Worst-Case Interdiction Analysis of Large-Scale Electric Power Grids

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***Abstract*—This paper generalizes Benders decomposition to maximize a nonconcave objective function and uses that decomposition to solve an "electric power grid interdiction problem." Under one empirically verified assumption, the solution to this bilevel optimization problem identifies a set of components, limited by cardinality or "interdiction resource," whose destruction maximizes economic losses to customers (and can thereby guide defensive measures). The decomposition subproblem typically incorporates a set of dc optimal power-flow models that cover various states of repair after an attack, along with a load-duration curve. Test problems describe a regional power grid in the United States with approximately 5000 buses, 6000 lines, and 500 generators. Solution time on a 2-GHz personal computer is approximately one hour.**

***Index Terms*—Failure analysis, load flow analysis, power system security.**

I. Introduction

A. Background on Interdiction Models for Power Grids

SALMERON *et al.* [1] develop a bilevel optimization model whose solution identifies critical components of an electric power transmission grid. Specifically, [1] posits an "interdictor" seeking an optimal subset of grid components which, if "interdicted" (i.e., disabled), would maximize "disruption" to the grid's customers. Attacked components are critical to the grid's functionality, and may warrant special defensive measures.

In this "electric power grid interdiction problem," we typically measure disruption in terms of long-term energy-shedding, but can also measure it in terms of medium-term, peak power-shedding. (As discussed below, we ignore the short-term shedding that might result from cascading outages immediately after an attack.) Either objective, medium- or long-term, may weight buses and/or customer sectors differently to account for differing economic losses.

Solution methods for this problem, except heuristics (see [1] and Bier *et al.* [2]), have been unable to solve large, real-world problems. This paper demonstrates a decomposition method that overcomes this difficulty. The method does require one essential assumption that does not hold in general, but we find no violations of the assumption in practice.

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Our ultimate goal is to help electric utility companies identify critical transmission-system components whose protection yields a system that is robust against attacks by an intelligent adversary. Protection can involve "hardening," adding security personnel, stockpiling spare parts, etc. A trilevel model can be defined to identify optimal protective measures given a limited budget for those measures (Brown *et al.* [3]). Unfortunately, we cannot yet solve such models at the scale necessary for real-world transmission systems. Using our bilevel model, however, will give utilities an objective means to compare how various, proposed, budget-feasible portfolios of defensive measures will protect against worst-case attacks.

Our approach contrasts with schemes that rank system components using ad hoc measures of "importance" or "criticality"; for example, see Albert *et al.* [4], Chassin and Posse [5], Espiritu *et al.* [6], and Qiang and Nagurney [7]. Such measures are not typically validated using standard engineering (i.e., power-flow) models. Bier *et al.* [2] do measure system functionality using a power-flow model but, in effect, rank components through a greedy heuristic.

We do not model short-term outages that may be caused by cascading failures (e.g., Mili *et al.* [9]). A cascading failure caused by an attack on a power grid may create widespread outages, and some economic distress. But, the distress caused by medium- and long-term outages is likely to be much greater. For instance, it may take weeks or months to replace transformers that are damaged by an attack, while the effects of an accompanying cascading failure may be overcome (as best possible given the damaged transformers) within a few hours or days.

Finally, we note that it will never be possible to identify an adversary's exact motivation, at least not before an attack. It is probably better to prepare for a worst-case attack, measured in some objective fashion, than to guess at an adversary's motivation and prepare defenses against that. An incorrect guess may overlook a devastating attack. (However, if the adversary's motives should become known with certainty, then the approach described by Motto *et al.* [8] may apply.)

B. Detailed Interdiction Model for an Electric Power Grid

As in [1] and [10], we measure the (medium- and long-term) functionality of a grid using a set of linear dc optimal power flow models (OPFs), each stated in terms of the vector of demands $\mathbf{d}$ (throughout, bold letters indicate vectors), and functioning (non-interdicted) components, i.e., lines, generators and buses. In addition to disregarding losses from cascading failures, we disregard details related to initial restoration after an attack, e.g., unit-commitment issues that arise as service is restored to different parts of the grid. The objective function for

OPF includes terms for generation costs and for penalty costs associated with shed power or energy. The penalties approximate both the direct and indirect costs of the corresponding unserved demand.

The ultimate goal is to measure how well the grid functions following the initial restoration after an attack, and how that functionality improves as interdicted components are repaired: this requires the solution of multiple instances of OPF. To simplify the presentation, however, we initially assume constant demands and a single, constant repair time for any interdicted component. Thus, only a single instance of OPF need be solved to evaluate the effects of a given attack. (We extend to a time-phased model in Section II-D.) The model follows.

Indices:

$i \in \mathcal{I}$	buses, with i_0 as the reference bus;
$g \in \mathcal{G}$	generating units, with $\mathcal{G}_i \in \mathcal{G}$ units connected to bus i;
$l \in \mathcal{L}$	AC transmission lines (including transformers, which are modeled as lines);
$l \in \mathcal{L}^{DC}$	DC transmission lines;
$o(l)$	origin bus for line l;
$d(l)$	destination bus for line l;
$c \in \mathcal{C}$	customer sectors.

Data [units]:

h_g	generation cost for unit g [\$/MWh];
q_{ic}	load-shedding cost for sector c at bus i [\$/MWh];
$\bar{P}_g^{\mathcal{G}}$	maximum output from generating unit g [MW];
$\bar{P}_l^{\mathcal{L}}$	transmission capacity for line l [MW];
B_l	series susceptance of AC line l [Ω^{-1}];
μ_l	loss coefficient for DC line l [unitless];
d_{ic}	load for consumer sector c at bus i [MW] (the vector of demands is $\mathbf{d}$).

Variables [units]:

θ_i	phase angle at bus i [radians].
$P_l^{\mathcal{L}}$	power flow on AC line l [MW];
$\mu_l U_l - V_l$	power flow on DC line l [MW];
$P_g^{\mathcal{G}}$	generation from unit g [MW];
S_{ic}	load shed by customer sector c at bus i [MW].

Formulation: $\mathrm{OPF}(\mathcal{L},\mathcal{G},\mathcal{I};\mathbf{d})$

$$z^* = \min_{\mathbf{P}^{\mathcal{G}},\mathbf{P}^{\mathcal{L}},\mathbf{S},\boldsymbol{\theta},\mathbf{U},\mathbf{V}} \sum_g h_g P_g^{\mathcal{G}} + \sum_i \sum_c q_{ic} S_{ic} \quad \text{(OPF.0)}$$

$$\text{s.t. } P_l^{\mathcal{L}} = B_l\left(\theta_{o(l)} - \theta_{d(l)}\right) \forall l \in \mathcal{L} \quad \text{(OPF.1)}$$

$$\sum_{g \in \mathcal{G}_i} P_g^{\mathcal{G}} - \sum_{\substack{l \in \mathcal{L} | \\ o(l)=i}} P_l^{\mathcal{L}} + \sum_{\substack{l \in \mathcal{L} | \\ d(l)=i}} P_l^{\mathcal{L}} + \sum_{\substack{l \in \mathcal{L}^{DC} | \\ o(l)=i}} (-U_l + \mu_l V_l) + \sum_{\substack{l \in \mathcal{L}^{DC} | \\ d(l)=i}} (\mu_l U_l - V_l) = \sum_c (d_{ic} - S_{ic}) \, \forall i \in \mathcal{I} \quad \text{(OPF.2)}$$

$$-\bar{P}_l^{\mathcal{L}} \le P_l^{\mathcal{L}} \le \bar{P}_l^{\mathcal{L}}, \quad \forall l \in \mathcal{L} \cup \mathcal{L}^{DC} \quad \text{(OPF.3)}$$

$$0 \le P_g^{\mathcal{G}} \le \bar{P}_g^{\mathcal{G}}, \quad \forall g \in \mathcal{G} \quad \text{(OPF.4)}$$

$$0 \le S_{ic} \le d_{ic}, \quad \forall i \in \mathcal{I}, c \in \mathcal{C} \quad \text{(OPF.5)}$$

$$U_l, V_l \ge 0, \quad \forall l \in \mathcal{L}^{DC} \quad \text{(OPF.6)}$$

$$\theta_{i_0} = 0. \quad \text{(OPF.7)}$$

(Note: All units above are converted into per-unit values for a base power of 100 MVA.)

The objective function (OPF.0) minimizes generation costs plus load-shedding costs in \$/h. (However, the former will typically be negligible in an interdiction problem with load-shedding.) Constraints (OPF.1) are linearized admittance constraints that approximate active power flows on AC lines. Constraints (OPF.2) maintain power-balance at the buses; for any DC line we assume a fixed loss rate for flow in either direction. Constraints (OPF.3) and (OPF.4) set maximum power flows for lines and maximum generating-unit outputs, respectively. Constraints (OPF.5) ensure that load-shedding does not exceed demand. Equation (OPF.6) requires nonnegative power flows on DC lines. Equation (OPF.7) sets the phase angle on the reference bus to 0.

OPF does not need to model substations, but the interdiction problem does, because the buses and transformers (and other equipment) that make up a substation could all be destroyed in a single attack. Consequently, we assume substations $s \in S$ have been defined, along with these derived subsets:

$\mathcal{I}_s \subset \mathcal{I}$	buses at substation s;
$\mathcal{L}_s^{\mathcal{S}} \subset \mathcal{L}$	lines connected to substation s;
$\mathcal{G}_s^{\mathcal{S}} \subset \mathcal{G}$	generators connected to substation s.

Other derived subsets used later in this paper are:

$\mathcal{L}_i^{\mathcal{I}} \subset \mathcal{L}$	lines connected to bus i;
$\mathcal{L}_l^{\parallel} \subset \mathcal{L}$	lines running in parallel to line l (multiple circuits on the same towers or in close proximity that would be interdicted by an attack on line l).

As shorthand, $\min_{\mathbf{p} \in \mathcal{P}(\mathbf{d})} \bar{f}(\mathbf{p})$ will denote OPF, with the vector $\mathbf{p}$ incorporating all decision variables, with $\mathcal{P}(\mathbf{d})$ representing (OPF.1)–(OPF.7), and with $\bar{f}(\cdot)$ representing (OPF.0). In turn, the "interdict power flow model" (IPF) can be stated as

$$\text{IPF}: z^* = \max_{\boldsymbol{\delta} \in \Delta} \min_{\mathbf{p} \in \mathcal{P}(\boldsymbol{\delta};\mathbf{d})} \bar{f}(\mathbf{p}) \quad \text{(IPF.1)}$$

where $\boldsymbol{\delta} \in \Delta \subseteq \{0,1\}^{|\mathcal{K}|}$ represents resource-limited, binary interdiction plans defined on generic components $k \in \mathcal{K}$, and $\mathbf{p} \in \mathcal{P}(\boldsymbol{\delta};\mathbf{d})$ represents feasible operation of the power grid with demand vector $\mathbf{d}$ and with operating components that are dictated by $\boldsymbol{\delta}$. More precisely, for $\boldsymbol{\delta} \in \Delta$, $\delta_k = 1$ if component k (i.e., line, bus, generator, substation) is attacked and disabled, and $\delta_k = 0$, otherwise. The constraint set Δ also includes 1) one or more interdiction-resource constraints (see [1]), and 2) logical constraints to prohibit "inefficient" interdiction plans that, for instance, separately interdict bus i and a line connected to i. The latter, logical constraints are optional, but tend to speed convergence of the solution algorithm.

In summary then, IPF takes the view of an interdictor who wishes to use his limited interdiction resources (with an assumed valuation of resources necessary to disable each component) to maximize "disruption" to the operations of the power grid. Nominally, disruption would measure the difference in operating costs, including penalties for unserved demand, after and before interdiction. Since the cost of operating the system before interdiction may be viewed as a constant, IPF simply seeks to maximize estimated post-interdiction operating costs; we consider the peak operating cost rate or total cost until the system is repaired.

For a fixed interdiction plan $\hat{\boldsymbol{\delta}}$, we evaluate $f(\hat{\boldsymbol{\delta}}) \equiv \min_{\mathbf{p}\in\mathcal{P}(\hat{\boldsymbol{\delta}};\mathbf{d})} \bar{f}(\mathbf{p})$ by first identifying the working components of the transmission grid, denoted $\mathcal{L}(\hat{\boldsymbol{\delta}})$, $\mathcal{I}(\hat{\boldsymbol{\delta}})$ and $\mathcal{G}(\hat{\boldsymbol{\delta}})$ for lines, buses, and generators, respectively. Then, we solve $\mathrm{OPF}(\mathcal{G}(\hat{\boldsymbol{\delta}}),\mathcal{I}(\hat{\boldsymbol{\delta}}),\mathcal{L}(\hat{\boldsymbol{\delta}});\mathbf{d})$, usually denoted here as $\mathrm{OPF}(\hat{\boldsymbol{\delta}})$ for brevity.

C. Existing Approaches for Solving IPF

To date, two basic approaches have been suggested to solve IPF: heuristics, and direct solution of equivalent mixed-integer programs (MIPs).

Reference [1] introduces a decomposition-based heuristic that solves a series of OPF models. (See also the greedy heuristic in [2].) At each iteration n of the heuristic, a tentative interdiction plan $\hat{\boldsymbol{\delta}}^n$ is evaluated, starting with $\hat{\boldsymbol{\delta}}^{\mathbf{0}} = 0$. $\mathrm{OPF}(\hat{\boldsymbol{\delta}}^n)$ provides power-flow patterns, which are used to assign a relative value v_k^n to each component k in the power grid. A MIP then solves $\max_{\boldsymbol{\delta}\in\Delta} \mathbf{v}^n\boldsymbol{\delta}$ to produce $\hat{\boldsymbol{\delta}}^{n+1}$. (Constraints are also added so that no solution repeats.)

The heuristic procedure continues for a fixed number of iterations or until a time limit is reached. The procedure can handle large-scale models because the OPF "subproblems" and MIP "master problem" all solve quickly, even at large scale. As with any heuristic, however, this approach lacks the foundation of a formal algorithm that guarantees convergence (except through total enumeration).

The difficulty of formulating IPF as a standard mixed-integer program stems from the non-convex, max-min nature of the problem. One may view $\boldsymbol{\delta}$ as simply modifying the original sets $\mathcal{L}$, $\mathcal{I}$, and $\mathcal{G}$ to $\mathcal{L}(\boldsymbol{\delta})$, $\mathcal{I}(\boldsymbol{\delta})$, and $\mathcal{G}(\boldsymbol{\delta})$, respectively, but representing this in a MIP is difficult. For example, suppose line $l \in \mathcal{L}$ can be interdicted as can its origin bus $o(l)$ and destination bus $d(l)$. Then, the original constraints (OPF.1) and (OPF.3) in OPF may be written in IPF as in (OPF.1′) and (OPF.3′) at the bottom of the page (with optional logical constraints omitted).

Note that the superscripts on the variables correspond to component types, and the subscripts correspond to individual components. (This convention is also used with coefficients α below.) Obviously, the product terms do not bode well for developing an efficient solution procedure based on mathematical programming.

IPF has been converted into a MIP using two separate methods, however. In [11], we 1) linearize any product term of the form $(1-\delta_{k_1})(1-\delta_{k_2})\ldots$, 2) take the dual of the modified inner minimization, and 3) linearize the product terms that then appear in the dualized objective function. This procedure leads to a large, difficult-to-solve MIP, even for a small power grid. For example, a 48-bus scenario created from the IEEE Reliability Test System (see [12] and [13]) takes up to three minutes to solve on a personal computer, depending on the amount of interdiction resource allocated. For a realistic power grid with a few thousand buses, the MIP equivalent for IPF cannot even be generated because of excessive requirements for computer memory. Alvarez [14] applies Benders decomposition (BD) [15] to this MIP but, again, can solve only small problems.

Motto *et al.* [8] describe a second transformation of IPF into a MIP. Their method also linearizes product terms of the form $(1-\delta_{k_1})(1-\delta_{k_2})\ldots$, but incorporates both primal and dual variables within the same model: an explicit constraint then enforces strong duality for the inner $\mathrm{OPF}(\boldsymbol{\delta})$ as a surrogate for that model. In fact, this elegant approach can model a more general situation in which the interdictor and system operator do not have diametrically opposed objective functions. Motto *et al.* solve the small test scenarios also solved in [11], and report similar computational times. Thus, this approach also seems unsuited for solving large-scale models.

In summary, to date, no formal optimization method has been devised that can solve large-scale electric power grid interdiction problems. A new approach is needed.

II. New Approach

A. Global Benders Decomposition

This section describes a new decomposition algorithm for solving IPF. As with BD applied to a maximizing MIP [15], the algorithm alternates between an integer-programming master problem and one or more linear-programming subproblems. And, like BD, the algorithm does build a concave, piecewise-linear approximating function to the function being maximized (which is the optimal objective value to the OPF as a function of variables $\boldsymbol{\delta}$). Unlike BD, however, the function being approximated need not be concave. Consequently, we refer to the algorithm as "global Benders decomposition" (GLBD). ("Generalized Benders decomposition," or "GBD," developed in [15], still requires the function being maximized to be concave. Thus, GLBD maybe be viewed as a further generalization of GBD.)

A key advantage of GLBD over the MIP approach to solving IPF is that the algorithm's subproblems represent simple, familiar instances of the primal linear program OPF. Thus, the user need not maintain a MIP that involves unfamiliar constructs

$$P_l^{\mathcal{L}} = B_l\left(\theta_{o(l)} - \theta_{d(l)}\right)\left(1-\delta_l^{\mathcal{L}}\right)\left(1-\delta_{o(l)}^{\mathcal{I}}\right)\left(1-\delta_{d(l)}^{\mathcal{I}}\right)\ \forall\, l \in \mathcal{L}_i \qquad \text{(OPF.1')}$$

$$-\bar{P}_l^{\mathcal{L}}\left(1-\delta_l^{\mathcal{L}}\right)\left(1-\delta_{o(l)}^{\mathcal{I}}\right)\left(1-\delta_{d(l)}^{\mathcal{I}}\right) \le P_l^{\mathcal{L}} \le \bar{P}_l^{\mathcal{L}}\left(1-\delta_l^{\mathcal{L}}\right)\left(1-\delta_i^{\mathcal{I}}\right)\left(1-\delta_{d(l)}^{\mathcal{I}}\right) \qquad \text{(OPF.3')}$$

from the dual of the OPF model that are complicated by interactions with binary variables as in [8], [11], and [14]. In fact, our linearized OPF model could be replaced by a nonlinear, ac OPF model (e.g., [16]) and could include security constraints explicitly, although the increased computational burden might be excessive.

The decomposition relies on a sequence of upper-bounding (i.e., optimistic) piecewise-linear functions for the interdictor's objective, $f(\boldsymbol{\delta})$. The maximum of those functions must converge to the optimal solution of IPF since only a finite number of interdiction plans exist; however, practical use of the decomposition rests on verifiably close-to-optimal solutions being found quickly.

Again, let $k \in \mathcal{K}$ index generic grid components. Then, for each $\hat{\boldsymbol{\delta}} \in \Delta$, we can find coefficients $\alpha_k(\hat{\boldsymbol{\delta}})$ such that the affine function

$$f(\hat{\boldsymbol{\delta}}) + \sum_{k \in \mathcal{K}} \alpha_k(\hat{\boldsymbol{\delta}})\,(\delta_k - \hat{\delta}_k)$$

bounds $f(\boldsymbol{\delta})$ from above for all feasible $\boldsymbol{\delta}$, i.e.,

$$f(\boldsymbol{\delta}) \le f(\hat{\boldsymbol{\delta}}) + \sum_{k \in \mathcal{K}} \alpha_k(\hat{\boldsymbol{\delta}})\,(\delta_k - \hat{\delta}_k)\ \forall\,\boldsymbol{\delta},\,\hat{\boldsymbol{\delta}} \in \Delta. \quad \text{(MP.1)}$$

Inequality (MP.1) leads to the following master problem:

$$\begin{aligned} \mathrm{MP}(\hat{\Delta}): \quad & z(\hat{\Delta}) = \max_{\boldsymbol{\delta} \in \Delta,\, z} z \\ \text{s.t.} \quad & z \le f(\hat{\boldsymbol{\delta}}) + \sum_{k \in \mathcal{K}} \alpha_k(\hat{\boldsymbol{\delta}})\,(\delta_k - \hat{\delta}_k), \quad \forall\,\hat{\boldsymbol{\delta}} \in \hat{\Delta} \end{aligned} \quad \text{(MP.2)}$$

where $\hat{\Delta} \subset \Delta$ denotes an enumerated set of feasible vectors in Δ. IPF is equivalent to $\mathrm{MP}(\hat{\Delta})$ when $\hat{\Delta} \equiv \Delta$. Otherwise, $z^* \le z(\hat{\Delta})$, i.e., $\mathrm{MP}(\hat{\Delta})$ provides an upper bound on the optimal objective function value of IPF.

The following algorithm implements GLBD for IPF, although the reader will see that it applies to a much broader class of problems.

Global Benders Decomposition Algorithm (GLBDA)

Input: Grid data, interdiction data, and optimality tolerance $\varepsilon \ge 0$.

Output: $\hat{\boldsymbol{\delta}}^*$, an ε-optimal solution to IPF, with cost z^*.

Initialization:

$\hat{\boldsymbol{\delta}} \leftarrow \mathbf{0}$; /* Initial interdiction plan, assumed feasible */

$\hat{\Delta} \leftarrow \{\hat{\boldsymbol{\delta}}\}$; /* Initial subset of feasible interdiction plans */

$\hat{\boldsymbol{\delta}}^* \leftarrow \hat{\boldsymbol{\delta}}$; /* Current best plan for the interdictor */

$z^* \leftarrow 0$; /* Lower bound on cost of best plan */

$\bar{z} \leftarrow \infty$; /* Upper bound on cost of best plan */

Subproblem:

Solve OPF($\hat{\boldsymbol{\delta}}$) for power-flow solution $\mathbf{p}(\hat{\boldsymbol{\delta}})$ and objective value $f(\hat{\boldsymbol{\delta}})$.

If $f(\hat{\boldsymbol{\delta}}) > z^*$ then $z^* \leftarrow f(\hat{\boldsymbol{\delta}})$ and $\hat{\boldsymbol{\delta}}^* \leftarrow \hat{\boldsymbol{\delta}}$;

If $\bar{z} - z^* \le \varepsilon z^*$, then report $(\hat{\boldsymbol{\delta}}^*, z^*)$ as the ε-optimal solution to IPF and halt;

$\hat{\Delta} \leftarrow \hat{\Delta} \cup \{\hat{\boldsymbol{\delta}}\}$;

Master Problem:

Use $\mathbf{p}(\hat{\boldsymbol{\delta}})$ to compute coefficients $\alpha_k(\hat{\boldsymbol{\delta}})\ \forall\ k \in \mathcal{K}$ satisfying (MP.1), and add the following generalized Benders cut to $\mathrm{MP}(\hat{\Delta})$:

$z \le f(\hat{\boldsymbol{\delta}}) + \sum_{k \in \mathcal{K}} \alpha_k(\hat{\boldsymbol{\delta}})\,(\boldsymbol{\delta}_k - \hat{\boldsymbol{\delta}}_k)$;

Solve $\mathrm{MP}(\hat{\Delta})$ for new interdiction plan $\hat{\boldsymbol{\delta}}$ and for objective value $z(\hat{\Delta})$, and set $\bar{z} \leftarrow z(\hat{\Delta})$;

If $\bar{z} - z^* \le \varepsilon z^*$, then report $(\hat{\boldsymbol{\delta}}^*, z^*)$ as the ε-optimal solution to IPF and halt;

Return to Subproblem step;

(End of Algorithm)

To better understand the need for GLBD versus standard BD, consider first the standard decomposition of a simple, profit-maximizing MIP. The following MIP represents a capacity-expansion problem in which variables $\boldsymbol{\delta}$ correspond to strategic capacity-expansion decisions, and variables $\mathbf{p}$ describe system operation:

$$\begin{aligned} \mathrm{CE}: \max_{\boldsymbol{\delta} \in \Delta} \max_{\mathbf{p} \ge \mathbf{0}}\ & \mathbf{hp} \\ \text{s.t. } & A\,\mathbf{p} = \mathbf{d} \\ & p_k \le u_k \delta_k,\ \forall k \in \mathcal{K}. \end{aligned}$$

The (linear-programming) Benders subproblem, for a specified $\hat{\boldsymbol{\delta}}$, is

$$\begin{aligned} \mathrm{CE}_{\mathrm{LP}}(\hat{\boldsymbol{\delta}}): \quad f(\hat{\boldsymbol{\delta}}) \equiv \max_{\mathbf{p} \ge \mathbf{0}}\ & \mathbf{hp} \\ \text{s.t. } & A\,\mathbf{p} = \mathbf{d} \\ & p_k \le u_k \hat{\delta}_k,\ \forall k \in \mathcal{K}. \quad \text{(LP.1)} \end{aligned}$$

For simplicity, suppose that $\mathrm{CE}_{\mathrm{LP}}(\hat{\boldsymbol{\delta}})$ is bounded and feasible for any $\hat{\boldsymbol{\delta}} \in \Delta$, and define $\beta_k(\hat{\boldsymbol{\delta}})$ as the optimal dual variables for constraints (LP.1). Then, defining $\alpha_k(\hat{\boldsymbol{\delta}}) = u_k \beta_k(\hat{\boldsymbol{\delta}})$ for all $k \in \mathcal{K}$, $\mathrm{MP}(\hat{\Delta})$ becomes the standard Benders master problem for this model. In this case, $f(\cdot)$ is a piecewise-linear, *concave* function (when its argument $\boldsymbol{\delta}$ is viewed as a continuous vector, $\mathbf{0} \le \boldsymbol{\delta} \le \mathbf{1}$), and $\mathrm{MP}(\hat{\Delta})$ defines a piecewise-linear, concave (outer) approximation of that function. Thus, GLBDA applied to CE can correspond to a standard BD algorithm.

IPF is not, however, a pure, maximizing optimization problem, but rather a bilevel, max-min optimization model for planning interdictions. The interdiction analog of CE, with a cost-minimizing operational model, is

$$\begin{aligned} \mathrm{CI}: \max_{\boldsymbol{\delta} \in \Delta} \min_{\mathbf{p} \ge \mathbf{0}}\ & \mathbf{hp} \\ \text{s.t. } & A\,\mathbf{p} = \mathbf{d} \\ & p_k \le u_k \delta_k,\ \forall k \in \mathcal{K}. \end{aligned}$$

(A linearized version of IPF is more complicated than CI, but that fact does not alter the conclusions here.) Accordingly, the natural subproblem becomes

$$\begin{aligned} \mathrm{CI}_{\mathrm{LP}}(\hat{\boldsymbol{\delta}}) : f(\hat{\boldsymbol{\delta}}) \equiv \min_{\mathbf{p}\geq \mathbf{0}} \ & \mathbf{hp} \\ \text{s.t. } A\,\mathbf{p} &= \mathbf{d} \\ p_k &\leq u_k(1-\hat{\delta}_k),\ \forall\, k \in \mathcal{K}. \end{aligned} \qquad \text{(LP.2)}$$

Now, $f(\cdot)$ is a piecewise-linear *convex* function which (generalized) Benders decomposition cannot maximize as it can the concave function in CE. But, GLBDA will still optimize this function as long as two requirements are met: 1) $f(\hat{\boldsymbol{\delta}})$ is easy to evaluate for fixed $\hat{\boldsymbol{\delta}}$, and 2) valid and useful cut coefficients $\alpha_k(\hat{\boldsymbol{\delta}})$ can be defined for $\mathrm{MP}(\hat{\Delta})$. Requirement 1) certainly holds, as evaluation of $f(\hat{\boldsymbol{\delta}})$ involves the solution of a straightforward LP, but requirement 2) demands more thought. We consider two cases in this development: $\hat{\delta}_k = 0$ and $\hat{\delta}_k = 1$.

If $\hat{\delta}_k = 1$, then δ_k can only change to 0, and this adds $\hat{p}_k$ units of capacity to the minimizing subproblem $\mathrm{CI}_{\mathrm{LP}}(\hat{\boldsymbol{\delta}})$. This implies a relaxation, so $f(\cdot)$ will not increase with such a change. Thus, $\alpha_k(\hat{\boldsymbol{\delta}}) = 0$ is a valid cut coefficient.

On the other hand, if $\hat{\delta}_k = 0$, then a change to $\delta_k = 1$ removes capacity in $\mathrm{CI}_{\mathrm{LP}}(\hat{\boldsymbol{\delta}})$, and $f(\cdot)$ may increase. Suppose that component k has "flow" $\hat{p}_k$ given $\boldsymbol{\delta} = \hat{\boldsymbol{\delta}}$, and problem structure implies that a loss of all capacity on component k can lead to at most $\hat{p}_k$ units of unserved demand. If the unit cost for unserved demand is at most $\bar{h}$, then $\alpha_k(\hat{\boldsymbol{\delta}}) = \bar{h}\hat{p}_k$ is a valid cut coefficient for GLBD.

More information about a problem could yield tighter cut coefficients, resulting in a tighter master-problem bound and a more efficient decomposition algorithm. Regardless, we have a theoretically convergent algorithm: computational tests will determine its practical efficiency.

B. Cut Coefficients for Decomposing IPF

We would like to use analogs of the cut coefficients just developed for CI when solving IPF by decomposition. Those analogs are not strictly valid, however, without the following two-part assumption.

Assumption 1: Ignoring short-term load-shedding due to interdiction-caused cascading failures, (a) the interdiction of a set of components $k \in \hat{\mathcal{K}}$, each carrying P_k MW of power, leads to the shedding of at most $\sum_{k\in\hat{\mathcal{K}}} P_k$ MW of demand, and (b) the restoration of an interdicted component does not increase load-shedding. ■

Assumption 1(a) does not hold in general. For example, Baldick [17] describes a dispatch condition in which a generator is producing 1250 MW, and is contributing to counterflow on a constrained line. Interdicting that generator could result in curtailment of more than 1250 MW of demand if the removal of the counterflow also requires other generators to reduce production.

Assumption 1(b) may not hold either. Although restoration relaxes OPF by adding capacity, it also restricts the model by enforcing an admittance constraint (OPF.1). For example, consider a system consisting of two buses joined by two transmission lines of equal admittance, but with capacities 10 MW and 100 MW. With the 100-MW line in service and the 10-MW line interdicted and out of service, the system can transmit 100 MW. But, if the 10-MW line is brought back into service, at most 20 MW can be transmitted.

Despite the counterexamples, we find Assumption 1 to hold in practice, and have empirical evidence of the validity of the cut coefficients it generates. In particular, we have run thousands of GLBDA iterations; have generated cuts based on Assumption 1; have solved the resulting OPFs (after interdictions derived from $\mathrm{MP}(\hat{\Delta})$); and have never found an optimal subproblem objective value that exceeds the master-problem upper bound. Consequently, we feel secure in using the cut coefficients for IPF described next.

To develop these coefficients, we first rewrite the generalized Benders cut (MP.1) to account for specific component types (where the superscripts on the variables again correspond to component types, and the subscripts correspond to individual components)

$$\begin{aligned} z \leq f(\hat{\boldsymbol{\delta}}) &+ \sum_{l\in\mathcal{L}} \alpha_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})\,(\delta_l^{\mathcal{L}} - \hat{\delta}_l^{\mathcal{L}}) + \sum_{i\in\mathcal{I}} \alpha_i^{\mathcal{I}}(\hat{\boldsymbol{\delta}})\,(\delta_i^{\mathcal{I}} - \hat{\delta}_i^{\mathcal{I}}) \\ &+ \sum_{g\in\mathcal{G}} \alpha_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}})\,(\delta_g^{\mathcal{G}} - \hat{\delta}_g^{\mathcal{G}}) + \sum_{s\in S} \alpha_s^{S}(\hat{\boldsymbol{\delta}})\,(\delta_s^{S} - \hat{\delta}_s^{S})\ \forall \hat{\boldsymbol{\delta}} \in \hat{\Delta}. \end{aligned}$$

Now, let $\mathcal{K}(\hat{\boldsymbol{\delta}})$ and $\bar{\mathcal{K}}(\hat{\boldsymbol{\delta}})$ denote, respectively, the functioning and non-functioning components in the grid given interdiction plan $\hat{\boldsymbol{\delta}}$; define analogous subsets, as functions of $\hat{\boldsymbol{\delta}}$, for $\mathcal{L}, \mathcal{G}, \mathcal{I}$ and S; and make these additional definitions:

$P_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}})$	power generation from generator g in solution to $\mathrm{OPF}(\hat{\boldsymbol{\delta}})$;
$P_l^{L}(\hat{\boldsymbol{\delta}})$	power flow on line l in solution to $\mathrm{OPF}(\hat{\boldsymbol{\delta}})$;
$\mathcal{L}_s^o, \mathcal{L}_s^d$	lines whose origin or destination, resp., is bus i;
$\mathcal{L}_s^o, \mathcal{L}_s^d$	"frontier lines" originating or terminating at substation s, respectively;
$\bar{C}$	upper bound on the cost, in \$/h, to shed 1 MW of demand; this paper uses $\bar{C} = \max_{i,c} q_{ic} - \min_g h_g$.

Given feasible interdiction plan $\hat{\boldsymbol{\delta}}$, cut coefficients can now be defined for (MP.1) that are valid under Assumption 1:

$$\alpha_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) = \begin{cases} \bar{C}\overbrace{\left(\left|P_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})\right| + \sum_{l'\in\mathcal{L}_l^{\parallel}} \left|P_{l'}^{\mathcal{L}}(\hat{\boldsymbol{\delta}})\right|\right)}^{\bar{P}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})} \\ \quad \text{if } \hat{\boldsymbol{\delta}} \text{ does not interdict line } l \\ 0 \text{ otherwise} \end{cases} \qquad \text{(IPFC.1)}$$

$$\alpha_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}}) = \begin{cases} \bar{C}\,P_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}}), & \text{if } \hat{\boldsymbol{\delta}} \text{ does not interdict generator } g, \\ 0 \text{ otherwise} \end{cases}$$

$$\alpha_i^{\mathcal{I}}(\hat{\boldsymbol{\delta}}) = \begin{cases} \bar{C}\left(\begin{array}{l} \sum_{g\in\mathcal{G}_i\cap\bar{\mathcal{G}}(\hat{\boldsymbol{\delta}})} P_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}}) + \\ \sum_{\substack{l\in\mathcal{L}_i^d\cap\bar{\mathcal{L}}(\hat{\boldsymbol{\delta}})\,|\\ P_l(\hat{\boldsymbol{\delta}})>0}} P_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) - \sum_{\substack{l\in\mathcal{L}_i^o\cap\bar{\mathcal{L}}(\hat{\boldsymbol{\delta}})\,|\\ P_l(\hat{\boldsymbol{\delta}})<0}} P_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) \end{array}\right) \\ \quad \text{if } \hat{\boldsymbol{\delta}} \text{ does not interdict bus } i, \\ 0 \text{ otherwise} \end{cases}$$

$$\alpha_s^{S}(\hat{\boldsymbol{\delta}}) = \begin{cases} \bar{C}\left(\begin{array}{l} \sum_{g\in\mathcal{G}_s\cap\bar{\mathcal{G}}(\hat{\boldsymbol{\delta}})} P_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}}) + \\ \sum_{\substack{l\in\mathcal{L}_s^d\cap\bar{\mathcal{L}}(\hat{\boldsymbol{\delta}})\,|\\ P_l(\hat{\boldsymbol{\delta}})>0}} P_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) - \sum_{\substack{l\in\mathcal{L}_s^o\cap\bar{\mathcal{L}}(\hat{\boldsymbol{\delta}})\,|\\ P_l(\hat{\boldsymbol{\delta}})<0}} P_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) \end{array}\right) \\ \quad \text{if } \hat{\boldsymbol{\delta}} \text{ does not interdict substation } s \\ 0 \text{ otherwise.} \end{cases}$$

To illustrate, suppose line l is not interdicted. Then coefficient $\alpha_l^L(\hat{\boldsymbol{\delta}})$ represents an optimistic bound (for the interdictor) on the cost of the load that would be shed if line l, currently in operation, were interdicted. This is computed as a function of a bound, $\bar{P}_l^L(\hat{\boldsymbol{\delta}})$, on the total disruption that interdiction of line l might cause, given Assumption 1:

$$\begin{aligned}\bar{P}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) = & \text{power across line } l + \text{power across any lines} \\ & \text{in parallel to } l \text{ that would necessarily be inter–} \\ & \text{dicted with an attack on line } l \text{ because of prox–} \\ & \text{imity} \\ = & \left|P_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})\right| + \sum_{l' \in \mathcal{L}_l^{\parallel}} \left|P_{l'}^{\mathcal{L}}(\hat{\boldsymbol{\delta}})\right|.\end{aligned}$$

Clearly then, an upper bound on the cost of disruption is $\alpha_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) = \bar{C} \times \bar{P}_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}})$. Similarly, $\alpha_g^{\mathcal{G}}(\hat{\boldsymbol{\delta}})$ is an optimistic bound on the cost of unmet demand incurred by interdicting generator g. Computing $\alpha_i^{\mathcal{I}}(\hat{\boldsymbol{\delta}})$ involves all generators and lines injecting power at bus i (thus, the need to distinguish the direction of power flow). We compute $\alpha_s^S(\hat{\boldsymbol{\delta}})$ (for substations) in a similar fashion, except that only frontier lines are involved, to avoid double-counting.

C. Logical Constraints in Δ

Other constraints can be added to Δ in order to speed convergence, although they are unnecessary from a theoretical point of view. We have already mentioned one type of constraint in Section I-A: constraints that forbid interdiction plans that are clearly "inefficient" for the interdictor. As another example, consider

$$\sum_{k \in \mathcal{K} \mid \hat{\delta}_k = 1} \delta_k + \sum_{k \in \mathcal{K} \mid \hat{\delta}_k = 0} (1 - \delta_k) \le |\mathcal{K}| - 1$$

where $\hat{\boldsymbol{\delta}}$ is an already-evaluated interdiction plan, and $\mathcal{K}$ again indexes all components. This constraint forces any new solution $\boldsymbol{\delta}$ to differ from $\hat{\boldsymbol{\delta}}$ in at least one component; such a "supervalid inequality" [18] guarantees not to eliminate any ϵ-optimal solution unless the incumbent solution is already ϵ-optimal.

D. Time-Phased Model and Algorithm

Two considerations motivate the extension of IPF to multiple time periods.

1) Repair Times: As a power grid is repaired after interdiction, its functionality, measured through OPF, will tend to improve.

2) Demand Variation Over Time: OPF should accommodate intra-day demand variations. We use a three-segment staircase function to represent a daily load-duration curve (LDC). (For simplicity of presentation, we assume no inter-day variations in demands.)

IPF extends to handle these temporal issues (not including unit commitment) through simple numerical integration involving the solution of a set of independent OPF models for each fixed $\hat{\boldsymbol{\delta}}$. We make the following definitions for this purpose:

$j \in J$	ordered set $\{1, 2, \ldots\}$ that indexes distinct repair times for components;
t_j	distinct repair times for components, $t_j < t_{j+1}$, each measured in days, with $t_0 \equiv 0$;
$e_k(t_j)$	1 if component $k \in \mathcal{K}$, having been interdicted at time $t_0 = 0$, would be operating by time t_j; and $e_k(t_j) = 0$ otherwise;
$n \in \mathcal{N}$	segments of daily LDC;
$\mathbf{d}_n$	demand vector for segment n of the LDC;
ρ_n	fraction of time during a day that the LDC applies $\mathbf{d}_n$ as the relevant demand vector ;
$f(\hat{\boldsymbol{\delta}}, \mathbf{d})$	the optimal objective-function value for $\mathrm{OPF}(\mathcal{G}(\hat{\boldsymbol{\delta}}), \mathcal{I}(\hat{\boldsymbol{\delta}}), L(\hat{\boldsymbol{\delta}}); \mathbf{d})$, i.e., $\min_{\mathbf{p} \in P(\hat{\boldsymbol{\delta}}; \mathbf{d})} \bar{f}(\mathbf{p})$.

The time-dependent variant of IPF can now be written as

$$\begin{aligned}\mathrm{IPFT}: z^* &= \max_{\boldsymbol{\delta} \in \Delta} z(\boldsymbol{\delta}) \\ &= \max_{\boldsymbol{\delta} \in \Delta} \sum_{j \in J} \sum_{n \in N} \rho_n \times (t_j - t_{j-1}) \\ &\quad \times f\left([\boldsymbol{\delta} - \mathbf{e}(t_j)]^+, \mathbf{d}_n\right)\end{aligned}$$

where $[\mathbf{v}]^+$ denotes the component-wise maximum of $\mathbf{v}$ and $\mathbf{0}$.

The notation used for IPFT deemphasizes the minimization involved in evaluating various instances of $f\left([\boldsymbol{\delta} - \mathbf{e}(t_j)]^+, \mathbf{d}_n\right)$, but it should be clear that evaluating $z(\boldsymbol{\delta})$ for fixed $\boldsymbol{\delta} = \hat{\boldsymbol{\delta}}$ involves the solution of a set of (at most $|\mathcal{N}| \times |J|$) separate instances of OPF. That is, GLBDA applied to IPFT will solve subproblems that decompose by repair time and LDC segment.

To completely define GLBDA for IPFT, we need only define appropriate cut coefficients. This involves straightforward integration as with the objective function. For instance, for a non-interdicted line l, we compute

$$\begin{aligned}\alpha_l^{\mathcal{L}}(\hat{\boldsymbol{\delta}}) = &\sum_{j \in J} \sum_{n \in N} \rho_n \times (t_j - t_{j-1}) \\ &\times \alpha_l'^{\mathcal{L}}\left(\left[\hat{\boldsymbol{\delta}} - \mathbf{e}(t_j)\right]^+, \mathbf{d}_n\right), \ \forall\, l \in \mathcal{L} \backslash \mathcal{L}(\hat{\boldsymbol{\delta}})\end{aligned}$$

where $\alpha_l'^{\mathcal{L}}\left(\hat{\boldsymbol{\delta}}, \mathbf{d}\right)$ denotes the cut coefficient from (IPFC.1), for interdiction plan $\hat{\boldsymbol{\delta}}$ and constant demand $\mathbf{d}$.

III. RESULTS

A. Implementation

We have implemented GLBDA using the Xpress-MP 2006 optimization suite [19] on a 2-GHz laptop computer with 2 GB of RAM. Some key aspects of our implementation are that:

- At every iteration of GLBDA, the first OPF model (for t_1 and $\mathbf{d}_1$) is solved using Xpress-MP's Newton barrier algorithm: this is substantially faster than primal or dual algorithms for this problem. Subsequent OPFs are solved using an advanced starting basis and the dual simplex algorithm in Xpress-MP.
- Master problems are generated with Xpress-MP, but solved using the CPLEX 9.0 [20], because of notable computational savings over the Xpress-MP solver.

TABLE I
OPTIMIZED DISRUPTION VALUES FOR THE RTS-96 TWO-AREA SYSTEM[1]

	Medium-term scenarios			Long-term scenarios		
M	Peak Shed (MW)	Total Shed (GWh)	Iters.	Peak Shed (MW)	Total Shed (GWh)	Iters.
2	309	22.2	43	309	22.2	51
4	842	60.6	28	620	341.1	17
6	1,373	98.8	34	769	611.3	8
8	1,804	129.9	47	1,134	870.9	8
10	2,275	163.8	59	1,416	1,087.5	10
12	2,781	276.5	64	1,804	1,109.7	77

B. Testing on IEEE Reliability Test System RTS-96

The IEEE Two-Area 1996 Reliability Test System (RTS-96; see [12] and [13]) defines our first test grid.

For purposes of comparison, we use the interdiction data presented in [1]: an overhead line requires one unit of interdiction resource to attack, a transformer two, and a bus or substation three; generators and underground lines are assumed invulnerable. Also as in [1], constant demands are assumed. We solve multiple *scenarios* that depend on amount of interdiction resource M, and whether the interdictor seeks medium-term or long-term effects. In a "medium-term scenario," the interdictor seeks to maximize peak power-shedding and solves IPF with a single vector of peak demands. (For consistency with earlier work, the constant demand vector $\mathbf{d}$ is also interpreted as the peak demand vector.) In a "long-term scenario," the interdictor seeks to maximize energy-shedding and takes repair times into account by solving IPFT. We assume that an interdicted line requires 72 h (three days) to repair, a bus 360 h (15 days), and a transformer or substation 768 h (32 days). Table I shows results for a number of scenarios. Note that "disruption" corresponds to amount of power or energy shed, and represents a simple surrogate for objective-function value.

Naturally, for some values of interdiction resource, solutions to GLBDA improve upon earlier ones identified by the heuristic in [1], and match those produced by the exact MIPs in [11] and [18]. (Comparisons with [8] for some medium-term scenarios show minor discrepancies in disruption values, possibly due to transcription errors in data.)

A long-term scenario solves more quickly than its medium-term counterpart because the effective solution space, both for the overall problem and for any individual master problem, is smaller in the former case. For example, we implicitly have $\delta_l^{\mathcal{L}} \equiv 0$ for all lines l in a long-term scenario, because lines have such short repair times.

[1]"M" denotes interdiction resource, and "Iters." denotes the number of major iterations in GLBDA. The optimality tolerance is 1%. Each medium-term scenario solves in at most 170 s; each long-term scenario solves in at most 30 s.

C. U.S. Regional Test System

We have created a realistic test system, which we call "U.S. Regional Grid" (USRG), from data provided by one of NERC's reliability councils. USRG comprises over 5000 buses, 5000 lines, and 1000 transformers. Total system load is close to 70 000 MW, and generating capacity exceeds 90 000 MW from more than 500 generating units.

These data omit substation information, so we define a substation as a maximal group of transformers that share one or more buses. This produces over 500 substations, with one to fifteen transformers each.

The original data give total peak demand by bus. We use these values to create a three-segment LDC at each bus, consisting of 1) a peak period, with demand as given, covering 20% of each day, 2) a normal period with demand equaling 75% of peak demand and covering 50% of each day, and 3) a valley period with demand equaling 45% of peak demand and covering 30% of each day. We also set penalties to depend on the LDC segment, with values 1000, 800, and 500 \$/MWh, respectively. Both the LDC and penalties apply to each of about 3000 buses with load.

A complete scenario also requires definition of these interdiction data pairs for each component type: (amount of interdiction resource required, repair time in hours). These values are (1, 48) for lines, (2, 96) for transformers, (2, 180) for buses, and (3, 360) for substations; generators are assumed invulnerable. We realize these repair times are generally optimistic, and use them only for demonstration.

We run GLBDA for 60 minutes or until an optimality gap of 1% is achieved, and report the best solution found. All long-term scenarios achieve the 1% gap, but medium-term scenarios leave an average gap of 6.5%, with a maximum of 10.9%. Table II shows detailed results for a medium-term "baseline scenario," defined as having $M=15$; Table III shows results for the analogous long-term baseline scenario. We list both medium-term and long-term effects for both scenarios to emphasize how the solutions differ with the interdictor's intentions. Table IV summarizes results for $M \in \{3, 6, \ldots, 18\}$.

The medium-term baseline scenario (Table II) shows these interdicted components: six lines, all with thermal ratings between 1000 and 2000 MW; three buses (with an average of eight lines connected to each bus, with each line averaging 1400 MW of capacity); and, one substation with four transformers. Load-shedding is nearly 8.4 GW. Long-term effects are ignored, as seen by the facts that: after 48 h, disruption has dropped substantially, and is limited to peak hours; and after 180 h, disruption vanishes completely, even though the interdicted substation is still offline. This contrasts with the solution to the long-term, baseline case (Table III), which interdicts five substations. That results in the shedding of 1095 GWh of energy, substantially higher than the

[2]With $M=15$, GLBDA makes 164 iterations in 60 min and halts with an optimality gap of 10.9%. The table shows strong medium-term effects but weak long-term effects. Compare with the long-term-scenario results in Table III.

[3]With $M=15$, GLBDA halts after reaching an optimality gap of 1%, taking 389 iterations in 57 min.

[4]"$[L, I, S]$" represents the number of interdicted lines, buses and substations, respectively.

TABLE II
USRG RESULTS, MEDIUM-TERM BASELINE SCENARIO[2]

Unrepaired components	Period	LDC hours (segment)	Shed (MW)	Shed (GWh)
6 lines, 3 buses, 1 substation	0–48 h	9.6 (peak)	8,395	80.6
		24.0 (normal)	4,211	101.1
		14.4 (valley)	127	1.8
3 buses, 1 substation	48–180h	26.4 (peak)	1,826	43.8
		66.0 (normal)	75	4.5
		39.6 (valley)	16	0.6
1 substation	180–360h	36.0 (peak)	0	0.0
		90.0 (normal)	0	0.0
		54.0 (valley)	0	0.0
Overall	0–360h	n/a	645 (avg.)	232.3 (total)

TABLE III
USRG RESULTS, LONG-TERM BASELINE SCENARIO[3]

Unrepaired components	Period	LDC hours (segment)	Shed (MW)	Shed (GWh)
5 sub-stations	0–360 h	72 (peak)	5,537	398.6
		180 (normal)	3,339	601.0
		108 (valley)	884	95.4
Overall	0–360 h	n/a	3,042 (avg.)	1,095.0 (total)

TABLE IV
RESULTS FOR USRG WITH INTERDICTION RESOURCE M VARIED[4]

	Medium-term scenarios			Long-term Scenarios		
M	[*L*,*I*,*S*]	Peak (MW)	Total (GWh)	[*L*,*I*,*S*]	Peak (MW)	Total (GWh)
3	[1,1,0]	1,411	150.7	[0,0,1]	1,202	307.2
6	[2,2,0]	2,257	219.8	[0,0,2]	1,746	446.3
9	[1,4,0]	4,557	295.2	[0,0,3]	2,230	570.0
12	[5,2,1]	6,332	148.2	[0,0,4]	4,399	777.6
15	[6,3,1]	8,395	232.3	[0,0,5]	5,537	1,095.0
18	[7,4,1]	9,589	375.6	[0,0,6]	6,725	1,328.7

232.3 GWh from the medium-term scenario, even though peak power-shedding is about 33% higher in that scenario.

Starting with $M = 3$ (see Table IV), the medium-term scenarios reveal marginal increases of 60%, 102%, 39%, 33%, and 14% for each additional three units of interdiction resource, respectively. These figures become 45%, 28%, 36%, 41%, and 21% for the long-term scenarios. As with the RTS-96 scenarios, we see that an interdictor seeking to maximize medium-term harm would choose a different strategy than one looking to cause long-term harm.

D. Results With Stronger Assumptions

Given the empirical validity of Assumption 1, one is tempted to make an even stronger assumption, which should lead to stronger cut coefficients and shorter solution times for GLBDA.

Assumption 2: Ignoring short-term load-shedding due to interdiction-caused cascading failures, the interdiction of a set of components $k \in \hat{\mathcal{K}}$, each carrying P_k MW of power, leads to the shedding of at most $\gamma \sum_{k\in\hat{\mathcal{K}}} P_k$ MW of demand, where γ is chosen such that $0 < \gamma \leq 1$. ■

If Assumption 2 can replace Assumption 1(a), valid cut coefficients result from simply multiplying the coefficients defined in Section II-B by γ. For example, for $\gamma = 0.8$, the results for the long-term, baseline USRG scenario described in Table III are reproduced exactly, but solution time reduces by an order of magnitude. Unfortunately, as we make γ smaller and smaller ($\gamma = 0.75$ suffices here), the bounding function described by (MP.1) will eventually become invalid. Furthermore, it may or may not be obvious when γ has become "too small"; see [18].

IV. CONCLUSIONS AND FUTURE WORK

This paper has introduced *global Benders decomposition* (GLBD) for solving large-scale, electric power grid interdiction problems. The purpose in solving these problems is to identify components in a power grid that are the most critical to the grid's functionality. We demonstrate that a GLBD algorithm can solve problems defined on a real-world transmission grid with thousands of components.

Future work will attempt to improve computational efficiency for GLBD. In particular, we also hope to improve the master-problem bound by identifying nonzero cut coefficients for interdicted components. This may be possible by exploiting special problem structures. Similarly, we may be able to create tighter cuts by reducing currently defined nonzero coefficients.

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