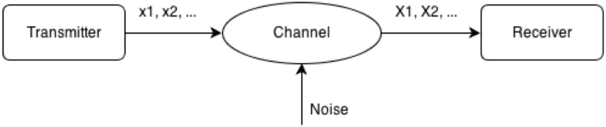
Channel Coding

October 2018

Content

- Error-free transmission problem
- Channel Coding
- Erasure Code

Transmission over noisy channel



- Noisy channel
- $X_i = x_i + noise(i)$
- We cannot guarantee $X_1 = x_1$!!!

QUESTION:

Is there a solution to send $\{x_i\}$ to receiving side so that receiving side can receive $\{x_i\}$ without error?

Noisy channel modeling

- Binary Symetric Channel (BSC)

$$x \longrightarrow (x||!x)$$

where, $x \in \{0, 1\}$

- Binary Erasure Channel (BEC)

$$x \longrightarrow (x||?)$$

where, $x \in \{0, 1\}$

- Packet Erasure Channel

$$P \longrightarrow (P||?)$$

where, P is a packet

Noisy channel cause data loss

Error-free transmission - Solution

The basic idea: "Transmit original data with redundant data":

1. Encode data by adding *redundancy*

$$C(\mathbf{x}) = \mathbf{x} + redundant$$

2. Then send $C(\mathbf{x})$. We will get $C'(\mathbf{x})$ the receiving side

$$C'(\mathbf{x}) = C(\mathbf{x}) - loss = \mathbf{x} + (redundant - loss)$$

3. Try to recover $\mathbf{x}$ from $C'(\mathbf{x})$. (When will it be possible?)

Erasure codes

Is a coding scheme that can add redundant data to original data

- Origin blocks (input to erasure encoder): $\{x_i | i = 1..n\}$
- Coded blocks (output of erasure encoder): $\{X_i | i = 1..m\}, m > n$
- All blocks (both original and coded) are of the same size
- After transmission (because of loss): $\{X_i | i = 1..k\}, k \leq m$
- If $k \geq n$, we can decode to get $\{x_i\}$ from $\{X_i | i = 1..k\}$

Erasure codes (cont.)

- Encoding

$$\mathbf{X} = \mathbf{C}\mathbf{x}$$

  $\mathbf{C}$ is $n \times m$

- Encoding example: 2-time repitition code 1

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

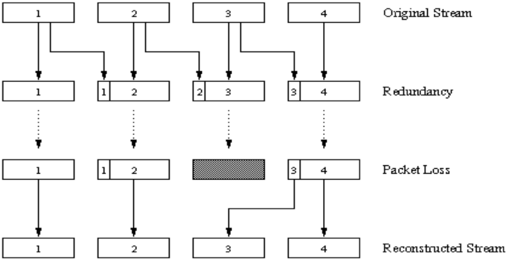
Erasure code example: $(n+1)\ FEC$

$(n+1)\ FEC$ for 3 original blocks

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Decoding of erasure code is equivalent to solving a system of linear equations

Other possible schemes: Multi-description



Other possible schemes: Interleaving