In plasma and certain areas of astrophysics, it is often of interest to understand the orbits of charged particles moving in complicated magnetic fields. Of course, the motion of such a particle in a uniform magnetic field is quite easy to solve algebraically (i.e., helical motion about the magnetic field lines), but for arbitrary fields, the orbits can grow intricate and can only be solved numerically. Furthermore, understanding these complex orbits is desirable in many contexts, for example they can cause interesting instabilities in plasmas that are difficult to anticipate a priori, like the so-called banana orbits in tokamaks. To that extent, when modeling plasmas with a kinetic model (i.e., tracing individual particle tracks), it is crucial to have a numerical method for integrating the motions of charged particles that is both accurate and stable. The Boris algorithm is one such algorithm, frequently used for its simplicity, phase-space preserving properties, and accuracy.

The basis for the Boris algorithm is in the leapfrog algorithm, a symplectic differential equation solver. The basic idea is to calculate a velocity at a half time step, use the velocity to update the position, and then use the acceleration (i.e., force) at the new position to finish the velocity update. The leapfrog algorithm is a variant of the Verlet algorithm, and as such is symplectic. Because of this, it has many nice properties including conservation of energy and angular momentum. However, the leapfrog algorithm also requires that the acceleration term (really the forcing term) in the equations of motion be dependent only on the particle's position. Recalling that the Lorentz force has a velocity cross magnetic field term, this condition is clearly not met for a system consisting of a charged particle moving a magnetic field. Importantly, naïvely applying the leapfrog algorithm results in a solution that is unstable. The Boris algorithm modifies the leapfrog algorithm by realizing that using a clever redefinition of terms and doing some geometry, the velocity update can be re-written as applying a rotation. With this step, the Boris algorithm is no longer symplectic like the leapfrog algorithm, but remarkably maintains its phase-space preserving properties with an applied magnetic field term, such that the conservation of physical quantities still occurs [1].

Because of its stability over long time periods, the Boris algorithm is currently somewhat standard in many kinetic models of plasmas. Such models are often useful in experimental plasma physics and the pursuit of nuclear fusion. Even with the Boris algorithm, plasmas are still notoriously difficult to model, and some recent research has resulted in improvements to the algorithm. For example, in 2023 Decyk et. al. published a relativistic version of the Boris algorithm, which performs calculations of the Lorentz factor at each time step [2]. For a second example, a 2024 article by Yanyan Shi compared the results of the Boris algorithm with a related variational integrator, which yielded much more accurate results [3]. Thinking of translational applications, it would also be interesting to test the performance of the Boris algorithm on a set of differential equations of the same or similar form (i.e., one with crossed first derivative and quantity terms) that are unrelated to electromagnetism.

Works Cited:

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