Q1

(a) $L^+ = LMNOPQRS$, so L is a superkey and L → NO does not violate BCNF

$M^+ = MP$, so M → P violates BCNF

$N^+ = MNPQR$ (But not LOS), so N → MQR violates BCNF

$O^+ = OS$, so O → S violates BCNF

(b)

- Decompose R using FD M → P, $M^+ = MP$, so this yields two relations:

  R1 = LMNOQRS and R2 = MP.

- Project the FDs onto R2 = MP.

M	P	closure	FDs
√		$M^+ = MP$	M → P, M is a superkey of R2.
	√	$P^+ = P$	Nothing

this relation satisfies BCNF.

- Project the FDs onto R1 = LMNOQRS.

L	M	N	O	Q	R	S	closure	FDs
√							$L^+ = LMNOPQRS$	L → MNOQRS: L is a superkey of R1
	√						$M^+ = MP$	Nothing
		√					$N^+ = MNPQR$	N → MQR: violates BCNF, abort the projection

We must decompose R1 further.

- Decompose R1 using FD N → MQR, so this yields two relations:

  R3 = LNOS and R4 = MNQR.

- Project the FDs onto R3 = LNOS.

L	N	O	S	closure	FDs
√				$L^+ = LMNOPQRS$	L → NOS: L is a superkey of R3
	√			$N^+ = MNPQR$	Nothing
		√		$O^+ = OS$	O → S: violates BCNF, abort the projection

We must decompose R3 further.

- Decompose R3 using FD O → OS, so this yields two relations:

  R5 = LNO and R6 = OS.

- Project the FDs onto R5 = LNO.

L	N	O	closure	FDs
√			$L^+ = LMNOPQRS$	L → NO: L is a superkey of R5
	√		$N^+ = MNPQR$	Nothing
		√	$O^+ = OS$	Nothing

this relation satisfies BCNF.

- Project the FDs onto R6 = OS.

O	S	closure	FDs
√		$O^+ = OS$	O → S: O is a superkey of R6
	√	$S^+ = S$	Nothing

this relation satisfies BCNF.

- Return to R4 = MNQR, and project the FDs onto R4.

M	N	Q	R	closure	FDs
√				$M^+ = MP$	Nothing
	√			$N^+ = MNPQR$	N → MQR: N is a superkey of R4
		√		$Q^+ = Q$	Nothing
			√	$R^+ = R$	Nothing
Subsets of N				Irrelevant	Can only generate weaker FDs than what we already have
√		√		$MQ^+ = MPQ$	Nothing
√			√	$MR^+ = MPR$	Nothing
		√	√	$QR^+ = QR$	Nothing
√		√	√	$MQR^+ = MPQR$	Nothing

this relation satisfies BCNF.

Thus, final decomposition:

1. R2 = MP with FD M → P,
2. R5 = LNO with FD L → NO,
3. R6 = OS with FD O → S,
4. R4 = MNQR with FD N → MQR.

(c) Yes, our schema preserves dependencies because every FD, there is a relation that includes all of the FD's attributes, such as

R2 = MP with FD M → P,
R5 = LNO with FD L → NO,
R6 = OS with FD O → S,
R4 = MNQR with FD N → MQR.

(d) Since we decompose into relations LNO, OS, MNQR, MP, the chase test demonstrates that it is a lossless-join decomposition.

We start with:

L	M	N	O	P	Q	R	S
1	1	n	o	2	3	4	5
6	7	8	o	9	10	11	s
12	m	n	13	13	q	r	15
16	m	17	18	p	19	20	21

then, because $O \rightarrow S$, we make these changes:

L	M	N	O	P	Q	R	S
l	1	n	o	2	3	4	~~5~~ s
6	7	8	o	9	10	11	s
12	m	n	13	13	q	r	15
16	m	17	18	p	19	20	21

then, because $N \rightarrow MQR$, we make these changes:

L	M	N	O	P	Q	R	S
l	~~1~~ m	n	o	2	~~3~~ q	~~4~~ r	~~5~~ s
6	7	8	o	9	10	11	s
12	m	n	13	13	q	r	15
16	m	17	18	p	19	20	21

then, because $M \rightarrow P$, we make these changes:

L	M	N	O	P	Q	R	S
l	**~~1~~ m**	**n**	**o**	**~~2~~ p**	**~~3~~ q**	**~~4~~ r**	**~~5~~ s**
6	7	8	o	9	10	11	s
12	m	n	13	13	q	r	15
16	m	17	18	p	19	20	21

We observe that the tuple < l, m, n, o, p, q, r, s > does occur. The Chase Test has succeeded.

2.

a.

We will find the minimal basis by following these steps:

1. Simplify FDs to singleton right-hand sides. Name the simplified set of FDs as S1.
   (a) $ACD \to E$
   (b) $B \to C$
   (c) $B \to D$
   (d) $BE \to A$
   (e) $BE \to C$
   (f) $BE \to F$
   (g) $D \to A$
   (h) $D \to B$
   (i) $E \to A$
   (j) $E \to C$
2. Perform the LHS reduction on FDs with multiple attributes on the LHS.
   (a) $ACD \to E$
   Since we have $D^+ = ABCDEF$, which includes E, the LHS can be reduced to D.
   (b) $BE \to A$
   Since we have $B^+ = ABCDE$, which includes A, the LHS can be reduced to B
   (c) $BE \to C$
   Since we have $B^+ = ABCDE$, which includes C, the LHS can be reduced to B, which already exists in S1.
   Since we have $E^+ = ACE$, which includes C, the LHS can be reduced to E, which already exists in S1.
   (d) $BE \to F$
   Since we have $B^+ = ABCDEF$, which includes F, so we can reduce LHS to B.

   After reducing LHS, the new set of FDs S2 is:
   (a) $D \to E$
   (b) $B \to C$
   (c) $B \to D$
   (d) $B \to A$

(e) $B \to F$
(f) $D \to A$
(g) $D \to B$
(h) $E \to A$
(i) $E \to C$

3. Then, find redundant FDs by not considering each of the FDs and observe the closure.

   (a) Not considering $D \to E$
   Then $D^+ = ABCDF$, which doesn't include E. Thus, this FD is kept.
   (b) Not considering $B \to C$
   Then $B^+ = ABDECF$, which includes C. Thus, this FD is redundant.
   (c) Not considering $B \to D$
   Then $B^+ = ABCF$, which doesn't include D. Thus, this FD is kept.
   (d) Not considering $B \to A$
   Then, $B^+ = CDABEF$, which includes A. Thus, this FD is redundant.
   (e) Not considering $B \to F$
   Then, $B^+ = ABCDE$, which doesn't include F. Thus, this FD is kept.
   (f) Not considering $D \to A$
   Then, $D^+ = DBECAF$, which includes A. Thus, this FD is redundant.
   (g) Not considering $D \to B$
   Then, $D^+ = ADEC$, which doesn't include B. Thus, this FD is kept.
   (h) Not considering $E \to A$
   Then, $E^+ = CE$, which doesn't include A. Thus, this FD is kept.
   (i) Not considering $E \to C$
   Then, $E^+ = AE$, which doesn't include C. Thus, this FD is kept.

   After removing the redundant ones and order alphabetically, the set of FDs become

   (a) $B \to D$
   (b) $B \to F$
   (c) $D \to B$
   (d) $D \to E$
   (e) $E \to A$
   (f) $E \to C$

   Since no further simplification is possbile, this is the minimal basis.

b.

First of all, notice that H and G are not in any of the functional dependencies in the minimal basis. Thus, they need to be in every of the keys for P.
Other than H, G, we still need to determine A,B,C,D,E,F. From the minimal basis, $D^+ = ABCDEF$, i.e. D determines all of A,B,C,D,E,F.
Since we need at least 3 attributes in the key (H, G, and at least one other attributes to determine A,B,C,D,E,F), DHG is a valid key.
Now we know that keys have 3 attributes and must contain H, G. To find other keys, we only need to verify the closures of AHG, BHG, CHG, EHG, FHG.

1. $AHG^+ = AHG$
   Since AHG doesn't determine all attributes, AHG is not a valid key.
2. $BHG^+ = BHGDEFAC$ Since BHG determines all attributes, BHG is a valid key.
3. $CHG^+ = CHG$
   Since CHG doesn't determine all attributes, CHG is not a valid key.
4. $EHG^+ = EHGAC$
   Since EHG doesn't determine all attributes, EHG is not a valid key.
5. $FHG^+ = FHG$
   Since FHG doesn't determine all attributes, FHG is not a valid key.

Thus, there are two keys for P: BHG and DHG.

c.

First, by merge the RHS of minimal basis, we get

1. $B \to DF$
2. $D \to BE$
3. $E \to AC$

Following the 3NF synthesis algorithm, for each of the FDs $X \to Y$, define a new relation with schema $X \cup Y$.
The resulted set of relations is:

$$R1(D, B, E), R2(B, D, F), R3(E, A, C)$$

Since none of the relations have all its attributes occur within another relation, we don't need to remove any of the relations.

Since the only two keys are BHG and DHG, and none of R1,R2,R3 is a superkey for P, we need to add a new relation whose schema is some key. Since BHG is a key, a new relation with attributes B,H,G would work.

Thus, the final set of relations is:

$$R1(D, B, E), R2(B, D, F), R3(E, A, C), R4(B, H, G)$$

d.

Since all three FDs are used to form the relations, all of the FDs' LHSs are superkeys for their relations, and therefore no redundancy is allowed in R1, R2, R3. Since no FD is related to R4, there is no redundancy allowed in R4 as well. Thus, the schema doesn't allow redundancy.