Proof of Linear Regression Project

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1 Proof of Singular Matrix

For square matrix, $A = \begin{bmatrix} I & X \\ Z & Y \end{bmatrix}$, assume I is $m \times m$ identity matrix, X is $m \times n$ matrix, and A is $(m+n) \times (m+n)$ matrix. As the first column of Y is zero and Z is all zero, we can get the $(m+1)$th column of A, $A[:][m]$ is the combination of former m columns, i.e.,

$$A[:][m] = \sum_{j=0}^{m-1} A[j][m] \cdot A[:][j]$$

Considering solving the inverse matrix of A, for $[A \mid I] = \begin{bmatrix} I & X & I \\ Z & Y & I \end{bmatrix}$, we can make the mth column is all zero by Guass-jordan method:

$$A[:][m] \leftarrow \sum_{j=0}^{m-1} -A[j][m] \cdot A[:][j] + A[:][m]$$

So the former $m+n$ columns of $[A \mid I]$ cannot be converted to identity matrix. Therefore, it doesn't exist inverse matrix for A. So A is singular matrix.

2 Proof of Loss Function

$$E(m,b) = \sum_{i=1}^{n}[(y_i - b)^2 - 2(y_i - b)mx_i + m^2x_i^2]$$

so we can get

$$\begin{aligned}\frac{\partial E}{\partial m} &= \sum_{i=1}^{n}[-2(y_i - b)x_i + 2mx_i^2] \\ &= \sum_{i=1}^{n} -2x_i(y_i - mx_i - b)\end{aligned}$$

And also,

$$E(m,b) = \sum_{i=1}^{n}[(y_i - mx_i)^2 - 2(y_i - mx_i)b + b^2]$$

so we can get:

$$\begin{aligned}\frac{\partial E}{\partial b} &= \sum_{i=1}^{n}[-2(y_i - mx_i) + 2b] \\ &= \sum_{i=1}^{n} -2(y_i - mx_i - b)\end{aligned}$$

On the other side:

$$\begin{aligned}X^TXh &= \begin{bmatrix} x_1 & x_2 & ... & x_n \\ 1 & 1 & ... & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ ... & ... \\ x_n & 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ b \end{bmatrix} \\ &= \begin{bmatrix} \sum\limits_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum\limits_{i=1}^{n} x_i & \sum_{i=1}^{n} 1 \end{bmatrix} \cdot \begin{bmatrix} m \\ b \end{bmatrix} \\ &= \begin{bmatrix} \sum\limits_{i=1}^{n}(mx_i^2 + bx_i) \\ \sum\limits_{i=1}^{n}(mx_i + b) \end{bmatrix}\end{aligned}$$

$$X^TY = \begin{bmatrix} x_1 & x_2 & ... & x_n \\ 1 & 1 & ... & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ ... \\ y_3 \end{bmatrix} = \begin{bmatrix} \sum\limits_{i=1}^{n} x_iy_i \\ \sum\limits_{i=1}^{n} y_i \end{bmatrix}$$

Therefore,

$$2X^TXh - 2X^TY = \begin{bmatrix} \sum_{i=1}^{n}(2mx_i^2 + 2bx_i - 2x_iy_i) \\ \sum_{i=1}^{n}(2mx_i + 2b - 2y_i) \end{bmatrix} = \begin{bmatrix} \frac{\partial E}{\partial m} \\ \frac{\partial E}{\partial b} \end{bmatrix}$$