RflySim bottom flight control algorithm development series course 10th lecture fixed position controller design experiment

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Outline

Part 2 of 3: Experimental Principles 1 Experiment principles

2. Basic experiment

3. Analytical experiments

4. Design the experiment

5. Summary

Principles of Experimentation

Basic concepts

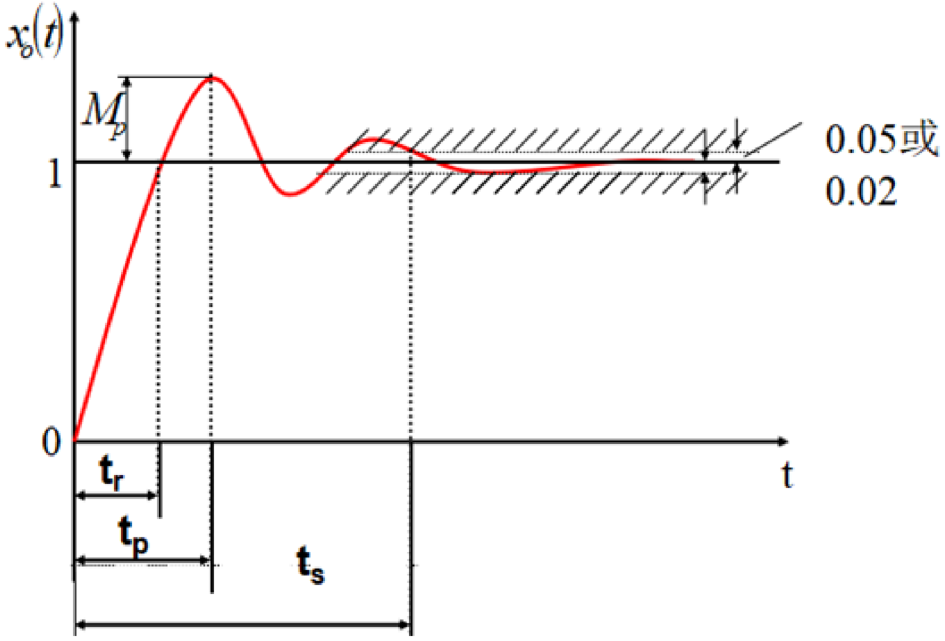


Figure. Second order system step response

(1) Time domain characteristics of the system

For second-order system

$$G(s)=\frac{\omega_n^2}{s^2+2\varsigma\omega_n s+\omega_n^2}$$

$0<\xi<1$ Where the step response curve is shown on the left

1) Amount of overshoot $M_p=\frac{x_o(t_p)-x_o(\infty)}{x_o(\infty)}*100\%=e^{-\xi\pi/\sqrt{1-\xi^2}}*100\%$

2) Adjust the time

In the initial analysis of the system, the following formula is often used to calculate the adjustment time. When the damping ratio $t_s<0.8$

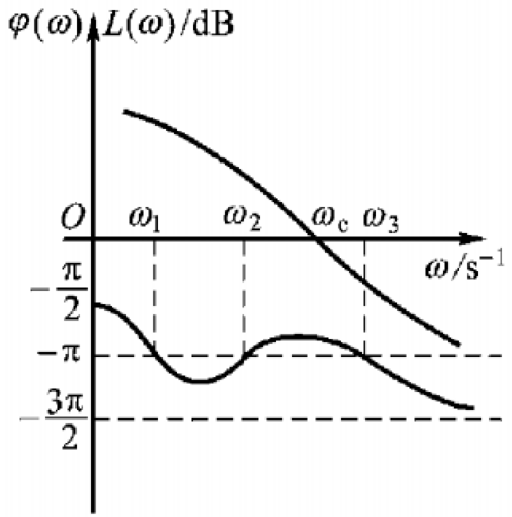
$$t_s=\frac{3.5}{\xi\omega_n}(\text{取5\%误差带}) \qquad t_s=\frac{4.5}{\xi\omega_n}(\text{取2\%误差带})$$

Experimental principles

□ Basic concepts

(2) Bode diagram and stability margin

Bode diagram is also called logarithmic frequency characteristic curve, which draws the open-loop amplitude-phase characteristic on logarithmic coordinates. The logarithmic stability criterion is based on the relationship between the open-loop amplitude-frequency curve and log-phase-frequency curve to judge the closed-loop system stability.



Graph. Stability margin

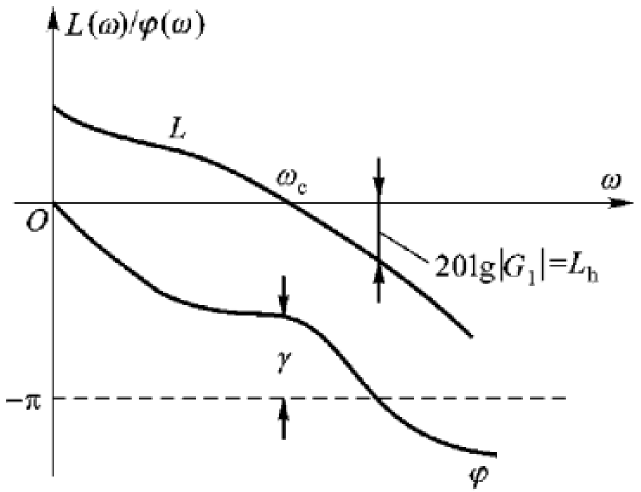


Figure. Logarithmic frequency characteristic curve

Phase Angle margin γ: $L(\omega)=0\text{dB}$ the difference between the phase frequency at $-\pi$:

$$\gamma=\angle G(j\omega_c)H(j\omega_c)-(-180^\circ)$$

Where: ω_c indicates the cut-off frequency. $L(\omega_c)=0\text{dB}$

Mode stability margin h: when, corresponding $\angle G(j\omega_1)H(j\omega_1)$

The absolute value $-\pi$ of logarithmic amplitude frequency, i.e. :

$$h(\text{dB})=20\lg\left|\frac{1}{G(j\omega_1)H(j\omega_1)}\right|=-20\lg\left|G(j\omega_1)H(j\omega_1)\right|$$

Under the condition of closed-loop system stability, the greater the sum of the system, the higher the stability of the reaction system. The stability margin also indirectly reflects the stationarity of the dynamic process of the system, and the general meaning of the margin is small overshoot, weak oscillation, and large "damping". General requirements:

$$\gamma>40^\circ$$

$$h>6\text{dB}$$

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Experimental principle

Underlying control framework

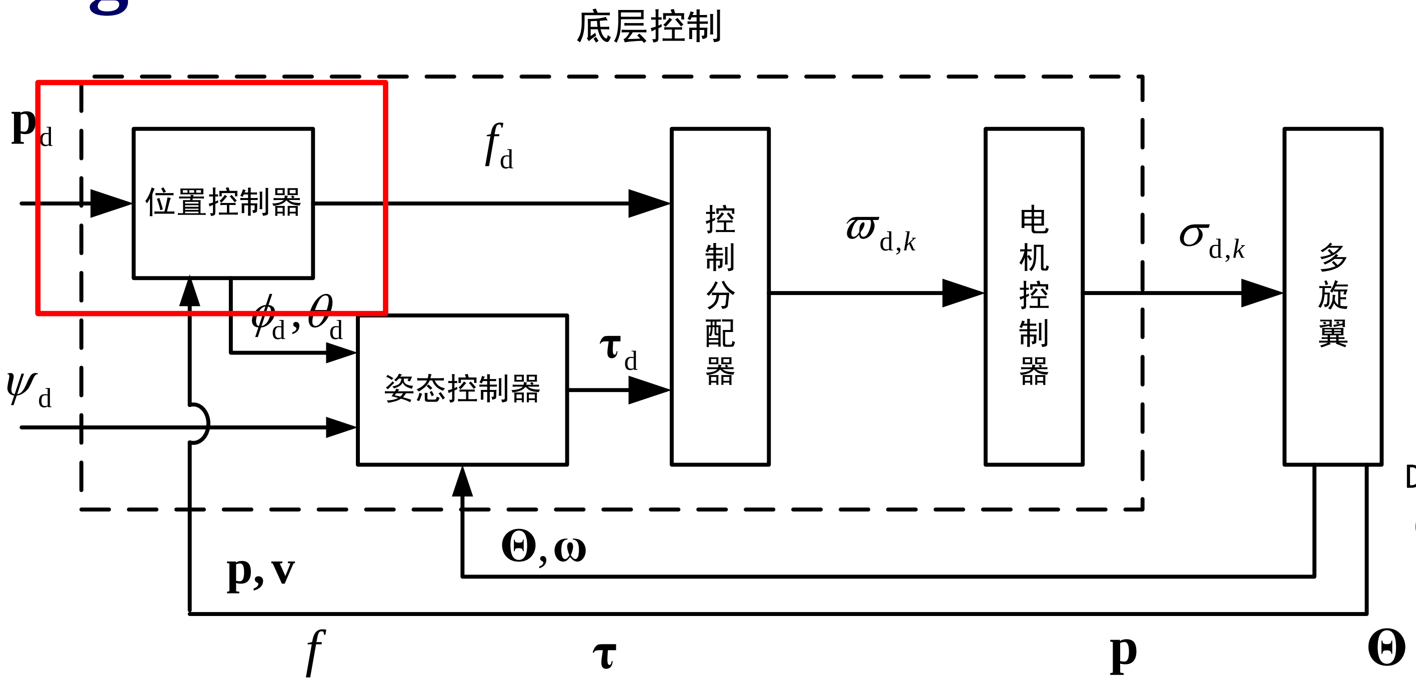

Diagram. Multi-rotor full autonomy Control closed loop block diagram

Underdrive system: 4 inputs (total pull and three-axis torque) control 6 outputs (position and attitude Angle). When designing the multi-rotor flight controller, the control strategy of the inner and outer rings can be adopted, in which the inner ring controls the attitude Angle of the multi-rotor vehicle, while the outer ring controls the position of the multi-rotor vehicle. The lift, hover and side flight modes of the multi-rotor aircraft can be realized by the control of the inner and outer rings.

Experimental principle

□ Position control

(1) Traditional PID design

■ Horizontal channel model

$$\dot{\mathbf{p}}_{\mathrm{h}} = \mathbf{v}_{\mathrm{h}}$$
$$\dot{\mathbf{v}}_{\mathrm{h}} = -g\mathbf{A}_{\psi}\mathbf{\Theta}_{\mathrm{h}}$$

■ Expect horizontal position dynamics

$$\ddot{\mathbf{p}}_{\mathrm{h}} = \ddot{\mathbf{p}}_{\mathrm{hd}} - \mathbf{K}_{\mathbf{p}_{\mathrm{h}}\mathrm{d}}\left(\dot{\mathbf{p}}_{\mathrm{h}} - \dot{\mathbf{p}}_{\mathrm{hd}}\right) - \mathbf{K}_{\mathbf{p}_{\mathrm{h}}\mathrm{p}}\left(\mathbf{p}_{\mathrm{h}} - \mathbf{p}_{\mathrm{hd}}\right)$$

$$-\mathrm{g}\mathbf{A}_{\psi}\mathbf{\Theta}_{\mathrm{hd}} = \ddot{\mathbf{p}}_{\mathrm{hd}} - \mathbf{K}_{\mathbf{p}_{\mathrm{h}}\mathrm{d}}\left(\dot{\mathbf{p}}_{\mathrm{h}} - \dot{\mathbf{p}}_{\mathrm{hd}}\right) - \mathbf{K}_{\mathbf{p}_{\mathrm{h}}\mathrm{p}}\left(\mathbf{p}_{\mathrm{h}} - \mathbf{p}_{\mathrm{hd}}\right)$$

When considering point control

$$\dot{\mathbf{p}}_{\mathrm{hd}} = \ddot{\mathbf{p}}_{\mathrm{hd}} = \mathbf{0}_{2\times 1}$$

$$\boxed{\mathbf{\Theta}_{\mathrm{hd}} = -\mathrm{g}^{-1}\mathbf{A}_{\psi}^{-1}\left(\ddot{\mathbf{p}}_{\mathrm{hd}} - \mathbf{K}_{\mathbf{p}_{\mathrm{h}}\mathrm{d}}\left(\dot{\mathbf{p}}_{\mathrm{h}} - \dot{\mathbf{p}}_{\mathrm{hd}}\right) - \mathbf{K}_{\mathbf{p}_{\mathrm{h}}\mathrm{p}}\left(\mathbf{p}_{\mathrm{h}} - \mathbf{p}_{\mathrm{hd}}\right)\right)}$$

$\mathbf{K}_{(\cdot)}$ Where the parameters are represented.

Experimental principle

Position control

(1) Traditional PID design

- **Height channel model**

$$\dot{p}_z = v_z$$

$$\dot{v}_z = g - \frac{f}{m}$$

- **Desired height dynamics**

$$\ddot{p}_z = \ddot{p}_{z_d} - k_{p_z d}\left(\dot{p}_z - \dot{p}_{z_d}\right) - k_{p_z p}\left(p_z - p_{z_d}\right)$$

$$\boxed{f_d = mg - m\left(\ddot{p}_{z_d} - k_{p_z d}\left(\dot{p}_z - \dot{p}_{z_d}\right) - k_{p_z p}\left(p_z - p_{z_d}\right)\right)}$$

When considering point control, The above formula becomes $\dot{p}_{z_d} = \ddot{p}_{z_d} = 0$

$$f_d = mg - m\left(-k_{p_z d}\dot{p}_z - k_{p_z p}\left(p_z - p_{z_d}\right)\right)$$

Experimental principle

☐ Position control

(2) Open source self-drive PID design

1) Horizontal channel model $\lim_{t\to\infty}\left\|\mathbf{e}_{\mathbf{p}_\mathrm{h}}(t)\right\|=0$

In order to make, we first target

Desired Speed

$$\dot{\mathbf{p}}_\mathrm{h}=\mathbf{v}_\mathrm{h} \Longrightarrow \boxed{\mathbf{v}_\mathrm{hd}=\mathbf{K}_{\mathbf{p}_\mathrm{h}}\left(\mathbf{p}_\mathrm{hd}-\mathbf{p}_\mathrm{h}\right)}$$

$\dot{\mathbf{p}}_\mathrm{hd}=0$ On the premise of if

$$\lim_{t\to\infty}\left\|\mathbf{e}_{\mathbf{v}_\mathrm{h}}(t)\right\|=0$$

then

$$\lim_{t\to\infty}\left\|\mathbf{e}_{\mathbf{p}_\mathrm{h}}(t)\right\|=0$$

Where speed meets expectations, so does location

Among them. $\mathbf{e}_{\mathbf{v}_\mathrm{h}}\triangleq\mathbf{v}_\mathrm{h}-\mathbf{v}_\mathrm{hd}$

$$\sigma_k=\sigma_{k,\mathrm{d}}$$

$$\lim_{t\to\infty}\left|\tilde{\varpi}_k\right|=0$$

$$f_\mathrm{d} \qquad \boldsymbol{\tau}_\mathrm{d}$$

$$\lim_{t\to\infty}\left|e_{v_z}\right|=0 \qquad \lim_{t\to 0}\left\|\mathbf{e}_{\boldsymbol{\omega}}(t)\right\|=0$$

$$\lim_{t\to 0}\left|e_z(t)\right|=0 \qquad \lim_{t\to 0}\left\|\mathbf{e}_{\boldsymbol{\Theta}}(t)\right\|=0$$

高度达到期望

$$\lim_{t\to 0}\left\|\mathbf{e}_{\mathbf{v}_\mathrm{h}}(t)\right\|=0 \qquad \lim_{t\to 0}\left\|\psi(t)-\psi_\mathrm{d}(t)\right\|=0$$

$$\boxed{\lim_{t\to 0}\left\|\mathbf{e}_{\mathbf{p}_\mathrm{h}}(t)\right\|=0}$$

水平位置达到期望

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Principle of experiment

☐ Position control

(2) Open source self-drive PID design

1) Horizontal channel model $\lim_{t\to\infty}\left\|\mathbf{e}_{\mathbf{v}_\mathrm{h}}(t)\right\|=0$

In order to make, we first target

$$\dot{\mathbf{v}}_\mathrm{h}=-g\mathbf{A}_\psi\mathbf{\Theta}_\mathrm{h}$$

$$-g\mathbf{A}_\psi\mathbf{\Theta}_\mathrm{hd}=-\mathbf{K}_{\mathbf{v}_\mathrm{h}\mathrm{p}}\mathbf{e}_{\mathbf{v}_\mathrm{h}}-\mathbf{K}_{\mathbf{v}_\mathrm{h}\mathrm{i}}\int\mathbf{e}_{\mathbf{v}_\mathrm{h}}-\mathbf{K}_{\mathbf{v}_\mathrm{h}\mathrm{d}}\dot{\mathbf{e}}_{\mathbf{v}_\mathrm{h}}$$

$$\mathbf{\Theta}_\mathrm{hd}=g^{-1}\mathbf{A}_\psi^{-1}\left(\mathbf{K}_{\mathbf{v}_\mathrm{h}\mathrm{p}}\mathbf{e}_{\mathbf{v}_\mathrm{h}}+\mathbf{K}_{\mathbf{v}_\mathrm{h}\mathrm{i}}\int\mathbf{e}_{\mathbf{v}_\mathrm{h}}+\mathbf{K}_{\mathbf{v}_\mathrm{h}\mathrm{d}}\dot{\mathbf{e}}_{\mathbf{v}_\mathrm{h}}\right)$$

if $\lim_{t\to\infty}\left\|\mathbf{\Theta}_\mathrm{h}(t)-\mathbf{\Theta}_\mathrm{hd}(t)\right\|=0$

then $\lim_{t\to\infty}\left\|\mathbf{e}_{\mathbf{v}_\mathrm{h}}(t)\right\|=0$

If the Angle can meet the expectation, the speed can meet the expectation

$\sigma_k=\sigma_{k,\mathrm{d}}$

$\lim_{t\to\infty}\left|\tilde{\varpi}_k\right|=0$

f_d

$\lim_{t\to\infty}\left|e_{v_z}\right|=0$

$\lim_{t\to0}\left|e_z(t)\right|=0$

高度达到期望

$\boldsymbol{\tau}_\mathrm{d}$

$\lim_{t\to0}\left\|\mathbf{e}_{\boldsymbol{\omega}}(t)\right\|=0$

$\lim_{t\to0}\left\|\mathbf{e}_{\mathbf{\Theta}}(t)\right\|=0$

$\boxed{\lim_{t\to0}\left\|\mathbf{e}_{\mathbf{v}_\mathrm{h}}(t)\right\|=0}$

$\lim_{t\to0}\left\|\mathbf{e}_{\mathbf{p}_\mathrm{h}}(t)\right\|=0$

水平位置达到期望

$\lim_{t\to0}\left\|\psi(t)-\psi_\mathrm{d}(t)\right\|=0$

Experimental principle

□ Position control

(2) Open source self-drive PID design

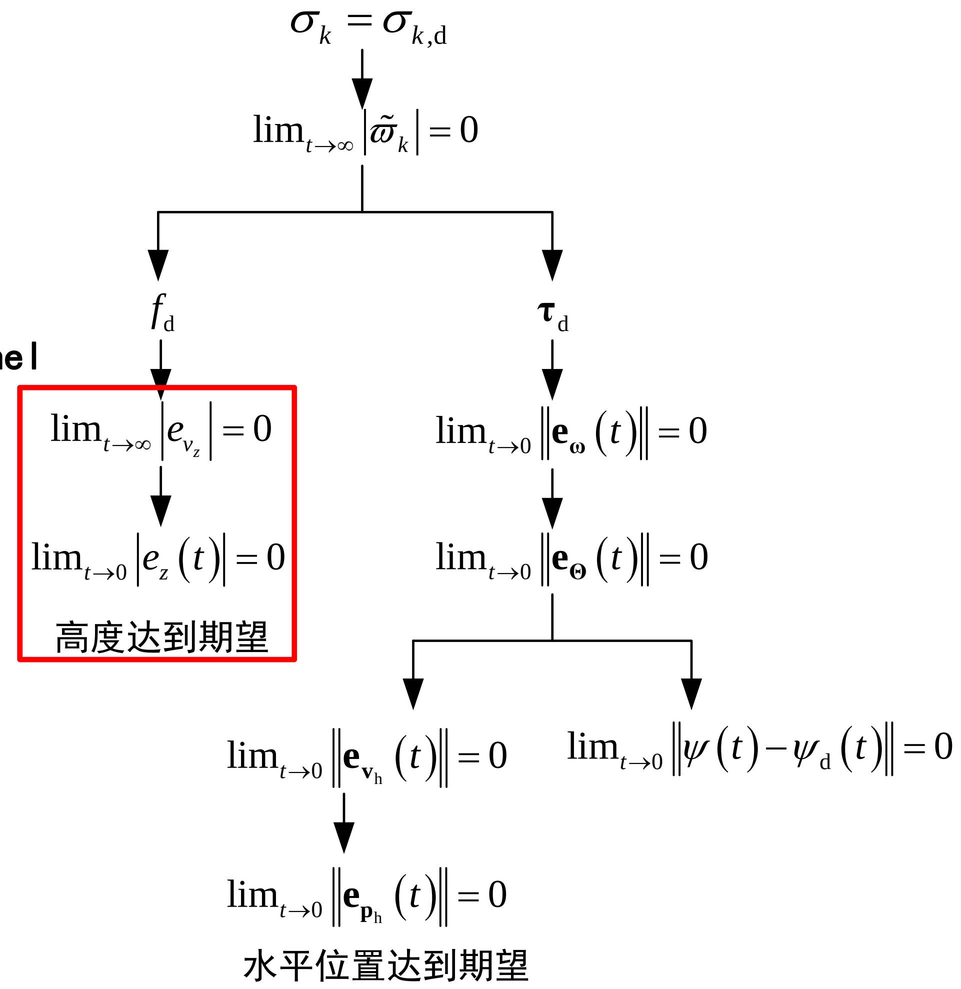
2) Height channel model

Similar to the horizontal channel design, the height channel

The controller design is as follows

$$v_{zd} = -k_{p_z}\left(p_z - p_{zd}\right)$$

$$f_d = m\left(g + k_{v_z p} e_{v_z} + k_{v_z i}\int e_{v_z} + k_{v_z d}\dot{e}_{v_z}\right)$$



If the vertical speed can meet the expectation, the height can also meet the expectation

Principles of Experimentation

Position control

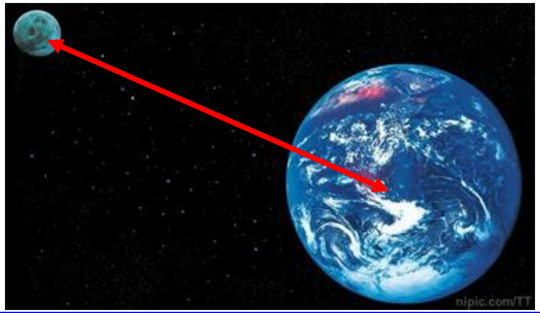
(3) Add saturated PID control

Traditional PID $\boldsymbol{\Theta}_{\mathrm{hd}} = -g^{-1}\mathbf{A}_{\psi}^{-1}\left(\ddot{\mathbf{p}}_{\mathrm{hd}} - \mathbf{K}_{\mathbf{p}_{\mathrm{h}}\mathrm{d}}\left(\dot{\mathbf{p}}_{\mathrm{h}} - \dot{\mathbf{p}}_{\mathrm{hd}}\right) - \mathbf{K}_{\mathbf{p}_{\mathrm{h}}\mathrm{p}}\left(\mathbf{p}_{\mathrm{h}} - \mathbf{p}_{\mathrm{hd}}\right)\right)$

Autopilot PID $\boldsymbol{\Theta}_{\mathrm{hd}} = g^{-1}\mathbf{A}_{\psi}^{-1}\left(\mathbf{K}_{\mathbf{v}_{\mathrm{h}}\mathrm{p}}\mathbf{e}_{\mathbf{v}_{\mathrm{h}}} + \mathbf{K}_{\mathbf{v}_{\mathrm{h}}\mathrm{i}}\int \mathbf{e}_{\mathbf{v}_{\mathrm{h}}} + \mathbf{K}_{\mathbf{v}_{\mathrm{h}}\mathrm{d}}\dot{\mathbf{e}}_{\mathbf{v}_{\mathrm{h}}}\right)$

Controller Design
Doesn't make sense

Position error is large



Large Angle error

$$\boldsymbol{\Theta}_{\mathrm{hd}} \gg 2\pi$$

Small Angle hypothesis broken!

Saturate is very necessary

Experimental Principles

Position control

(3) Add saturated PID control

e_{v_h}

Similarly, in the open source autopilot PID design, we add clipping to the right end of the pair and controller

$$\mathbf{e}_{\mathrm{v_h}} = \mathrm{sat_{gd}}\left(\mathbf{v}_{\mathrm{h}} - \mathbf{v}_{\mathrm{hd}}, a_1\right)$$

$$\mathbf{\Theta}_{\mathrm{hd}} = \mathrm{sat_{gd}}\left(\mathrm{g}^{-1}\mathbf{A}_{\psi}^{-1}\left(\mathbf{K}_{\mathrm{v_h p}}\mathbf{e}_{\mathrm{v_h}} + \mathbf{K}_{\mathrm{v_h i}}\int \mathbf{e}_{\mathrm{v_h}} + \mathbf{K}_{\mathrm{v_h d}}\dot{\mathbf{e}}_{\mathrm{v_h}}\right), a_2\right)$$

$a_1, a_2 \geq 0$ Where. The directional saturation function is defined as follows $\mathrm{sat_{gd}}(\mathbf{u}, a)$

$$\mathrm{sat_{gd}}(\mathbf{u}, a) \triangleq \begin{cases} \mathbf{u}, & \|\mathbf{u}\|_{\infty} \leq a \\ a\dfrac{\mathbf{u}}{\|\mathbf{u}\|_{\infty}}, & \|\mathbf{u}\|_{\infty} > a \end{cases}$$

$\mathrm{sat_{gd}}(\mathbf{u}, a)$ $\mathrm{sat}(\mathbf{u}, a)$

The difference between the direction-preserving saturation function and the traditional saturation function: The saturation function limits the absolute value of each component to no greater than a, but it may have a different direction from x; And the direction-preserving saturation function can not only limit the absolute value of each component of the final component to no greater than a, but also guarantee that its direction is the same as x.

Experimental principle

☐ Position control

(3) Add saturated PID control

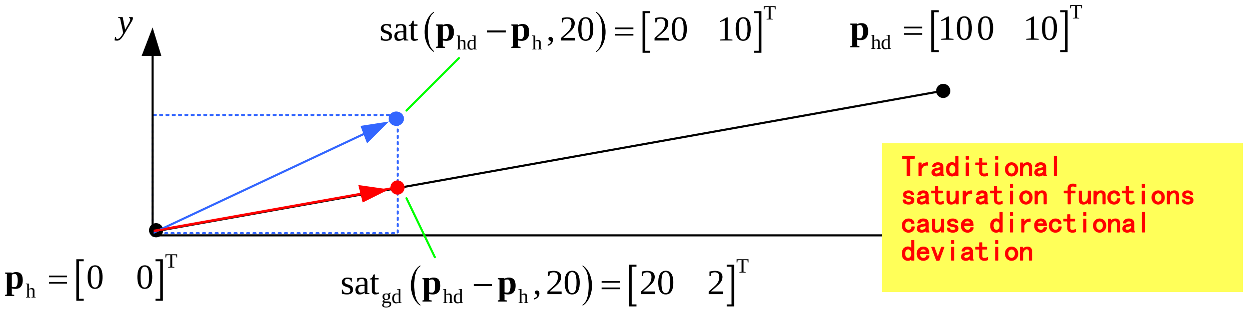


Figure. Comparison of the effect of two saturation functions

Conventional saturation And function

$$\mathrm{sat}(\mathbf{x},a)\triangleq\begin{bmatrix}\mathrm{sat}(x_1,a)\\ \vdots\\ \mathrm{sat}(x_n,a)\end{bmatrix},\mathrm{sat}(x_k,a)\triangleq\begin{cases}x_k & ,|x_k|\le a\\ a\cdot\mathrm{sign}(x_k), & |x_k|>a\end{cases}$$

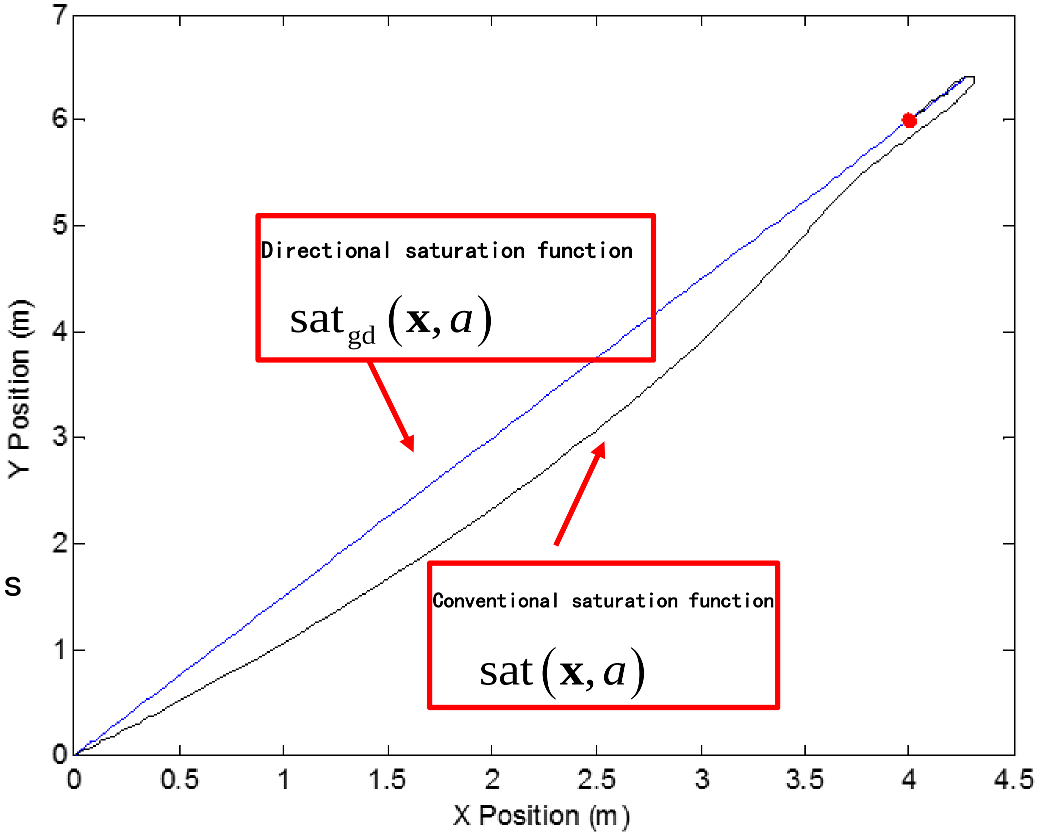


Figure. Multi-rotor fixed-point tracking of horizontal plane motion trajectory unde

The directional saturation function can guarantee straight flight of multiple rotors, while the traditional saturation function cannot

Experimental principle

□ Position control that produces the desired Euler Angle

(3) Add saturated PID control

In order to avoid over-throttle, it is also necessary to add saturation. In the traditional PID controller design, we need to apply a lower limit to the right end of the controller

$$f_{\mathrm{d}} = \mathrm{sat}_{\mathrm{gd}}\left(m\left(g + k_{p_z\mathrm{d}}\dot{p}_z + k_{p_z\mathrm{p}}\left(p_z - p_{z_\mathrm{d}}\right)\right), a_3\right)$$

Where,. Similarly, in the open source autopilot PID design, we add clipping to the right end of the pair and controller $a_3 > 0$

$$e_{v_z} = \mathrm{sat}_{\mathrm{gd}}\left(v_z - v_{z\mathrm{d}}, a_4\right)$$

e_{v_z}

$$f_{\mathrm{d}} = \mathrm{sat}_{\mathrm{gd}}\left(m\left(g + k_{v_z\mathrm{p}}e_{v_z} + k_{v_z\mathrm{i}}\int e_{v_z} + k_{v_z\mathrm{d}}\dot{e}_{v_z}\right), a_5\right)$$

Where. For one-dimensional variables, the direction-preserving saturation function acts the same as the traditional saturation function. $\mathrm{sat}_{\mathrm{gd}}$

$a_4, a_5 > 0$ sat

Experimental principles

☐ Control system correction

Series calibration is mainly introduced below. The system structure diagram of adding series correction is shown in the figure. $G(s)$Where represents the transfer function of the series correction device, $G_c(s)$ represents the transfer function of the invariant part of the system. The series correction commonly used in engineering practice are lead correction, lag correction and lag - lead correction.

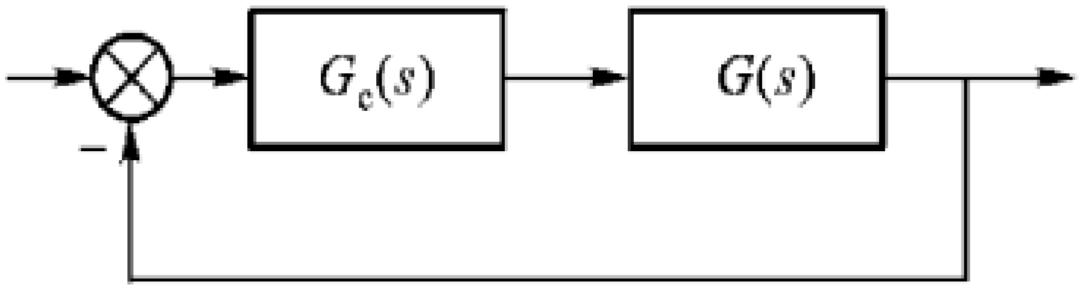


Figure. System series correction

Experimental principle

Control system correction

(1) Phase lead correction

$$G_c(s) = \frac{1 + aTs}{1 + Ts}(a > 1)$$

Features: Provide positive phase shift.

Phase lead correction mainly occurs in the frequency band $\left(\frac{1}{aT}, \frac{1}{T}\right)$

And the maximum value of the lead is:

$$\varphi_m = \arcsin\frac{a-1}{a+1}$$

This maximum occurs at the geometric center of the logarithmic frequency characteristic curve, and the corresponding angular frequency is: $\omega_m = \frac{1}{\sqrt{aT}}$

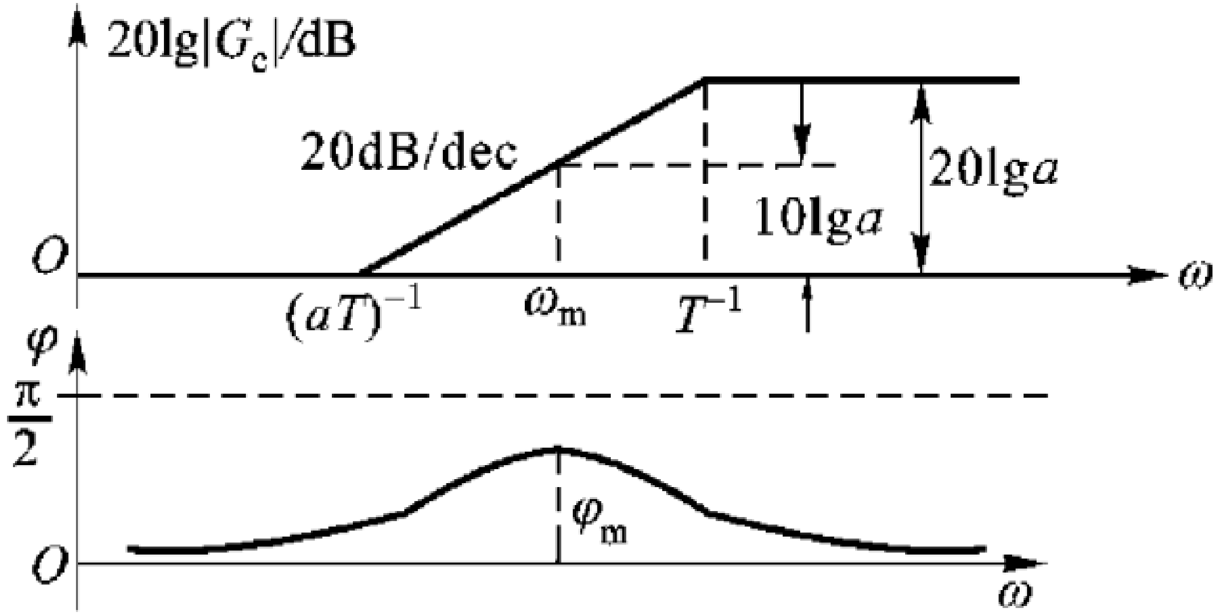


Figure. Amplitude-frequency curve of advance correction

Experimental principle

Control system correction

(2) Phase lag correction

$$G_c(s)=\frac{1+bTs}{1+Ts}(b<1)$$

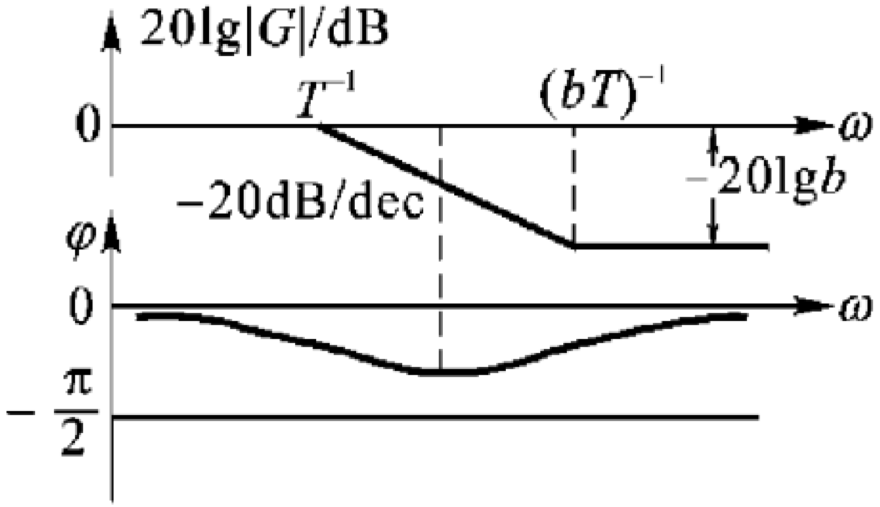


Figure. Lag correction amplitude-frequency curve

(3) Phase lag correction

$$G_c(s)=\frac{(1+bT_1s)(1+aT_2s)}{(1+T_1s)(1+T_2s)}$$

其中$a>1, b>1$，且有$bT_1>aT_2$

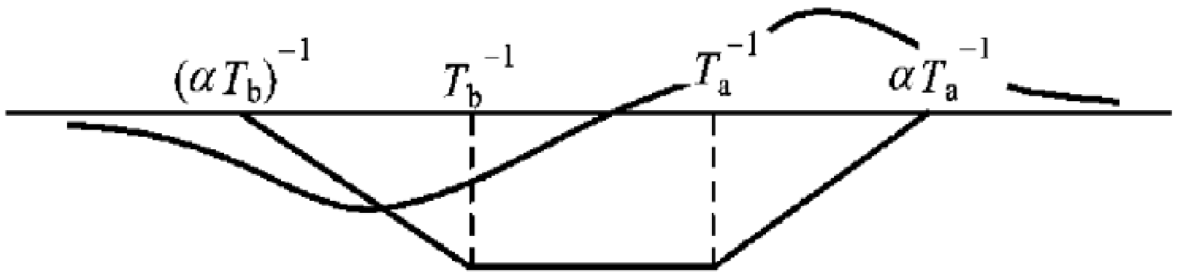


Figure. Amplitude-frequency curve of lag lead correction

Principle of Experiment

See "Quan Quan. Introduction to Multicopter Design and Control. Springer, Singapore, 2017" or "Discretionary. Du Guangxun, Zhao Zhiyao, Dai Xunhua, Ren Jinrui, Deng Hengyi. Design and Control of Multi-rotor Aircraft, Publishing House of Electronics Industry, 2018, Chapter 11.

Basic experiments

☐ Experimental objectives

■ Known

(1) Hardware: Pixhawk Autopilot system, multi-rotor hardware system.

(2) software: MATALB 2017 b or above and, based on Simulink platform for the controller design and simulation, hardware in the loop simulation platform, the experiment guidance package "e6.1" (download address: https://rflysim.com/course).

■ Goal

(1) Reproduce the quadrotor Simulink simulation to analyze the control action on the shaft and the decoupling of the shaft.

(2) Sweep the system to draw bode diagram and analyze the stability margin of the closed-loop position control system;

$o_b x_b$ $o_b y_b$

(3) Complete the hardware-in-the-loop simulation.

Basic experiment

Experimental steps

See e6.1\Readme.pdf

Figure. Screenshot of position control sim

Analyze the experiment

◻ Experimental objectives

■ Known

Software: (1) 2017 b or above and MATLAB, controller design and simulation based on Simulink platform, experiment guidance package "e6.2" (download address: https://rflysim.com/course).

■ Goal

(1) Adjust the relevant parameters of PID controller to improve the system control performance, record the overshoot and adjustment time, and get a set of satisfactory parameters.

(2) After obtaining satisfactory parameters, the system is swept to draw the Bode diagram, observe the amplitude-frequency response and phase-frequency response curves of the system, and analyze its stability margin.

Analyze the experiment

□ Experiment steps

See e6.2\Readme.pdf

Figure. Screenshot of position control sim

Analyze the experiment

□ Notes

(1) The premise of the frequency response curve is that the quadrotor is in a balanced state, that is, the quadrotor keeps hovering, so to get the correct frequency response curve must keep the quadrotor in a balanced state.

(2) Clarify the input and output of the system, correctly select the input and Output points. When testing the Open-loop system, select the output point type as "open-loop output", and use "Output Measurement" when testing the closed loop. https://ww2.mathworks.cn/help/slcontrol/ug/specify-portion-of-model-to-linearize-in-simulink-model.html introduces can refer to the official document.

Design experiment

□ Experiment objectives

■ **Known**

(1) Hardware: Pixhawk Autopilot system, multi-rotor hardware system.

(2) Software: MATLAB 2017b or above, controller design and simulation platform based on Simulink, hardware-in-the-loop simulation platform, experiment instruction package "e6.3" and "e6.4" (download address: https://rflysim.com/course).

■ **Goals**

1) Establish the transfer function model of the position control channel, use MATLAB "ControlSystemDesigner" to design the correction controller, so that after adding the correction link, the step response of the system speed control ring is steady state error, and the phase margin is >75 ° . Cutoff frequency >2.0rad/s $e_{rss} \leq 0.01$. Position control ring cutoff frequency > 1rad/s, phase margin >60° .

2) Software-in-the-loop simulation experiment and hardware-in-the-loop simulation experiment were carried out with the controller designed by ourselves

3) Use the self-designed controller to carry out the real flight experiment.

Design experiment

☐ Experimental design

(1) Step 1: Simplify the overall structure

First, only one channel is analyzed, here channel x is selected. The simplified model is shown below.

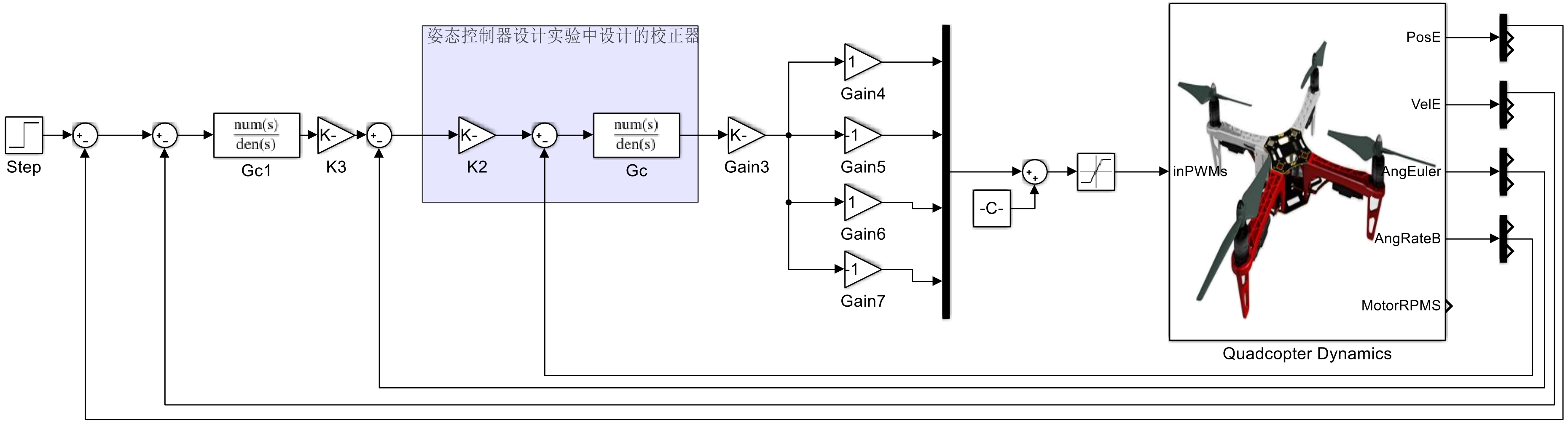


Figure. Simplified model of X-axis position control system

Design experiment

□ Experimental design

(2) Step Two: Velocity loop analysis

The input is the desired speed and the output is the actual speed. The steps are the same as for generating the Bode diagram. Set the input and output points as shown.

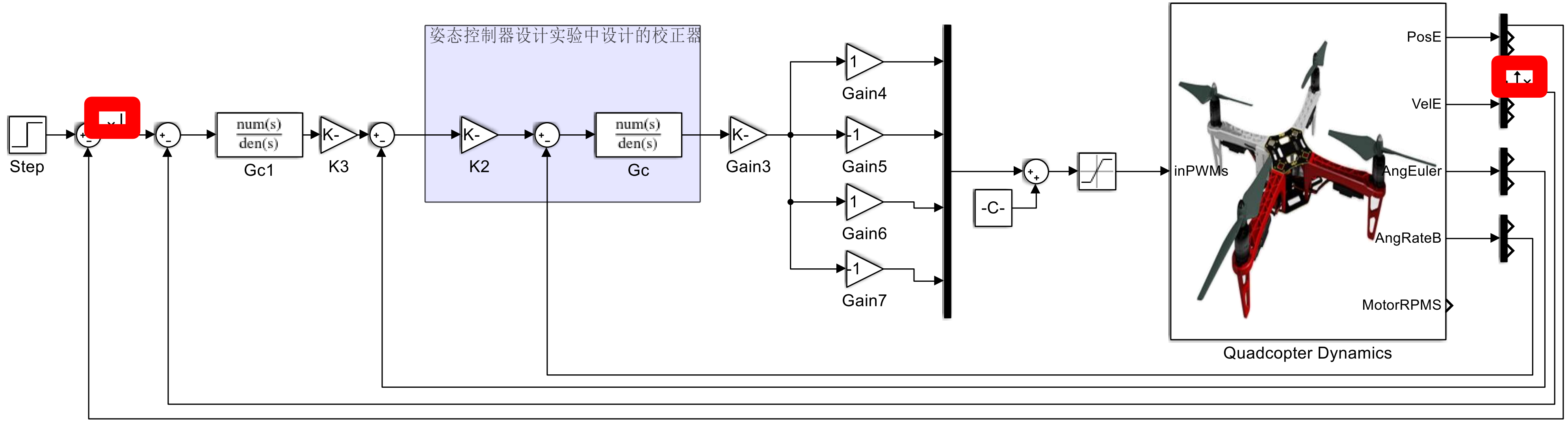


Figure. Set input/output points

Design the experiment

Experiment Steps

See e6.3\Readme.pdf and e6.4\Readme.pdf

Figure. Screenshot of position control sim

Summary

(1) Based on the position model of the multi-rotor, the common PID control method is established, and the design of the position controller is completed in MATLAB/Simulink, and the simulation effect is displayed in RflySim3D.

(2) Use the code generated by PSP toolbox in Simulink and download it to Pixhawk autopilot for hardware-in-the-loop simulation experiment.

(3) Adjust the parameters of PID controller, try to get a satisfactory set of parameters, and use MATLAB system analysis tool to get the Bode diagram of the entire open loop system, and check the phase margin and amplitude margin of the corresponding closed loop system.

(4) Using the system correction method in the automatic control principle to correct the multi-rotor system, the lead and lag lead links are designed to control the position ring and the speed ring respectively, and achieve the design index. After the completion of the design, the connected hardware is simulated in the ring and the real flight experiment verifies the design effect.

If in doubt, please visit https://doc.rflysim.com for more information.

Thanks！