Rebound is a N-body integrator widely used in astrophysics for simulating gravitational systems. It offers a variety of numerical integration methods that balance accuracy and efficiency, making it well-suited for studying planetary dynamics, asteroid trajectories, and long-term system evolution. The core of Rebound's functionality lies in its multitude of integrators, each optimized for different scenarios.

One of the key symplectic integrators in Rebound is WHFast, a Wisdom-Holman mapping method designed for efficiently handling nearly Keplerian systems. It separates the Hamiltonian into a dominant Keplerian term and a perturbative component, allowing large time steps while maintaining long-term energy conservation. This makes WHFast ideal for planetary systems where the primary force is a central gravitational attraction with minor perturbations from additional bodies. Another widely used method is IAS15, a high-order implicit integrator ideal for chaotic systems and close encounters. Unlike WHFast, IAS15 is adaptive, dynamically adjusting time steps to maintain precision when bodies approach one another. This is particularly useful for modeling situations where gravitational interactions vary significantly over time. Rebound also supports other integrators such as Leapfrog, which is symplectic and commonly used for simple, evenly spaced time steps, and Runge-Kutta-based methods like RK4, which are more general but less suitable for long-term stability in Hamiltonian systems. The choice of integrator depends on the specific problem, so for example, symplectic methods like WHFast and Leapfrog preserve energy over long timescales, while high-accuracy adaptive methods like IAS15 do well in handling unpredictable interactions.

The connection between these integrators and Lyapunov exponents lies in their ability to assess system stability. The Lyapunov exponent quantifies the rate at which nearby trajectories in phase space diverge, indicating whether a system behaves chaotically. Using Rebound, we can track the separation of initially close orbits and compute the Lyapunov time, which serves as a measure of stability. This is particularly relevant for predicting the long-term evolution of planetary systems and identifying what type of systems are prone to instability.

Sources:

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