
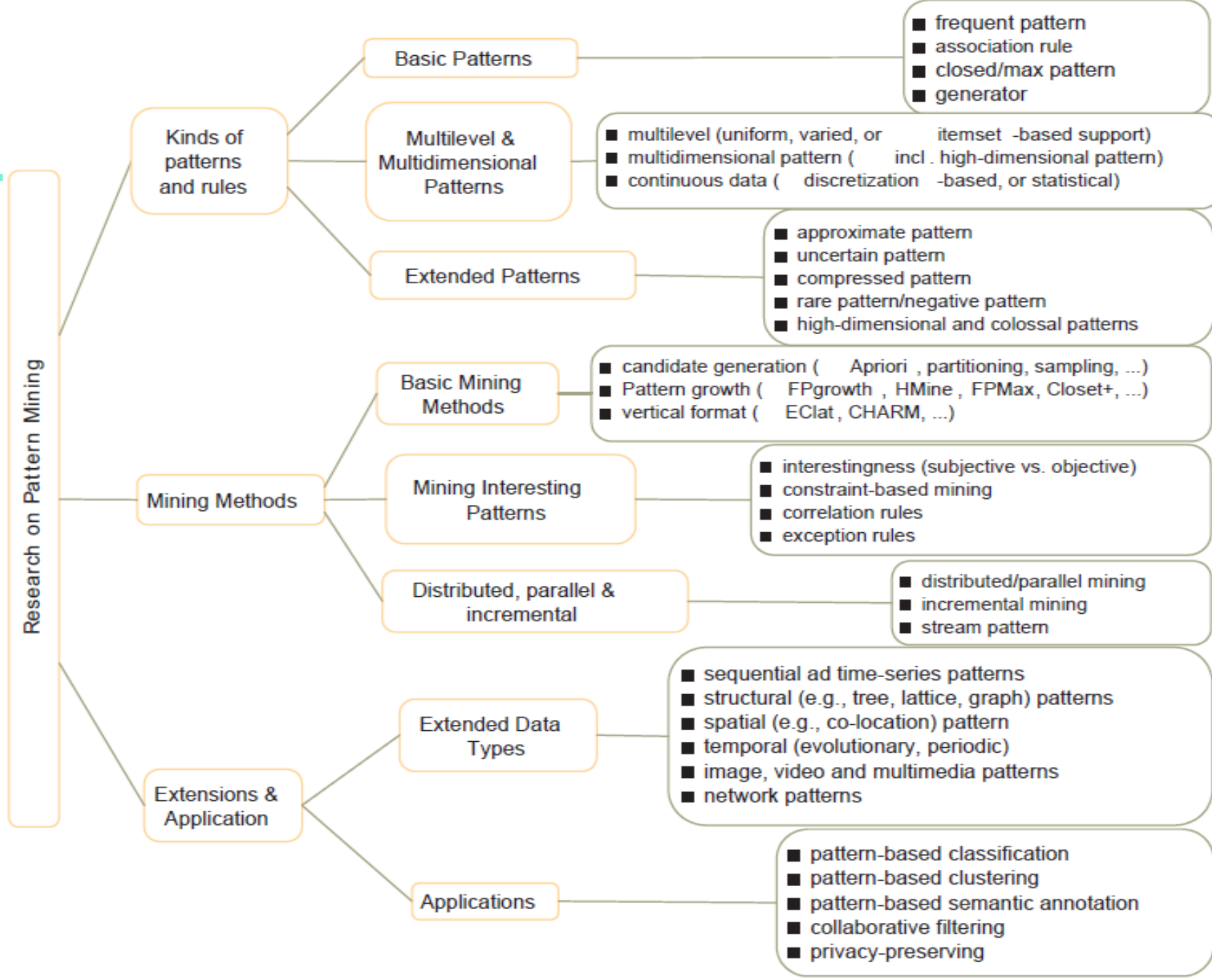

Data Mining:

Concepts and Techniques

Chapter 7 : Advanced Frequent Pattern Mining

- Pattern Mining: A Road Map 
- Pattern Mining in Multi-Level, Multi-Dimensional Space
- Constraint-Based Frequent Pattern Mining
- Mining High-Dimensional Data and Colossal Patterns
- Mining Compressed or Approximate Patterns
- Pattern Exploration and Application
- Summary

Research on Pattern Mining: A Road Map



Advanced Frequent Pattern Mining

- Pattern Mining: A Road Map
- Pattern Mining in Multi-Level, Multi-Dimensional Space 
 - Mining Multi-Level Association 
 - Mining Multi-Dimensional Association
 - Mining Quantitative Association Rules
 - Mining Rare Patterns and Negative Patterns
- Constraint-Based Frequent Pattern Mining
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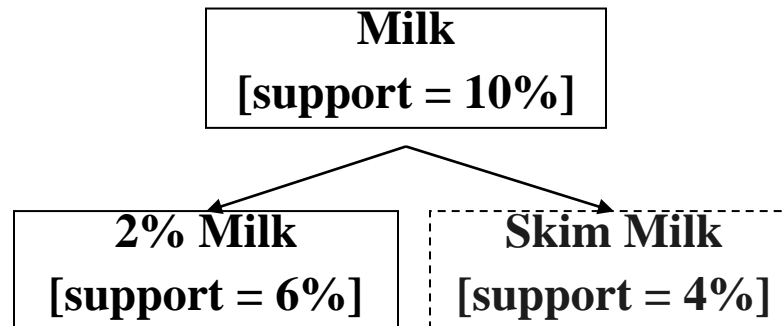
Mining Multiple-Level Association Rules

- Items often form hierarchies
- Flexible support settings
 - Items at the lower level are expected to have lower support
- Exploration of *shared* multi-level mining (Agrawal & Srikant@VLB'95, Han & Fu@VLDB'95)

uniform support

Level 1
min_sup = 5%

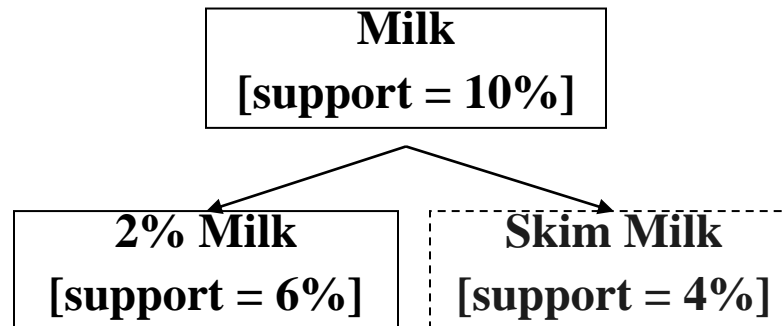
Level 2
min_sup = 5%



reduced support

Level 1
min_sup = 5%

Level 2
min_sup = 3%





Multi-level Association: Flexible Support and Redundancy filtering

- Flexible min-support thresholds: Some items are more valuable but less frequent
 - Use non-uniform, group-based min-support
 - E.g., {diamond, watch, camera}: 0.05%; {bread, milk}: 5%; ...
- Redundancy Filtering: Some rules may be redundant due to “ancestor” relationships between items
 - milk \Rightarrow wheat bread [support = 8%, confidence = 70%]
 - 2% milk \Rightarrow wheat bread [support = 2%, confidence = 72%]

The first rule is an ancestor of the second rule
- A rule is *redundant* if its support is close to the “expected” value, based on the rule’s ancestor

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Mining Multi-Dimensional Association

- Single-dimensional rules:

$\text{buys}(X, \text{"milk"}) \Rightarrow \text{buys}(X, \text{"bread"})$

- Multi-dimensional rules: ≥ 2 dimensions or predicates

- Inter-dimension assoc. rules (*no repeated predicates*)

$\text{age}(X, \text{"19-25"}) \wedge \text{occupation}(X, \text{"student"}) \Rightarrow \text{buys}(X, \text{"coke"})$

- hybrid-dimension assoc. rules (*repeated predicates*)

$\text{age}(X, \text{"19-25"}) \wedge \text{buys}(X, \text{"popcorn"}) \Rightarrow \text{buys}(X, \text{"coke"})$

- Categorical Attributes: finite number of possible values, no ordering among values—data cube approach
- Quantitative Attributes: Numeric, implicit ordering among values—discretization, clustering, and gradient approaches

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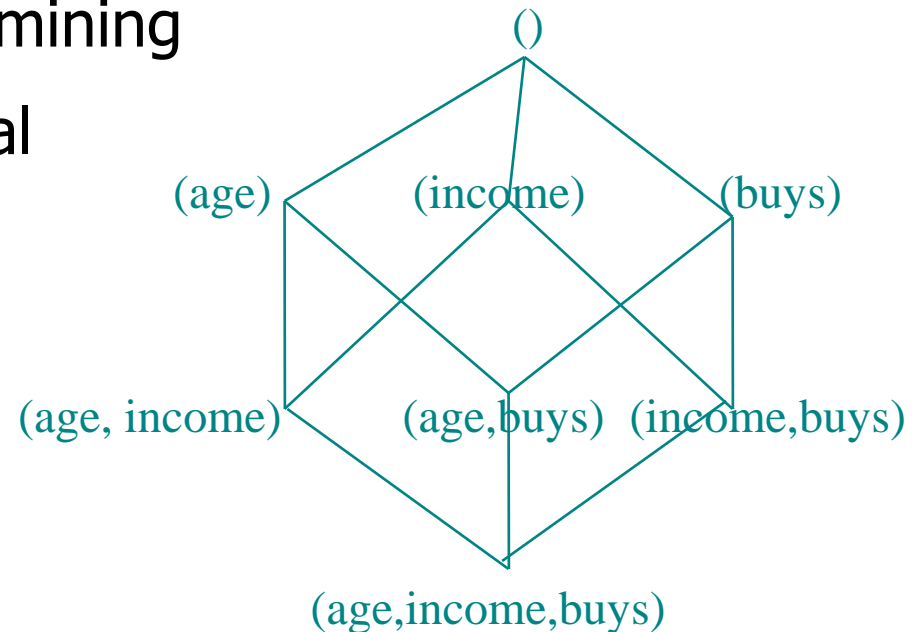
Mining Quantitative Associations

Techniques can be categorized by how numerical attributes, such as **age** or **salary** are treated

1. Static discretization based on predefined concept hierarchies (data cube methods)
2. Dynamic discretization based on data distribution (quantitative rules, e.g., Agrawal & Srikant@SIGMOD96)
3. Clustering: Distance-based association (e.g., Yang & Miller@SIGMOD97)
 - One dimensional clustering then association
4. Deviation: (such as Aumann and Lindell@KDD99)
Sex = female => Wage: mean=\$7/hr (overall mean = \$9)

Static Discretization of Quantitative Attributes

- Discretized prior to mining using concept hierarchy.
- Numeric values are replaced by ranges
- In relational database, finding all frequent k -predicate sets will require k or $k+1$ table scans
- Data cube is well suited for mining
- The cells of an n -dimensional cuboid correspond to the predicate sets
- Mining from data cubes can be much faster



Quantitative Association Rules Based on Statistical Inference Theory [Aumann and Lindell@DMKD'03]

- Finding extraordinary and therefore interesting phenomena, e.g.,
(Sex = female) \Rightarrow Wage: mean=\$7/hr (overall mean = \$9)
 - LHS: a subset of the population
 - RHS: an extraordinary behavior of this subset
- The rule is accepted only if a statistical test (e.g., Z-test) confirms the inference with high confidence
- Subrule: highlights the extraordinary behavior of a subset of the pop. of the super rule
 - E.g., (Sex = female) \wedge (South = yes) \Rightarrow mean wage = \$6.3/hr
- Two forms of rules
 - Categorical \Rightarrow quantitative rules, or Quantitative \Rightarrow quantitative rules
 - E.g., Education in [14-18] (yrs) \Rightarrow mean wage = \$11.64/hr
- Open problem: Efficient methods for LHS containing two or more quantitative attributes

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Negative and Rare Patterns

- Rare patterns: Very low support but interesting
 - E.g., buying Rolex watches
 - Mining: Setting individual-based or special group-based support threshold for valuable items
- Negative patterns
 - Since it is unlikely that one buys Ford Expedition (an SUV car) and Toyota Prius (a hybrid car) together, Ford Expedition and Toyota Prius are likely negatively correlated patterns
- Negatively correlated patterns that are infrequent tend to be more interesting than those that are frequent

Defining Negative Correlated Patterns (I)

- Definition 1 (support-based)
 - If itemsets X and Y are both frequent but rarely occur together, i.e.,
$$\text{sup}(X \cup Y) < \text{sup}(X) * \text{sup}(Y)$$
 - Then X and Y are negatively correlated
- Problem: A store sold two needle 100 packages A and B, only one transaction containing both A and B.
 - When there are in total 200 transactions, we have
$$s(A \cup B) = 0.005, s(A) * s(B) = 0.25, s(A \cup B) < s(A) * s(B)$$
 - When there are 10^5 transactions, we have
$$s(A \cup B) = 1/10^5, s(A) * s(B) = 1/10^3 * 1/10^3, s(A \cup B) > s(A) * s(B)$$
 - Where is the problem? —Null transactions, i.e., the support-based definition is not null-invariant!

Defining Negative Correlated Patterns (II)

- Definition 2 (negative itemset-based)


- X is a *negative itemset* if (1) $X = \bar{A} \cup B$, where B is a set of positive items, and \bar{A} is a set of negative items, $|\bar{A}| \geq 1$, and (2) $s(X) \geq \mu$
- Itemsets X is negatively correlated, if

$$s(X) < \prod_{i=1}^k s(x_i), \text{ where } x_i \in X, \text{ and } s(x_i) \text{ is the support of } x_i$$

- This definition suffers a similar null-invariant problem
- Definition 3 (Kulczynski measure-based) If itemsets X and Y are frequent, but $(P(X|Y) + P(Y|X))/2 < \epsilon$, where ϵ is a negative pattern threshold, then X and Y are negatively correlated.
- Ex. For the same needle package problem, when no matter there are 200 or 10^5 transactions, if $\epsilon = 0.01$, we have

$$(P(A|B) + P(B|A))/2 = (0.01 + 0.01)/2 < \epsilon$$

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Constraint-based (Query-Directed) Mining

- Finding **all** the patterns in a database **autonomously**? — unrealistic!
 - The patterns could be too many but not focused!
- Data mining should be an **interactive** process
 - User directs what to be mined using a **data mining query language** (or a graphical user interface)
- Constraint-based mining
 - User flexibility: provides **constraints** on what to be mined
 - Optimization: explores such constraints for efficient mining — **constraint-based mining**: constraint-pushing, similar to push selection first in DB query processing
 - Note: still find all the answers satisfying constraints, not finding some answers in “heuristic search”

Constraints in Data Mining

- Knowledge type constraint:
 - classification, association, etc.
- Data constraint — using SQL-like queries
 - find product pairs sold together in stores in Chicago this year
- Dimension/level constraint
 - in relevance to region, price, brand, customer category
- Rule (or pattern) constraint
 - small sales (price < \$10) triggers big sales (sum > \$200)
- Interestingness constraint
 - strong rules: $\text{min_support} \geq 3\%$, $\text{min_confidence} \geq 60\%$

Meta-Rule Guided Mining

- Meta-rule can be in the rule form with partially instantiated predicates and constants

$$P_1(X, Y) \wedge P_2(X, W) \Rightarrow \text{buys}(X, \text{"iPad"})$$

- The resulting rule derived can be

$$\text{age}(X, \text{"15-25"}) \wedge \text{profession}(X, \text{"student"}) \Rightarrow \text{buys}(X, \text{"iPad"})$$

- In general, it can be in the form of

$$P_1 \wedge P_2 \wedge \dots \wedge P_l \Rightarrow Q_1 \wedge Q_2 \wedge \dots \wedge Q_r$$

- Method to find meta-rules

- Find frequent (l+r) predicates (based on min-support threshold)
- Push constants deeply when possible into the mining process (see the remaining discussions on constraint-push techniques)
- Use confidence, correlation, and other measures when possible

Constraint-Based Frequent Pattern Mining

- Pattern space pruning constraints
 - **Anti-monotonic**: If constraint c is violated, its further mining can be terminated
 - **Monotonic**: If c is satisfied, no need to check c again
 - **Succinct**: c must be satisfied, so one can start with the data sets satisfying c
 - **Convertible**: c is not monotonic nor anti-monotonic, but it can be converted into it if items in the transaction can be properly ordered
- Data space pruning constraint
 - **Data succinct**: Data space can be pruned at the initial pattern mining process
 - **Data anti-monotonic**: If a transaction t does not satisfy c , t can be pruned from its further mining

Pattern Space Pruning with Anti-Monotonicity Constraints

- A constraint C is *anti-monotone* if the super pattern satisfies C , all of its sub-patterns do so too
- In other words, *anti-monotonicity*: If an itemset S **violates** the constraint, so does any of its superset
- Ex. 1. $\text{sum}(S.\text{price}) \leq v$ is **anti-monotone**
- Ex. 2. $\text{range}(S.\text{profit}) \leq 15$ is **anti-monotone**
 - Itemset ab violates C
 - So does every superset of ab
- Ex. 3. $\text{sum}(S.\text{Price}) \geq v$ is **not anti-monotone**
- Ex. 4. *support count* is anti-monotone: core property used in Apriori

TDB (min_sup=2)

| TID | Transaction |
|-----|------------------|
| 10 | a, b, c, d, f |
| 20 | b, c, d, f, g, h |
| 30 | a, c, d, e, f |
| 40 | c, e, f, g |

| Item | Profit |
|------|--------|
| a | 40 |
| b | 0 |
| c | -20 |
| d | 10 |
| e | -30 |
| f | 30 |
| g | 20 |
| h | -10 |

Pattern Space Pruning with Monotonicity Constraints

- A constraint C is *monotone* if the pattern satisfies C , we do not need to check C in subsequent mining
- Alternatively, monotonicity: *If an itemset S **satisfies** the constraint, so does any of its superset*
- Ex. 1. $\text{sum}(S.\text{Price}) \geq v$ is **monotone**
- Ex. 2. $\text{min}(S.\text{Price}) \leq v$ is **monotone**
- Ex. 3. $C: \text{range}(S.\text{profit}) \geq 15$
 - Itemset ab satisfies C
 - So does every superset of ab

TDB (min_sup=2)

| TID | Transaction |
|-----|------------------|
| 10 | a, b, c, d, f |
| 20 | b, c, d, f, g, h |
| 30 | a, c, d, e, f |
| 40 | c, e, f, g |

| Item | Profit |
|------|--------|
| a | 40 |
| b | 0 |
| c | -20 |
| d | 10 |
| e | -30 |
| f | 30 |
| g | 20 |
| h | -10 |

Data Space Pruning with Data Anti-monotonicity

- A constraint c is *data anti-monotone* if for a pattern p cannot satisfy a transaction t under c , p 's superset cannot satisfy t under c either
- The key for data anti-monotone is *recursive data reduction*
- Ex. 1. $sum(S.Price) \geq v$ is data anti-monotone
- Ex. 2. $min(S.Price) \leq v$ is data anti-monotone
- Ex. 3. $C: range(S.profit) \geq 25$ is data anti-monotone
 - Itemset $\{b, c\}$'s projected DB:
 - T10': $\{d, f, h\}$, T20': $\{d, f, g, h\}$, T30': $\{d, f, g\}$
 - since C cannot satisfy T10', T10' can be pruned

TDB (min_sup=2)

| TID | Transaction |
|-----|------------------|
| 10 | a, b, c, d, f, h |
| 20 | b, c, d, f, g, h |
| 30 | b, c, d, f, g |
| 40 | c, e, f, g |

| Item | Profit |
|------|--------|
| a | 40 |
| b | 0 |
| c | -20 |
| d | -15 |
| e | -30 |
| f | -10 |
| g | 20 |
| h | -5 |

Pattern Space Pruning with Succinctness

- Succinctness:
 - Given A_1 , the set of items satisfying a succinctness constraint C , then any set S satisfying C is based on A_1 , i.e., S contains a subset belonging to A_1
 - Idea: Without looking at the transaction database, whether an itemset S satisfies constraint C can be determined based on the selection of items
 - $\min(S.Price) \leq v$ is succinct
 - $\sum(S.Price) \geq v$ is not succinct
- Optimization: If C is succinct, C is pre-counting pushable

Naïve Algorithm: Apriori + Constraint

Database D

| TID | Items |
|-----|---------|
| 100 | 1 3 4 |
| 200 | 2 3 5 |
| 300 | 1 2 3 5 |
| 400 | 2 5 |

C_1

| itemset | sup. |
|---------|------|
| {1} | 2 |
| {2} | 3 |
| {3} | 3 |
| {4} | 1 |
| {5} | 3 |

Scan D

L_1

| itemset | sup. |
|----------------|--------------|
| {1} | 2 |
| {2} | 3 |
| {3} | 3 |
| {5} | 3 |

C_2

| itemset | sup |
|---------|-----|
| {1 2} | 1 |
| {1 3} | 2 |
| {1 5} | 1 |
| {2 3} | 2 |
| {2 5} | 3 |
| {3 5} | 2 |

Scan D

C_2

| itemset |
|---------|
| {1 2} |
| {1 3} |
| {1 5} |
| {2 3} |
| {2 5} |
| {3 5} |

L_2

| itemset | sup |
|------------------|--------------|
| {1 3} | 2 |
| {2 3} | 2 |
| {2 5} | 3 |
| {3 5} | 2 |

C_3

| itemset |
|---------|
| {2 3 5} |

L_3

| itemset | sup |
|--------------------|--------------|
| {2 3 5} | 2 |

Constraint:
 $\text{Sum}\{S.\text{price}\} < 5$

Constrained Apriori : Push a Succinct Constraint

Deep

Database D

| TID | Items |
|-----|---------|
| 100 | 1 3 4 |
| 200 | 2 3 5 |
| 300 | 1 2 3 5 |
| 400 | 2 5 |

Scan D

C_1

| itemset | sup. |
|---------|------|
| {1} | 2 |
| {2} | 3 |
| {3} | 3 |
| {4} | 1 |
| {5} | 3 |

L_1

| itemset | sup. |
|---------|------|
| {1} | 2 |
| {2} | 3 |
| {3} | 3 |
| {5} | 3 |

C_2

| itemset | sup |
|------------------|-----|
| {1 2} | 1 |
| {1 3} | 2 |
| {1 5} | 1 |
| {2 3} | 2 |
| {2 5} | 3 |
| {3 5} | 2 |

Scan D

C_2

| itemset |
|------------------|
| {1 2} |
| {1 3} |
| {1 5} |
| {2 3} |
| {2 5} |
| {3 5} |

not immediately
to be used

L_2

| itemset | sup |
|------------------|-----|
| {1 3} | 2 |
| {2 3} | 2 |
| {2 5} | 3 |
| {3 5} | 2 |

C_3

| itemset |
|--------------------|
| {2 3 5} |

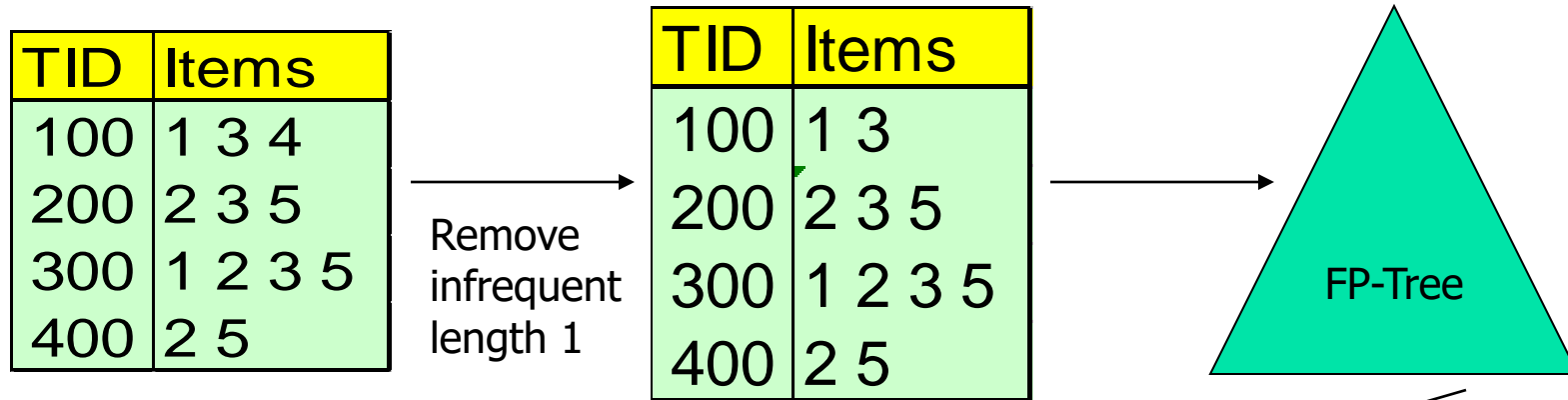
Scan D

L_3

| itemset | sup |
|--------------------|-----|
| {2 3 5} | 2 |

Constraint:
 $\min\{S.price\} \leq 1$

Constrained FP-Growth: Push a Succinct Constraint Deep



1-Projected DB

| TID | Items |
|-----|-------|
| 100 | 3 4 |
| 300 | 2 3 5 |

No Need to project on 2, 3, or 5

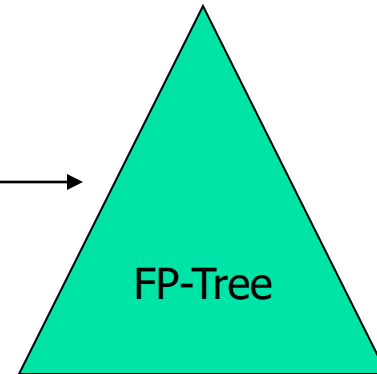
Constraint:
 $\min\{S.\text{price}\} \leq 1$

Constrained FP-Growth: Push a Data Anti-monotonic Constraint Deep

Remove from data

| TID | Items |
|----------------|------------------|
| 100 | 1 3 4 |
| 200 | 2 3 5 |
| 300 | 1 2 3 5 |
| 400 | 2 5 |

| TID | Items |
|-----|-------|
| 100 | 1 3 |
| 300 | 1 3 |



Single branch, we are done

Constraint:

$\min\{S.\text{price}\} \leq 1$

Constrained FP-Growth: Push a Data Anti-monotonic Constraint Deep

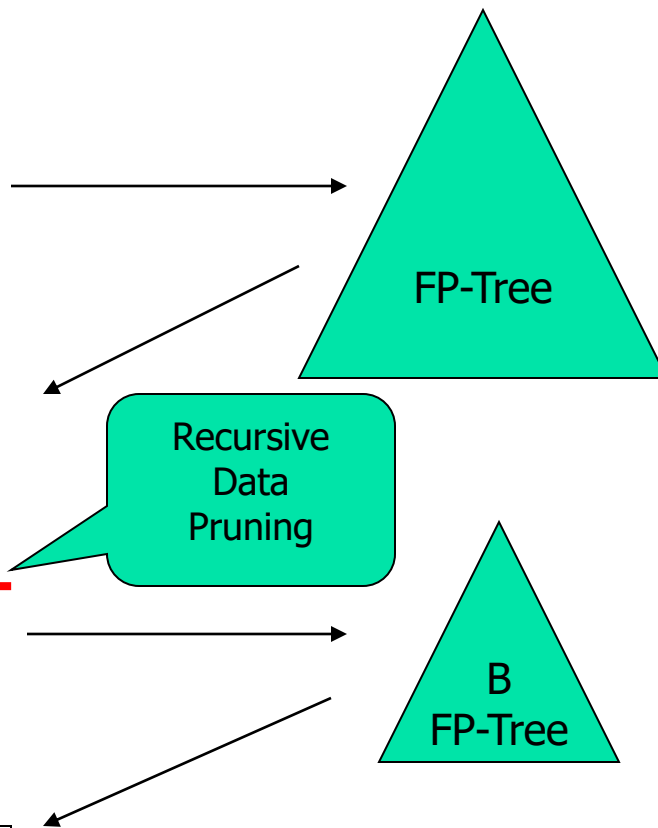
| TID | Transaction |
|-----|------------------|
| 10 | a, b, c, d, f, h |
| 20 | b, c, d, f, g, h |
| 30 | b, c, d, f, g |
| 40 | a, c, e, f, g |

| TID | Transaction |
|-----|------------------|
| 10 | a, b, c, d, f, h |
| 20 | b, c, d, f, g, h |
| 30 | b, c, d, f, g |
| 40 | a, c, e, f, g |

B-Projected DB

| TID | Transaction |
|---------------|--------------------------|
| 10 | a, c, d, f, h |
| 20 | c, d, f, g, h |
| 30 | c, d, f, g |

Single branch:
bcdfg: 2



| Item | Profit |
|------|--------|
| a | 40 |
| b | 0 |
| c | -20 |
| d | -15 |
| e | -30 |
| f | -10 |
| g | 20 |
| h | -5 |

Constraint:
 $\text{range}\{S.\text{price}\} > 25$
 $\text{min_sup} \geq 2$

Convertible Constraints: Ordering Data in Transactions

- Convert tough constraints into anti-monotone or monotone by properly ordering items
- Examine C: $\text{avg}(S.\text{profit}) \geq 25$
 - Order items in value-descending order
 - $\langle a, f, g, d, b, h, c, e \rangle$
 - If an itemset afb violates C
 - So does $afbh, afb^*$
 - It becomes **anti-monotone!**

TDB (min_sup=2)

| TID | Transaction |
|-----|------------------|
| 10 | a, b, c, d, f |
| 20 | b, c, d, f, g, h |
| 30 | a, c, d, e, f |
| 40 | c, e, f, g |

| Item | Profit |
|------|--------|
| a | 40 |
| b | 0 |
| c | -20 |
| d | 10 |
| e | -30 |
| f | 30 |
| g | 20 |
| h | -10 |

Strongly Convertible Constraints

- $\text{avg}(X) \geq 25$ is convertible anti-monotone w.r.t. item **value descending** order R : $\langle a, f, g, d, b, h, c, e \rangle$
 - If an itemset af violates a constraint C , so does every itemset with af as prefix, such as afd
- $\text{avg}(X) \geq 25$ is convertible monotone w.r.t. item **value ascending** order R^{-1} : $\langle e, c, h, b, d, g, f, a \rangle$
 - If an itemset d satisfies a constraint C , so do itemsets df and dfa , which have d as a prefix
- Thus, $\text{avg}(X) \geq 25$ is **strongly convertible**

| Item | Profit |
|------|--------|
| a | 40 |
| b | 0 |
| c | -20 |
| d | 10 |
| e | -30 |
| f | 30 |
| g | 20 |
| h | -10 |

Can Apriori Handle Convertible Constraints?

- A convertible, not monotone nor anti-monotone nor succinct constraint cannot be pushed deep into the an Apriori mining algorithm
 - Within the level wise framework, no direct pruning based on the constraint can be made
 - Itemset df violates constraint $C: \text{avg}(X) \geq 25$
 - Since adf satisfies C , Apriori needs df to assemble adf , df cannot be pruned
- But it can be pushed into frequent-pattern growth framework!

| Item | Value |
|------|-------|
| a | 40 |
| b | 0 |
| c | -20 |
| d | 10 |
| e | -30 |
| f | 30 |
| g | 20 |
| h | -10 |

Pattern Space Pruning w. Convertible Constraints

- C: $\text{avg}(X) \geq 25$, $\text{min_sup}=2$
- List items in every transaction in value descending order R: $\langle a, f, g, d, b, h, c, e \rangle$
 - C is convertible anti-monotone w.r.t. R
- Scan TDB once
 - remove infrequent items
 - Item h is dropped
 - Itemsets a and f are good, ...
- Projection-based mining
 - Imposing an appropriate order on item projection
 - Many tough constraints can be converted into (anti)-monotone

| Item | Value |
|------|-------|
| a | 40 |
| f | 30 |
| g | 20 |
| d | 10 |
| b | 0 |
| h | -10 |
| c | -20 |
| e | -30 |

TDB ($\text{min_sup}=2$)

| TID | Transaction |
|-----|---------------|
| 10 | a, f, d, b, c |
| 20 | f, g, d, b, c |
| 30 | a, f, d, c, e |
| 40 | f, g, h, c, e |

Handling Multiple Constraints

- Different constraints may require different or even conflicting item-ordering
- If there exists an order R s.t. both C_1 and C_2 are convertible w.r.t. R , then there is no conflict between the two convertible constraints
- If there exists conflict on order of items
 - Try to satisfy one constraint first
 - Then using the order for the other constraint to mine frequent itemsets in the corresponding projected database


What Constraints Are Convertible?

| Constraint | Convertible anti-monotone | Convertible monotone | Strongly convertible |
|---|---------------------------|----------------------|----------------------|
| $\text{avg}(S) \leq, \geq v$ | Yes | Yes | Yes |
| $\text{median}(S) \leq, \geq v$ | Yes | Yes | Yes |
| $\text{sum}(S) \leq v$ (items could be of any value, $v \geq 0$) | Yes | No | No |
| $\text{sum}(S) \leq v$ (items could be of any value, $v \leq 0$) | No | Yes | No |
| $\text{sum}(S) \geq v$ (items could be of any value, $v \geq 0$) | No | Yes | No |
| $\text{sum}(S) \geq v$ (items could be of any value, $v \leq 0$) | Yes | No | No |
| | | | |

Constraint-Based Mining — A General Picture

| Constraint | Anti-monotone | Monotone | Succinct |
|--|---------------|-------------|----------|
| $v \in S$ | no | yes | yes |
| $S \supseteq V$ | no | yes | yes |
| $S \subseteq V$ | yes | no | yes |
| $\min(S) \leq v$ | no | yes | yes |
| $\min(S) \geq v$ | yes | no | yes |
| $\max(S) \leq v$ | yes | no | yes |
| $\max(S) \geq v$ | no | yes | yes |
| $\text{count}(S) \leq v$ | yes | no | weakly |
| $\text{count}(S) \geq v$ | no | yes | weakly |
| $\text{sum}(S) \leq v \ (a \in S, a \geq 0)$ | yes | no | no |
| $\text{sum}(S) \geq v \ (a \in S, a \geq 0)$ | no | yes | no |
| $\text{range}(S) \leq v$ | yes | no | no |
| $\text{range}(S) \geq v$ | no | yes | no |
| $\text{avg}(S) \theta v, \theta \in \{=, \leq, \geq\}$ | convertible | convertible | no |
| $\text{support}(S) \geq \xi$ | yes | no | no |
| $\text{support}(S) \leq \xi$ | no | yes | no |

Advanced Frequent Pattern Mining

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Mining Colossal Frequent Patterns

- F. Zhu, X. Yan, J. Han, P. S. Yu, and H. Cheng, “Mining Colossal Frequent Patterns by Core Pattern Fusion”, ICDE'07.
- We have many algorithms, but can we mine large (i.e., colossal) patterns? — such as just size around 50 to 100? Unfortunately, not!
- Why not? — the curse of “downward closure” of frequent patterns
 - The “downward closure” property
 - Any sub-pattern of a frequent pattern is frequent.
 - Example. If $(a_1, a_2, \dots, a_{100})$ is frequent, then $a_1, a_2, \dots, a_{100}, (a_1, a_2), (a_1, a_3), \dots, (a_1, a_{100}), (a_1, a_2, a_3), \dots$ are all frequent! There are about 2^{100} such frequent itemsets!
 - No matter using breadth-first search (e.g., Apriori) or depth-first search (FPgrowth), we have to examine so many patterns
- Thus the downward closure property leads to explosion!

Colossal Patterns: A Motivating Example

Let's make a set of 40 transactions

T1 = 1 2 3 4 39 40

T2 = 1 2 3 4 39 40

⋮

.

⋮

.

⋮

.

⋮

.

T40=1 2 3 4 39 40

Then delete the items on the diagonal

T1 = 2 3 4 39 40

T2 = 1 3 4 39 40

⋮

.

⋮

.

⋮

.

⋮

.

T40=1 2 3 4 39

Closed/maximal patterns may partially alleviate the problem but not really solve it: We often need to mine scattered large patterns!

Let the minimum support threshold $\sigma = 20$

There are $\binom{40}{20}$ frequent patterns of size 20

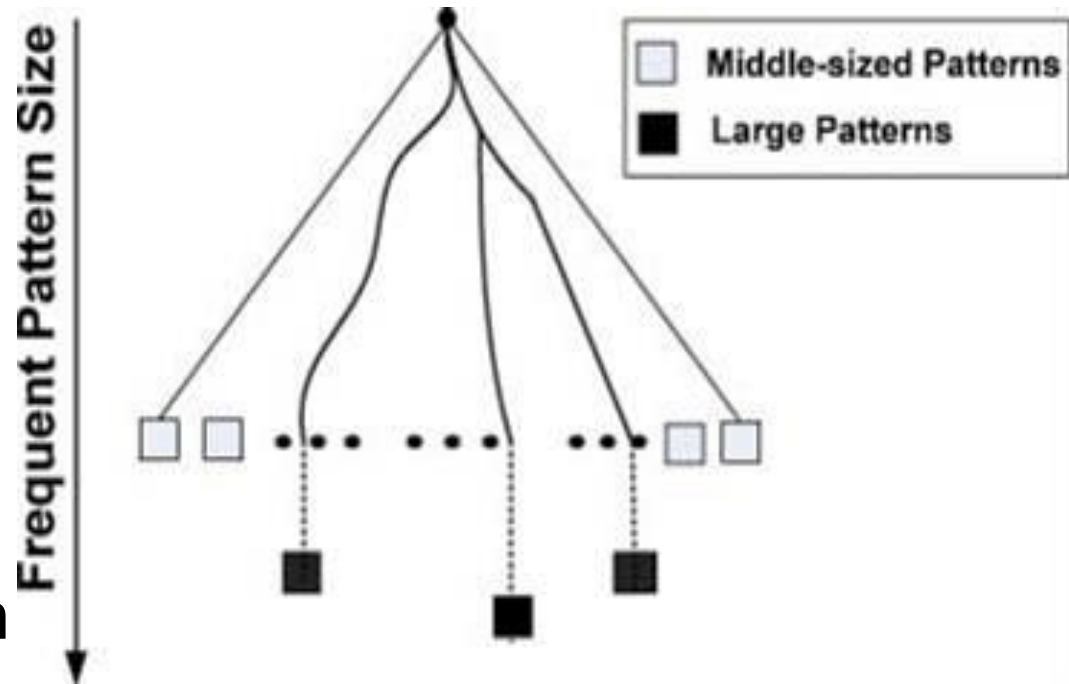
Each is closed and maximal

$$\# \text{ patterns} = \binom{n}{n/2} \approx \sqrt{2/\pi} \frac{2^n}{\sqrt{n}}$$

The size of the answer set is exponential to n

Colossal Pattern Set: Small but Interesting

- It is often the case that only a small number of patterns are colossal, i.e., of large size
- Colossal patterns are usually attached with greater importance than those of small pattern sizes



Mining Colossal Patterns: Motivation and Philosophy

- Motivation: Many real-world tasks need mining colossal patterns
 - Micro-array analysis in bioinformatics (when support is low)
 - Biological sequence patterns
 - Biological/sociological/information graph pattern mining
- *No hope for completeness*
 - If the mining of mid-sized patterns is explosive in size, there is no hope to find colossal patterns efficiently by insisting “complete set” mining philosophy
- *Jumping out of the swamp of the mid-sized results*
 - What we may develop is a philosophy that may jump out of the swamp of mid-sized results that are explosive in size and jump to reach colossal patterns
- *Striving for mining almost complete colossal patterns*
 - The key is to develop a mechanism that may quickly reach colossal patterns and discover most of them

Alas, A Show of Colossal Pattern Mining!

T₁ = 2 3 4 39 40
T₂ = 1 3 4 39 40
:
:
:
:
:
T₄₀ = 1 2 3 4 39
T₄₁ = 41 42 43 79
T₄₂ = 41 42 43 79
:
:
:
T₆₀ = 41 42 43 ... 79

Let the min-support threshold $\sigma = 20$

Then there are $\binom{40}{20}$ closed/maximal frequent patterns of size 20

However, there is only one with size greater than 20, (*i.e.*, colossal):

$\alpha = \{41, 42, \dots, 79\}$ of size 39

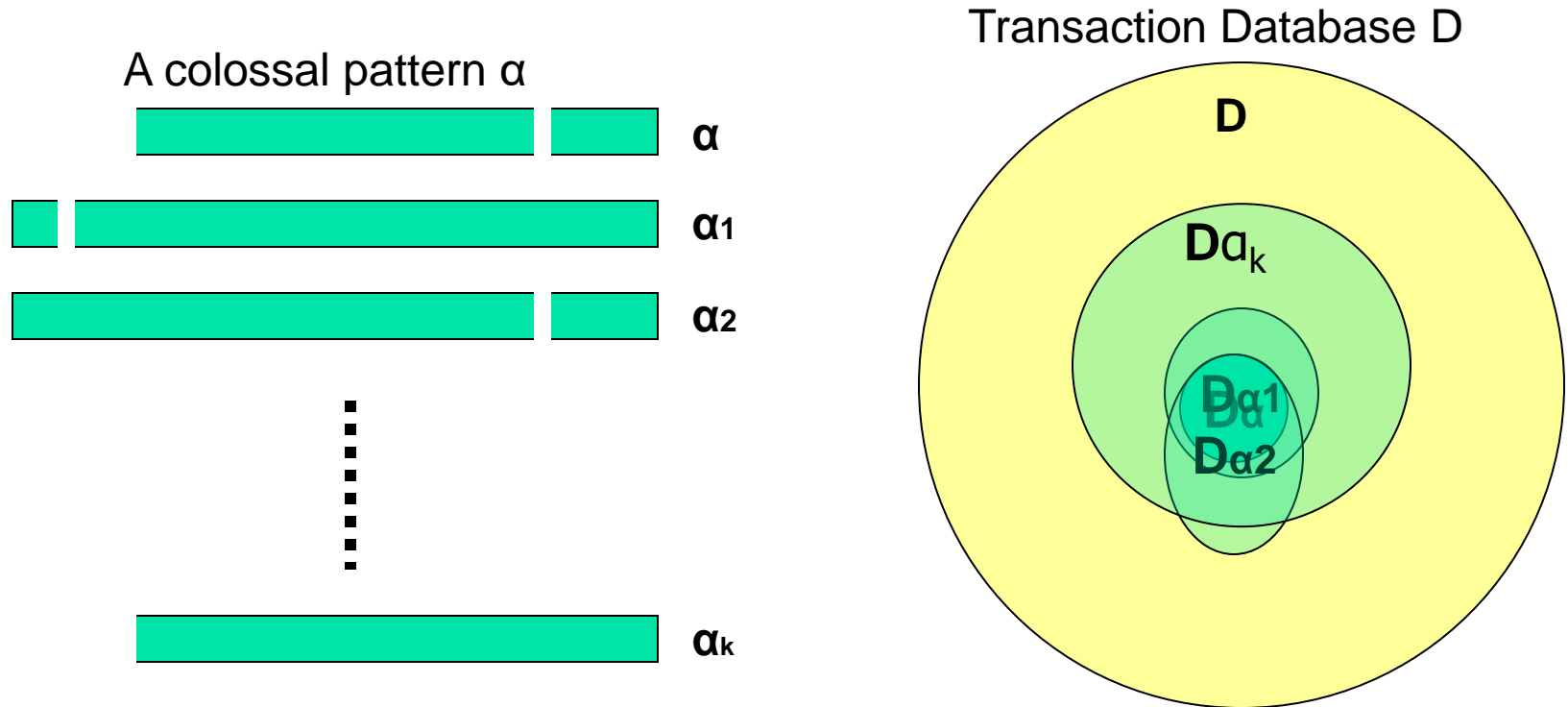
The existing fastest mining algorithms (*e.g.*, FPClose, LCM) fail to complete running

Our algorithm outputs this colossal pattern in seconds

Methodology of Pattern-Fusion Strategy

- Pattern-Fusion traverses the tree in a bounded-breadth way
 - Always pushes down a frontier of a bounded-size candidate pool
 - Only a fixed number of patterns in the current candidate pool will be used as the starting nodes to go down in the pattern tree — thus avoids the exponential search space
- Pattern-Fusion identifies “shortcuts” whenever possible
 - Pattern growth is not performed by single-item addition but by leaps and bounded: agglomeration of multiple patterns in the pool
 - These shortcuts will direct the search down the tree much more rapidly towards the colossal patterns

Observation: Colossal Patterns and Core Patterns



Subpatterns α_1 to α_k cluster tightly around the colossal pattern α by sharing a similar support. We call such subpatterns *core patterns* of α

Robustness of Colossal Patterns

- Core Patterns

Intuitively, for a frequent pattern α , a subpattern β is a τ -core pattern of α if β shares a similar support set with α , i.e.,

$$\frac{|D_{\alpha}|}{|D_{\beta}|} \geq \tau \quad 0 < \tau \leq 1$$

where τ is called the core ratio

- Robustness of Colossal Patterns

A colossal pattern is robust in the sense that it tends to have much more core patterns than small patterns

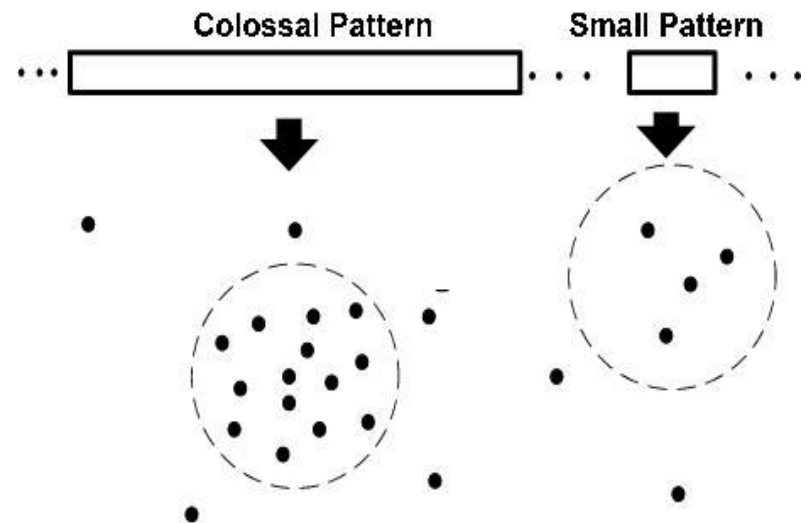
Example: Core Patterns

- A colossal pattern has far more core patterns than a small-sized pattern
- A colossal pattern has far more core descendants of a smaller size c
- A random draw from a complete set of pattern of size c would more likely to pick a core descendant of a colossal pattern
- A colossal pattern can be generated by merging a set of core patterns

| Transaction (# of Ts) | Core Patterns ($\tau = 0.5$) |
|-----------------------|--|
| (abe) (100) | (abe), (ab), (be), (ae), (e) |
| (bcf) (100) | (bcf), (bc), (bf) |
| (acf) (100) | (acf), (ac), (af) |
| (abcef) (100) | (ab), (ac), (af), (ae), (bc), (bf), (be), (ce), (fe), (e), (abc), (abf), (abe), (ace), (acf), (afe), (bcf), (bce), (bfe), (cfe), (abcf), (abce), (bcfe), (acfe), (abfe), (abcef) |

Colossal Patterns Correspond to Dense Balls

- Due to their robustness, colossal patterns correspond to dense balls
 - $\Omega(2^d)$ in population
- A random draw in the pattern space will hit somewhere in the ball with high probability



Idea of Pattern-Fusion Algorithm

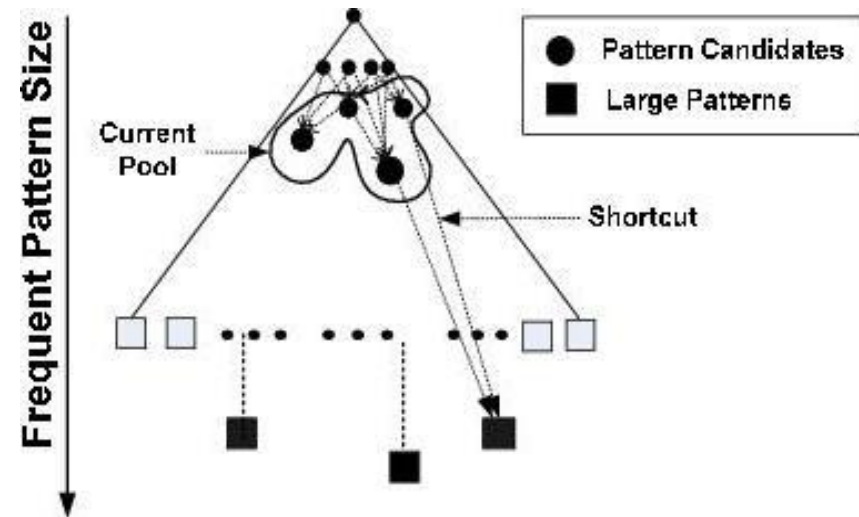
- Generate a complete set of frequent patterns up to a small size
- Randomly pick a pattern β , and β has a high probability to be a core-descendant of some colossal pattern α
- Identify all α 's descendants in this complete set, and merge all of them — This would generate a much larger core-descendant of α
- In the same fashion, we select K patterns. This set of larger core-descendants will be the candidate pool for the next iteration

Pattern-Fusion: The Algorithm

- Initialization (Initial pool): Use an existing algorithm to mine all frequent patterns up to a small size, e.g., 3
- Iteration (Iterative Pattern Fusion):
 - At each iteration, k seed patterns are randomly picked from the current pattern pool
 - For each seed pattern thus picked, we find all the patterns within a bounding ball centered at the seed pattern
 - All these patterns found are fused together to generate a set of super-patterns. All the super-patterns thus generated form a new pool for the next iteration
- Termination: when the current pool contains no more than K patterns at the beginning of an iteration

Why Is Pattern-Fusion Efficient?

- A bounded-breadth pattern tree traversal
 - It avoids explosion in mining mid-sized ones
 - Randomness comes to help to stay on the right path
- Ability to identify “short-cuts” and take “leaps”
 - fuse small patterns together in one step to generate new patterns of significant sizes
 - Efficiency



Pattern-Fusion Leads to Good Approximation

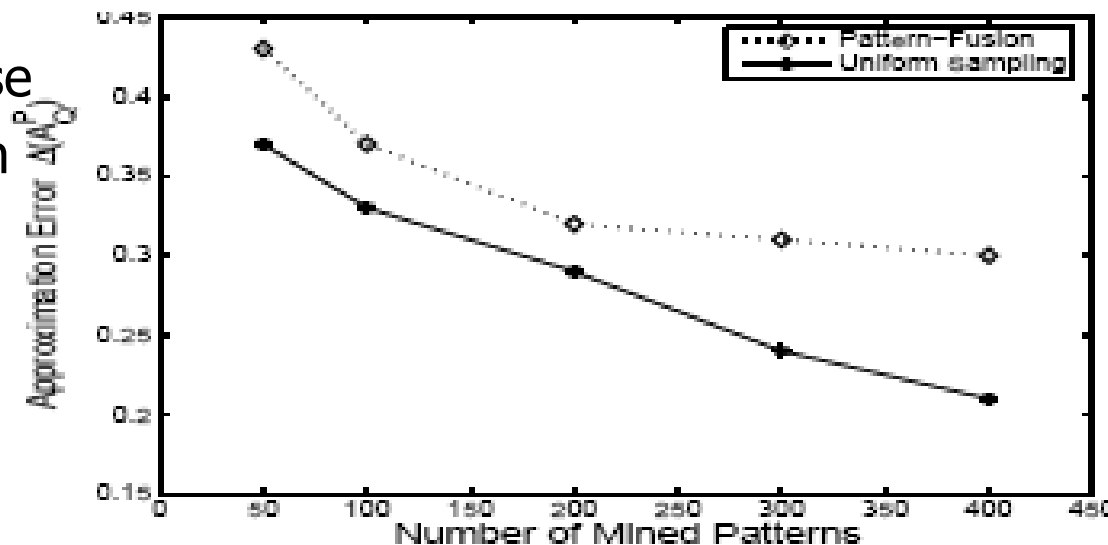
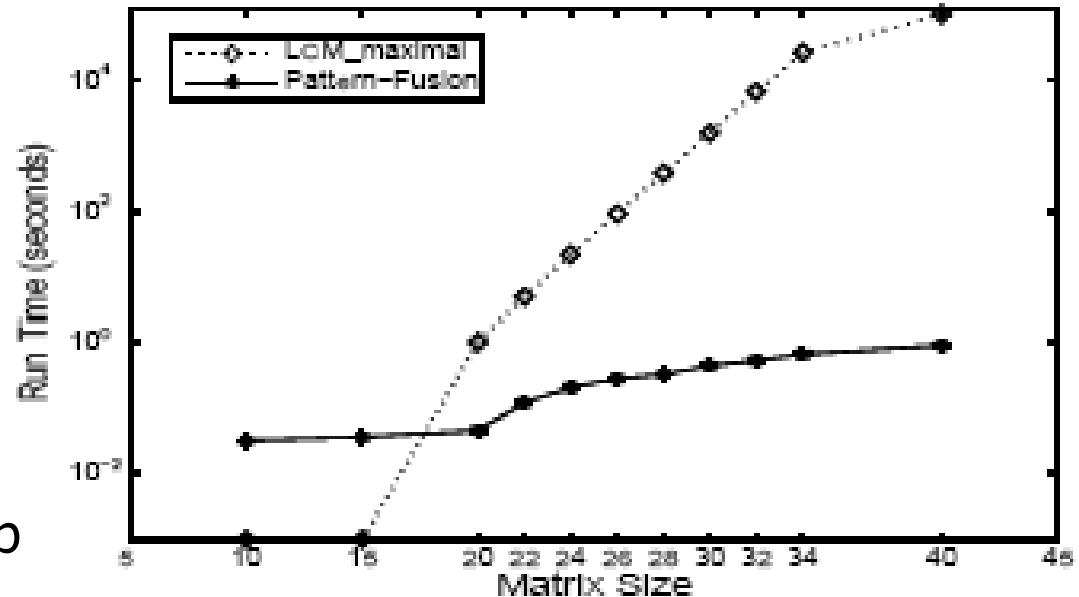
- Gearing toward colossal patterns
 - The larger the pattern, the greater the chance it will be generated
- Catching outliers
 - The more distinct the pattern, the greater the chance it will be generated

Experimental Setting

- Synthetic data set
 - Diag_n an $n \times (n-1)$ table where i^{th} row has integers from 1 to n except i . Each row is taken as an itemset. min_support is $n/2$.
- Real data set
 - Replace: A program trace data set collected from the “replace” program, widely used in software engineering research
 - ALL: A popular gene expression data set, a clinical data on ALL-AML leukemia (www.broad.mit.edu/tools/data.html).
 - Each item is a column, representing the activity level of gene/protein in the same
 - Frequent pattern would reveal important correlation between gene expression patterns and disease outcomes

Experiment Results on Diag_n

- LCM run time increases exponentially with pattern size n
- Pattern-Fusion finishes efficiently
- The approximation error of Pattern-Fusion (with min-sup 20) in comparison with the complete set) is rather close to uniform sampling (which randomly picks K patterns from the complete answer set)

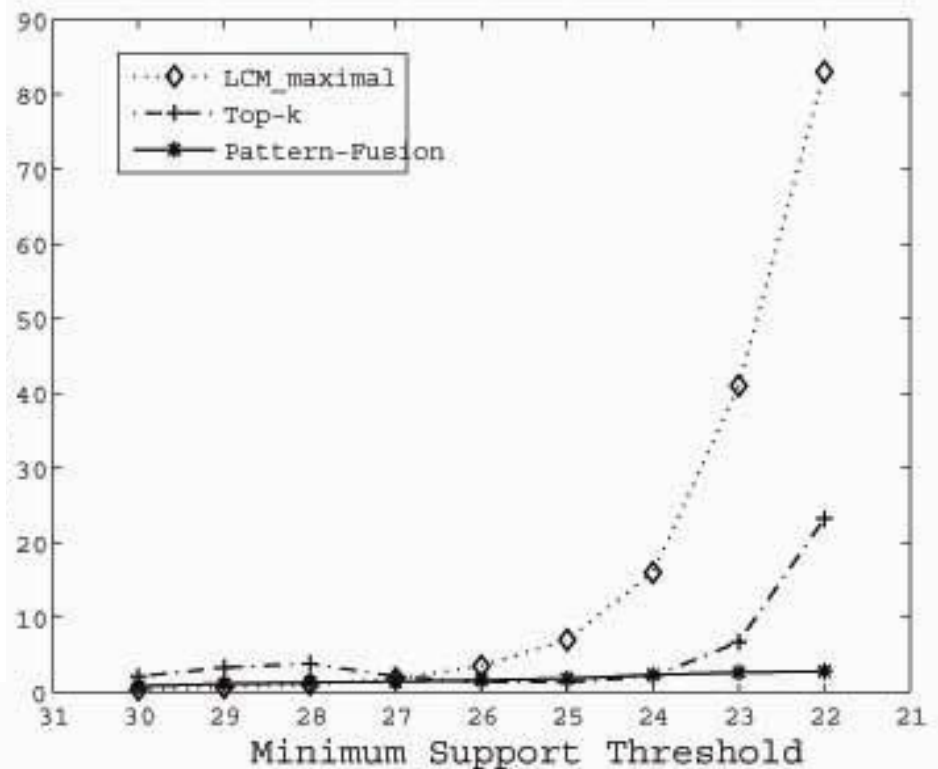


Experimental Results on ALL

- ALL: A popular gene expression data set with 38 transactions, each with 866 columns
 - There are 1736 items in total
 - The table shows a high frequency threshold of 30

| | | | | | | | |
|------------------|-----|-----|-----|----|----|----|----|
| Pattern Size | 110 | 107 | 102 | 91 | 86 | 84 | 83 |
| The complete set | 1 | 1 | 1 | 1 | 1 | 2 | 6 |
| Pattern-Fusion | 1 | 1 | 1 | 1 | 1 | 1 | 4 |

| | | | | | | | |
|------------------|----|----|----|----|----|----|----|
| Pattern Size | 82 | 77 | 76 | 75 | 74 | 73 | 71 |
| The complete set | 1 | 2 | 1 | 1 | 1 | 2 | 1 |
| Pattern-Fusion | 0 | 2 | 0 | 1 | 1 | 1 | 1 |



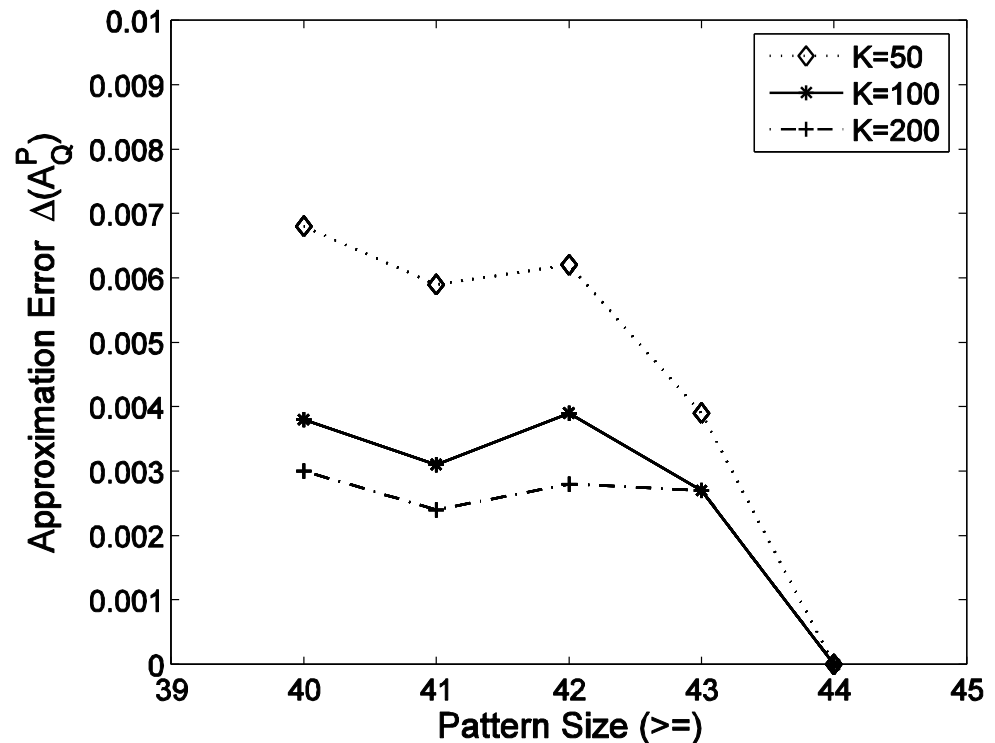
Experimental Results on REPLACE

■ REPLACE


- A program trace data set, recording 4395 calls and transitions
- The data set contains 4395 transactions with 57 items in total
- With support threshold of 0.03, the largest patterns are of size 44
- They are all discovered by Pattern-Fusion with different settings of K and τ , when started with an initial pool of 20948 patterns of size ≤ 3

Experimental Results on REPLACE

- Approximation error when compared with the complete mining result
- Example. Out of the total 98 patterns of size ≥ 42 , when $K=100$, Pattern-Fusion returns 80 of them
- A good approximation to the colossal patterns in the sense that any pattern in the complete set is on average at most 0.17 items away from one of these 80 patterns



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Mining Compressed Patterns: δ -clustering

- Why compressed patterns?
 - too many, but less meaningful
- Pattern distance measure

$$D(P_1, P_2) = 1 - \frac{|T(P_1) \cap T(P_2)|}{|T(P_1) \cup T(P_2)|}$$

- δ -clustering: For each pattern P, find all patterns which can be expressed by P and their distance to P are within δ (δ -cover)
- All patterns in the cluster can be represented by P
- Xin et al., "Mining Compressed Frequent-Pattern Sets", VLDB'05

| ID | Item-Sets | Support |
|----|---------------------|---------|
| P1 | {38,16,18,12} | 205227 |
| P2 | {38,16,18,12,17} | 205211 |
| P3 | {39,38,16,18,12,17} | 101758 |
| P4 | {39,16,18,12,17} | 161563 |
| P5 | {39,16,18,12} | 161576 |

- Closed frequent pattern
 - Report P1, P2, P3, P4, P5
 - Emphasize too much on support
 - no compression
- Max-pattern, P3: info loss
- A desirable output: P2, P3, P4

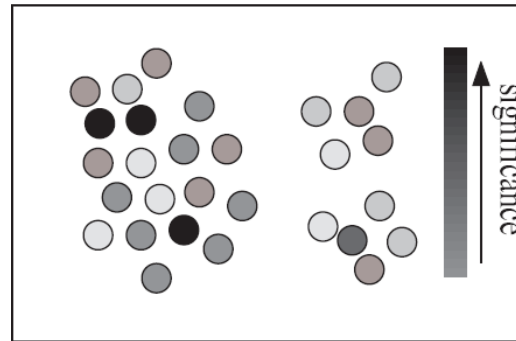
Redundancy-Award Top-k Patterns

- Why redundancy-aware top-k patterns?

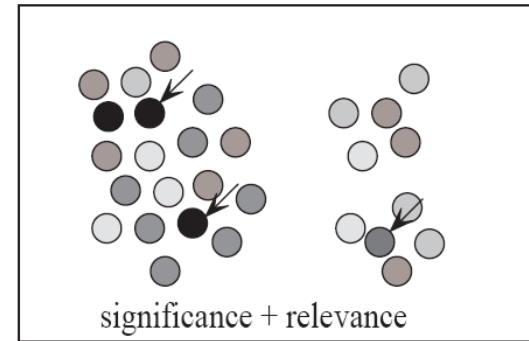
- Desired patterns: high significance & low redundancy

- Propose the MMS (Maximal Marginal Significance) for measuring the combined significance of a pattern set

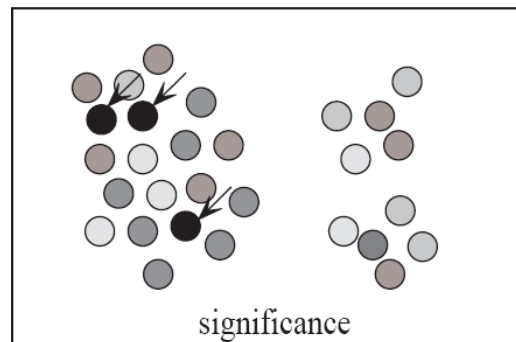
- Xin et al., Extracting Redundancy-Aware Top-K Patterns, KDD'06



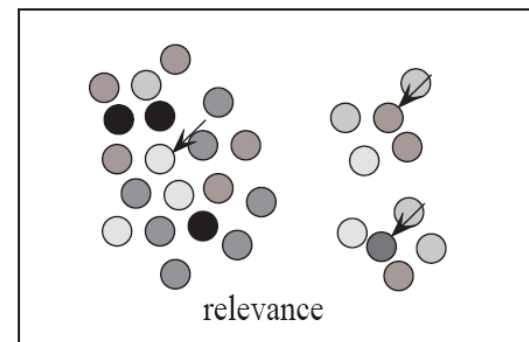
(a) a set of patterns



(b) redundancy-aware top-k




(c) traditional top-k

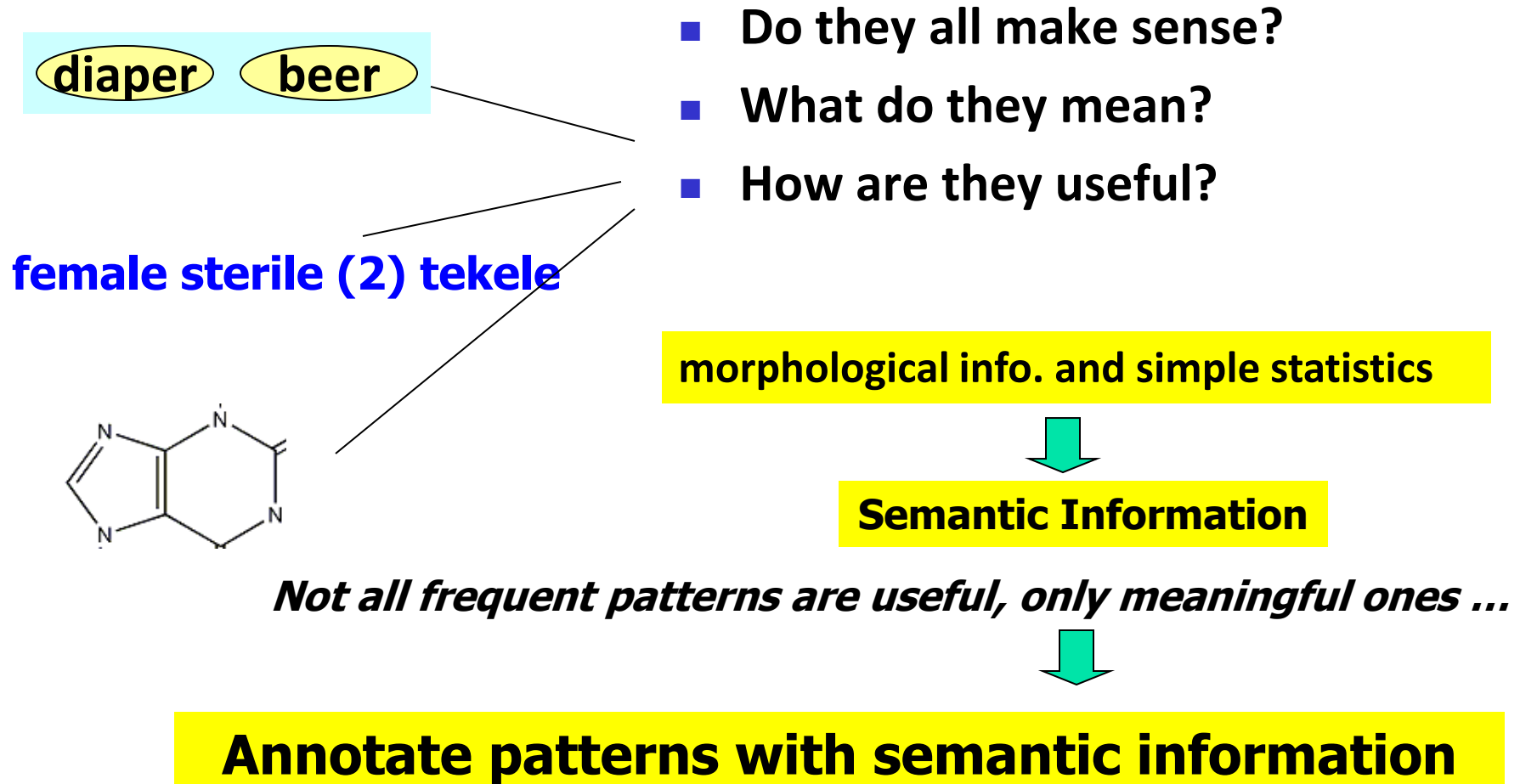


(d) summarization

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How to Understand and Interpret Patterns?



A Dictionary Analogy

Word: "pattern" – from Merriam-Webster

Main Entry: **1** *pat-tern*

Pronunciation: 'pa-tern

Function: *noun*

Etymology: Middle English *patron*, from Middle French, from Latin *patronus*

Date: 14th century

Non-semantic info.

Definitions indicating semantics

1 : a form

2 : something

pattern

3 : a model

4 : an artist

5 : a nature

Main Entry:

pattern

Function:

noun

1

Synonyms

MODEL 2, archetype, beau ideal, ensample, example, exemplar, idea,

error, paradigm, standard

Related Word

original

2

Synonyms

FIGURE 3, design, device, motif, motive

Related Word

patterning

Synonyms

ORDER 8, method, orderliness, plan, system

Related Word

arrangement constellation

Synonyms

Related Words

Semantic Analysis with Context Models

- Task1: Model the context of a frequent pattern

Based on the Context Model...

- Task2: Extract strongest context indicators
- Task3: Extract representative transactions
- Task4: Extract semantically similar patterns

Annotating DBLP Co-authorship & Title Pattern

Database:

| Authors | Title |
|-------------------------------------|---|
| X.Yan , P. Yu, J. Han | Substructure Similarity Search in Graph Databases |
| ... | ... |
| ... | ... |

Frequent Patterns

$P_1: \{x_yan, j_han\}$

Frequent Itemset

$P_2: \text{"substructure search"}$

Semantic Annotations

| Pattern | $\{x_yan, j_han\}$ |
|---------|---|
| Non | Sup = ... |
| CI | $\{p_yu\}$, graph pattern, ... |
| Trans. | gSpan : graph-base..... |
| SSPs | $\{j_wang\}$, $\{j_han, p_yu\}$, ... |

Context Units

$\langle \{p_yu, j_han\}, \{d_xin\}, \dots, \text{"graph pattern"}, \dots \text{"substructure similarity"}, \dots \rangle$

Pattern = {xifeng_yan, jiawei_han}

Annotation Results:

| | |
|-------------------------------------|---|
| Context Indicator (CI) | graph; {philip_yu}; mine close; graph pattern; sequential pattern; ... |
| Representative Transactions (Trans) | > gSpan: graph-base substructure pattern mining; > mining close relational graph connect constraint; ... |
| Semantically Similar Patterns (SSP) | {jiawei_han, philip_yu}; {jian_pei, jiawei_han}; {jiong_yang, philip_yu, wei_wang}; ... |

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Summary

- Roadmap: Many aspects & extensions on pattern mining
- Mining patterns in multi-level, multi dimensional space
- Mining rare and negative patterns
- Constraint-based pattern mining
- Specialized methods for mining high-dimensional data and colossal patterns
- Mining compressed or approximate patterns
- Pattern exploration and understanding: Semantic annotation of frequent patterns

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