It is sometimes useful to distinguish between forward and inverse problems. Roughly, forward problems involve determining the effects from some known cause. For example, if a dynamical system is in some state, determining what signal to expect some time later if one were to measure it. On the other hand, an inverse problem would ask, given some measurement of the system, what information could be obtained about the system that produced that signal? Tomographic imaging is one such sort of inverse problem commonly used in medical physics among other disciplines, primarily as a tool to help make diagnoses of different diseases. Essentially, a set of detectors and an x-ray source are rotated and translated around some target, and used to reconstruct a 2-D or 3-D map of the object's attenuations (which roughly translates an image of density values). Creating this reconstruction is a very computationally expensive process, involving a fair bit of math and x-ray physics.

The mathematical foundation for computed tomography lies in the Radon transform. The Radon transform of some object (represented as a scalar function) is the set of all line integrals through the object. Importantly, given the complete set of line integrals, it can be shown that it is possible to completely recover the image [1]. In practice, the complete set is not possible to obtain, but enough samples allow an image to be reconstructed, which ultimately makes CT possible. This is because X-rays attenuate through an object mostly according to Beer-Lambert's law, such that after a log operation, the attenuation of a thin x-ray beam is a very good approximation of a line integral of some object's density. A CT machine then collects many such "line integrals" and reconstructs this information into an image. One computational recipe for reconstruction notes an equivalency between the 1-D Fourier transform of a projection of the image along some line (a subset of the line integrals) and a particular line in the 2-D Fourier transform of the image [1]. This is called the central slice theorem. Then by finding the correct projections and filling in the 2-D Fourier transform, the original image can be found using the inverse Fourier transform. This process is called (filtered) back-projection, and is the basis for most model-based CT reconstruction algorithms. There are also many iterative CT reconstruction algorithms that represent the problem as a linear system, with a response matrix for each detector at each position, a vector representing the signal at each position, and a vector representing each pixel in the reconstructed image. In principle, all that must be done is invert the response matrix, but this is not feasible (the matrix is very, very large, and due to noise might not be invertible) [1]. This motivates several families of iterative approaches.

There are many applications for computed tomography across different areas of physics. As far as algorithm development goes, there is a great deal of interest in using various sorts of artificial intelligence to improve image quality, which may translate generally to other areas of optical physics. The advancement of CT is also tied to other areas of physics, for example, the image reconstruction algorithms and image processing techniques have some overlap with optics and the study of point spread functions in astronomy. As a more broad example, CT has resulted in the development of improving X-ray sources and detectors. This in turn will be useful for experiments that use X-rays to make measurements, like certain kinds of plasma diagnostics. Indeed, photon counting X-ray detectors were initially developed for CT, and provide much richer information by resolving the energies of single photons in the X-ray regime (as opposed to a scintillator). It is straightforward to see how photon counting could likewise contribute to much richer measurements in experiments using X-rays.

Works Cited:

[1] A. C. Kak and M. Slaney, “Algorithms for Reconstruction with Nondiffracting Sources,” in *Principles of Computerized Tomographic Imaging*, Philadelphia: Society for Industrial and Applied Mathematics, 2001