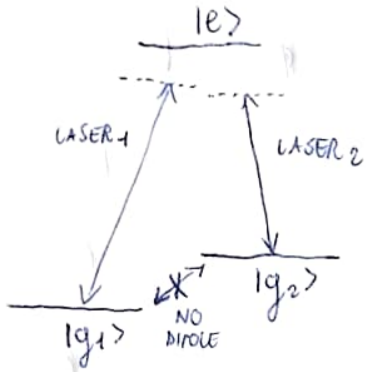
RAMAN COUPLING: THREE-LEVEL ATOM



$$H_{ATOM} = E_1 |g_1\rangle\langle g_1| + \overset{-\hbar\omega_{eg_2}}{E_2} |g_2\rangle\langle g_2| + \overset{0}{E_e} |e\rangle\langle e|$$

$$H_{\substack{ATOM\\ LIGHT\\ INTERACT.}} = -\hat{\vec{d}} \cdot \hat{\vec{E}}$$ WHERE

DIPOLE TRANSITIONS

$$\hat{\vec{d}} = -e\,\hat{\vec{r}} \begin{pmatrix}\text{ACTING ON}\\ g_1\, g_2\, e\end{pmatrix} = -e\,|g_1\rangle\langle g_1|\vec{r}|e\rangle\langle e| + $$
$$-e\,|g_2\rangle\langle g_2|\vec{r}|e\rangle\langle e| + h.c. =$$
$$\hat{\vec{d}} = \vec{d}^{\,*}_{eg_1} |g_1\rangle\langle e| + \vec{d}^{\,*}_{eg_2} |g_2\rangle\langle e| + h.c.$$

ELECTRIC FIELD

$$\hat{\vec{E}} = \sum_{k\lambda} \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0}}\, \vec{\varepsilon}_\lambda\, 2\,\mathrm{Im}\left(u_{k\lambda}(r_{ATOM})\, \hat{a}_{k\lambda}\right)$$ WHERE $\mathrm{Im}(A) = \frac{A - A^+}{2i}$

SCHRÖDINGER PICTURE (TIME-DEP. HIDDEN)

TWO LASERS → k_1, λ_1 IN $|\alpha_1\rangle$; $k_2 \lambda_2$ IN $|\alpha_2\rangle$; REST IN $|0\rangle$

$$|\Psi_{LASERS}\rangle = |0\rangle|0\rangle|0\rangle \cdots \underbrace{|\alpha_1\rangle}_{k_1\lambda_1} |0\rangle \cdots \underbrace{|\alpha_2\rangle}_{k_2\lambda_2} |0\rangle \cdots |0\rangle$$

$$\langle\Psi_{LASERS}|\vec{E}|\Psi_{LASERS}\rangle = \sqrt{\frac{\hbar c k_1}{2\varepsilon_0}}\, \vec{\varepsilon}_{k_1\lambda_1}\, 2\,\mathrm{Im}\left(u_{k_1\lambda_1} \langle\alpha_1|a|\alpha_1\rangle e^{-i(ck_1)t}\right) + (\text{SAME})_{k_2\lambda_2}$$

$$= \underbrace{\sqrt{\frac{2\hbar c k_1}{\varepsilon_0}}\, \vec{\varepsilon}_{k_1\lambda_1} |u|_{k_1\lambda_1} |\alpha_1| \cos\left((ck_1)t + \phi_1\right)}_{} + (\text{SAME})_{k_2\lambda_2}$$

($\langle\vec{E}\rangle$ OF TWO LASERS IS JUST SUM OF THE TWO $\langle\vec{E}\rangle$ FROM EACH LASER (OPTICS IS LINEAR))

$$\langle\vec{E}\rangle = \vec{\mathcal{E}}_1 \cos(\omega_1 t + \phi_1) + \vec{\mathcal{E}}_2 \cos(\omega_2 t + \phi_2)$$

$$H_{\substack{ATOM\\ LIGHT\\ INTERACTION}} \simeq -\hat{\vec{d}} \cdot \langle\vec{E}\rangle =$$
$$= -\left(\vec{d}_{eg_1} |e\rangle\langle g_1| + \vec{d}_{eg_2} |e\rangle\langle g_2| + \vec{d}^{\,*}_{eg_1} |g_1\rangle\langle e| + \vec{d}^{\,*}_{eg_2} |g_2\rangle\langle e|\right) \times$$
$$\left(\vec{\mathcal{E}}_1 \cos(\omega_1 t + \phi_1) + \vec{\mathcal{E}}_2 \cos(\omega_2 t + \phi_2)\right) =$$

$$\Omega_1 = \frac{\vec{d}_{eg_1} \cdot \vec{\mathcal{E}}_1}{\hbar} \quad \Omega_2 = \frac{\vec{d}_{eg_2} \cdot \vec{\mathcal{E}}_2}{\hbar} \quad \tilde{\Omega}_1 = \frac{\vec{d}_{eg_1} \cdot \vec{\mathcal{E}}_2}{\hbar} \quad \tilde{\Omega}_2 = \frac{\vec{d}_{eg_2} \cdot \vec{\mathcal{E}}_1}{\hbar}$$

$$H_{\text{ATOM-LIGHT}} = -\hbar\Big[|e\rangle\langle g_1|\big(\Omega_1\cos(\omega_1 t+\phi_1)+\cancel{\Omega}_1\cos(\omega_2 t+\phi_2)\big)+ |e\rangle\langle g_2|\big(\Omega_2\cos(\omega_2 t+\phi_2)+\cancel{\Omega}_2\cos(\omega_1 t+\phi_1)\big)+h.c.\Big]$$

$$H_{\text{FULL}}^{\binom{\text{LAB}}{\text{FRAME}}} = -\hbar \begin{array}{c} \begin{array}{ccc} \quad|g_1\rangle\quad & \quad|e\rangle\quad & \quad|g_2\rangle\quad \end{array} \\ \begin{pmatrix} +\omega_{eg_1} & c.c. & 0 \\ \Omega_1\cos(\omega_1 t+\phi_1)+\cancel{\Omega}_1\cos(\omega_2 t+\phi_2) & 0 & c.c. \\ 0 & \Omega_2^*\cos(\omega_2 t+\phi_2)+\cancel{\Omega}_2\cos(\omega_1 t+\phi_1) & +\omega_{eg_2} \end{pmatrix}\end{array}$$

CHANGE OF REFERENCE FRAME

$$U(t) = \begin{pmatrix} e^{-i(\omega_1 t+\phi_1)} & & \\ & 1 & \\ & & e^{-i(\omega_2 t+\phi_2)} \end{pmatrix}$$

$$H\binom{\text{ROTAT.}}{\text{FRAME}} = U\,H\binom{\text{LAB}}{\text{FRAME}}\,U^\dagger + i\hbar\,\dot{U}U^\dagger$$

$$i\hbar\,\dot{U}U^\dagger = +\hbar\begin{pmatrix} \omega_1 & & \\ & 0 & \\ & & \omega_2 \end{pmatrix}$$

$$H\binom{\text{ROTATING}}{\text{FRAME}} = -\hbar\begin{pmatrix} \omega_{eg_1}-\omega_1 & & \\ \Omega_1 e^{i(\omega_1 t+\phi_1)}\cos(\omega_1 t+\phi_1)+\dots & 0 & \\ 0 & \Omega_2^* e^{-i(\omega_2 t+\phi_2)}\cos(\omega_2 t+\phi_2)+\dots & \omega_{eg_2}-\omega_2 \end{pmatrix}$$

$$\delta_1 = \omega_{eg_1}-\omega_1 \qquad \delta_2 = \omega_{eg_2}-\omega_2 \quad ; \quad H(t) = H_0 + V(t)$$

$$H_0 = -\hbar\begin{pmatrix} \delta_1 & \Omega_1^*/2 & 0 \\ \Omega_1/2 & 0 & \Omega_2/2 \\ 0 & \Omega_2^*/2 & \delta_2 \end{pmatrix}$$

$$V(t) = -\hbar\begin{pmatrix} 0 & c.c. & 0 \\ \frac{\Omega_1}{2}e^{2i(\omega_1 t+\phi_1)}+\frac{\cancel{\Omega}_1}{2}e^{i(\omega_1+\omega_2)t}e^{i(\phi_1+\phi_2)}+\frac{\cancel{\Omega}_1}{2}e^{i(\omega_1-\omega_2)t}e^{i(\phi_1-\phi_2)} & 0 & c.c. \\ 0 & \text{SOMETHING SIMILAR} & 0 \end{pmatrix}$$

$$H_{eff} \simeq H_0 + \underbrace{\frac{1}{\tilde{\omega}}}_{\text{OSCILLATION FREQUENCIES OF } V(t)} [V, V^+] + \mathcal{O}\left(\frac{1}{\tilde{\omega}^2}\right)$$

<u>RWA ?</u>

$\frac{1}{\tilde{\omega}}[V, V^+]$ IS NEGLIGIBLE WHEN H_0-TIMESCALES $\gg$ $\frac{1}{\tilde{\omega}}$

OR $\underbrace{H_0}_{\text{AND } V}$-ENERGYSCALES $\ll \hbar\tilde{\omega}$

<u>BASICALLY</u>

$$\underbrace{\Omega_1, \Omega_2, |\delta_1|, |\delta_2|}_{\text{AND ALSO } \Omega_1\ \Omega_2} \ll \omega_1, \omega_2, \underline{\underline{|\omega_1 - \omega_2|}}$$

<u>RWA!</u>

THE TWO LASERS CAN NOT BE TOO CLOSE IN ω

<u>AND NOW</u>

ADDED CONSTANT

TURN $\Omega_1\ \Omega_2$ <u>REAL</u>

$$H_{eff} = H_0 = \hbar \begin{array}{c} \begin{array}{ccc} |g_1\rangle & |e\rangle & |g_2\rangle \end{array} \\ \begin{pmatrix} 0 & \left|\frac{\Omega_1}{2}\right| & 0 \\ +\left|\frac{\Omega_1}{2}\right| & \delta_1 & \left|\frac{\Omega_2}{2}\right| \\ 0 & \left|\frac{\Omega_2}{2}\right| & \Delta = \delta_1 - \delta_2 \end{pmatrix} \end{array}$$

<u>ALMOST</u> LIKE THE 3-LEVEL PROBLEM WE SOLVED EARLIER IN THE COURSE

let's say $\boxed{\Delta, \Omega_1 \ll \delta_1}$.

$$H_{UNP} = \begin{array}{c} \begin{array}{ccc} |g_1\rangle & |e\rangle & |g_2\rangle \end{array} \\ \begin{pmatrix} 0 & & \\ & \hbar\delta_1 \simeq \hbar\delta_2 & \\ & & 0 \end{pmatrix} \end{array} \qquad \underbrace{H^{(A)}_{PERT} = \begin{pmatrix} 0 & \Omega_1 & \\ \Omega_1 & 0 & \Omega_2 \\ & \Omega_2 & 0 \end{pmatrix} \frac{\hbar}{2}}_{\text{II ORDER}} \qquad \underbrace{H^{(B)}_{PERT} = \begin{pmatrix} 0 & & \\ & 0 & \\ & & \hbar\Delta \end{pmatrix}}_{\text{I ORDER (EXACT } \infty)}$$

PERTURBATION THEORY ON $\{|g_1\rangle, |g_2\rangle\}$

$$H^{(2)}_A = \Pi_0 H^{(A)} R_0 H^{(A)} \Pi_0 = \hbar \begin{pmatrix} 1 & & \\ & 0 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 0 & \Omega_1 & \\ \Omega_1 & 0 & \Omega_2 \\ & \Omega_2 & 0 \end{pmatrix} \begin{pmatrix} 0 & & \\ & -1/\delta & \\ & & 0 \end{pmatrix} \begin{pmatrix} 0 & \Omega_1 & \\ \Omega_1 & 0 & \Omega_2 \\ & \Omega_2 & 0 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

$$H^{\text{(ROTATING FRAME)}}_{FINAL} = -\frac{\hbar}{4\delta} \begin{pmatrix} \Omega_1^2 & \Omega_1\Omega_2 \\ \Omega_1\Omega_2 & \Omega_2^2 \end{pmatrix} + \hbar \begin{pmatrix} 0 & 0 \\ 0 & \Delta \end{pmatrix}$$

$$= \hbar\left(-\frac{\Omega_1\Omega_2}{4\delta}\right)\sigma^x + \hbar\left(\frac{\Omega_2^2 - \Omega_1^2}{8\delta} - \Delta\right)\sigma^z + \text{const.}$$

$\boxed{\text{FULL QUBIT CONTROL}}$