

On the first rectangular domain

$$\begin{aligned}
\psi(x + \Delta x_1, y) &= \psi(x, y) + \frac{\partial \psi}{\partial x} \Delta x_1 + \frac{\partial^2 \psi}{\partial x^2} \frac{\Delta x_1^2}{2} \\
\psi(x - \Delta x_1, y) &= \psi(x, y) - \frac{\partial \psi}{\partial x} \Delta x_1 + \frac{\partial^2 \psi}{\partial x^2} \frac{\Delta x_1^2}{2} \\
\psi(x, y + \Delta y_2) &= \psi(x, y) + \frac{\partial \psi}{\partial y} \Delta y_2 + \frac{\partial^2 \psi}{\partial y^2} \frac{\Delta y_2^2}{2} \\
\psi(x, y - \Delta y_2) &= \psi(x, y) - \frac{\partial \psi}{\partial y} \Delta y_2 + \frac{\partial^2 \psi}{\partial y^2} \frac{\Delta y_2^2}{2}
\end{aligned}$$

Therefore

$$\begin{aligned}
\Delta \psi &= \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\psi(x + \Delta x_1, y) + \psi(x - \Delta x_1, y) - 2\psi(x, y)}{\Delta x_1^2} \\
&\quad + \frac{\psi(x, y + \Delta y_2) + \psi(x, y - \Delta y_2) - 2\psi(x, y)}{\Delta y_2^2}
\end{aligned}$$

$$\Delta \psi = \frac{\psi(i + 1, j) + \psi(i - 1, j) - 2\psi(i, j)}{\Delta x_1^2} + \frac{\psi(i, j + 1) + \psi(i, j - 1) - 2\psi(i, j)}{\Delta y_2^2}$$

$$\gamma_1 \psi(i, j) + \beta_1^2 (\psi(i + 1, j) + \psi(i - 1, j)) + (\psi(i, j + 1) + \psi(i, j - 1)) = 0. \quad (1)$$

On the second rectangular domain, using the same argument with the definition $\beta_3 = \Delta y_3 / \Delta x_3$

$$\gamma_3 \psi(i, j) + \beta_3^2 (\psi(i + 1, j) + \psi(i - 1, j)) + (\psi(i, j + 1) + \psi(i, j - 1)) = 0. \quad (2)$$

On the oblique domain: