$$\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial t} + R\frac{\partial u^2}{\partial x} + R\frac{\partial uv}{\partial y} + R\frac{\partial uw}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + R\frac{\partial vu}{\partial x} + R\frac{\partial v^2}{\partial y} + R\frac{\partial vw}{\partial z} = -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial w}{\partial t} + R\frac{\partial wu}{\partial x} + R\frac{\partial wv}{\partial y} + R\frac{\partial w^2}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}$$

$$p = \rho/\delta$$

$$\begin{aligned}
\frac{u^{n+1}_{i,j,k} - u^n_{i,j,k}}{\Delta t} =& - R\frac{(u^n_{i+1,j,k})^2 - (u^n_{i,j,k})^2}{\Delta x} \\
& - R\frac{u^n_{i,j+1,k}v^n_{i,j+1,k} - u^n_{i,j,k}v^n_{i,j,k}}{\Delta y} \\
& - R\frac{u^n_{i,j,k+1}w^n_{i,j,k+1} - u^n_{i,j,k}w^n_{i,j,k}}{\Delta z} \\
& + \frac{u^n_{i+1,j,k} + u^n_{i-1,j,k} - u^n_{i,j,k} - u^{n+1}_{i,j,k}}{\Delta x^2} \\
& + \frac{u^n_{i,j+1,k} + u^n_{i,j-1,k} - u^n_{i,j,k} - u^{n+1}_{i,j,k}}{\Delta y^2} \\
& + \frac{u^n_{i,j,k-1} + u^n_{i,j,k-1} - u^n_{i,j,k} - u^{n+1}_{i,j,k}}{\Delta z^2} \\
& - \frac{1}{\delta \Delta x}(\rho^n_{i+1,j} - \rho^n_{i,j})
\end{aligned}$$

$$\begin{aligned}
\frac{v^{n+1}_{i,j,k} - v^n_{i,j,k}}{\Delta t} =& - R\frac{v^n_{i+1,j,k}u^n_{i+1,j,k} - v^n_{i,j,k}u^n_{i,j,k}}{\Delta x} \\
& - R\frac{(v^n_{i,j+1,k})^2 - (v^n_{i,j,k})^2}{\Delta y} \\
& - R\frac{v^n_{i,j,k+1}w^n_{i,j,k+1} - v^n_{i,j,k}w^n_{i,j,k}}{\Delta z} \\
& - \frac{\rho^n_{i,j+1,k} - \rho^n_{i,j,k}}{\delta \Delta y} \\
& + \frac{v^n_{i+1,j,k} + v^n_{i-1,j,k} - v^n_{i,j,k} - v^{n+1}_{i,j,k}}{\Delta x^2} \\
& + \frac{v^n_{i,j+1,k} + v^n_{i,j-1,k} - v^n_{i,j,k} - v^{n+1}_{i,j,k}}{\Delta y^2} \\
& + \frac{v^n_{i,j,k+1} + v^n_{i,j,k-1} - v^n_{i,j,k} - v^{n+1}_{i,j,k}}{\Delta z^2}.
\end{aligned}$$

$$\begin{aligned}
\frac{w^{n+1}_{i,j,k} - w^n_{i,j,k}}{\Delta t} =& - R\frac{w^n_{i+1,j,k}u^n_{i+1,j,k} - w^n_{i,j,k}u^n_{i,j,k}}{\Delta x} \\
& - R\frac{w^n_{i,j+1,k}v^n_{i,j+1,k} - w^n_{i,j,k}v^n_{i,j,k}}{\Delta y} \\
& - R\frac{(w^n_{i,j,k+1})^2 - (w^n_{i,j,k})^2}{\Delta z} \\
& - \frac{\rho^n_{i,j,k+1} - \rho^n_{i,j,k}}{\delta \Delta z} \\
& + \frac{w^n_{i+1,j,k} + w^n_{i-1,j,k} - w^n_{i,j,k} - w^{n+1}_{i,j,k}}{\Delta x^2} \\
& + \frac{w^n_{i,j+1,k} + w^n_{i,j-1,k} - w^n_{i,j,k} - w^{n+1}_{i,j,k}}{\Delta y^2} \\
& + \frac{w^n_{i,j,k+1} + w^n_{i,j,k-1} - w^n_{i,j,k} - w^{n+1}_{i,j,k}}{\Delta z^2}.
\end{aligned}$$

$$\left[1+2\left(\frac{\Delta t}{\Delta x^2}+\frac{\Delta t}{\Delta y^2}+\frac{\Delta t}{\Delta z^2}\right)\right]u_{i,j,k}^{n+1}+\frac{\Delta t}{\Delta x^2}\left[u_{i+1,j,k}^{n+1}+u_{i-1,j,k}^{n+1}\right]+\frac{\Delta t}{\Delta y^2}\left[u_{i,j+1,k}^{n+1}+u_{i,j-1,k}^{n+1}\right]+\frac{\Delta t}{\Delta z^2}\left[u_{i,j,k-1}^{n+1}+u_{i,j,k-1}^{n+1}\right]$$
$$=u_{i,j,k}^{n}-\frac{R\Delta t}{\Delta x}\left[(u_{i+1,j,k}^{n})^2-(u_{i,j,k}^{n})^2\right]-\frac{R\Delta t}{\Delta y}\left[u_{i,j+1,k}^{n}v_{i,j+1,k}^{n}-u_{i,j,k}^{n}v_{i,j,k}^{n}\right]-\frac{R\Delta t}{\Delta z}\left[u_{i,j,k+1}^{n}w_{i,j,k+1}^{n}-u_{i,j,k}^{n}w_{i,j,k}^{n}\right]-\frac{\Delta t}{\delta\Delta x}(\rho_{i+1,j}^{n}-\rho_{i,j}^{n})$$

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$$\left[1+2\left(\frac{\Delta t}{\Delta x^2}+\frac{\Delta t}{\Delta y^2}+\frac{\Delta t}{\Delta z^2}\right)\right]v_{i,j,k}^{n+1}+\frac{\Delta t}{\Delta x^2}\left[v_{i+1,j,k}^{n+1}+v_{i-1,j,k}^{n+1}\right]+\frac{\Delta t}{\Delta y^2}\left[v_{i,j+1,k}^{n+1}+v_{i,j-1,k}^{n+1}\right]+\frac{\Delta t}{\Delta z^2}\left[v_{i,j,k+1}^{n+1}+v_{i,j,k-1}^{n+1}\right]$$
$$=v_{i,j,k}^{n}-\frac{R\Delta t}{\Delta x}\left[v_{i+1,j,k}^{n}u_{i+1,j,k}^{n}-v_{i,j,k}^{n}u_{i,j,k}^{n}\right]-\frac{R\Delta t}{\Delta y}\left[(v_{i,j+1,k}^{n})^2-(v_{i,j,k}^{n})^2\right]-\frac{R\Delta t}{\Delta z}\left[v_{i,j,k+1}^{n}w_{i,j,k+1}^{n}-v_{i,j,k}^{n}w_{i,j,k}^{n}\right]-\frac{\Delta t}{\delta\Delta y}\left[\rho_{i,j+1,k}^{n}-\rho_{i,j,k}^{n}\right]$$

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$$\left[1+2\left(\frac{\Delta t}{\Delta x^2}+\frac{\Delta t}{\Delta y^2}+\frac{\Delta t}{\Delta z^2}\right)\right]w_{i,j,k}^{n+1}+\frac{\Delta t}{\Delta x^2}\left[w_{i+1,j,k}^{n+1}+w_{i-1,j,k}^{n+1}\right]+\frac{\Delta t}{\Delta y^2}\left[w_{i,j+1,k}^{n+1}+w_{i,j-1,k}^{n+1}\right]+\frac{\Delta t}{\Delta z^2}\left[w_{i,j,k+1}^{n+1}+w_{i,j,k-1}^{n+1}\right]$$
$$=w_{i,j,k}^{n}-\frac{R\Delta t}{\Delta x}\left[w_{i+1,j,k}^{n}u_{i+1,j,k}^{n}-w_{i,j,k}^{n}u_{i,j,k}^{n}\right]-\frac{R\Delta t}{\Delta y}\left[w_{i,j+1,k}^{n}v_{i,j+1,k}^{n}-w_{i,j,k}^{n}v_{i,j,k}^{n}\right]-\frac{R\Delta t}{\Delta z}\left[(w_{i,j,k+1}^{n})^2-(w_{i,j,k}^{n})^2\right]-\frac{\Delta t}{\delta\Delta z}\left[\rho_{i,j,k+1}^{n}-\rho_{i,j,k}^{n}\right].$$

Fully-explicit Scheme

$$\begin{aligned}\left(1+\frac{\Delta t}{\Delta x^2}+\frac{\Delta t}{\Delta y^2}+\frac{\Delta t}{\Delta z^2}\right)u_{i,j,k}^{n+1} =& u_{i,j,k}^{n}-\frac{R\Delta t}{\Delta x}\left[(u_{i+1,j,k}^{n})^2-(u_{i,j,k}^{n})^2\right]-\frac{R\Delta t}{\Delta y}\left[u_{i,j+1,k}^{n}v_{i,j+1,k}^{n}-u_{i,j,k}^{n}v_{i,j,k}^{n}\right]-\frac{R\Delta t}{\Delta z}\left[u_{i,j,k+1}^{n}w_{i,j,k+1}^{n}-u_{i,j,k}^{n}w_{i,j,k}^{n}\right]\\ &+\frac{\Delta t}{\Delta x^2}\left[u_{i+1,j,k}^{n}+u_{i-1,j,k}^{n}-u_{i,j,k}^{n}\right]+\frac{\Delta t}{\Delta y^2}\left[u_{i,j+1,k}^{n}+u_{i,j-1,k}^{n}-u_{i,j,k}^{n}\right]+\frac{\Delta t}{\Delta z^2}\left[u_{i,j,k-1}^{n}+u_{i,j,k-1}^{n}-u_{i,j,k}^{n}\right]\\ &-\frac{\Delta t}{\delta\Delta x}(\rho_{i+1,j}^{n}-\rho_{i,j}^{n})\end{aligned}$$

$$\begin{aligned}\left(1+\frac{\Delta t}{\Delta x^2}+\frac{\Delta t}{\Delta y^2}+\frac{\Delta t}{\Delta z^2}\right)v_{i,j,k}^{n+1} =& v_{i,j,k}^{n}--\frac{R\Delta t}{\Delta x}\left[v_{i+1,j,k}^{n}u_{i+1,j,k}^{n}-v_{i,j,k}^{n}u_{i,j,k}^{n}\right]-\frac{R\Delta t}{\Delta y}\left[(v_{i,j+1,k}^{n})^2-(v_{i,j,k}^{n})^2\right]-\frac{R\Delta t}{\Delta z}\left[v_{i,j,k+1}^{n}w_{i,j,k+1}^{n}-v_{i,j,k}^{n}w_{i,j,k}^{n}\right]\\ &+\frac{\Delta t}{\Delta x^2}\left[v_{i+1,j,k}^{n}+v_{i-1,j,k}^{n}-v_{i,j,k}^{n}\right]+\frac{\Delta t}{\Delta y^2}\left[v_{i,j+1,k}^{n}+v_{i,j-1,k}^{n}-v_{i,j,k}^{n}\right]+\frac{\Delta t}{\Delta z^2}\left[v_{i,j,k+1}^{n}+v_{i,j,k-1}^{n}-v_{i,j,k}^{n}\right]\\ &-\frac{\Delta t}{\delta\Delta y}\left(\rho_{i,j+1,k}^{n}-\rho_{i,j,k}^{n}\right)\end{aligned}$$

$$\begin{aligned}\left(1+\frac{\Delta t}{\Delta x^2}+\frac{\Delta t}{\Delta y^2}+\frac{\Delta t}{\Delta z^2}\right)w_{i,j,k}^{n+1} =& w_{i,j,k}^{n}-\frac{R\Delta t}{\Delta x}\left[w_{i+1,j,k}^{n}u_{i+1,j,k}^{n}-w_{i,j,k}^{n}u_{i,j,k}^{n}\right]-\frac{R\Delta t}{\Delta y}\left[w_{i,j+1,k}^{n}v_{i,j+1,k}^{n}-w_{i,j,k}^{n}v_{i,j,k}^{n}\right]-\frac{R\Delta t}{\Delta z}\left[(w_{i,j,k+1}^{n})^2-(w_{i,j,k}^{n})^2\right]\\ &+\frac{\Delta t}{\Delta x^2}\left[w_{i+1,j,k}^{n}+w_{i-1,j,k}^{n}-w_{i,j,k}^{n}\right]+\frac{\Delta t}{\Delta y^2}\left[w_{i,j+1,k}^{n}+w_{i,j-1,k}^{n}-w_{i,j,k}^{n}\right]+\frac{\Delta t}{\Delta z^2}\left[w_{i,j,k+1}^{n}+w_{i,j,k-1}^{n}-w_{i,j,k}^{n}\right]\\ &-\frac{\Delta t}{\delta\Delta z}\left(\rho_{i,j,k+1}^{n}-\rho_{i,j,k}^{n}\right)\end{aligned}$$

$$\rho^{n+1}=\rho^{n}-\Delta t\left[\frac{u_{i+1,j,k}^{n}-u_{i,j,k}^{n}}{\Delta x}+\frac{v_{i,j+1,k}^{n}-v_{i,j,k}^{n}}{\Delta y}+\frac{w_{i,j,k+1}-w_{i,j,k}}{\Delta z}\right]$$

For boundary nodes, we have: