyName) {
      return this.name.equals(toyName);
   }
```

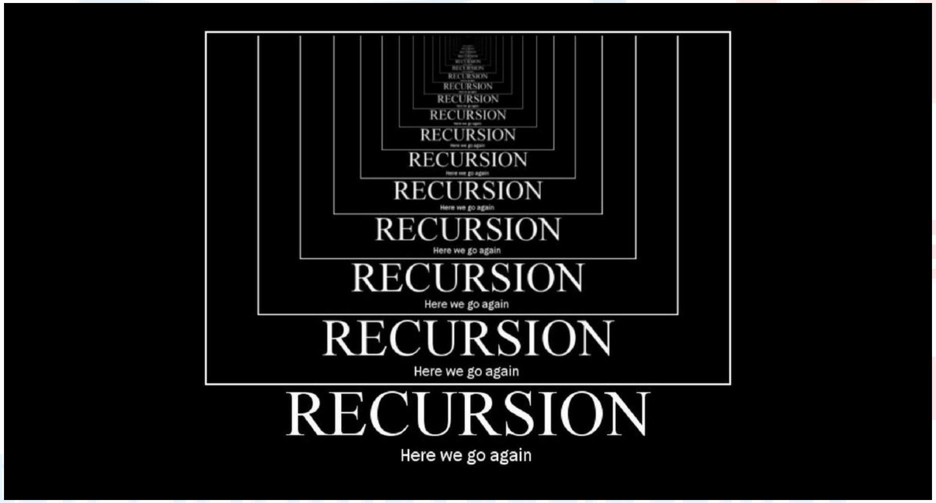
Recursion

What is recursion?

- A method of solving a problem where the solution depends on solutions to smaller instances of the same problem.
- Recursion is the process of defining something in terms of itself.
- An algorithm is recursive if it calls itself to do part of its work. It includes 2 parts:
  - The base case handling a simple input that can be solved without resorting to a recursive call
  - The recursive part containing one or more recursive calls to the algorithm

What is recursion?

- A recursive algorithm must eventually terminate.
- A recursive algorithm must have at least one base case, or stopping case.
- A base case does not execute a recursive call.



Situation

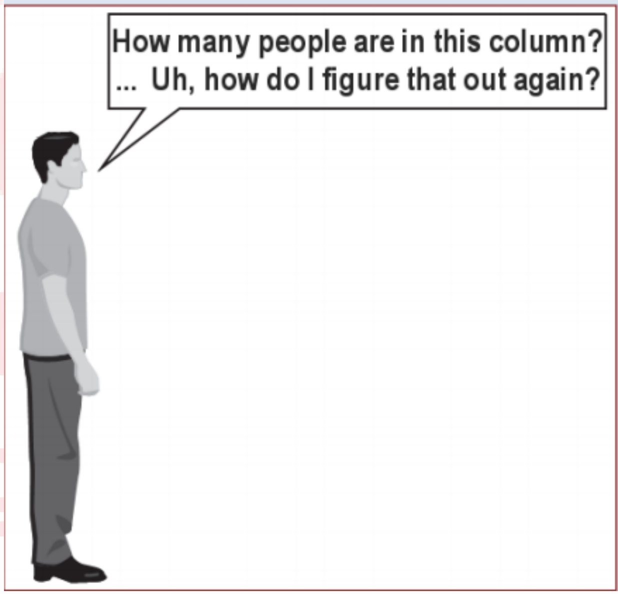
- How many students total are directly behind you in your "column" of the classroom?
- You have poor vision ➔ you can see only the people right next to you. So, you can't just look back and count.
- But you are allowed to

ask questions of the person

next to you.

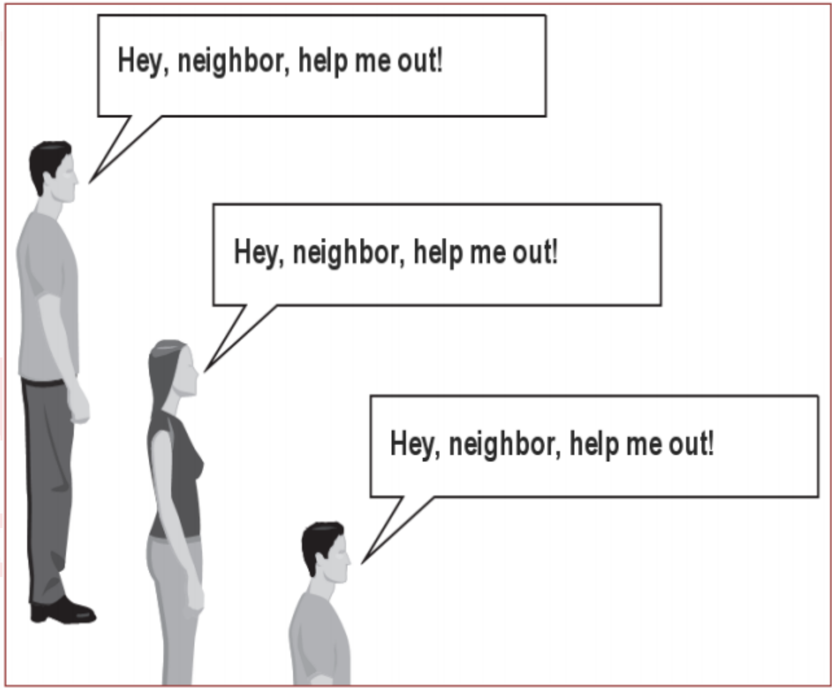
- How can we solve this problem?

(recursively!)



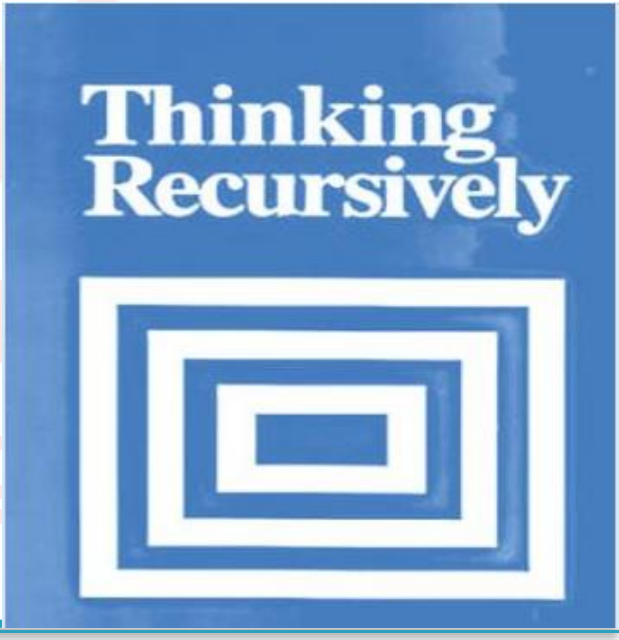
The recursion idea

- Recursion is all about breaking a big problem into smaller occurrences of that same problem.
  - Each person can solve a small part of the problem.
  - What is a small version of the problem that would be easy to answer?
  - What information from a neighbor might help me?



Recursive algorithm

- Number of people behind me:
  - If there is someone behind me, ask him/her how many people are behind him/her.
  - When they respond with a value N, then I will answer N + 1.
  - If there is nobody behind me, then I will answer 1.



Example

```java
package lab2_recursion;

public class RecursionExample1 {
    static void p() {
        System.out.println("hello");
        p();
    }

    public static void main(String[] args) {
        p();
    }
}
```

```
hello
hello
...
java.lang.StackOverflowError
```

Types of recursion

Types of recursion

- Linear recursion: makes at most one recursive call each time it is invoked.
- Binary recursion: algorithm makes two recursive calls.
- Multiple recursion: method may make (potentially more than two) recursive calls.

Linear recursion

Example:

```
public int linearSum(int[] array, n) {
    if (n == 1)
     return array[0];
   else
      return linearSum(array, n-1) + array[n-1];
  }
```

Binary recursion

Algorithm:

```
public int binarySum(int[] array, int i, int n) {
 if (n == 1)
  return array[i];
else
  return binarySum(array, i, [n/2]) +
            binarySum(array, i+[n/2], [n/2]);
}
```

Factorial

- **Factorial**: the factorial of a positive integer **n**, denoted by **n!**, is the product of all positive integers less than or equal to **n**:

$$\mathrm{fact}(n) = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot \mathrm{fact}(n-1) & \text{if } n > 0 \end{cases}$$

Pseudocode (recursive):

```
function factorial is:

input: integer n such that n >= 0

output: [n × (n-1) × (n-2) × … × 1]


    1. if n is 0, return 1
    2. otherwise, return [ n × factorial(n-1) ]


end factorial
```

Base case

Recursive part

Factorial (cont.)

- The function can also be written as a recurrence relation:

$$b_n = nb_{n-1}$$
$$b_0 = 1$$

Computing the recurrence relation for n = 4:

```
b4    = 4 * b3

      = 4 * (3 * b2)
      = 4 * (3 * (2 * b1))
      = 4 * (3 * (2 * (1 * b0)))
      = 4 * (3 * (2 * (1 * 1)))
      = 4 * (3 * (2 * 1))
      = 4 * (3 * 2)
      = 4 * 6
      = 24
```

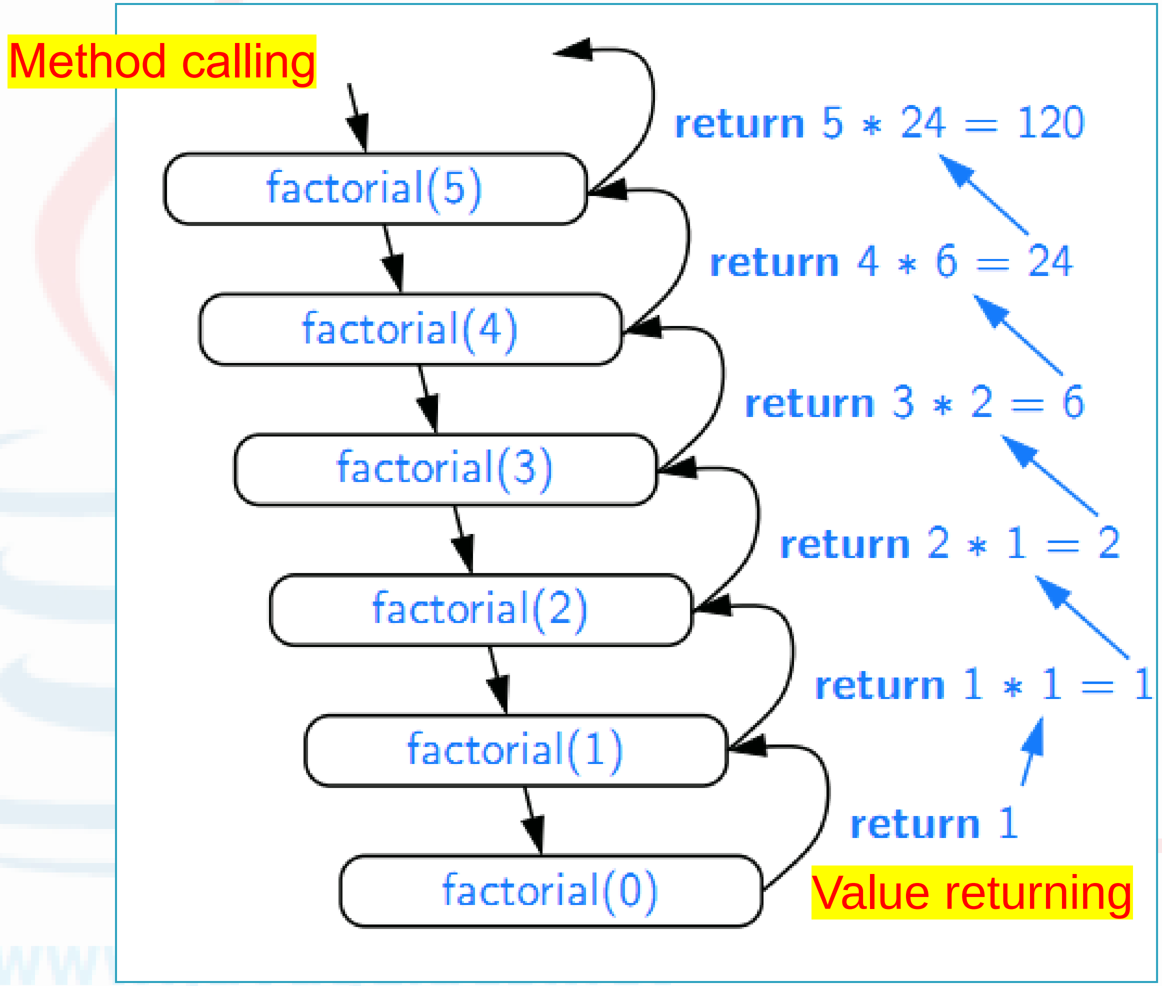
Factorial (cont.)

- Implemented Java code:

```
public static int factorial(int n) throws IllegalArgumentException {
  if (n < 0)
    throw new IllegalArgumentException( );      // argument must be nonnegative
  else if (n == 0)
    return 1;                                   // base case
  else
    return n * factorial(n−1);                  // recursive case
}
```

Factorial (cont.)

- A recursion trace for the call factorial(5)



Factorial (cont.)

- This factorial function can also be described **without using recursion** :

Pseudocode (iterative):

```
function factorial is:

input: integer n such that n >= 0

output: [n × (n-1) × (n-2) × … × 1]


    1. create new variable called running_total with a value of 1


    2. begin loop
          1. if n is 0, exit loop
          2. set running_total to (running_total × n)
          3. decrement n
          4. repeat loop


    3. return running_total


end factorial
```

Recursive version
Or
Iterative version?

Why recursion?

- Avoidance of unnecessary calling of functions.
- A substitute for iteration where the iterative solution is very complex.
- Extremely useful when applying the same solution.
- Leads to elegant, simplistic, short Java code (when used well).

When to Use Recursion Rather Than Iteration?

When to Use Recursion Rather Than Iteration

- Common reasons for using recursion:
  - The **problem** is naturally recursive (e.g. Fibonacci)
  - The **data** is naturally recursive (e.g. filesystem)
  - Take more advantage of immutability:
    - all variables are final, all data is immutable, and the recursive methods are all pure functions ➔ they do not mutate anything.
- Recursion is that it may take more space than an iterative solution (downside)

Recursive Problems vs. Recursive Data

- The problem structure lends itself naturally to a recursive definition.
  - Ex.: factorial, Fibonacci, …
- The data you are operating on is inherently recursive in structure.
  - Ex.: A filesystem consists of named *files*. Some files are *folders*, which can contain other files

Common Mistakes in Recursive

- The base case is missing entirely, or
  - the problem needs more than one base case but not all the base cases are covered.

```java
public static int factorial(int n) {
    return n * factorial(n - 1);
}
```


Common Mistakes in Recursive

- The recursive step doesn't reduce to a smaller subproblem, so the recursion doesn't converge.

```
public static int getFibonacci(int n) {
    if (n == 0) {
        return 0;
    }
    if (n == 1) {
        return 1;
    }
    int f_n2 = getFibonacci(n);
    int f_n1 = getFibonacci(n);
    return f_n2 + f_n1;
}
```

Helper Methods

- For a given array of integers, implement a method, called reversePrint to display the array reversely.

```java
public static void reversePrint(int[] arr) {
      //TODO
}
```

- Ex. arr= { 1, 2, 3, 4, 5 } ➔ { 5, 4, 3, 2, 1 }
- How to implement reversePrint method?
  - It’s easy if using **iterative approach**
  - How about **recursive approach**?

?

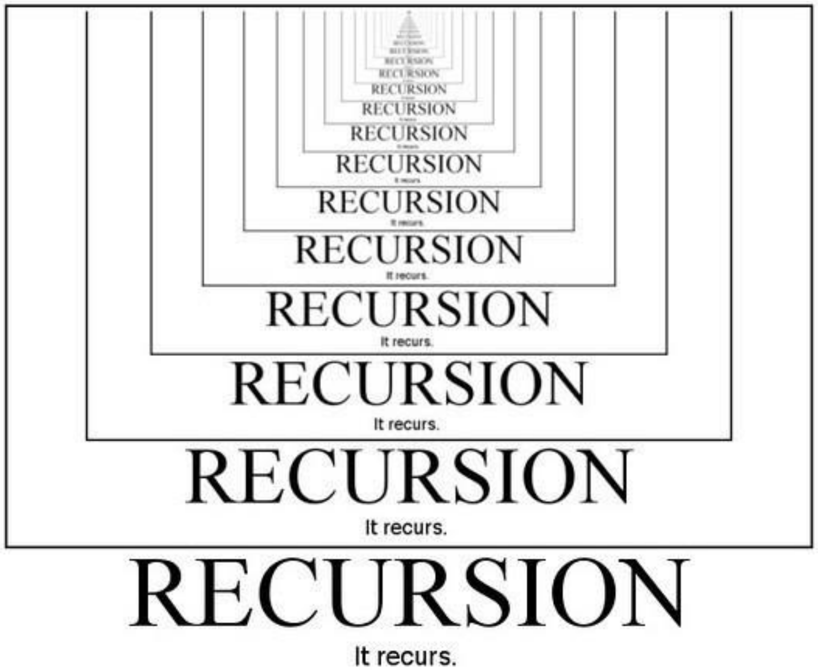
Helper Methods

- For a given array of integers, implement a method, called reversePrint to display the array reversely.
- Hint: using a helper method

```
public static void reversePrint(int[] arr) {
    reversePrintHelp(arr, arr.length);
}
```

```
public static void reversePrintHelp(int[] arr, int n) {
    //TODO
}
```

Other recursive problems



Recursive sum

- Calculation arithmetic series (sigma) recursive Sum

$$\sum_{x=1}^{n} x$$

```java
public int sigma(int n) {
    // TODO
    return 0;
}
```

```java
public static int sigma(int n) {
    if (n <= 1)
        return n;
    else
        return n + sigma(n - 1);
} // sigma
```

Iterative approach???

Calculation power

- How to calculate power:

$$x^y = \underbrace{x * x * \ldots * x}_{\text{y times}}$$

```java
public static int power(int x, int y) {
    //TODO
    return 0;
} // power
```

```java
public static int power(int x, int y) {
    if (y == 0)
        return 1;
    else
        return x * power(x, y - 1);
} // power
```

Iterative approach???

Calculation product

- How to calculate product?

$$x * y$$

$$x * y = \underbrace{x + x + x + x + \ldots + x}_{\text{y times}}$$

```java
public static int recMult(int x, int y) {
    // TODO
    return 0;
} // recMult
```

Printing stars

- If the method stars1 is called with the value 3, is it equivalent to the method stars2?

```
public static void stars1(int n) {
    if (n < 1)
        return;
    System.out.print(" * ");
    stars1(n - 1);
} // stars1

public static void stars2(int n) {
    if (n > 1)
        stars2(n - 1);
    System.out.print(" * ");
} // stars2
```

Explain!

Reverse Print

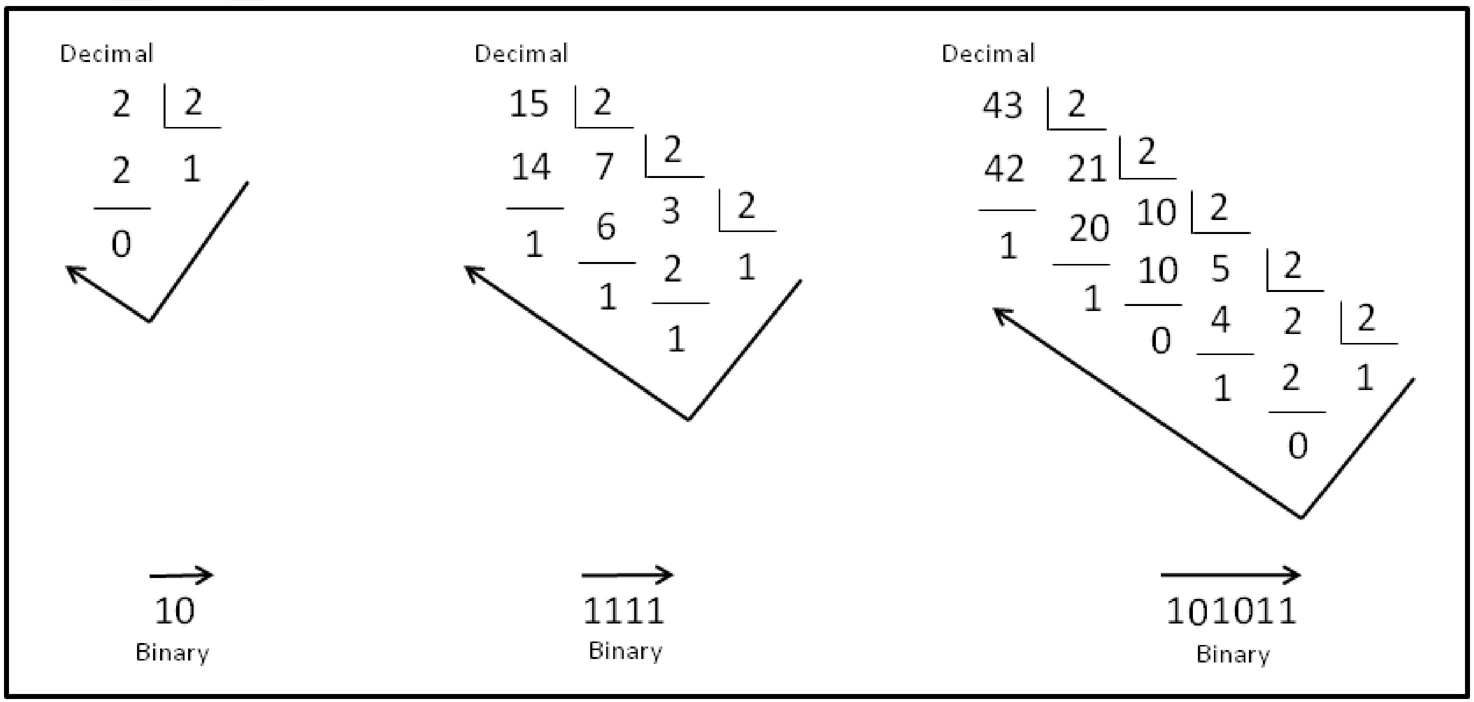
- Apply the recursive approach to reversely print elements in a given array:

```
public static void reversePrint(int[] arr)
        //TODO
        return null;
}
```

Input={1,2,3} → Output: 3 2 1

Decimal to binary number using recursion

- Given a decimal number as input, how to convert the given decimal number into equivalent binary number.



Decimal to binary number using recursion

- Given a decimal number as input, how to convert the given decimal number into equivalent binary number.

```
findBinary(decimal)
   if (decimal == 0)
      binary = 0
   else
      binary = decimal % 2 + 10 * (findBinary(decimal / 2)
```

Decimal to binary number

- Other approaches:
  - Using **Integer.toBinaryString(number):** returns a string representation of the integer argument as an unsigned integer in binary
  - Using <u>**iterative method**</u>?

Drawing an English Ruler

- How to draw the markings of a typical English ruler?
- The length of the tick designating a whole inch as the major tick length.
- Between the marks for whole inches, the ruler contains a series of minor ticks, placed at intervals of 1/2 inch, 1/4 inch, and so on.
- As the size of the interval decreases by half, the tick length decreases by one.

Drawing an English Ruler (cont.)

```
---- 0
-
--
-
---
-
--
-
---- 1
-
--
-
---
-
--
-
---- 2
```

(a)

```
----- 0
-
--
-
---
-
--
-
----
-
--
-
---
-
--
-
----- 1
```

(b)

```
--- 0
-
--
-
--- 1
-
--
-
--- 2
-
--
-
--- 3
```

(c)

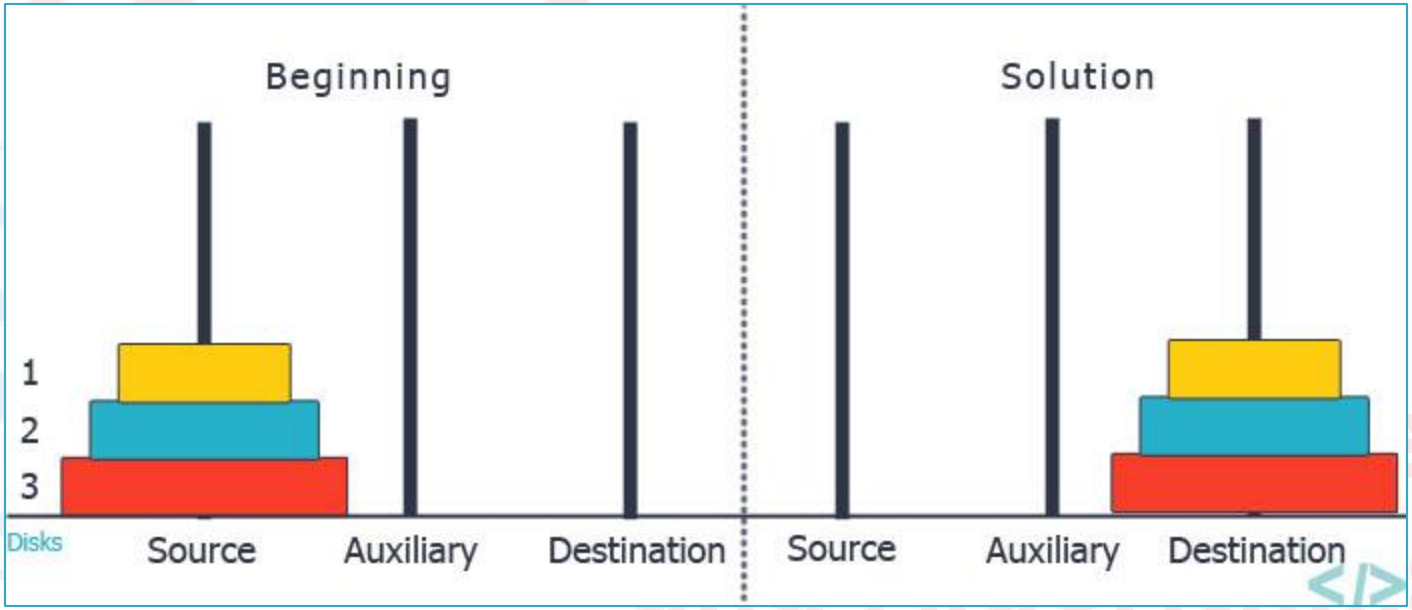
Three sample outputs of an English ruler drawing: (a) a 2-inch ruler with major tick length 4; (b) a 1-inch ruler with major tick length 5; (c) a 3-inch ruler with major tick length 3.

Towers of Hanoi

- A mathematical puzzle where we have **three rods** and **n disks**. Rules:
  - Only one disk can be moved at a time.
  - Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack i.e. a disk can only be moved if it is the uppermost disk on a stack.
  - No disk may be placed on top of a smaller disk.

Towers of Hanoi (cont.)

- The minimum number of moves required to solve is $2^n - 1$, where n is the number of discs.

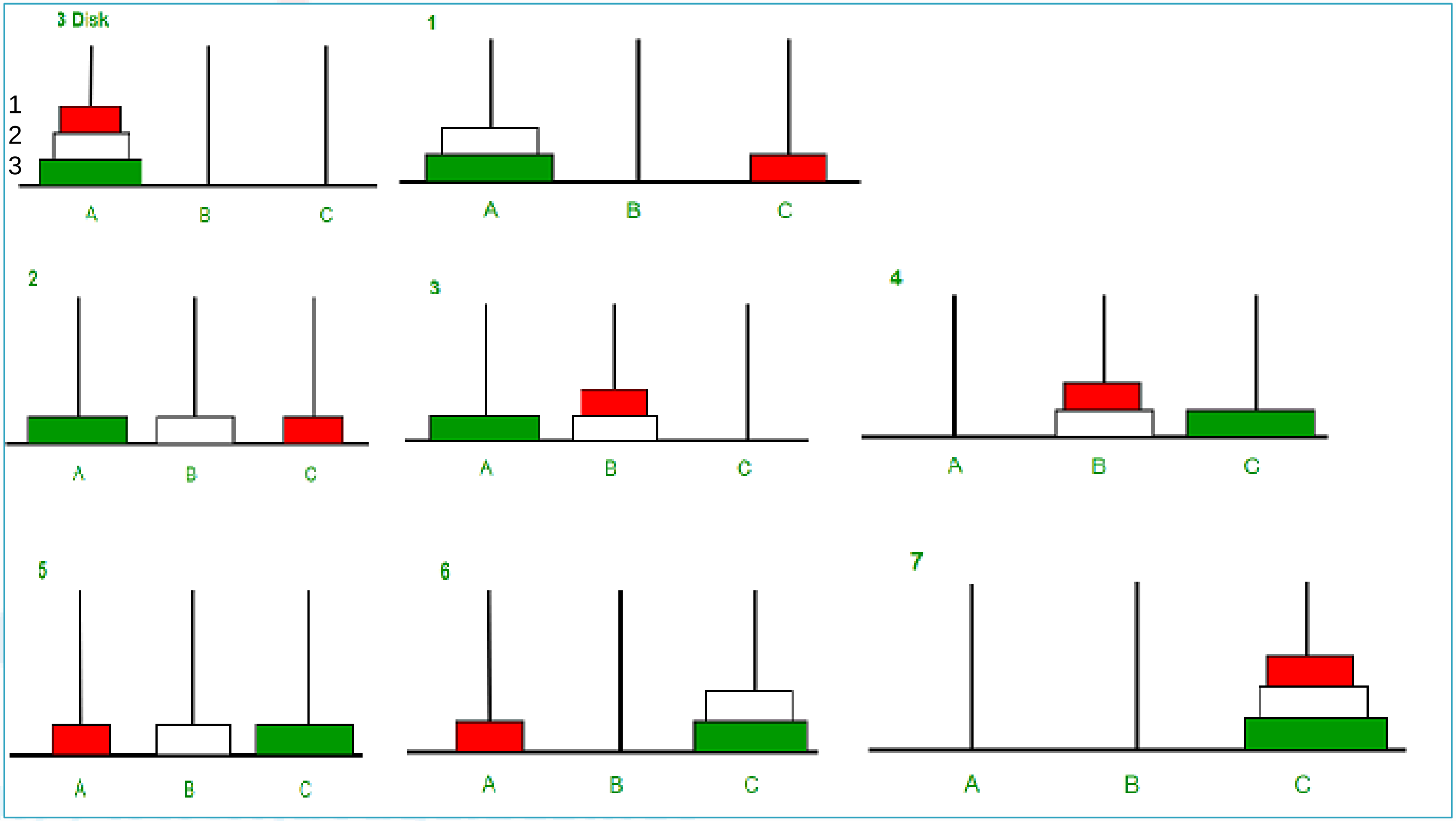


Towers of Hanoi (cont.)

- Label the pegs A, B, C
- Let n be the total number of discs
- Number the discs from 1 (smallest, topmost) to n (largest, bottommost)
- To move n discs from rod A to rod C:
  - **Step 1**. move **n−1** discs from A to B. This leaves disc **n** alone on peg A
  - **Step 2**. move disc **n** from A to C
  - **Step 3**. move **n−1** discs from B to C

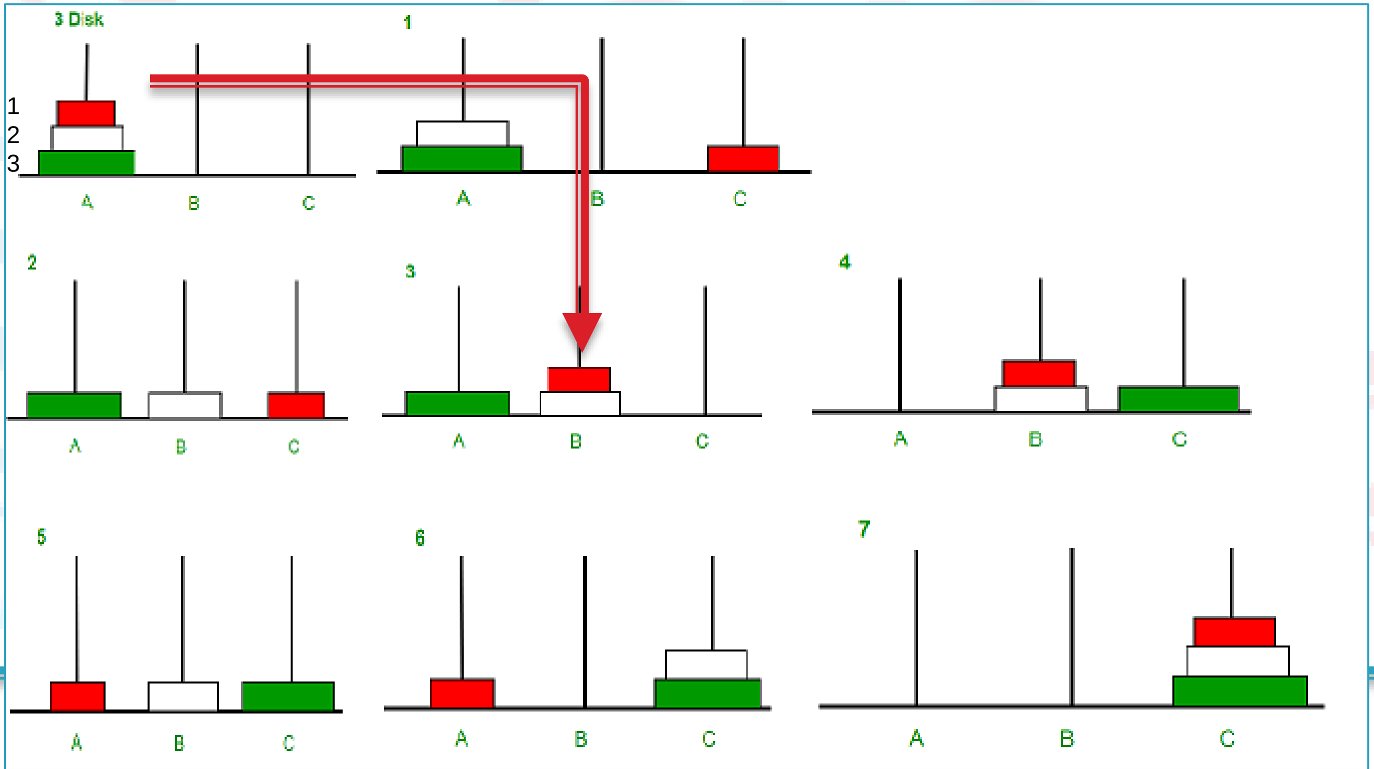
Towers of Hanoi (cont.)

- Image illustration for 3 discs :



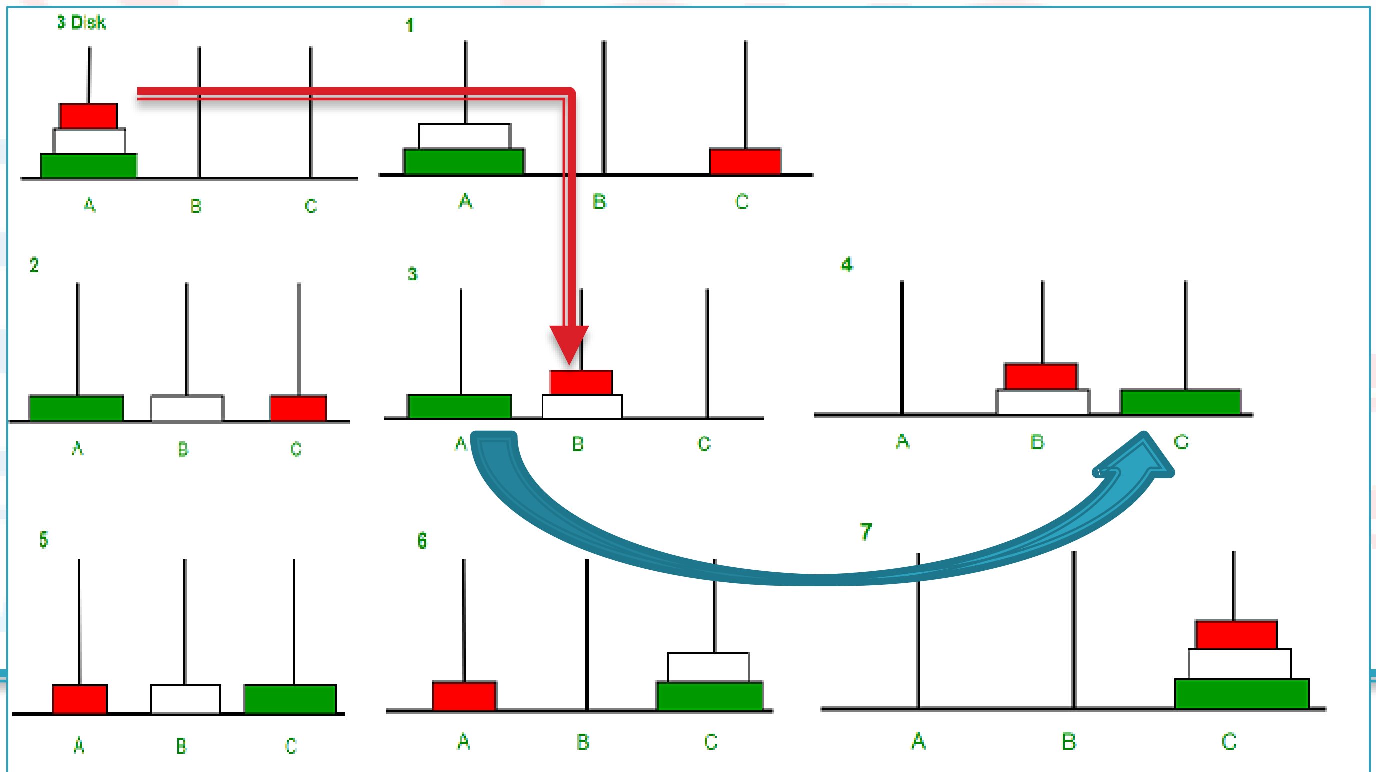
Towers of Hanoi: Think recursively

- For given 3 discs with order: disk 1 < disk 2 < disk 3. How to move all disks to C from A?
  - Step 1: Move discs 2 and smaller from peg A (*source*) to peg B (*spare*), using peg C (*dest*) as a spare



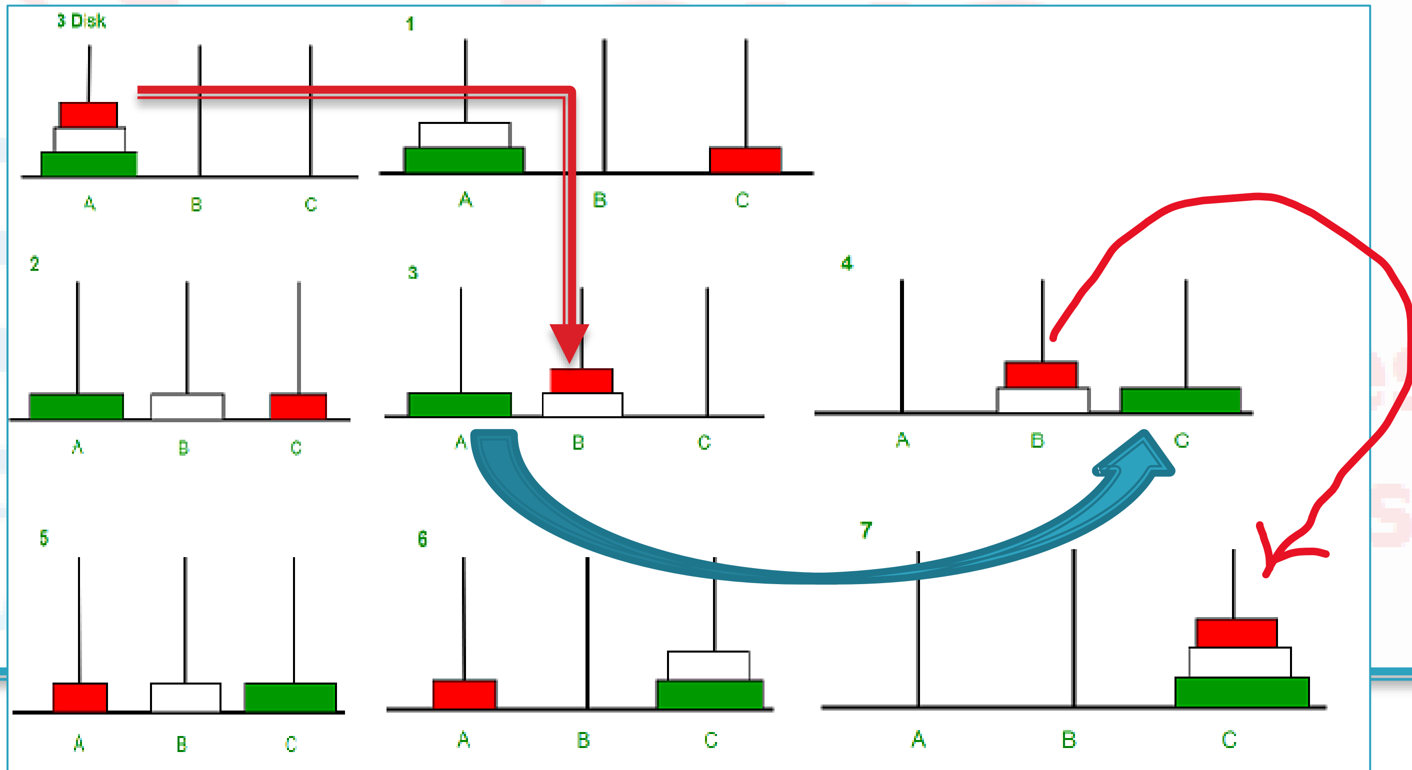
Towers of Hanoi (cont.): Think recursively

- For given 3 discs with order: disk 1 < disk 2 < disk 3. How to move all discs to C from A?
  - Step 2: With all the smaller discs on the spare peg, we can move disk 3 from peg A (*source*) to peg C (*dest*).



Towers of Hanoi (cont.): Think recursively

- For given 3 discs with order: disk 1 < disk 2 < disk 3. How to move all disks to C from A?
  - Step 3: We want discs 1 and smaller moved from peg C (*spare*) to peg B (*dest*)



Towers of Hanoi (cont.)

- Suppose *source*=A, *dest*=C, and *spare*=B, *disk* represents the number of disks.

```
FUNCTION MoveTower(disk, source, dest, spare):
IF disk == 1, THEN:
    move disk from source to dest
ELSE:
    MoveTower(disk - 1, source, spare, dest)   // Step 1 above
    move disk from source to dest              // Step 2 above
    MoveTower(disk - 1, spare, dest, source)   // Step 3 above
END IF
```

Fibonacci

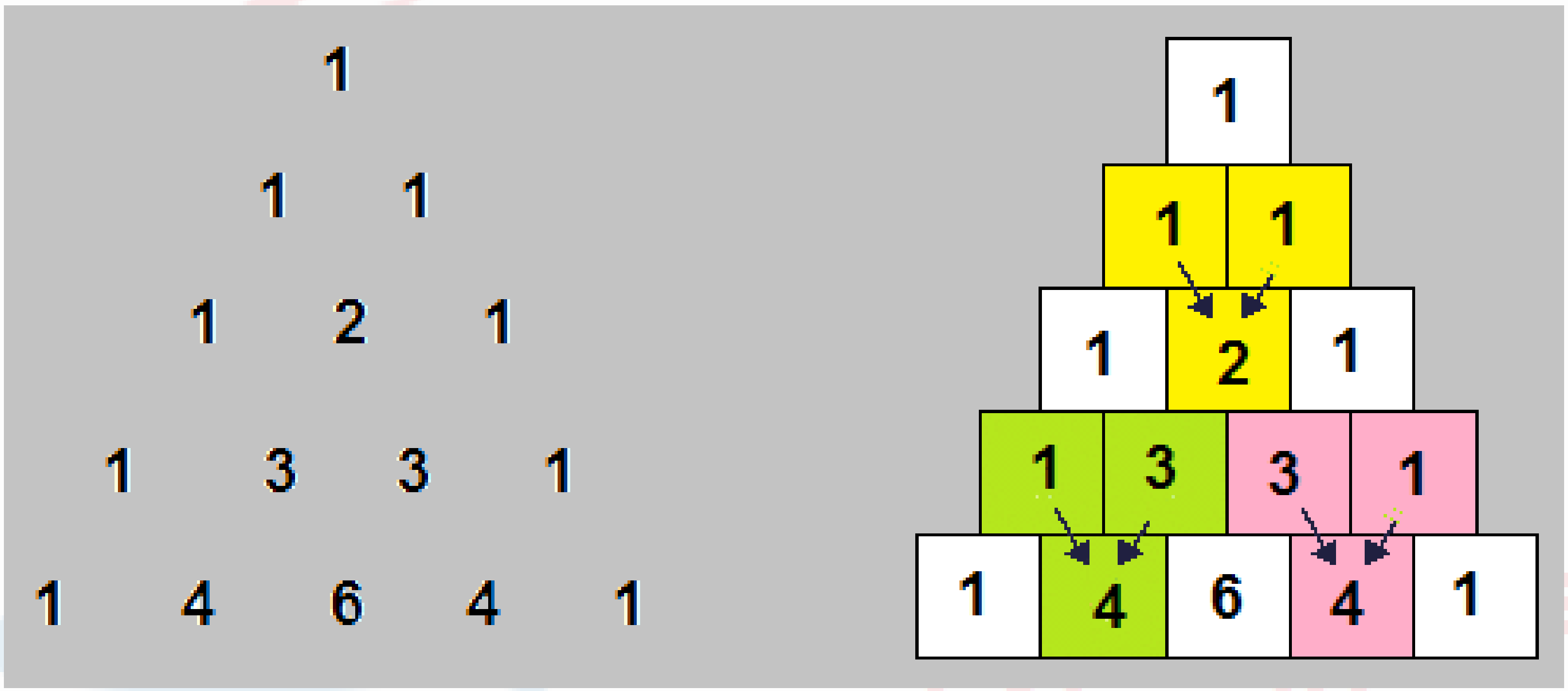
- Fibonacci: next number is the sum of previous two numbers
- Ex. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

$$\begin{aligned} F_0 &= 0 \\ F_1 &= 1 \\ F_n &= F_{n-2} + F_{n-1} \quad \text{for } n > 1. \end{aligned}$$

- Two approaches:
  - Fibonacci Series **without using recursion**
  - Fibonacci Series **using recursion**

Pascal's triangle

- Pascal's triangle: a triangular array of the binomial coefficients



Pascal's triangle (cont.)

It is commonly called "n choose k" and written like this: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

Notation: "n choose k" can also be written **C(n,k)**, $^{\mathbf{n}}\mathbf{C}_{\mathbf{k}}$ or even $_{\mathbf{n}}\mathbf{C}_{\mathbf{k}}$.

$$\binom{0}{0} = 1$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$
$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

Pyramid

```
        1
       2 2
      3 3 3
     4 4 4 4
    5 5 5 5 5
   6 6 6 6 6 6
  7 7 7 7 7 7 7
 8 8 8 8 8 8 8 8
9 9 9 9 9 9 9 9 9
```

Pyramid Pattern-1

```
        1
       1 2
      1 2 3
     1 2 3 4
    1 2 3 4 5
   1 2 3 4 5 6
  1 2 3 4 5 6 7
 1 2 3 4 5 6 7 8
1 2 3 4 5 6 7 8 9
```

Pyramid Pattern-2

```
        *
       * *
      * * *
     * * * *
    * * * * *
   * * * * * *
  * * * * * * *
 * * * * * * * *
* * * * * * * * *
```

Pyramid Pattern-3

```
                1
              1 2 1
            1 2 3 2 1
          1 2 3 4 3 2 1
        1 2 3 4 5 4 3 2 1
      1 2 3 4 5 6 5 4 3 2 1
    1 2 3 4 5 6 7 6 5 4 3 2 1
  1 2 3 4 5 6 7 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 8 7 6 5 4 3 2 1
```

Pyramid Pattern-4

```
                9
              8 9 8
            7 8 9 8 7
          6 7 8 9 8 7 6
        5 6 7 8 9 8 7 6 5
      4 5 6 7 8 9 8 7 6 5 4
    3 4 5 6 7 8 9 8 7 6 5 4 3
  2 3 4 5 6 7 8 9 8 7 6 5 4 3 2
1 2 3 4 5 6 7 8 9 8 7 6 5 4 3 2 1
```

Pyramid Pattern-5

```
* * * * * * * * *
 * * * * * * * *
  * * * * * * *
   * * * * * *
    * * * * *
     * * * *
      * * *
       * *
        *
```

Inverted Pyramid Pattern-6

```
9 9 9 9 9 9 9 9 9
 8 8 8 8 8 8 8 8
  7 7 7 7 7 7 7
   6 6 6 6 6 6
    5 5 5 5 5
     4 4 4 4
      3 3 3
       2 2
        1
```

Inverted Pyramid Pattern-7

Pyramid Pattern Programs in Java

Christmas tree

```
     *
    ***
   *****
  *******
    ***
   *****
  *******
 *********
   *****
  *******
 *********
***********
     *
     *
  *******
```

```
     *
    ***
   *****
  *******
 *********
    ***
   *****
  *******
 *********
***********
     *
     *
  *******
```

Algebra problems

1. $S(n)=1-2+3-4+\ldots+((-1)^{(n+1)}).n$, $n>0$

2. $S(n)=1+1.2+1.2.3+\ldots+1.2.3\ldots n$, $n>0$

3. $S(n)=1^2+2^2+3^2+\ldots.+n^2$, $n>0$

4. $S(n)=1+1/2+1/(2.4)+1/(2.4.6)+\ldots+1/(2.4.6.2n)$, $n>=0$

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