

# **Phylogenetic Biology**

## **Week 2**

Biology 1425

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2013

# Front matter...

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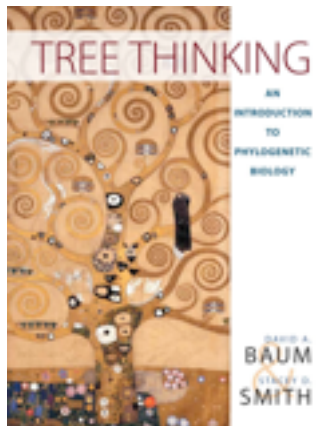


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# Sources

Some non-original content is drawn from:



Baum, D and S. Smith (2012) Tree Thinking: and Introduction to Phylogenetic Biology. Roberts and Company Publishers. ISBN 9781936221165

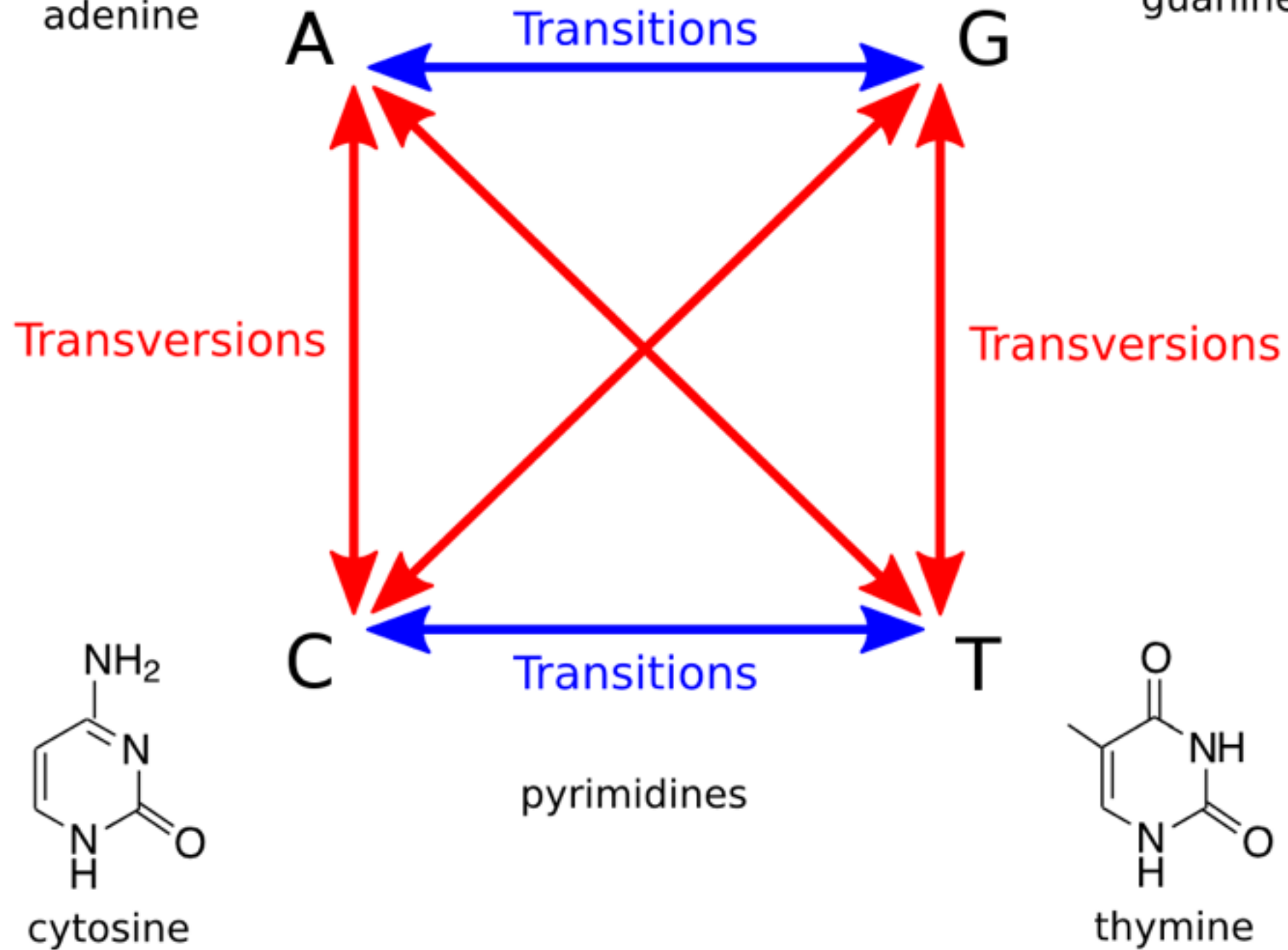
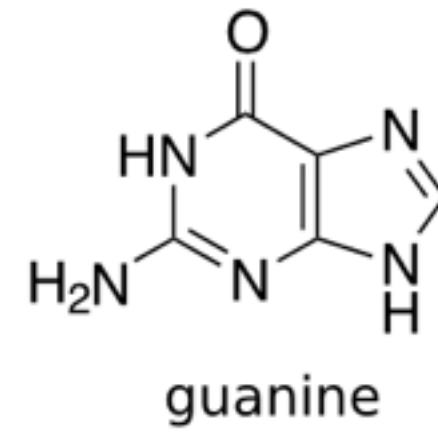
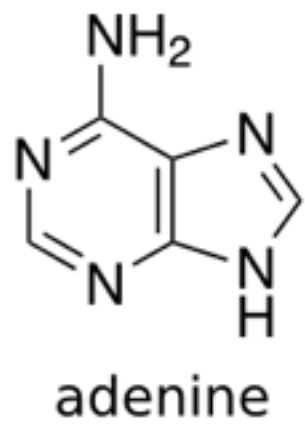
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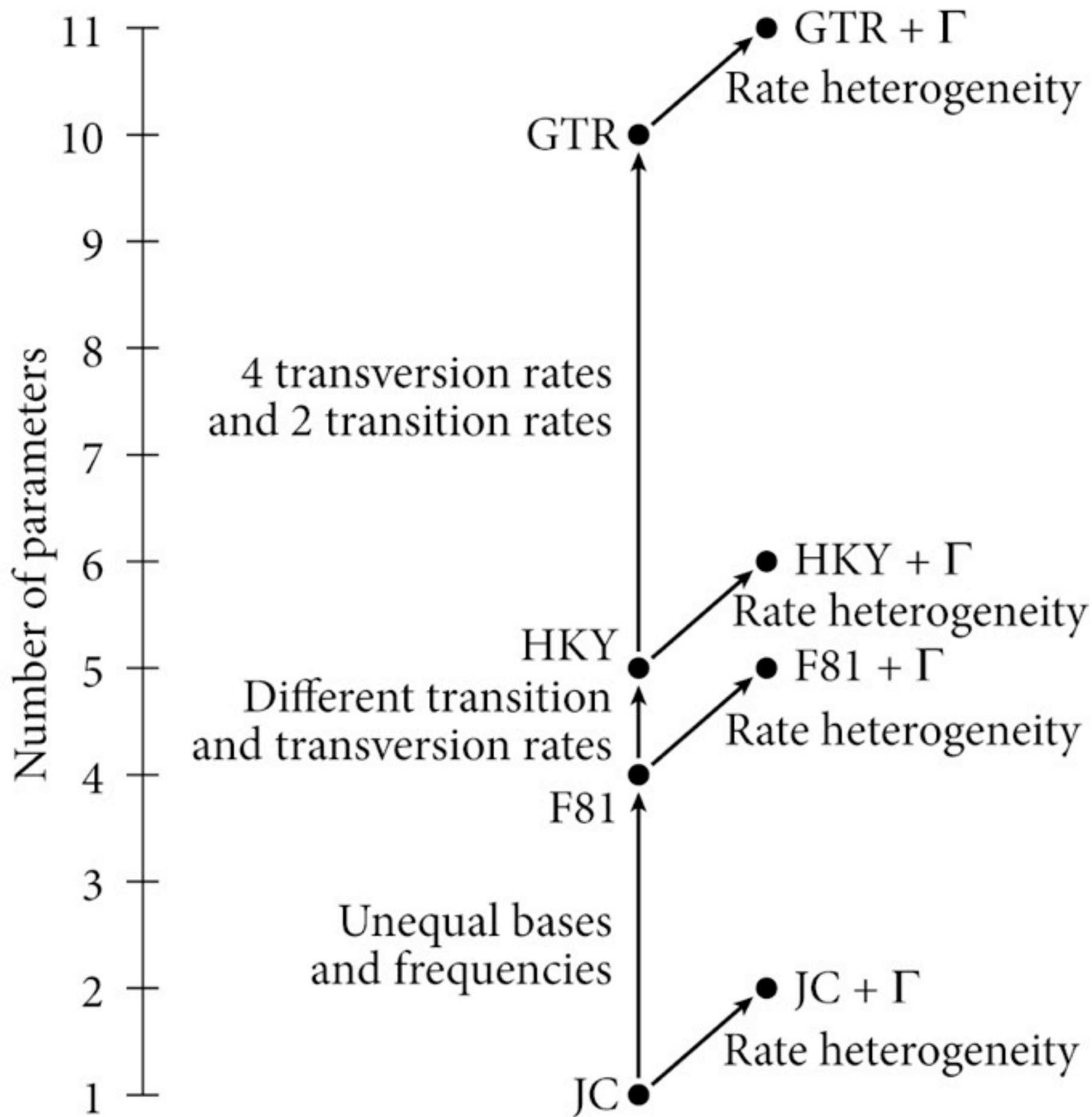
# Other resources

These slides supplement the following excellent presentations from the Wood's Hole Workshop in Molecular Evolution:

Paul Lewis - [http://www.eeb.uconn.edu/people/plewis/downloads/wh2012/Likelihood\\_WoodsHole\\_24July2012\\_1-per-page.pdf](http://www.eeb.uconn.edu/people/plewis/downloads/wh2012/Likelihood_WoodsHole_24July2012_1-per-page.pdf)

John Huelsenbeck - <https://molevol.mbl.edu/wiki/images/3/37/WoodsHoleHandout.pdf>





Baum and Smith 2012, Figure 8.10

# Rate matrix

The instantaneous rate of a given substitutions

$Q$  - Rate matrix

# Substitution probability matrix

The probability of a given substitution occurring in a given interval (branch length). Because of reversals, there are an infinite number of histories that could have given rise to the particular substitution. Can be derived from the rate matrix.

$P$  - Substitution probability matrix



# Substitution probability matrix

Substitution  
probability  
matrix

Rate matrix

The diagram illustrates the equation  $P(v) = e^{Qv}$ . Three arrows point to the components of the equation: one from 'Substitution probability matrix' to  $P$ , one from 'Rate matrix' to  $Q$ , and one from 'Branch length' to  $v$ .

$$P(v) = e^{Qv}$$

This is called matrix exponentiation

# F81 model

$Q$  - Rate matrix

		To:			
		A (freq = $\pi_A$ )	C (freq = $\pi_C$ )	G (freq = $\pi_G$ )	T (freq = $\pi_T$ )
From:	A (freq = $\pi_A$ )	$-m(\pi_C + \pi_G + \pi_T)$	$\pi_C m$	$\pi_G m$	$\pi_T m$
	C (freq = $\pi_C$ )	$\pi_A m$	$-m(\pi_A + \pi_G + \pi_T)$	$\pi_G m$	$\pi_T m$
	G (freq = $\pi_G$ )	$\pi_A m$	$\pi_C m$	$-m(\pi_A + \pi_C + \pi_T)$	$\pi_T m$
	T (freq = $\pi_T$ )	$\pi_A m$	$\pi_C m$	$\pi_G m$	$-m(\pi_A + \pi_C + \pi_G)$

$P$  - Substitution probability matrix

		To:			
		A	C	G	T
From:	A	$\pi_A + (1 - \pi_A)e^{-mt}$	$\pi_C(1 - e^{-mt})$	$\pi_G(1 - e^{-mt})$	$\pi_T(1 - e^{-mt})$
	C	$\pi_A(1 - e^{-mt})$	$\pi_C + (1 - \pi_C)e^{-mt}$	$\pi_G(1 - e^{-mt})$	$\pi_T(1 - e^{-mt})$
	G	$\pi_A(1 - e^{-mt})$	$\pi_C(1 - e^{-mt})$	$\pi_G + (1 - \pi_G)e^{-mt}$	$\pi_T(1 - e^{-mt})$
	T	$\pi_A(1 - e^{-mt})$	$\pi_C(1 - e^{-mt})$	$\pi_G(1 - e^{-mt})$	$\pi_T + (1 - \pi_T)e^{-mt}$

Baum and Smith 2012, Figures 8.7, 8.8

# F81 model

As the branch length goes to 0, **P** becomes a diagonal matrix

	A	C	G	T
A	1	0	0	0
C	0	1	0	0
G	0	0	1	0
T	0	0	0	1

***P*** - Substitution probability matrix

		To:			
		A	C	G	T
From:	A	$\pi_A + (1 - \pi_A)e^{-mt}$	$\pi_C(1 - e^{-mt})$	$\pi_G(1 - e^{-mt})$	$\pi_T(1 - e^{-mt})$
	C	$\pi_A(1 - e^{-mt})$	$\pi_C + (1 - \pi_C)e^{-mt}$	$\pi_G(1 - e^{-mt})$	$\pi_T(1 - e^{-mt})$
	G	$\pi_A(1 - e^{-mt})$	$\pi_C(1 - e^{-mt})$	$\pi_G + (1 - \pi_G)e^{-mt}$	$\pi_T(1 - e^{-mt})$
	T	$\pi_A(1 - e^{-mt})$	$\pi_C(1 - e^{-mt})$	$\pi_G(1 - e^{-mt})$	$\pi_T + (1 - \pi_T)e^{-mt}$

Baum and Smith 2012, Figures 8.7, 8.8