Chapter 1

Theoretical Part

1.1 Equation of Motions

We will consider the equation of motion, there are only two force that are important

$$m\vec{a} = m\vec{g} + \vec{F}_{aerodynamics} + \vec{F}_{inertia} \tag{1.1}$$

$$I\ddot{\omega} = \vec{M}_{gravity} + \vec{M}_{aerodynamics} + \vec{M}_{inertia} \tag{1.2}$$

The inertia force relate to this motion is due to the acceleration of the airplane only, therefore

$$\vec{F}_{inertia} = -m\vec{a}_{airplane} \tag{1.3}$$

The aerodynamic force could be projected into two components, the lift and the drag; however, in this general configuration, we will project them generally into three compments along three axes. First, we have the components of aerodynamic force based on lift, drag and lateral directions:

$$\vec{F}_{aerodynamics} = \frac{1}{2}\rho_\infty \|\vec{v}_{ice} - \vec{v}_{air}\|(\vec{v}_{ice} - \vec{v}_{air})C_{aerodynamics}, \tag{1.4}$$

where $C_{aerodynamics}$ are all aerodynamic coefficients.

For the sake of simplicity, we will employ C_x, C_y and C_z the aerodynamic force coefficients along axes x, y and z of fixed-body coordinates system could be written:

$$\vec{F}_{A,x} = \frac{1}{2}\rho_\infty \left[(\dot{x} - u)^2 + (\dot{y} - v)^2 + (\dot{z} - w)^2\right] C_x \tag{1.5}$$

$$\vec{F}_{A,y} = \frac{1}{2}\rho_\infty \left[(\dot{x} - u)^2 + (\dot{y} - v)^2 + (\dot{z} - w)^2\right] C_y \tag{1.6}$$

$$\vec{F}_{A,z} = \frac{1}{2}\rho_\infty \left[(\dot{x} - u)^2 + (\dot{y} - v)^2 + (\dot{z} - w)^2\right] C_z \tag{1.7}$$

The aerodynamic moment can be written with the same manner:

$$\vec{M}_{A,x} = \frac{1}{2}\rho_\infty \left[(\dot{x}-u)^2 + (\dot{y}-v)^2 + (\dot{z}-w)^2\right] HC_x \tag{1.8}$$

$$\vec{M}_{A,y} = \frac{1}{2}\rho_\infty \left[(\dot{x}-u)^2 + (\dot{y}-v)^2 + (\dot{z}-w)^2\right] HC_y \tag{1.9}$$

$$\vec{M}_{A,z} = \frac{1}{2}\rho_\infty \left[(\dot{x}-u)^2 + (\dot{y}-v)^2 + (\dot{z}-w)^2\right] HC_z \tag{1.10}$$

H is a reference length from ice rectangular cuboid; u, v and w are the velocity of flow field with respect to fixed-body coordinate system.

An arbitrary velocity vector is transformed from a fixed coordinates system to body-fixed coordinates system by the following matrix multiplication:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ -c_\phi s_\psi + s_\phi s_\theta c_\psi & c_\phi c_\psi + s_\phi s_\theta s_\psi & s_\phi c_\theta \\ s_\phi s_\psi + c_\phi s_\theta c_\psi & -s_\phi c_\psi + c_\phi s_\theta s_\psi & c_\phi c_\theta \end{bmatrix} \cdot \begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{bmatrix}, \tag{1.11}$$

where (ψ, θ, ϕ) are Euler's angle: rotating the fixed coordinates system successively angle ψ, θ, ϕ about the axes Z, y' and x''. We denote this matrix as $\underline{\underline{R}}_{\psi,\theta,\phi}$, then we could write the transformation of flow field velocity:

$$\begin{bmatrix}u\\ v\\ w\end{bmatrix} = \underline{\underline{R}}_{\psi,\theta,\phi} \cdot \begin{bmatrix}U\\ V\\ W\end{bmatrix} \tag{1.12}$$

Then we could write the equation of motion with the following notation for the rotation of ice cuboid $\boldsymbol{\omega} = \omega_x\vec{e}_x + \omega_y\vec{e}_y + \omega_z\vec{e}_z$:

$$F_x = m(\dot{x} + \omega_y\dot{z} - \omega_z\dot{y}) \tag{1.13}$$

$$F_y = m(\dot{y} + \omega_z\dot{x} - \omega_x\dot{x}) \tag{1.14}$$

$$F_z = m(\dot{z} + \omega_x\dot{y} - \omega_y\dot{z}) \tag{1.15}$$

$$M_x = I_x\dot{\omega}_x + I_z\omega_z\omega_y - I_y\omega_y\omega_z \tag{1.16}$$

$$M_y = I_y\dot{\omega}_y + I_x\omega_x\omega_z - I_z\omega_z\omega_x \tag{1.17}$$

$$M_z = I_z\dot{\omega}_z + I_y\omega_y\omega_x - I_x\omega_x\omega_y \tag{1.18}$$

where we choose the origin is at the center of mass, then $I_{xy} = I_{yz} = I_{zx} = 0$. This leads to the last three equations of moment.

$$F_i = F_{A,i} + mg_i \quad \text{and} \quad M_i = M_{A,i} \tag{1.19}$$

where

$$\begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = \underline{\underline{R}}_{\psi,\theta,\phi} \cdot \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{1.20}$$

The rest of the problem is to determine aerodynamic coefficients. This is the suject of the following section.